# 8 Quadrilaterals



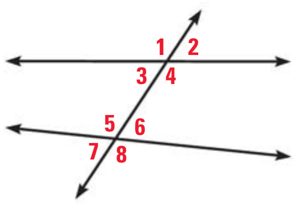
## Before

In previous chapters, you learned the following skills, which you'll use in Chapter 8: identifying angle pairs, using the Triangle Sum Theorem, and using parallel lines.

## Prerequisite Skills

### VOCABULARY CHECK

**Copy and complete the statement.**

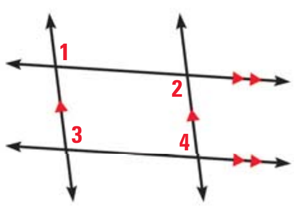



1. $\angle 1$ and _?_ are vertical angles.
2. $\angle 3$ and _?_ are consecutive interior angles.
3. $\angle 7$ and _?_ are corresponding angles.
4. $\angle 5$ and _?_ are alternate interior angles.

### SKILLS AND ALGEBRA CHECK

5. In $\triangle ABC$, $m\angle A = x°$, $m\angle B = 3x°$, and $m\angle C = (4x - 12)°$. Find the measures of the three angles. *(Review p. 217 for 8.1.)*

**Find the measure of the indicated angle.** *(Review p. 154 for 8.2–8.5.)*




6. If $m\angle 3 = 105°$, then $m\angle 2 =$ _?_.
7. If $m\angle 1 = 98°$, then $m\angle 3 =$ _?_.
8. If $m\angle 4 = 82°$, then $m\angle 1 =$ _?_.
9. If $m\angle 2 = 102°$, then $m\angle 4 =$ _?_.

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## Now

In Chapter 8, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 559. You will also use the key vocabulary listed below.

# Big Ideas

1. **Using angle relationships in polygons**
2. **Using properties of parallelograms**
3. **Classifying quadrilaterals by their properties**

**KEY VOCABULARY**

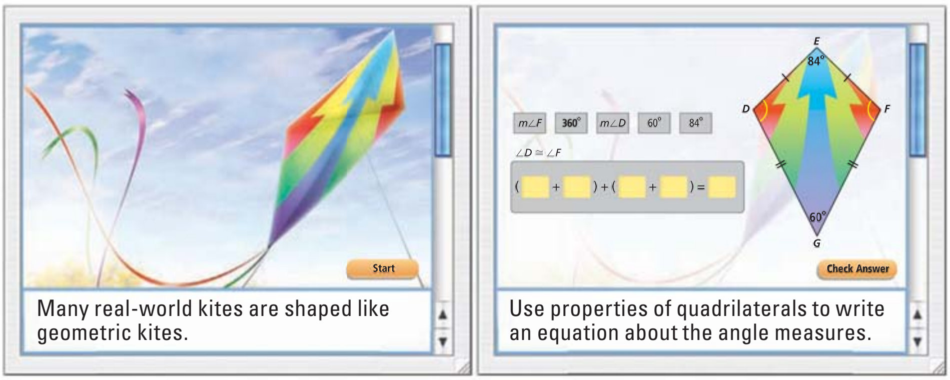
- diagonal, *p. 507*
- parallelogram, *p. 515*
- rhombus, *p. 533*
- rectangle, *p. 533*
- square, *p. 533*
- trapezoid, *p. 542*
  bases, base angles, legs
- isosceles trapezoid, *p. 543*
- midsegment of a trapezoid, *p. 544*
- kite, *p. 545*

## Why?

You can use properties of quadrilaterals and other polygons to find side lengths and angle measures.

## Animated Geometry

The animation illustrated below for Example 4 on page 545 helps you answer this question: How can classifying a quadrilateral help you draw conclusions about its sides and angles?




**Animated Geometry** at classzone.com

**Other animations for Chapter 8:** pages 509, 519, 527, 535, 551, and 553

Investigating Geometry **ACTIVITY** *Use before Lesson 8.1*

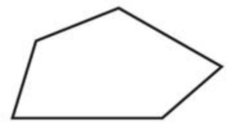
# 8.1 Investigate Angle Sums in Polygons

**MATERIALS** • straightedge • ruler

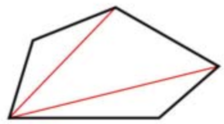
**QUESTION** **What is the sum of the measures of the interior angles of a convex *n*-gon?**

Recall from page 43 that an *n*-gon is a polygon with *n* sides and *n* vertices.

**EXPLORE** **Find sums of interior angle measures**



**STEP 1** ***Draw polygons*** Use a straightedge to draw convex polygons with three sides, four sides, five sides, and six sides. An example is shown.



**STEP 2** ***Draw diagonals*** In each polygon, draw all the diagonals from one vertex. A *diagonal* is a segment that joins two nonconsecutive vertices. Notice that the diagonals divide the polygon into triangles.

**STEP 3** ***Make a table*** Copy the table below. By the Triangle Sum Theorem, the sum of the measures of the interior angles of a triangle is 180°. Use this theorem to complete the table.

| Polygon | Number of sides | Number of triangles | Sum of measures of interior angles |
|---|---|---|---|
| Triangle | 3 | 1 | $1 \cdot 180° = 180°$ |
| Quadrilateral | ? | ? | $2 \cdot 180° = 360°$ |
| Pentagon | ? | ? | ? |
| Hexagon | ? | ? | ? |

**DRAW CONCLUSIONS** **Use your observations to complete these exercises**

1. Look for a pattern in the last column of the table. What is the sum of the measures of the interior angles of a convex heptagon? a convex octagon? *Explain* your reasoning.
2. Write an expression for the sum of the measures of the interior angles of a convex *n*-gon.
3. Measure the side lengths in the hexagon you drew. Compare the lengths with those in hexagons drawn by other students. Do the side lengths affect the sum of the interior angle measures of a hexagon? *Explain*.

# 8.1 Find Angle Measures in Polygons

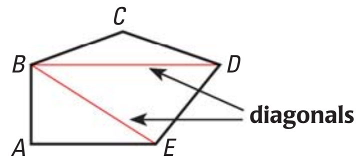
**Before** You classified polygons.

**Now** You will find angle measures in polygons.

**Why?** So you can describe a baseball park, as in Exs. 28–29.

**Key Vocabulary**
- **diagonal**
- **interior angle,** *p. 218*
- **exterior angle,** *p. 218*

In a polygon, two vertices that are endpoints of the same side are called *consecutive vertices*. A **diagonal** of a polygon is a segment that joins two *nonconsecutive vertices*. Polygon $ABCDE$ has two diagonals from vertex $B$, $\overline{BD}$ and $\overline{BE}$.




As you can see, the diagonals from one vertex form triangles. In the Activity on page 506, you used these triangles to find the sum of the interior angle measures of a polygon. Your results support the following theorem and corollary.

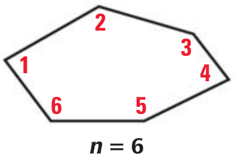
**THEOREMS** *For Your Notebook*

**THEOREM 8.1 Polygon Interior Angles Theorem**

The sum of the measures of the interior angles of a convex $n$-gon is $(n - 2) \cdot 180°$.

$m\angle 1 + m\angle 2 + \cdots + m\angle n = (n - 2) \cdot 180°$

*Proof:* Ex. 33, p. 512 (for pentagons)



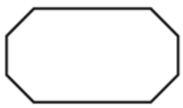

**COROLLARY TO THEOREM 8.1 Interior Angles of a Quadrilateral**

The sum of the measures of the interior angles of a quadrilateral is 360°.

*Proof:* Ex. 34, p. 512

**EXAMPLE 1 Find the sum of angle measures in a polygon**

**Find the sum of the measures of the interior angles of a convex octagon.**



**Solution**

An octagon has 8 sides. Use the Polygon Interior Angles Theorem.

$(n - 2) \cdot 180° = (8 - 2) \cdot 180°$ **Substitute 8 for *n*.**

$= 6 \cdot 180°$ **Subtract.**

$= 1080°$ **Multiply.**

▶ The sum of the measures of the interior angles of an octagon is 1080°.

## EXAMPLE 2 Find the number of sides of a polygon

**The sum of the measures of the interior angles of a convex polygon is 900°. Classify the polygon by the number of sides.**

### Solution

Use the Polygon Interior Angles Theorem to write an equation involving the number of sides $n$. Then solve the equation to find the number of sides.

$(n - 2) \cdot 180° = 900°$ **Polygon Interior Angles Theorem**

$n - 2 = 5$ **Divide each side by 180°.**

$n = 7$ **Add 2 to each side.**

▶ The polygon has 7 sides. It is a heptagon.

## ✓ GUIDED PRACTICE for Examples 1 and 2

1. The coin shown is in the shape of a regular 11-gon. Find the sum of the measures of the interior angles.


2. The sum of the measures of the interior angles of a convex polygon is 1440°. Classify the polygon by the number of sides.

## EXAMPLE 3 Find an unknown interior angle measure

**xy ALGEBRA Find the value of $x$ in the diagram shown.**


### Solution

The polygon is a quadrilateral. Use the Corollary to the Polygon Interior Angles Theorem to write an equation involving $x$. Then solve the equation.

$x° + 108° + 121° + 59° = 360°$ **Corollary to Theorem 8.1**

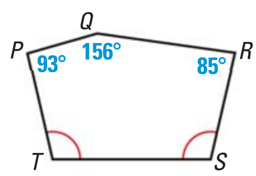
$x + 288 = 360$ **Combine like terms.**

$x = 72$ **Subtract 288 from each side.**

▶ The value of $x$ is 72.

## ✓ GUIDED PRACTICE for Example 3

3. Use the diagram at the right. Find $m\angle S$ and $m\angle T$.

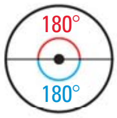


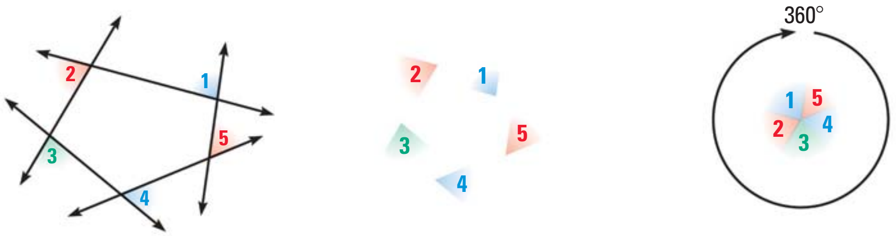

4. The measures of three of the interior angles of a quadrilateral are 89°, 110°, and 46°. Find the measure of the fourth interior angle.

**EXTERIOR ANGLES** Unlike the sum of the interior angle measures of a convex polygon, the sum of the exterior angle measures does *not* depend on the number of sides of the polygon. The diagrams below suggest that the sum of the measures of the exterior angles, one at each vertex, of a pentagon is 360°. In general, this sum is 360° for any convex polygon.

**VISUALIZE IT**

A circle contains two straight angles. So, there are 180° + 180°, or 360°, in a circle.




***STEP 1*** **Shade** one exterior angle at each vertex.

***STEP 2*** **Cut** out the exterior angles.

***STEP 3*** **Arrange** the exterior angles to form 360°.

*Animated* **Geometry** at classzone.com

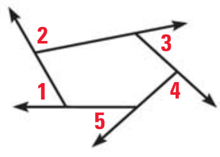
## THEOREM *For Your Notebook*

**THEOREM 8.2 Polygon Exterior Angles Theorem**

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360°.

$m\angle 1 + m\angle 2 + \cdots + m\angle n = 360°$

*Proof:* Ex. 35, p. 512



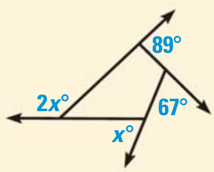

$n = 5$

## ★ EXAMPLE 4 Standardized Test Practice

**What is the value of $x$ in the diagram shown?**

Ⓐ 67 Ⓑ 68

Ⓒ 91 Ⓓ 136




**ELIMINATE CHOICES**

You can quickly eliminate choice *D*. If $x$ were equal to 136, then the sum of only two of the angle measures ($x°$ and $2x°$) would be greater than 360°.

### Solution

Use the Polygon Exterior Angles Theorem to write and solve an equation.

$x° + 2x° + 89° + 67° = 360°$ **Polygon Exterior Angles Theorem**

$3x + 156 = 360$ **Combine like terms.**

$x = 68$ **Solve for $x$.**

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

## ✓ GUIDED PRACTICE for Example 4

**5.** A convex hexagon has exterior angles with measures 34°, 49°, 58°, 67°, and 75°. What is the measure of an exterior angle at the sixth vertex?

## EXAMPLE 5 Find angle measures in regular polygons

**READ VOCABULARY**
Recall that a *dodecagon* is a polygon with 12 sides and 12 vertices.

**TRAMPOLINE** The trampoline shown is shaped like a regular dodecagon. Find (a) the measure of each interior angle and (b) the measure of each exterior angle.

**Solution**

**a.** Use the Polygon Interior Angles Theorem to find the sum of the measures of the interior angles.

$$(n - 2) \cdot 180° = (12 - 2) \cdot 180° = 1800°$$

Then find the measure of one interior angle. A regular dodecagon has 12 congruent interior angles. Divide 1800° by 12: 1800° ÷ 12 = 150°.

▶ The measure of each interior angle in the dodecagon is 150°.

**b.** By the Polygon Exterior Angles Theorem, the sum of the measures of the exterior angles, one angle at each vertex, is 360°. Divide 360° by 12 to find the measure of one of the 12 congruent exterior angles: 360° ÷ 12 = 30°.

▶ The measure of each exterior angle in the dodecagon is 30°.

### ✓ GUIDED PRACTICE for Example 5

**6.** An interior angle and an adjacent exterior angle of a polygon form a linear pair. How can you use this fact as another method to find the exterior angle measure in Example 5?

# 8.1 EXERCISES

**HOMEWORK KEY**
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 11, and 29
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, 23, and 37
◆ = **MULTIPLE REPRESENTATIONS** Ex. 36

## SKILL PRACTICE

**1. VOCABULARY** Sketch a convex hexagon. Draw all of its diagonals.

**2.** ★ **WRITING** How many exterior angles are there in an $n$-gon? Are all the exterior angles considered when you use the Polygon Exterior Angles Theorem? *Explain.*

**EXAMPLES 1 and 2**
on pp. 507–508 for Exs. 3–10

**INTERIOR ANGLE SUMS** **Find the sum of the measures of the interior angles of the indicated convex polygon.**

**3.** Nonagon **4.** 14-gon **5.** 16-gon **6.** 20-gon

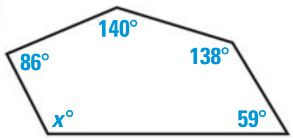
**FINDING NUMBER OF SIDES** **The sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides.**

**7.** 360° **8.** 720° **9.** 1980° **10.** 2340°

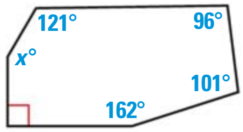
**EXAMPLES 3 and 4**
on pp. 508–509
for Exs. 11–18

**ALGEBRA** **Find the value of $x$.**

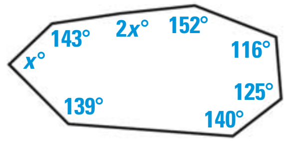
**11.**



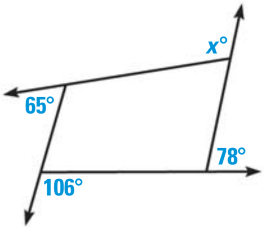

**12.**



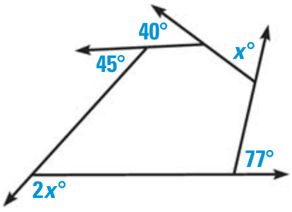

**13.**



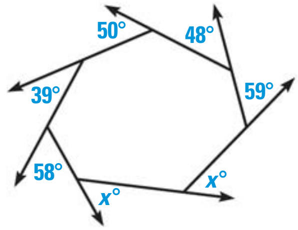

**14.**




**15.**




**16.**




**17. ERROR ANALYSIS** A student claims that the sum of the measures of the exterior angles of an octagon is greater than the sum of the measures of the exterior angles of a hexagon. The student justifies this claim by saying that an octagon has two more sides than a hexagon. *Describe* and correct the error the student is making.

**18.** ★ **MULTIPLE CHOICE** The measures of the interior angles of a quadrilateral are $x°$, $2x°$, $3x°$, and $4x°$. What is the measure of the largest interior angle?

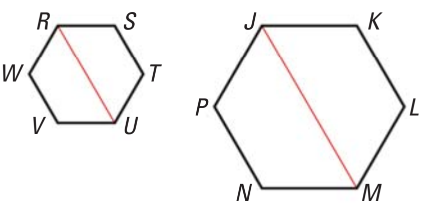
(A) 120° (B) 144° (C) 160° (D) 360°

**EXAMPLE 5**
on p. 510
for Exs. 19–21

**REGULAR POLYGONS** **Find the measures of an interior angle and an exterior angle of the indicated regular polygon.**

**19.** Regular pentagon **20.** Regular 18-gon **21.** Regular 90-gon

**22. DIAGONALS OF SIMILAR FIGURES** Hexagons *RSTUVW* and *JKLMNP* are similar. $\overline{RU}$ and $\overline{JM}$ are diagonals. Given $ST = 6$, $KL = 10$, and $RU = 12$, find $JM$.




**23.** ★ **SHORT RESPONSE** *Explain* why any two regular pentagons are similar.

**REGULAR POLYGONS** **Find the value of $n$ for each regular $n$-gon described.**

**24.** Each interior angle of the regular $n$-gon has a measure of 156°.

**25.** Each exterior angle of the regular $n$-gon has a measure of 9°.

**26. POSSIBLE POLYGONS** Determine if it is possible for a regular polygon to have an interior angle with the given angle measure. *Explain* your reasoning.

**a.** 165° **b.** 171° **c.** 75° **d.** 40°

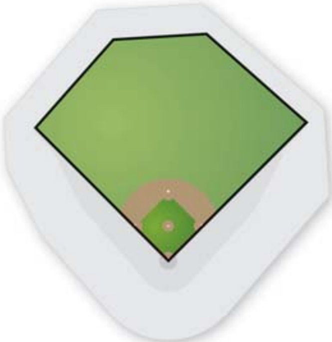
**27. CHALLENGE** Sides are added to a convex polygon so that the sum of its interior angle measures is increased by 540°. How many sides are added to the polygon? *Explain* your reasoning.

## PROBLEM SOLVING

**EXAMPLE 1** on p. 507 for Exs. 28–29

**BASEBALL The outline of the playing field at a baseball park is a polygon, as shown. Find the sum of the measures of the interior angles of the polygon.**

**28.**



**29.**

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**EXAMPLE 5** on p. 510 for Exs. 30–31

**30. JEWELRY BOX** The base of a jewelry box is shaped like a regular hexagon. What is the measure of each interior angle of the hexagon?

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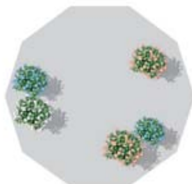
**31. GREENHOUSE** The floor of the greenhouse shown is a shaped like a regular decagon. Find the measure of an interior angle of the regular decagon. Then find the measure of an exterior angle.



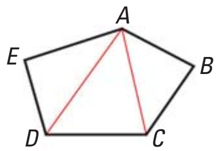
**32. MULTI-STEP PROBLEM** In pentagon $PQRST$, $\angle P$, $\angle Q$, and $\angle S$ are right angles, and $\angle R \cong \angle T$.

**a. Draw a Diagram** Sketch pentagon $PQRST$. Mark the right angles and the congruent angles.

**b. Calculate** Find the sum of the interior angle measures of $PQRST$.

**c. Calculate** Find $m\angle R$ and $m\angle T$.

**33. PROVING THEOREM 8.1 FOR PENTAGONS** The Polygon Interior Angles Theorem states that the sum of the measures of the interior angles of an $n$-gon is $(n - 2) \cdot 180°$. Write a paragraph proof of this theorem for the case when $n = 5$.




**34. PROVING A COROLLARY** Write a paragraph proof of the Corollary to the Polygon Interior Angles Theorem.

**35. PROVING THEOREM 8.2** Use the plan below to write a paragraph proof of the Polygon Exterior Angles Theorem.

**Plan for Proof** In a convex $n$-gon, the sum of the measures of an interior angle and an adjacent exterior angle at any vertex is 180°. Multiply by $n$ to get the sum of all such sums at each vertex. Then subtract the sum of the interior angles derived by using the Polygon Interior Angles Theorem.

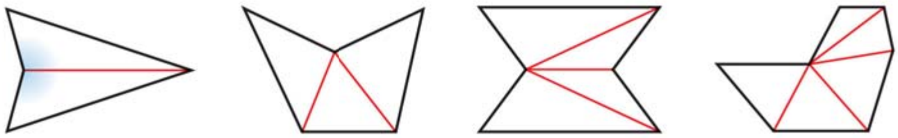
**36.** **MULTIPLE REPRESENTATIONS** The formula for the measure of each interior angle in a regular polygon can be written in function notation.

**a.** **Writing a Function** Write a function $h(n)$, where $n$ is the number of sides in a regular polygon and $h(n)$ is the measure of any interior angle in the regular polygon.

**b.** **Using a Function** Use the function from part (a) to find $h(9)$. Then use the function to find $n$ if $h(n) = 150°$.

**c.** **Graphing a Function** Graph the function from part (a) for $n = 3, 4, 5, 6, 7,$ and $8$. Based on your graph, *describe* what happens to the value of $h(n)$ as $n$ increases. *Explain* your reasoning.

**37.** ★ **EXTENDED RESPONSE** In a concave polygon, at least one interior angle measure is greater than 180°. For example, the measure of the shaded angle in the concave quadrilateral below is 210°.



**a.** In the diagrams above, the interiors of a concave quadrilateral, pentagon, hexagon, and heptagon are divided into triangles. Make a table like the one in the Activity on page 506. For each of the polygons shown above, record the number of sides, the number of triangles, and the sum of the measures of the interior angles.

**b.** Write an algebraic expression that you can use to find the sum of the measures of the interior angles of a concave polygon. *Explain*.

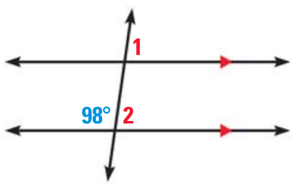
**38.** **CHALLENGE** Polygon *ABCDEFGH* is a regular octagon. Suppose sides $\overline{AB}$ and $\overline{CD}$ are extended to meet at a point *P*. Find $m\angle BPC$. *Explain* your reasoning. Include a diagram with your answer.

## MIXED REVIEW

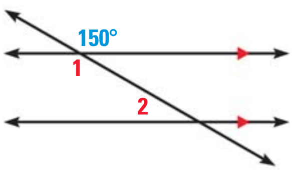
**PREVIEW**
Prepare for Lesson 8.2 in Exs. 39–41.

**Find $m\angle 1$ and $m\angle 2$. *Explain* your reasoning.** *(p. 154)*

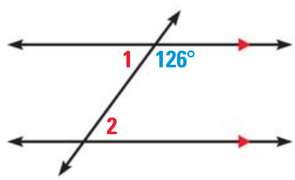
**39.**




**40.**




**41.**




**42.** Quadrilaterals *JKLM* and *PQRS* are similar. If $JK = 3.6$ centimeters and $PQ = 1.2$ centimeters, find the scale factor of *JKLM* to *PQRS*. *(p. 372)*

**43.** Quadrilaterals *ABCD* and *EFGH* are similar. The scale factor of *ABCD* to *EFGH* is 8 : 5, and the perimeter of *ABCD* is 90 feet. Find the perimeter of *EFGH*. *(p. 372)*

**Let $\angle A$ be an acute angle in a right triangle. Approximate the measure of $\angle A$ to the nearest tenth of a degree.** *(p. 483)*

**44.** $\sin A = 0.77$ **45.** $\sin A = 0.35$ **46.** $\cos A = 0.81$ **47.** $\cos A = 0.23$

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Keystrokes

# 8.2 Investigate Parallelograms

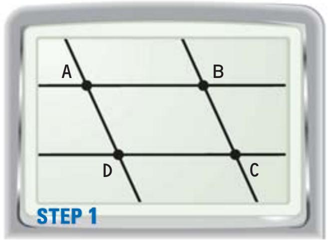
**MATERIALS** • graphing calculator or computer

**QUESTION** What are some of the properties of a parallelogram?

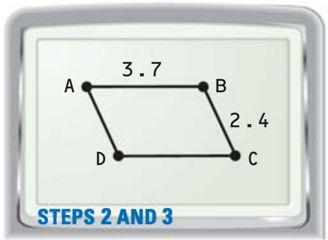
You can use geometry drawing software to investigate relationships in special quadrilaterals.

**EXPLORE** Draw a quadrilateral

**STEP 1** ***Draw parallel lines*** Construct $\overleftrightarrow{AB}$ and a line parallel to $\overleftrightarrow{AB}$ through point $C$. Then construct $\overleftrightarrow{BC}$ and a line parallel to $\overleftrightarrow{BC}$ through point $A$. Finally, construct a point $D$ at the intersection of the line drawn parallel to $\overleftrightarrow{AB}$ and the line drawn parallel to $\overleftrightarrow{BC}$.




**STEP 2** ***Draw quadrilateral*** Construct segments to form the sides of quadrilateral $ABCD$. After you construct $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, hide the parallel lines that you drew in Step 1.




**STEP 3** ***Measure side lengths*** Measure the side lengths $AB$, $BC$, $CD$, and $DA$. Drag point $A$ or point $B$ to change the side lengths of $ABCD$. What do you notice about the side lengths?

**STEP 4** ***Measure angles*** Find the measures of $\angle A$, $\angle B$, $\angle C$, and $\angle D$. Drag point $A$ or point $B$ to change the angle measures of $ABCD$. What do you notice about the angle measures?

**DRAW CONCLUSIONS** Use your observations to complete these exercises

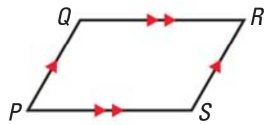
1. The quadrilateral you drew in the Explore is called a *parallelogram*. Why do you think this type of quadrilateral has this name?
2. Based on your observations, make a conjecture about the side lengths of a parallelogram and a conjecture about the angle measures of a parallelogram.
3. **REASONING** Draw a parallelogram and its diagonals. Measure the distance from the intersection of the diagonals to each vertex of the parallelogram. Make and test a conjecture about the diagonals of a parallelogram.

# 8.2 Use Properties of Parallelograms


| | |
|---|---|
| **Before** | You used a property of polygons to find angle measures. |
| **Now** | You will find angle and side measures in parallelograms. |
| **Why?** | So you can solve a problem about airplanes, as in Ex. 38. |

**Key Vocabulary**
- **parallelogram**

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. The term "parallelogram $PQRS$" can be written as $\square PQRS$. In $\square PQRS$, $\overline{PQ} \parallel \overline{RS}$ and $\overline{QR} \parallel \overline{PS}$ by definition. The theorems below describe other properties of parallelograms.

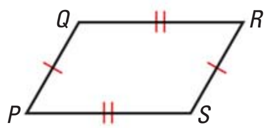



## THEOREMS — *For Your Notebook*

### THEOREM 8.3

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If $PQRS$ is a parallelogram, then $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{PS}$.

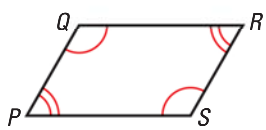



*Proof:* p. 516

### THEOREM 8.4

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

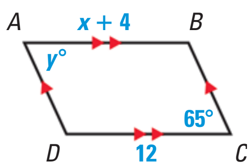
If $PQRS$ is a parallelogram, then $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.




*Proof:* Ex. 42, p. 520

## EXAMPLE 1 Use properties of parallelograms

**xy ALGEBRA Find the values of $x$ and $y$.**




$ABCD$ is a parallelogram by the definition of a parallelogram. Use Theorem 8.3 to find the value of $x$.

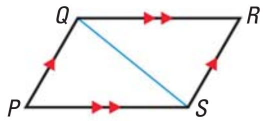
| | |
|---|---|
| $AB = CD$ | **Opposite sides of a $\square$ are $\cong$.** |
| $x + 4 = 12$ | **Substitute $x + 4$ for $AB$ and 12 for $CD$.** |
| $x = 8$ | **Subtract 4 from each side.** |

By Theorem 8.4, $\angle A \cong \angle C$, or $m\angle A = m\angle C$. So, $y° = 65°$.

▶ In $\square ABCD$, $x = 8$ and $y = 65$.

## PROOF Theorem 8.3

If a quadrilateral is a parallelogram, then its opposite sides are congruent.




**GIVEN** ▶ $PQRS$ is a parallelogram.

**PROVE** ▶ $\overline{PQ} \cong \overline{RS}$, $\overline{QR} \cong \overline{PS}$

**Plan for Proof**

**a.** Draw diagonal $\overline{QS}$ to form $\triangle PQS$ and $\triangle RSQ$.

**b.** Use the ASA Congruence Postulate to show that $\triangle PQS \cong \triangle RSQ$.

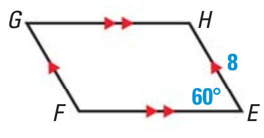
**c.** Use congruent triangles to show that $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{PS}$.

**Plan in Action**

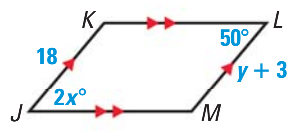
| | STATEMENTS | REASONS |
|---|---|---|
| **a.** | **1.** $PQRS$ is a ▱. | **1.** Given |
| | **2.** Draw $\overline{QS}$. | **2.** Through any 2 points there exists exactly 1 line. |
| | **3.** $\overline{PQ} \parallel \overline{RS}$, $\overline{QR} \parallel \overline{PS}$ | **3.** Definition of parallelogram |
| **b.** | **4.** $\angle PQS \cong \angle RSQ$, $\angle PSQ \cong \angle RQS$ | **4.** Alternate Interior Angles Theorem |
| | **5.** $\overline{QS} \cong \overline{QS}$ | **5.** Reflexive Property of Congruence |
| | **6.** $\triangle PQS \cong \triangle RSQ$ | **6.** ASA Congruence Postulate |
| **c.** | **7.** $\overline{PQ} \cong \overline{RS}$, $\overline{QR} \cong \overline{PS}$ | **7.** Corresp. parts of $\cong$ ⚠ are $\cong$. |

## ✓ GUIDED PRACTICE for Example 1

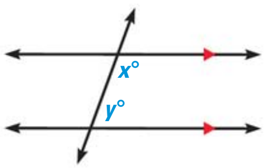
**1.** Find $FG$ and $m\angle G$.




**2.** Find the values of $x$ and $y$.




**INTERIOR ANGLES** The Consecutive Interior Angles Theorem (page 155) states that if two parallel lines are cut by a transversal, then the pairs of consecutive interior angles formed are supplementary.



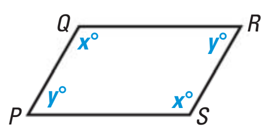

$x° + y° = 180°$

A pair of consecutive angles in a parallelogram are like a pair of consecutive interior angles between parallel lines. This similarity suggests Theorem 8.5.

## THEOREM *For Your Notebook*

### THEOREM 8.5

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.




If $PQRS$ is a parallelogram, then $x° + y° = 180°$.

*Proof:* Ex. 43, p. 520

## EXAMPLE 2 Use properties of a parallelogram

**DESK LAMP** As shown, part of the extending arm of a desk lamp is a parallelogram. The angles of the parallelogram change as the lamp is raised and lowered. Find $m\angle BCD$ when $m\angle ADC = 110°$.


**Solution**

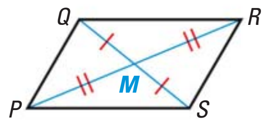
By Theorem 8.5, the consecutive angle pairs in $\square ABCD$ are supplementary. So, $m\angle ADC + m\angle BCD = 180°$. Because $m\angle ADC = 110°$, $m\angle BCD = 180° - 110° = 70°$.

### THEOREM — *For Your Notebook*

**THEOREM 8.6**

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

*Proof:* Ex. 44, p. 521



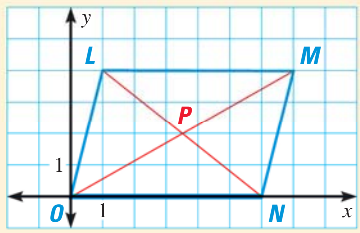

$\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$

## ★ EXAMPLE 3 Standardized Test Practice

**The diagonals of $\square LMNO$ intersect at point $P$. What are the coordinates of $P$?**

Ⓐ $\left(\frac{7}{2}, 2\right)$ Ⓑ $\left(2, \frac{7}{2}\right)$

Ⓒ $\left(\frac{5}{2}, 2\right)$ Ⓓ $\left(2, \frac{5}{2}\right)$




**SIMPLIFY CALCULATIONS**

In Example 3, you can use either diagonal to find the coordinates of $P$. Using $\overline{OM}$ simplifies calculations because one endpoint is (0, 0).

**Solution**

By Theorem 8.6, the diagonals of a parallelogram bisect each other. So, $P$ is the midpoint of diagonals $\overline{LN}$ and $\overline{OM}$. Use the Midpoint Formula.

Coordinates of midpoint $P$ of $\overline{OM} = \left(\frac{7+0}{2}, \frac{4+0}{2}\right) = \left(\frac{7}{2}, 2\right)$

▶ The correct answer is A. Ⓐ Ⓑ Ⓒ Ⓓ

## ✓ GUIDED PRACTICE for Examples 2 and 3

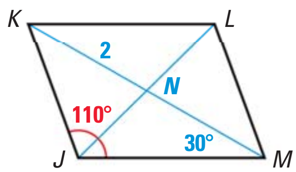
**Find the indicated measure in $\square JKLM$.**

**3.** $NM$

**4.** $KM$

**5.** $m\angle JML$

**6.** $m\angle KML$



# 8.2 EXERCISES

**HOMEWORK KEY**

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 13, and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 29, 35, and 41

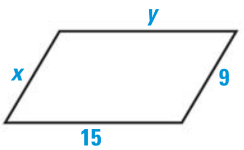
## SKILL PRACTICE

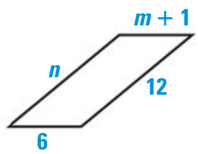
1. **VOCABULARY** What property of a parallelogram is included in the definition of a parallelogram? What properties are described by the theorems in this lesson?

2. ★ **WRITING** In parallelogram $ABCD$, $m\angle A = 65°$. *Explain* how you would find the other angle measures of ▱$ABCD$.

**EXAMPLE 1** on p. 515 for Exs. 3–8

**ALGEBRA** **Find the value of each variable in the parallelogram.**

3.



4.
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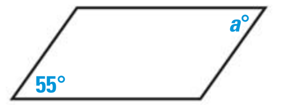


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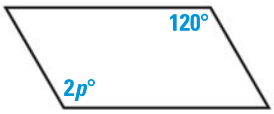
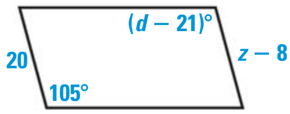
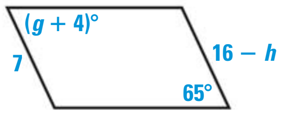


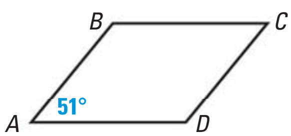

7.



8.



**EXAMPLE 2** on p. 517 for Exs. 9–12

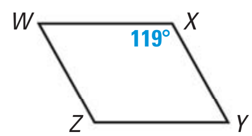
**FINDING ANGLE MEASURES** **Find the measure of the indicated angle in the parallelogram.**

9. Find $m\angle B$.




10. Find $m\angle L$.

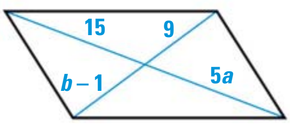

11. Find $m\angle Y$.

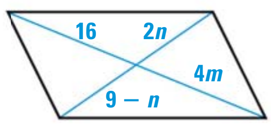


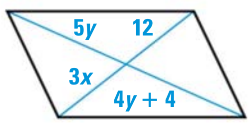

12. **SKETCHING** In ▱$PQRS$, $m\angle R$ is 24 degrees more than $m\angle S$. Sketch ▱$PQRS$. Find the measure of each interior angle. Then label each angle with its measure.

**EXAMPLE 3** on p. 517 for Exs. 13–16

**ALGEBRA** **Find the value of each variable in the parallelogram.**

13.


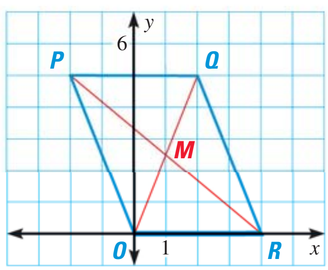

14.



15.



16. ★ **MULTIPLE CHOICE** The diagonals of parallelogram $OPQR$ intersect at point $M$. What are the coordinates of point $M$?

Ⓐ $\left(1, \frac{5}{2}\right)$ Ⓑ $\left(2, \frac{5}{2}\right)$

Ⓒ $\left(1, \frac{3}{2}\right)$ Ⓓ $\left(2, \frac{3}{2}\right)$



**REASONING** **Use the photo to copy and complete the statement. *Explain.***

**17.** $\overline{AD} \cong$ _?_

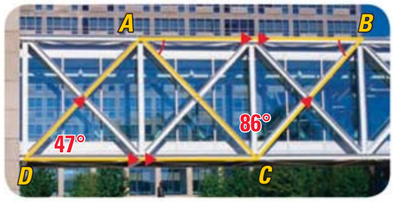
**18.** $\angle DAB \cong$ _?_

**19.** $\angle BCA \cong$ _?_

**20.** $m\angle ABC =$ _?_

**21.** $m\angle CAB =$ _?_

**22.** $m\angle CAD =$ _?_




**USING A DIAGRAM** **Find the indicated measure in ▱*EFGH*. *Explain.***

**23.** $m\angle EJF$

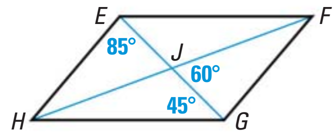
**24.** $m\angle EGF$

**25.** $m\angle HFG$

**26.** $m\angle GEF$

**27.** $m\angle HGF$

**28.** $m\angle EHG$

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**29.** ★ **MULTIPLE CHOICE** In parallelogram *ABCD*, $AB = 14$ inches and $BC = 20$ inches. What is the perimeter (in inches) of ▱*ABCD*?

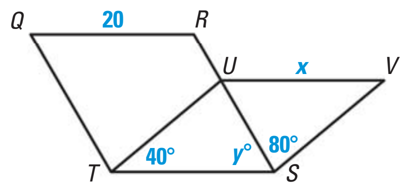
(A) 28 (B) 40 (C) 68 (D) 280

**30.** **ALGEBRA** The measure of one interior angle of a parallelogram is 0.25 times the measure of another angle. Find the measure of each angle.

**31.** **ALGEBRA** The measure of one interior angle of a parallelogram is 50 degrees more than 4 times the measure of another angle. Find the measure of each angle.

**32.** **ERROR ANALYSIS** In ▱*ABCD*, $m\angle B = 50°$. A student says that $m\angle A = 50°$. *Explain* why this statement is incorrect.

**33.** **USING A DIAGRAM** In the diagram, *QRST* and *STUV* are parallelograms. Find the values of $x$ and $y$. *Explain* your reasoning.

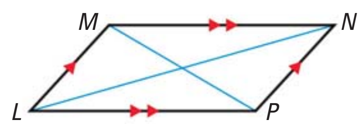



**34.** **FINDING A PERIMETER** The sides of ▱*MNPQ* are represented by the expressions below. Sketch ▱*MNPQ* and find its perimeter.

$MQ = -2x + 37$ $QP = y + 14$ $NP = x - 5$ $MN = 4y + 5$

**35.** ★ **SHORT RESPONSE** In *ABCD*, $m\angle B = 124°$, $m\angle A = 66°$, and $m\angle C = 124°$. *Explain* why *ABCD* cannot be a parallelogram.

**36.** **FINDING ANGLE MEASURES** In ▱*LMNP* shown at the right, $m\angle MLN = 32°$, $m\angle NLP = (x^2)°$, $m\angle MNP = 12x°$, and $\angle MNP$ is an acute angle. Find $m\angle NLP$.




**37.** **CHALLENGE** Points $A(1, 2)$, $B(3, 6)$, and $C(6, 4)$ are three vertices of ▱*ABCD*. Find the coordinates of each point that could be vertex *D*. Sketch each possible parallelogram in a separate coordinate plane. *Justify* your answers.

## PROBLEM SOLVING

**EXAMPLE 2** on p. 517 for Ex. 38

**38. AIRPLANE** The diagram shows the mechanism for opening the canopy on a small airplane. Two pivot arms attach at four pivot points $A$, $B$, $C$, and $D$. These points form the vertices of a parallelogram. Find $m\angle D$ when $m\angle C = 40°$. *Explain* your reasoning.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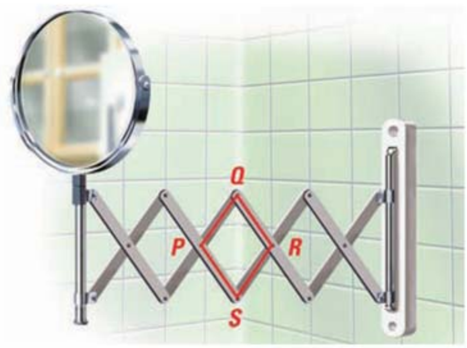
**39. MIRROR** The mirror shown is attached to the wall by an arm that can extend away from the wall. In the figure, points $P$, $Q$, $R$, and $S$ are the vertices of a parallelogram. This parallelogram is one of several that change shape as the mirror is extended.




**a.** If $PQ = 3$ inches, find $RS$.

**b.** If $m\angle Q = 70°$, what is $m\angle S$?

**c.** What happens to $m\angle P$ as $m\angle Q$ increases? What happens to $QS$ as $m\angle Q$ decreases? *Explain*.

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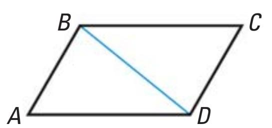
**40. USING RATIOS** In ▱$LMNO$, the ratio of $LM$ to $MN$ is $4:3$. Find $LM$ if the perimeter of $LMNO$ is 28.

**41. ★ OPEN-ENDED MATH** Draw a triangle. Copy the triangle and combine the two triangles to form a quadrilateral. Show that the quadrilateral is a parallelogram. Then show how you can make additional copies of the triangle to form a larger parallelogram that is similar to the first parallelogram. *Justify* your method.

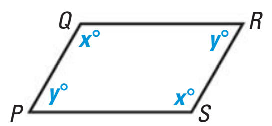
**42. PROVING THEOREM 8.4** Use the diagram of quadrilateral $ABCD$ with the auxiliary line segment drawn to write a two-column proof of Theorem 8.4.




**GIVEN** ▶ $ABCD$ is a parallelogram.

**PROVE** ▶ $\angle A \cong \angle C$, $\angle B \cong \angle D$

**43. PROVING THEOREM 8.5** Use properties of parallel lines to prove Theorem 8.5.




**GIVEN** ▶ $PQRS$ is a parallelogram.

**PROVE** ▶ $x° + y° = 180°$

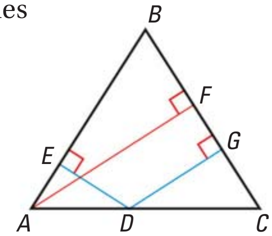


○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

44. **PROVING THEOREM 8.6** Theorem 8.6 states that if a quadrilateral is a parallelogram, then its diagonals bisect each other. Write a two-column proof of Theorem 8.6.

45. **CHALLENGE** Suppose you choose a point on the base of an isosceles triangle. You draw segments from that point perpendicular to the legs of the triangle. Prove that the sum of the lengths of those segments is equal to the length of the altitude drawn to one leg.




**GIVEN** ▶ $\triangle ABC$ is isosceles with base $\overline{AC}$,
$\overline{AF}$ is the altitude drawn to $\overline{BC}$,
$\overline{DE} \perp \overline{AB}$, $\overline{DG} \perp \overline{BC}$

**PROVE** ▶ For $D$ anywhere on $\overline{AC}$, $DE + DG = AF$.

## MIXED REVIEW

**PREVIEW**
Prepare for Lesson 8.3 in Exs. 46–48.

**Tell whether the lines through the given points are *parallel*, *perpendicular*, or *neither*. *Justify* your answer.** *(p. 171)*

46. **Line 1:** (2, 4), (4, 1)
**Line 2:** (5, 7), (9, 0)

47. **Line 1:** (−6, 7), (−2, 3)
**Line 2:** (9, −1), (2, 6)

48. **Line 1:** (−3, 0), (−6, 5)
**Line 2:** (3, −5), (5, −10)

**Decide if the side lengths form a triangle. If so, would the triangle be *acute*, *right*, or *obtuse*?** *(p. 441)*

49. 9, 13, and 6

50. 10, 12, and 7

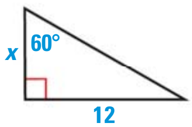
51. 5, 9, and $\sqrt{106}$

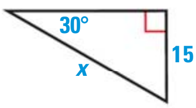
52. 8, 12, and 4

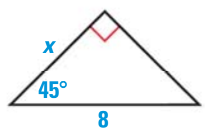
53. 24, 10, and 26

54. 9, 10, and 11

**Find the value of $x$. Write your answer in simplest radical form.** *(p. 457)*

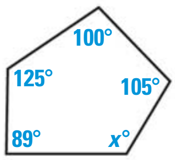
55.


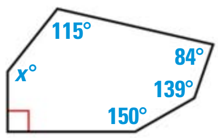

56.


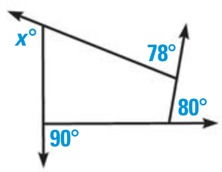

57.



## QUIZ *for Lessons 8.1–8.2*

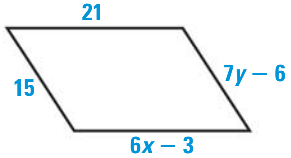
**Find the value of $x$.** *(p. 507)*

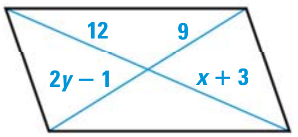
1.


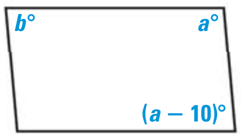

2.



3.



**Find the value of each variable in the parallelogram.** *(p. 515)*

4.



5.



6.


# 8.3 Show that a Quadrilateral is a Parallelogram

**Before** You identified properties of parallelograms.

**Now** You will use properties to identify parallelograms.

**Why?** So you can describe how a music stand works, as in Ex. 32.

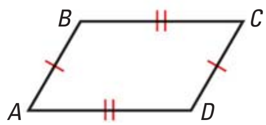
**Key Vocabulary**
- **parallelogram,** *p. 515*

Given a parallelogram, you can use Theorem 8.3 and Theorem 8.4 to prove statements about the angles and sides of the parallelogram. The converses of Theorem 8.3 and Theorem 8.4 are stated below. You can use these and other theorems in this lesson to prove that a quadrilateral with certain properties is a parallelogram.

## THEOREMS *For Your Notebook*

### THEOREM 8.7

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

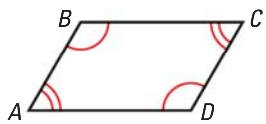



If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

*Proof:* below

### THEOREM 8.8

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



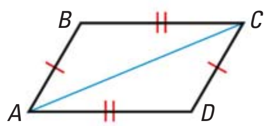

If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.

*Proof:* Ex. 38, p. 529

## PROOF Theorem 8.7

**GIVEN** ▶ $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{AD}$

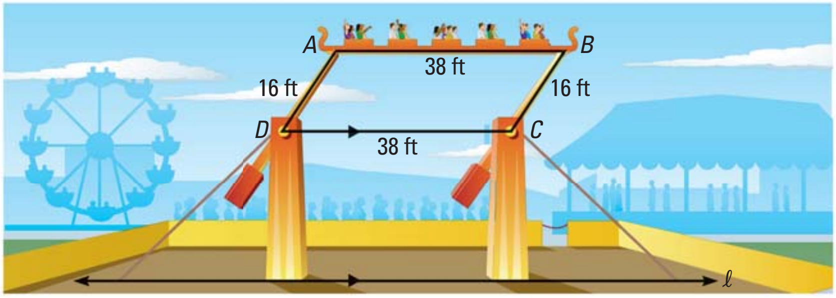
**PROVE** ▶ $ABCD$ is a parallelogram.




**Proof** Draw $\overline{AC}$, forming $\triangle ABC$ and $\triangle CDA$. You are given that $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$. Also, $\overline{AC} \cong \overline{AC}$ by the Reflexive Property of Congruence. So, $\triangle ABC \cong \triangle CDA$ by the SSS Congruence Postulate. Because corresponding parts of congruent triangles are congruent, $\angle BAC \cong \angle DCA$ and $\angle BCA \cong \overline{DAC}$. Then, by the Alternate Interior Angles Converse, $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$. By definition, $ABCD$ is a parallelogram.

## EXAMPLE 1 Solve a real-world problem

**RIDE** An amusement park ride has a moving platform attached to four swinging arms. The platform swings back and forth, higher and higher, until it goes over the top and around in a circular motion. In the diagram below, $\overline{AD}$ and $\overline{BC}$ represent two of the swinging arms, and $\overline{DC}$ is parallel to the ground (line $\ell$). *Explain* why the moving platform $\overline{AB}$ is always parallel to the ground.




**Solution**

The shape of quadrilateral $ABCD$ changes as the moving platform swings around, but its side lengths do not change. Both pairs of opposite sides are congruent, so $ABCD$ is a parallelogram by Theorem 8.7.

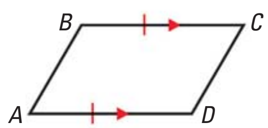
By the definition of a parallelogram, $\overline{AB} \parallel \overline{DC}$. Because $\overline{DC}$ is parallel to line $\ell$, $\overline{AB}$ is also parallel to line $\ell$ by the Transitive Property of Parallel Lines. So, the moving platform is parallel to the ground.

### ✓ GUIDED PRACTICE for Example 1

1. In quadrilateral $WXYZ$, $m\angle W = 42°$, $m\angle X = 138°$, $m\angle Y = 42°$. Find $m\angle Z$. Is $WXYZ$ a parallelogram? *Explain* your reasoning.

### THEOREMS *For Your Notebook*

**THEOREM 8.9**

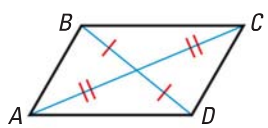



If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

*Proof:* Ex. 33, p. 528

**THEOREM 8.10**




If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

If $\overline{BD}$ and $\overline{AC}$ bisect each other, then $ABCD$ is a parallelogram.

*Proof:* Ex. 39, p. 529

# EXAMPLE 2 Identify a parallelogram

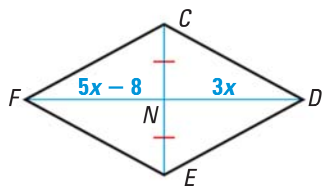
**ARCHITECTURE** The doorway shown is part of a building in England. Over time, the building has leaned sideways. *Explain* how you know that $SV = TU$.


**Solution**

In the photograph, $\overline{ST} \parallel \overline{UV}$ and $\overline{ST} \cong \overline{UV}$. By Theorem 8.9, quadrilateral $STUV$ is a parallelogram. By Theorem 8.3, you know that opposite sides of a parallelogram are congruent. So, $SV = TU$.

# EXAMPLE 3 Use algebra with parallelograms

**ALGEBRA For what value of $x$ is quadrilateral $CDEF$ a parallelogram?**




**Solution**

By Theorem 8.10, if the diagonals of $CDEF$ bisect each other, then it is a parallelogram. You are given that $\overline{CN} \cong \overline{EN}$. Find $x$ so that $\overline{FN} \cong \overline{DN}$.

| | |
|---|---|
| $FN = DN$ | Set the segment lengths equal. |
| $5x - 8 = 3x$ | Substitute $5x - 8$ for $FN$ and $3x$ for $DN$. |
| $2x - 8 = 0$ | Subtract $3x$ from each side. |
| $2x = 8$ | Add 8 to each side. |
| $x = 4$ | Divide each side by 2. |

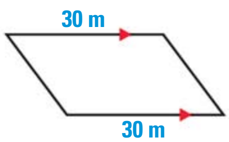
When $x = 4$, $FN = 5(4) - 8 = 12$ and $DN = 3(4) = 12$.

▶ Quadrilateral $CDEF$ is a parallelogram when $x = 4$.

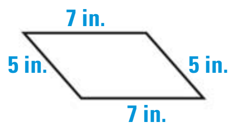
## ✓ GUIDED PRACTICE for Examples 2 and 3

**What theorem can you use to show that the quadrilateral is a parallelogram?**

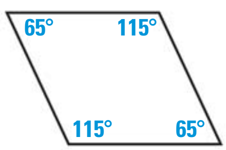
**2.**



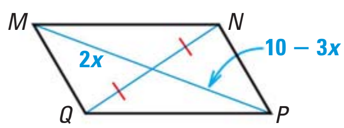

**3.**




**4.**



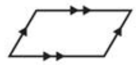

**5.** For what value of $x$ is quadrilateral $MNPQ$ a parallelogram? *Explain* your reasoning.



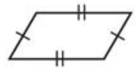
## CONCEPT SUMMARY *For Your Notebook*

### Ways to Prove a Quadrilateral is a Parallelogram

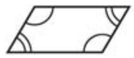
1. Show both pairs of opposite sides are parallel.
*(DEFINITION)*



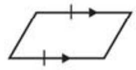
2. Show both pairs of opposite sides are congruent.
*(THEOREM 8.7)*



3. Show both pairs of opposite angles are congruent.
*(THEOREM 8.8)*



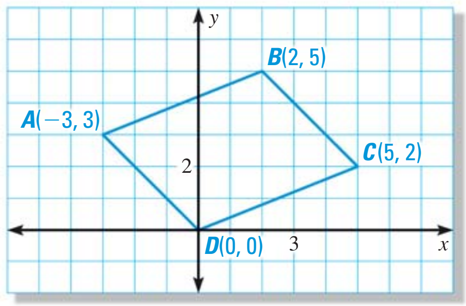
4. Show one pair of opposite sides are congruent and parallel.
*(THEOREM 8.9)*



5. Show the diagonals bisect each other.
*(THEOREM 8.10)*

## EXAMPLE 4 Use coordinate geometry

**Show that quadrilateral *ABCD* is a parallelogram.**




**ANOTHER WAY**
For alternative methods for solving the problem in Example 4, turn to page 530 for the **Problem Solving Workshop**.

### Solution

One way is to show that a pair of sides are congruent and parallel. Then apply Theorem 8.9.

First use the Distance Formula to show that $\overline{AB}$ and $\overline{CD}$ are congruent.

$AB = \sqrt{[2 - (-3)]^2 + (5 - 3)^2} = \sqrt{29}$  $CD = \sqrt{(5 - 0)^2 + (2 - 0)^2} = \sqrt{29}$

Because $AB = CD = \sqrt{29}$, $\overline{AB} \cong \overline{CD}$.

Then use the slope formula to show that $\overline{AB} \parallel \overline{CD}$.

Slope of $\overline{AB} = \dfrac{5 - (3)}{2 - (-3)} = \dfrac{2}{5}$  Slope of $\overline{CD} = \dfrac{2 - 0}{5 - 0} = \dfrac{2}{5}$

Because $\overline{AB}$ and $\overline{CD}$ have the same slope, they are parallel.

▶ $\overline{AB}$ and $\overline{CD}$ are congruent and parallel. So, *ABCD* is a parallelogram by Theorem 8.9.

## ✓ GUIDED PRACTICE for Example 4

6. Refer to the Concept Summary above. *Explain* how other methods can be used to show that quadrilateral *ABCD* in Example 4 is a parallelogram.

# 8.3 EXERCISES

**HOMEWORK KEY**

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 11, and 31

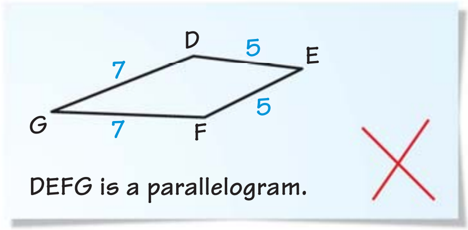
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 18, and 37

## SKILL PRACTICE

1. **VOCABULARY** *Explain* how knowing that $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$ allows you to show that quadrilateral $ABCD$ is a parallelogram.

2. ★ **WRITING** A quadrilateral has four congruent sides. Is the quadrilateral a parallelogram? *Justify* your answer.

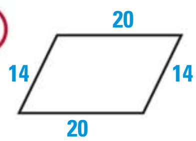
3. **ERROR ANALYSIS** A student claims that because two pairs of sides are congruent, quadrilateral $DEFG$ shown at the right is a parallelogram. *Describe* the error that the student is making.

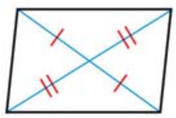



**EXAMPLES 1 and 2**
on pp. 523–524
for Exs. 4–7

**REASONING** **What theorem can you use to show that the quadrilateral is a parallelogram?**

4. 

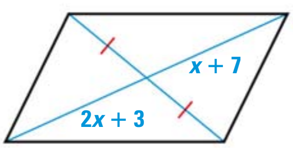

5. 

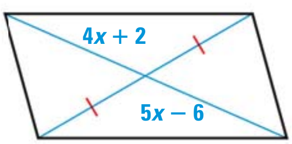

6. 

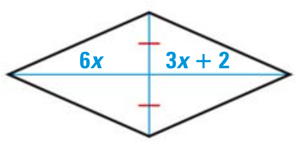
7. ★ **SHORT RESPONSE** When you shift gears on a bicycle, a mechanism called a *derailleur* moves the chain to a new gear. For the derailleur shown below, $JK = 5.5$ cm, $KL = 2$ cm, $ML = 5.5$ cm, and $MJ = 2$ cm. *Explain* why $\overline{JK}$ and $\overline{ML}$ are always parallel as the derailleur moves.


**EXAMPLE 3**
on p. 524
for Exs. 8–10

**ALGEBRA** **For what value of $x$ is the quadrilateral a parallelogram?**

8. 


9. 


10. 


**EXAMPLE 4**
on p. 525
for Exs. 11–14

**COORDINATE GEOMETRY** **The vertices of quadrilateral $ABCD$ are given. Draw $ABCD$ in a coordinate plane and show that it is a parallelogram.**

11. $A(0, 1)$, $B(4, 4)$, $C(12, 4)$, $D(8, 1)$

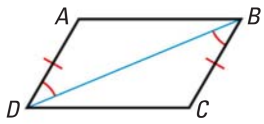
12. $A(-3, 0)$, $B(-3, 4)$, $C(3, -1)$, $D(3, -5)$

13. $A(-2, 3)$, $B(-5, 7)$, $C(3, 6)$, $D(6, 2)$

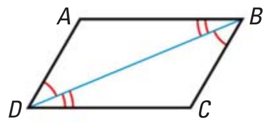
14. $A(-5, 0)$, $B(0, 4)$, $C(3, 0)$, $D(-2, -4)$

**REASONING** *Describe* **how to prove that** ***ABCD*** **is a parallelogram.**

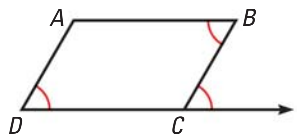
**15.**




**16.**




**17.**

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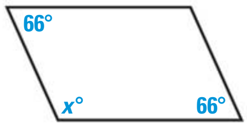
**18.** ★ **MULTIPLE CHOICE** In quadrilateral $WXYZ$, $\overline{WZ}$ and $\overline{XY}$ are congruent and parallel. Which statement below is not necessarily true?

Ⓐ $m\angle Y + m\angle W = 180°$ Ⓑ $\angle X \cong \angle Z$

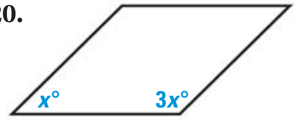
Ⓒ $\overline{WX} \cong \overline{ZY}$ Ⓓ $\overline{WX} \parallel \overline{ZY}$

**xy ALGEBRA** **For what value of** $x$ **is the quadrilateral a parallelogram?**

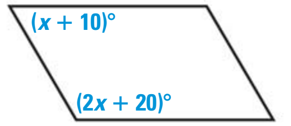
**19.**




**20.**




**21.**

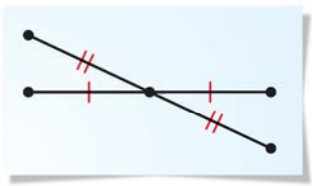



**BICONDITIONALS** **Write the indicated theorems as a biconditional statement.**

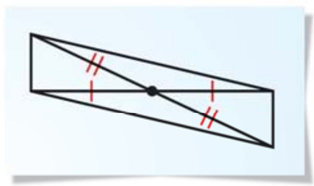
**22.** Theorem 8.3, page 515 and Theorem 8.7, page 522

**23.** Theorem 8.4, page 515 and Theorem 8.8, page 522

**24.** **REASONING** Follow the steps below to draw a parallelogram. *Explain* why this method works. State a theorem to support your answer.



***STEP 1*** Use a ruler to draw two segments that intersect at their midpoints.



***STEP 2*** Connect the endpoints of the segments to form a quadrilateral.

**COORDINATE GEOMETRY** **Three of the vertices of** $\square ABCD$ **are given. Find the coordinates of point** ***D*****. Show your method.**

**25.** $A(-2, -3)$, $B(4, -3)$, $C(3, 2)$, $D(x, y)$

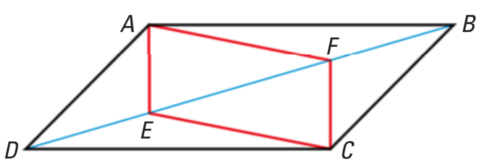
**26.** $A(-4, 1)$, $B(-1, 5)$, $C(6, 5)$, $D(x, y)$

**27.** $A(-4, 4)$, $B(4, 6)$, $C(3, -1)$, $D(x, y)$

**28.** $A(-1, 0)$, $B(0, -4)$, $C(8, -6)$, $D(x, y)$

**29.** **CONSTRUCTION** There is more than one way to use a compass and a straightedge to construct a parallelogram. *Describe* a method that uses Theorem 8.7 or Theorem 8.9. Then use your method to construct a parallelogram.

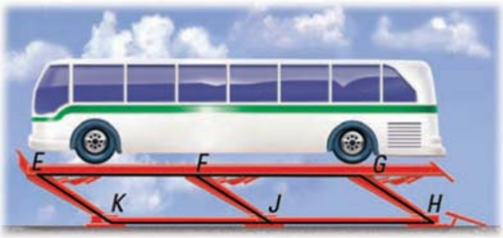
**30.** **CHALLENGE** In the diagram, $ABCD$ is a parallelogram, $BF = DE = 12$, and $CF = 8$. Find $AE$. *Explain* your reasoning.



## PROBLEM SOLVING

**EXAMPLES 1 and 2** on pp. 523–524 for Exs. 31–32

**31.** **AUTOMOBILE REPAIR** The diagram shows an automobile lift. A bus drives on to the ramp ($\overline{EG}$). Levers ($\overline{EK}$, $\overline{FJ}$, and $\overline{GH}$) raise the bus. In the diagram, $\overline{EG} \cong \overline{KH}$ and $EK = FJ = GH$. Also, $F$ is the midpoint of $\overline{EG}$, and $J$ is the midpoint of $\overline{KH}$.




**a.** Identify all the quadrilaterals in the automobile lift. *Explain* how you know that each one is a parallelogram.

**b.** *Explain* why $\overline{EG}$ is always parallel to $\overline{KH}$.

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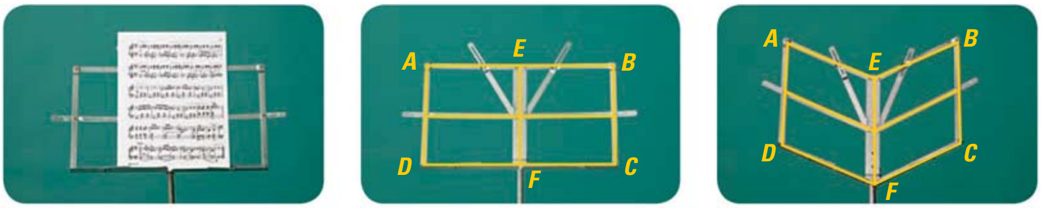
**32.** **MUSIC STAND** A music stand can be folded up, as shown below. In the diagrams, $\angle A \cong \angle EFD$, $\angle D \cong \angle AEF$, $\angle C \cong \angle BEF$, and $\angle B \cong \angle CFE$. *Explain* why $\overline{AD}$ and $\overline{BC}$ remain parallel as the stand is folded up. Which other labeled segments remain parallel?

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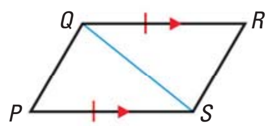
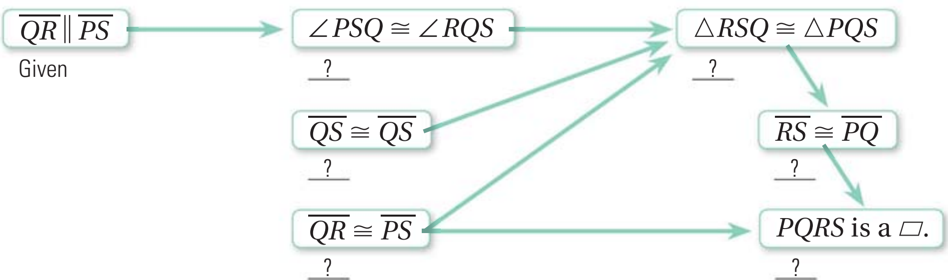
**33.** **PROVING THEOREM 8.9** Use the diagram of $PQRS$ with the auxiliary line segment drawn. Copy and complete the flow proof of Theorem 8.9.



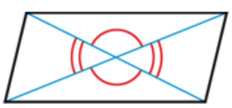

**GIVEN** ▶ $\overline{QR} \parallel \overline{PS}$, $\overline{QR} \cong \overline{PS}$

**PROVE** ▶ $PQRS$ is a parallelogram.

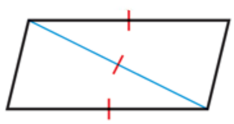



**REASONING** **A student claims incorrectly that the marked information can be used to show that the figure is a parallelogram. Draw a quadrilateral with the marked properties that is clearly *not* a parallelogram. *Explain*.**

**34.**



**35.**



**36.**

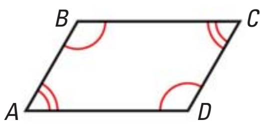



◯ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

37. ★ **EXTENDED RESPONSE** Theorem 8.5 states that if a quadrilateral is a parallelogram, then its consecutive angles are supplementary. Write the converse of Theorem 8.5. Then write a plan for proving the converse of Theorem 8.5. Include a diagram.

38. **PROVING THEOREM 8.8** Prove Theorem 8.8.

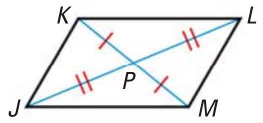



**GIVEN ▶** $\angle A \cong \angle C$, $\angle B \cong \angle D$

**PROVE ▶** $ABCD$ is a parallelogram.

*Hint:* Let $x°$ represent $m\angle A$ and $m\angle C$, and let $y°$ represent $m\angle B$ and $m\angle D$. Write and simplify an equation involving $x$ and $y$.

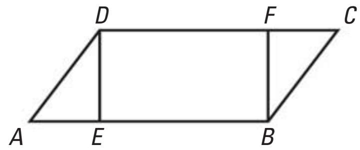
39. **PROVING THEOREM 8.10** Prove Theorem 8.10.




**GIVEN ▶** Diagonals $\overline{JL}$ and $\overline{KM}$ bisect each other.

**PROVE ▶** $JKLM$ is a parallelogram.

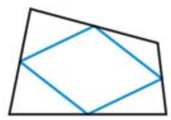
40. **PROOF** Use the diagram at the right.



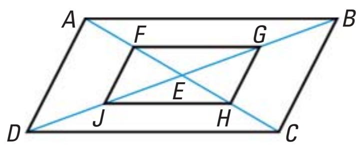

**GIVEN ▶** $DEBF$ is a parallelogram, $AE = CF$

**PROVE ▶** $ABCD$ is a parallelogram.

41. **REASONING** In the diagram, the midpoints of the sides of a quadrilateral have been joined to form what appears to be a parallelogram. Show that a quadrilateral formed by connecting the midpoints of the sides of any quadrilateral is *always* a parallelogram. (*Hint:* Draw a diagram. Include a diagonal of the larger quadrilateral. Show how two sides of the smaller quadrilateral are related to the diagonal.)



42. **CHALLENGE** Show that if $ABCD$ is a parallelogram with its diagonals intersecting at $E$, then you can connect the midpoints $F$, $G$, $H$, and $J$ of $\overline{AE}$, $\overline{BE}$, $\overline{CE}$, and $\overline{DE}$, respectively, to form another parallelogram, $FGHJ$.




## MIXED REVIEW

**PREVIEW**
Prepare for Lesson 8.4 in Exs. 43–45.

**In Exercises 43–45, draw a figure that fits the description.** *(p. 42)*

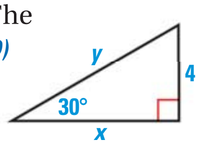
43. A quadrilateral that is equilateral but not equiangular

44. A quadrilateral that is equiangular but not equilateral

45. A quadrilateral that is concave

46. The width of a rectangle is 4 centimeters less than its length. The perimeter of the rectangle is 42 centimeters. Find its area. *(p. 49)*

47. Find the values of $x$ and $y$ in the triangle shown at the right. Write your answers in simplest radical form. *(p. 457)*



PROBLEM SOLVING WORKSHOP
LESSON 8.3

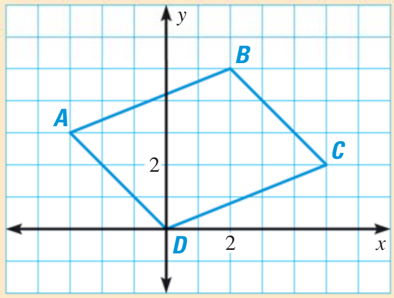
# Using ALTERNATIVE METHODS

## Another Way to Solve Example 4, page 525

**MULTIPLE REPRESENTATIONS** In Example 4 on page 525, the problem is solved by showing that one pair of opposite sides are congruent and parallel using the Distance Formula and the slope formula. There are other ways to show that a quadrilateral is a parallelogram.

**PROBLEM**

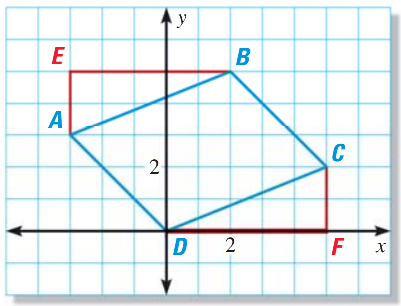
**Show that quadrilateral *ABCD* is a parallelogram.**




**METHOD 1**

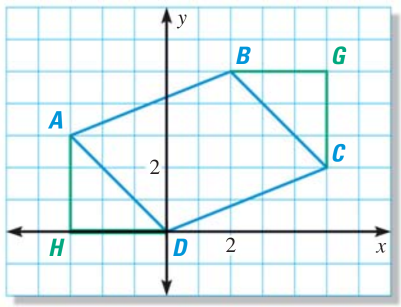
**Use Opposite Sides** You can show that both pairs of opposite sides are congruent.

*STEP 1* **Draw** two right triangles. Use $\overline{AB}$ as the hypotenuse of $\triangle AEB$ and $\overline{CD}$ as the hypotenuse of $\triangle CFD$.




*STEP 2* **Show** that $\triangle AEB \cong \triangle CFD$. From the graph, $AE = 2$, $BE = 5$, and $\angle E$ is a right angle. Similarly, $CF = 2$, $DF = 5$, and $\angle F$ is a right angle. So, $\triangle AEB \cong \triangle CFD$ by the SAS Congruence Postulate.

*STEP 3* **Use** the fact that corresponding parts of congruent triangles are congruent to show that $\overline{AB} \cong \overline{CD}$.




*STEP 4* **Repeat** Steps 1–3 for sides $\overline{AD}$ and $\overline{BC}$. You can prove that $\triangle AHD \cong \triangle CGB$. So, $\overline{AD} \cong \overline{CB}$.

▶ The pairs of opposite sides, $\overline{AB}$ and $\overline{CD}$ and $\overline{AD}$ and $\overline{CB}$, are congruent. So, *ABCD* is a parallelogram by Theorem 8.7.

**METHOD 2** **Use Diagonals** You can show that the diagonals bisect each other.

***STEP 1*** **Use** the Midpoint Formula to find the midpoint of diagonal $\overline{AC}$.

The coordinates of the endpoints of $\overline{AC}$ are $A(-3, 3)$ and $C(5, 2)$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3 + 5}{2}, \frac{3 + 2}{2}\right) = \left(\frac{2}{2}, \frac{5}{2}\right) = \left(1, \frac{5}{2}\right)$$

***STEP 2*** **Use** the Midpoint Formula to find the midpoint of diagonal $\overline{BD}$.

The coordinates of the endpoints of $\overline{BD}$ are $B(2, 5)$ and $D(0, 0)$.

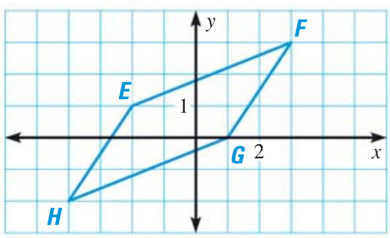
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + 0}{2}, \frac{5 + 0}{2}\right) = \left(\frac{2}{2}, \frac{5}{2}\right) = M\left(1, \frac{5}{2}\right)$$

▶ Because the midpoints of both diagonals are the same point, the diagonals bisect each other. So, *ABCD* is a parallelogram by Theorem 8.10.

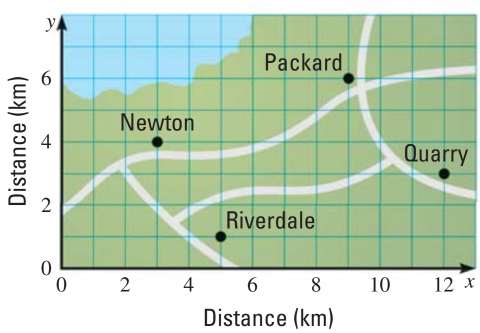
## PRACTICE

1. **SLOPE** Show that quadrilateral *ABCD* in the problem on page 530 is a parallelogram by showing that both pairs of opposite sides are parallel.

2. **PARALLELOGRAMS** Use two methods to show that *EFGH* is a parallelogram.




3. **MAP** Do the four towns on the map form the vertices of a parallelogram? *Explain.*




4. **QUADRILATERALS** Is the quadrilateral a parallelogram? *Justify* your answer.
   a. $A(1, 0)$, $B(5, 0)$, $C(7, 2)$, $D(3, 2)$
   b. $E(3, 4)$ $F(9, 5)$, $G(6, 8)$, $H(6, 0)$
   c. $J(-1, 0)$, $K(2, -2)$, $L(2, 2)$, $M(-1, 4)$

5. **ERROR ANALYSIS** Quadrilateral *PQRS* has vertices $P(2, 2)$, $Q(3, 4)$, $R(6, 5)$, and $S(5, 3)$. A student makes the conclusion below. *Describe* and correct the error(s) made by the student.

> $\overline{PQ}$ and $\overline{QR}$ are opposite sides, so they should be congruent.
>
> $PQ = \sqrt{(3-2)^2 + (4-2)^2} = \sqrt{5}$
>
> $QR = \sqrt{(6-3)^2 + (5-4)^2} = \sqrt{10}$
>
> But $\overline{PQ} \not\cong \overline{QR}$. So, PQRS is not a parallelogram.

6. **WRITING** Points $O(0, 0)$, $P(3, 5)$, and $Q(4, 0)$ are vertices of $\triangle OPQ$, and are also vertices of a parallelogram. Find all points *R* that could be the other vertex of the parallelogram. *Explain* your reasoning.

# MIXED REVIEW *of Problem Solving*



## Lessons 8.1–8.3

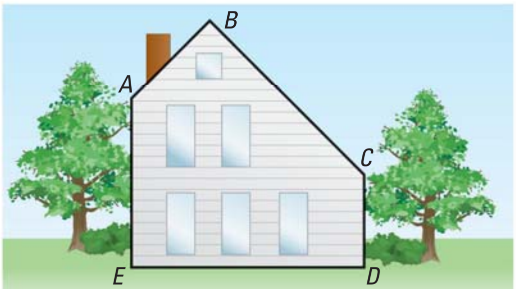
1. **MULTI-STEP PROBLEM** The shape of Iowa can be approximated by a polygon, as shown.


   a. How many sides does the polygon have? Classify the polygon.

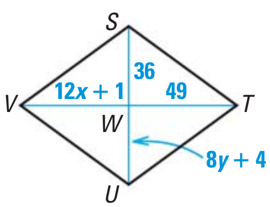
   b. What is the sum of the measures of the interior angles of the polygon?

   c. What is the sum of the measures of the exterior angles of the polygon?

2. **SHORT RESPONSE** A graphic designer is creating an electronic image of a house. In the drawing, $\angle B$, $\angle D$, and $\angle E$ are right angles, and $\angle A \cong \angle C$. *Explain* how to find $m\angle A$ and $m\angle C$.



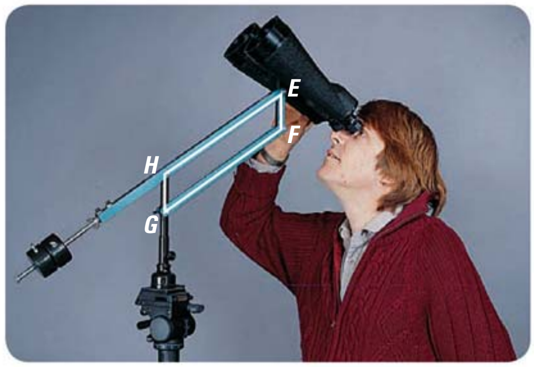

3. **SHORT RESPONSE** Quadrilateral *STUV* shown below is a parallelogram. Find the values of *x* and *y*. *Explain* your reasoning.




4. **GRIDDED ANSWER** A convex decagon has interior angles with measures 157°, 128°, 115°, 162°, 169°, 131°, 155°, 168°, $x°$, and $2x°$. Find the value of *x*.

5. **SHORT RESPONSE** The measure of an angle of a parallelogram is 12 degrees less than 3 times the measure of an adjacent angle. Explain how to find the measures of all the interior angles of the parallelogram.

6. **EXTENDED RESPONSE** A stand to hold binoculars in place uses a quadrilateral in its design. Quadrilateral *EFGH* shown below changes shape as the binoculars are moved. In the photograph, $\overline{EF}$ and $\overline{GH}$ are congruent and parallel.




   a. *Explain* why $\overline{EF}$ and $\overline{GH}$ remain parallel as the shape of *EFGH* changes. *Explain* why $\overline{EH}$ and $\overline{FG}$ remain parallel.

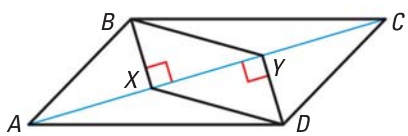
   b. As *EFGH* changes shape, $m\angle E$ changes from 55° to 50°. *Describe* how $m\angle F$, $m\angle G$, and $m\angle H$ will change. *Explain*.

7. **EXTENDED RESPONSE** The vertices of quadrilateral *MNPQ* are $M(-8, 1)$, $N(3, 4)$, $P(7, -1)$, and $Q(-4, -4)$.

   a. Use what you know about slopes of lines to prove that *MNPQ* is a parallelogram. *Explain* your reasoning.

   b. Use the Distance Formula to show that *MNPQ* is a parallelogram. *Explain*.

8. **EXTENDED RESPONSE** In $\square ABCD$, $\overline{BX} \perp \overline{AC}$, $\overline{DY} \perp \overline{AC}$. Show that *XBYD* is a parallelogram.



# 8.4 Properties of Rhombuses, Rectangles, and Squares

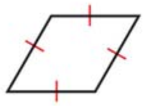
**Before** You used properties of parallelograms.

**Now** You will use properties of rhombuses, rectangles, and squares.

**Why?** So you can solve a carpentry problem, as in Example 4.

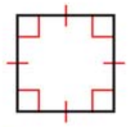
**Key Vocabulary**
- **rhombus**
- **rectangle**
- **square**

In this lesson, you will learn about three special types of parallelograms: *rhombuses*, *rectangles*, and *squares*.



**A rhombus is a parallelogram with four congruent sides.**

**A rectangle is a parallelogram with four right angles.**



**A square is a parallelogram with four congruent sides and four right angles.**

You can use the corollaries below to prove that a quadrilateral is a rhombus, rectangle, or square, without first proving that the quadrilateral is a parallelogram.

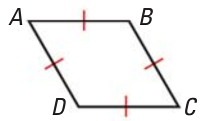
## COROLLARIES — *For Your Notebook*

### RHOMBUS COROLLARY

A quadrilateral is a rhombus if and only if it has four congruent sides.

$ABCD$ is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.

*Proof:* Ex. 57, p. 539

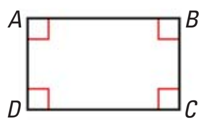



### RECTANGLE COROLLARY

A quadrilateral is a rectangle if and only if it has four right angles.

$ABCD$ is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

*Proof:* Ex. 58, p. 539

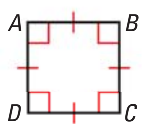



### SQUARE COROLLARY

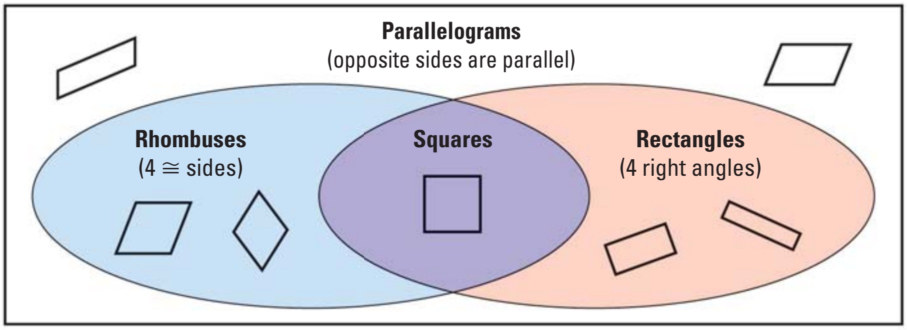
A quadrilateral is a square if and only if it is a rhombus and a rectangle.

$ABCD$ is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

*Proof:* Ex. 59, p. 539



The *Venn diagram* below illustrates some important relationships among parallelograms, rhombuses, rectangles, and squares. For example, you can see that a square is a rhombus because it is a parallelogram with four congruent sides. Because it has four right angles, a square is also a rectangle.




## EXAMPLE 1 Use properties of special quadrilaterals

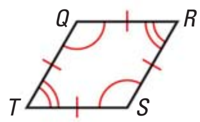
**For any rhombus *QRST*, decide whether the statement is *always* or *sometimes* true. Draw a sketch and explain your reasoning.**

**a.** $\angle Q \cong \angle S$

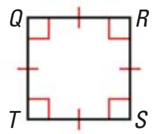
**b.** $\angle Q \cong \angle R$

### Solution

**a.** By definition, a rhombus is a parallelogram with four congruent sides. By Theorem 8.4, opposite angles of a parallelogram are congruent. So, $\angle Q \cong \angle S$. The statement is *always* true.

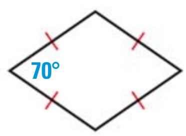



**b.** If rhombus *QRST* is a square, then all four angles are congruent right angles. So, $\angle Q \cong \angle R$ if *QRST* is a square. Because not all rhombuses are also squares, the statement is *sometimes* true.




## EXAMPLE 2 Classify special quadrilaterals

**Classify the special quadrilateral. Explain your reasoning.**




### Solution

The quadrilateral has four congruent sides. One of the angles is not a right angle, so the rhombus is not also a square. By the Rhombus Corollary, the quadrilateral is a rhombus.

## ✓ GUIDED PRACTICE for Examples 1 and 2

1. For any rectangle *EFGH*, is it *always* or *sometimes* true that $\overline{FG} \cong \overline{GH}$? *Explain* your reasoning.
2. A quadrilateral has four congruent sides and four congruent angles. Sketch the quadrilateral and classify it.

**DIAGONALS** The theorems below describe some properties of the diagonals of rhombuses and rectangles.

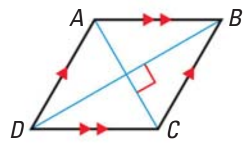
## THEOREMS *For Your Notebook*

### THEOREM 8.11

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

$\square ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

*Proof:* p. 536; Ex. 56, p. 539

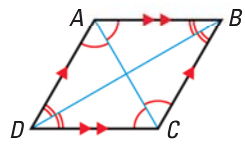



### THEOREM 8.12

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

$\square ABCD$ is a rhombus if and only if $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$ and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.

*Proof:* Exs. 60–61, p. 539

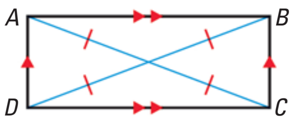



### THEOREM 8.13

A parallelogram is a rectangle if and only if its diagonals are congruent.

$\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

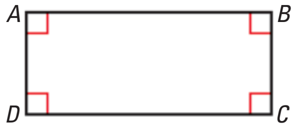
*Proof:* Exs. 63–64, p. 540




## EXAMPLE 3 List properties of special parallelograms

**Sketch rectangle *ABCD*. List everything that you know about it.**

### Solution




By definition, you need to draw a figure with the following properties:

- The figure is a parallelogram.
- The figure has four right angles.

Because *ABCD* is a parallelogram, it also has these properties:

- Opposite sides are parallel and congruent.
- Opposite angles are congruent. Consecutive angles are supplementary.
- Diagonals bisect each other.

By Theorem 8.13, the diagonals of *ABCD* are congruent.

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## ✓ GUIDED PRACTICE for Example 3

3. Sketch square *PQRS*. List everything you know about the square.

**BICONDITIONALS** Recall that biconditionals such as Theorem 8.11 can be rewritten as two parts. To prove a biconditional, you must prove both parts.

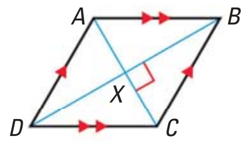
**Conditional statement** If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

**Converse** If a parallelogram is a rhombus, then its diagonals are perpendicular.

## PROOF Part of Theorem 8.11

**PROVE THEOREMS**
You will prove the other part of Theorem 8.11 in Exercise 56 on page 539.

If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.



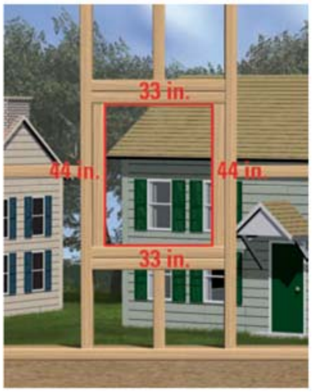

**GIVEN** ▶ $ABCD$ is a parallelogram; $\overline{AC} \perp \overline{BD}$

**PROVE** ▶ $ABCD$ is a rhombus.

**Proof** $ABCD$ is a parallelogram, so $\overline{AC}$ and $\overline{BD}$ bisect each other, and $\overline{BX} \cong \overline{DX}$. Also, $\angle BXC$ and $\angle CXD$ are congruent right angles, and $\overline{CX} \cong \overline{CX}$. So, $\triangle BXC \cong \triangle DXC$ by the SAS Congruence Postulate. Corresponding parts of congruent triangles are congruent, so $\overline{BC} \cong \overline{DC}$. Opposite sides of a $\square ABCD$ are congruent, so $\overline{AD} \cong \overline{BC} \cong \overline{DC} \cong \overline{AB}$. By definition, $ABCD$ is a rhombus.

## EXAMPLE 4 Solve a real-world problem

**CARPENTRY** You are building a frame for a window. The window will be installed in the opening shown in the diagram.




**a.** The opening must be a rectangle. Given the measurements in the diagram, can you assume that it is? *Explain.*

**b.** You measure the diagonals of the opening. The diagonals are 54.8 inches and 55.3 inches. What can you conclude about the shape of the opening?

### Solution

**a.** No, you cannot. The boards on opposite sides are the same length, so they form a parallelogram. But you do not know whether the angles are right angles.

**b.** By Theorem 8.13, the diagonals of a rectangle are congruent. The diagonals of the quadrilateral formed by the boards are not congruent, so the boards do not form a rectangle.

## ✓ GUIDED PRACTICE for Example 4

**4.** Suppose you measure only the diagonals of a window opening. If the diagonals have the same measure, can you conclude that the opening is a rectangle? *Explain.*

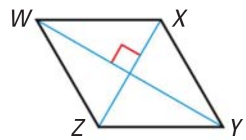
# 8.4 EXERCISES

**HOMEWORK KEY**

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 15, and 55

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 30, 31, and 62

## SKILL PRACTICE




1. **VOCABULARY** What is another name for an equilateral rectangle?

2. ★ **WRITING** Do you have enough information to identify the figure at the right as a rhombus? *Explain*.

**EXAMPLES 1, 2, and 3** on pp. 534–535 for Exs. 3–25

**RHOMBUSES For any rhombus *JKLM*, decide whether the statement is *always* or *sometimes* true. Draw a diagram and *explain* your reasoning.**

3. $\angle L \cong \angle M$
4. $\angle K \cong \angle M$
5. $\overline{JK} \cong \overline{KL}$
6. $\overline{JM} \cong \overline{KL}$
7. $\overline{JL} \cong \overline{KM}$
8. $\angle JKM \cong \angle LKM$

**RECTANGLES For any rectangle *WXYZ*, decide whether the statement is *always* or *sometimes* true. Draw a diagram and *explain* your reasoning.**

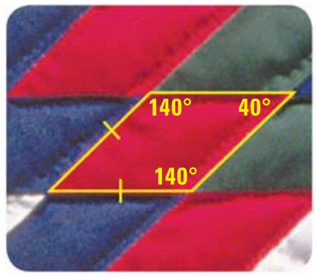
9. $\angle W \cong \angle X$
10. $\overline{WX} \cong \overline{YZ}$
11. $\overline{WX} \cong \overline{XY}$
12. $\overline{WY} \cong \overline{XZ}$
13. $\overline{WY} \perp \overline{XZ}$
14. $\angle WXZ \cong \angle YXZ$

**CLASSIFYING Classify the quadrilateral. *Explain* your reasoning.**

15.

16.

17.

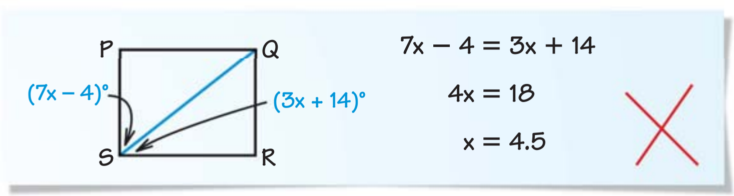



18. **USING PROPERTIES** Sketch rhombus *STUV*. *Describe* everything you know about the rhombus.

**USING PROPERTIES Name each quadrilateral—*parallelogram, rectangle, rhombus,* and *square*—for which the statement is true.**

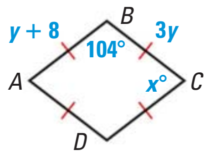
19. It is equiangular.
20. It is equiangular and equilateral.
21. Its diagonals are perpendicular.
22. Opposite sides are congruent.
23. The diagonals bisect each other.
24. The diagonals bisect opposite angles.

25. **ERROR ANALYSIS** Quadrilateral *PQRS* is a rectangle. *Describe* and correct the error made in finding the value of *x*.

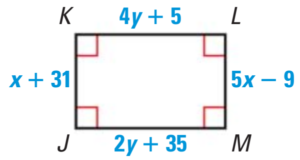


**ALGEBRA** **Classify the special quadrilateral. *Explain* your reasoning. Then find the values of $x$ and $y$.**

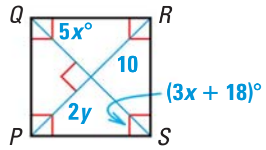
**26.**



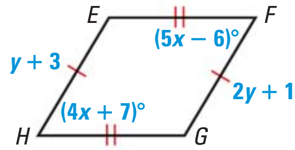

**27.**




**28.**




**29.**

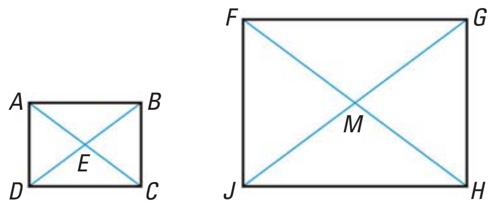



**30.** ★ **SHORT RESPONSE** The diagonals of a rhombus are 6 inches and 8 inches. What is the perimeter of the rhombus? *Explain*.

**31.** ★ **MULTIPLE CHOICE** Rectangle $ABCD$ is similar to rectangle $FGHJ$. If $AC = 5$, $CD = 4$, and $FM = 5$, what is $HJ$?

Ⓐ 4 Ⓑ 5

Ⓒ 8 Ⓓ 10

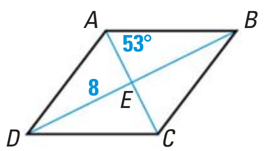



**RHOMBUS** **The diagonals of rhombus $ABCD$ intersect at $E$. Given that $m\angle BAC = 53°$ and $DE = 8$, find the indicated measure.**

**32.** $m\angle DAC$ **33.** $m\angle AED$

**34.** $m\angle ADC$ **35.** $DB$

**36.** $AE$ **37.** $AC$

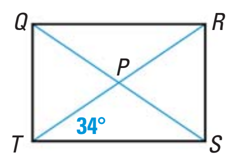



**RECTANGLE** **The diagonals of rectangle $QRST$ intersect at $P$. Given that $m\angle PTS = 34°$ and $QS = 10$, find the indicated measure.**

**38.** $m\angle SRT$ **39.** $m\angle QPR$

**40.** $QP$ **41.** $RP$

**42.** $QR$ **43.** $RS$

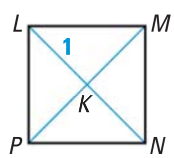



**SQUARE** **The diagonals of square $LMNP$ intersect at $K$. Given that $LK = 1$, find the indicated measure.**

**44.** $m\angle MKN$ **45.** $m\angle LMK$

**46.** $m\angle LPK$ **47.** $KN$

**48.** $MP$ **49.** $LP$



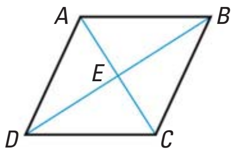

**COORDINATE GEOMETRY** **Use the given vertices to graph ▱$JKLM$. Classify ▱$JKLM$ and *explain* your reasoning. Then find the perimeter of ▱$JKLM$.**

**50.** $J(-4, 2)$, $K(0, 3)$, $L(1, -1)$, $M(-3, -2)$

**51.** $J(-2, 7)$, $K(7, 2)$, $L(-2, -3)$, $M(-11, 2)$

52. **REASONING** Are all rhombuses similar? Are all squares similar? *Explain* your reasoning.

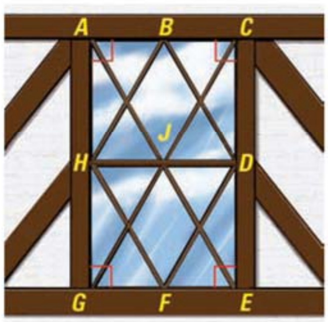
53. **CHALLENGE** Quadrilateral $ABCD$ shown at the right is a rhombus. Given that $AC = 10$ and $BD = 16$, find all side lengths and angle measures. *Explain* your reasoning.




## PROBLEM SOLVING

**EXAMPLE 2** on p. 534 for Ex. 54

54. **MULTI-STEP PROBLEM** In the window shown at the right, $\overline{BD} \cong \overline{DF} \cong \overline{BH} \cong \overline{HF}$. Also, $\angle HAB$, $\angle BCD$, $\angle DEF$, and $\angle FGH$ are right angles.




   **a.** Classify $HBDF$ and $ACEG$. *Explain* your reasoning.

   **b.** What can you conclude about the lengths of the diagonals $\overline{AE}$ and $\overline{GC}$? Given that these diagonals intersect at $J$, what can you conclude about the lengths of $\overline{AJ}$, $\overline{JE}$, $\overline{CJ}$, and $\overline{JG}$? *Explain*.

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**EXAMPLE 4** on p. 536 for Ex. 55

55. **PATIO** You want to mark off a square region in your yard for a patio. You use a tape measure to mark off a quadrilateral on the ground. Each side of the quadrilateral is 2.5 meters long. *Explain* how you can use the tape measure to make sure that the quadrilateral you drew is a square.

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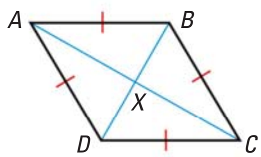
56. **PROVING THEOREM 8.11** Use the plan for proof below to write a paragraph proof for the converse statement of Theorem 8.11.




   **GIVEN** ▶ $ABCD$ is a rhombus.

   **PROVE** ▶ $\overline{AC} \perp \overline{BD}$

   **Plan for Proof** Because $ABCD$ is a parallelogram, its diagonals bisect each other at $X$. Show that $\triangle AXB \cong \triangle CXB$. Then show that $\overline{AC}$ and $\overline{BD}$ intersect to form congruent adjacent angles, $\angle AXB$ and $\angle CXB$.

**PROVING COROLLARIES** **Write the corollary as a conditional statement and its converse. Then *explain* why each statement is true.**

57. Rhombus Corollary

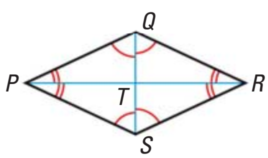
58. Rectangle Corollary

59. Square Corollary

**PROVING THEOREM 8.12** **In Exercises 60 and 61, prove both parts of Theorem 8.12.**

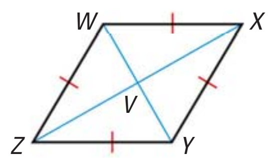
60. **GIVEN** ▶ $PQRS$ is a parallelogram.
$\overline{PR}$ bisects $\angle SPQ$ and $\angle QRS$.
$\overline{SQ}$ bisects $\angle PSR$ and $\angle RQP$.

   **PROVE** ▶ $PQRS$ is a rhombus.

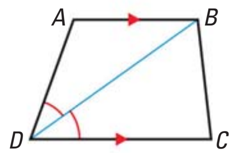



61. **GIVEN** ▶ $WXYZ$ is a rhombus.

   **PROVE** ▶ $\overline{WY}$ bisects $\angle ZWX$ and $\angle XYZ$.
$\overline{ZX}$ bisects $\angle WZY$ and $\angle YXW$.



**62.** ★ **EXTENDED RESPONSE** In $ABCD$, $\overline{AB} \parallel \overline{CD}$, and $\overline{DB}$ bisects $\angle ADC$.




**a.** Show that $\angle ABD \cong \angle CDB$. What can you conclude about $\angle ADB$ and $\angle ABD$? What can you conclude about $\overline{AB}$ and $\overline{AD}$? *Explain.*

**b.** Suppose you also know that $\overline{AD} \cong \overline{BC}$. Classify $ABCD$. *Explain.*

**63. PROVING THEOREM 8.13** Write a coordinate proof of the following statement, which is part of Theorem 8.13.

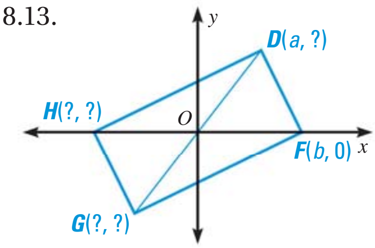
If a quadrilateral is a rectangle, then its diagonals are congruent.

**64. CHALLENGE** Write a coordinate proof of part of Theorem 8.13.

**GIVEN** ▶ $DFGH$ is a parallelogram, $\overline{DG} \cong \overline{HF}$

**PROVE** ▶ $DFGH$ is a rectangle.

**Plan for Proof** Write the coordinates of the vertices in terms of $a$ and $b$. Find and compare the slopes of the sides.



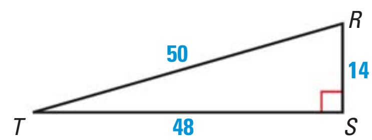

## MIXED REVIEW

**PREVIEW**
Prepare for Lesson 8.5 in Ex. 65.

**65.** In $\triangle JKL$, $KL = 54.2$ centimeters. Point $M$ is the midpoint of $\overline{JK}$ and $N$ is the midpoint of $\overline{JL}$. Find $MN$. *(p. 295)*

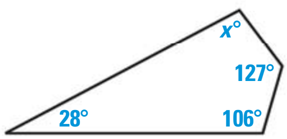
**Find the sine and cosine of the indicated angle. Write each answer as a fraction and a decimal.** *(p. 473)*

**66.** $\angle R$ **67.** $\angle T$

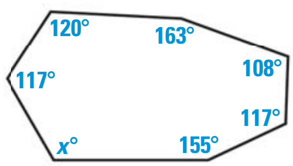



**Find the value of *x*.** *(p. 507)*

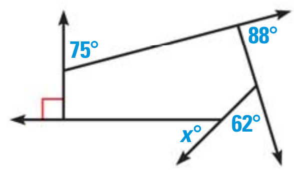
**68.**




**69.**



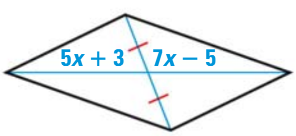

**70.**



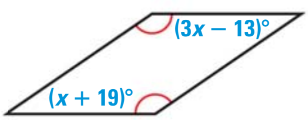

## QUIZ *for Lessons 8.3–8.4*

**For what value of *x* is the quadrilateral a parallelogram?** *(p. 522)*

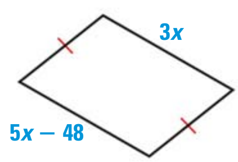
**1.**




**2.**

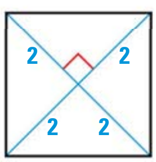



**3.**

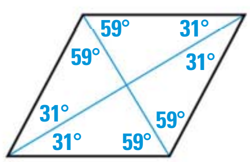



**Classify the quadrilateral. *Explain* your reasoning.** *(p. 533)*

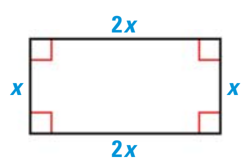
**4.**




**5.**




**6.**



**Investigating Geometry ACTIVITY** *Use before Lesson 8.5*

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Keystrokes

# 8.5 Midsegment of a Trapezoid

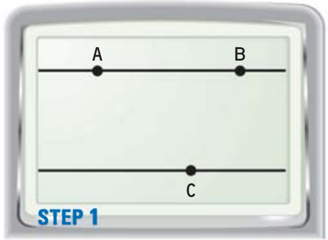
**MATERIALS** • graphing calculator or computer

**QUESTION** **What are the properties of the midsegment of a trapezoid?**

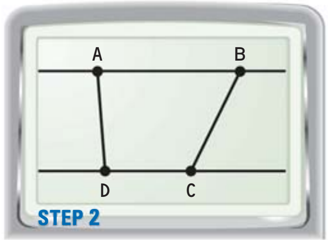
You can use geometry drawing software to investigate properties of trapezoids.

**EXPLORE** **Draw a trapezoid and its midsegment**

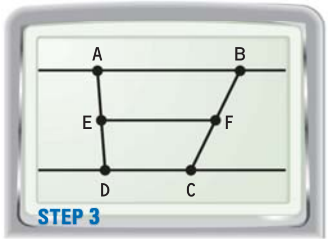
**STEP 1** ***Draw parallel lines*** Draw $\overleftrightarrow{AB}$. Draw a point $C$ not on $\overleftrightarrow{AB}$ and construct a line parallel to $\overleftrightarrow{AB}$ through point $C$.




**STEP 2** ***Draw trapezoid*** Construct a point $D$ on the same line as point $C$. Then draw $\overline{AD}$ and $\overline{BC}$ so that the segments are not parallel. Draw $\overline{AB}$ and $\overline{DC}$. Quadrilateral $ABCD$ is called a *trapezoid*. A trapezoid is a quadrilateral with exactly one pair of parallel sides.




**STEP 3** ***Draw midsegment*** Construct the midpoints of $\overline{AD}$ and $\overline{BC}$. Label the points $E$ and $F$. Draw $\overline{EF}$. $\overline{EF}$ is called a *midsegment* of trapezoid $ABCD$. The midsegment of a trapezoid connects the midpoints of its nonparallel sides.




**STEP 4** ***Measure lengths*** Measure $AB$, $DC$, and $EF$.

**STEP 5** ***Compare lengths*** The average of $AB$ and $DC$ is $\frac{AB + DC}{2}$. Calculate and compare this average to $EF$. What do you notice? Drag point $A$ or point $B$ to change the shape of trapezoid $ABCD$. Do not allow $\overline{AD}$ to intersect $\overline{BC}$. What do you notice about $EF$ and $\frac{AB + DC}{2}$?

**DRAW CONCLUSIONS** **Use your observations to complete these exercises**

1. Make a conjecture about the length of the midsegment of a trapezoid.
2. The midsegment of a trapezoid is parallel to the two parallel sides of the trapezoid. What measurements could you make to show that the midsegment in the *Explore* is parallel to $\overline{AB}$ and $\overline{CD}$? *Explain.*
3. In Lesson 5.1 (page 295), you learned a theorem about the midsegment of a triangle. How is the midsegment of a trapezoid similar to the midsegment of a triangle? How is it different?

# 8.5 Use Properties of Trapezoids and Kites

**Before** You used properties of special parallelograms.

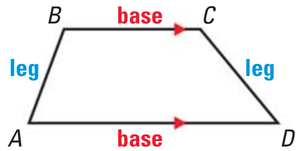
**Now** You will use properties of trapezoids and kites.

**Why?** So you can measure part of a building, as in Example 2.

**Key Vocabulary**
- **trapezoid**
  bases, base angles, legs
- **isosceles trapezoid**
- **midsegment of a trapezoid**
- **kite**

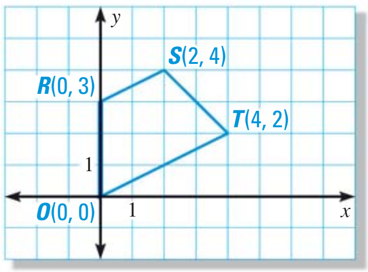
A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the **bases**.

A trapezoid has two pairs of **base angles**. For example, in trapezoid *ABCD*, $\angle A$ and $\angle D$ are one pair of base angles, and $\angle B$ and $\angle C$ are the second pair. The nonparallel sides are the **legs** of the trapezoid.




## EXAMPLE 1 Use a coordinate plane

**Show that *ORST* is a trapezoid.**




**Solution**

Compare the slopes of opposite sides.

$$\text{Slope of } \overline{RS} = \frac{4-3}{2-0} = \frac{1}{2}$$

$$\text{Slope of } \overline{OT} = \frac{2-0}{4-0} = \frac{2}{4} = \frac{1}{2}$$

The slopes of $\overline{RS}$ and $\overline{OT}$ are the same, so $\overline{RS} \parallel \overline{OT}$.

$$\text{Slope of } \overline{ST} = \frac{2-4}{4-2} = \frac{-2}{2} = -1$$

$$\text{Slope of } \overline{OR} = \frac{3-0}{0-0} = \frac{3}{0}, \text{ which is undefined.}$$

The slopes of $\overline{ST}$ and $\overline{OR}$ are not the same, so $\overline{ST}$ is not parallel to $\overline{OR}$.

▶ Because quadrilateral *ORST* has exactly one pair of parallel sides, it is a trapezoid.

## ✓ GUIDED PRACTICE for Example 1

1. **WHAT IF?** In Example 1, suppose the coordinates of point *S* are (4, 5). What type of quadrilateral is *ORST*? *Explain*.

2. In Example 1, which of the interior angles of quadrilateral *ORST* are supplementary angles? *Explain* your reasoning.

**ISOSCELES TRAPEZOIDS** If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid**.

isosceles trapezoid

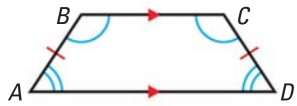
## THEOREMS *For Your Notebook*

### THEOREM 8.14

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid $ABCD$ is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

*Proof:* Ex. 37, p. 548




### THEOREM 8.15

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid $ABCD$ is isosceles.

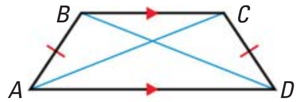
*Proof:* Ex. 38, p. 548


### THEOREM 8.16

A trapezoid is isosceles if and only if its diagonals are congruent.

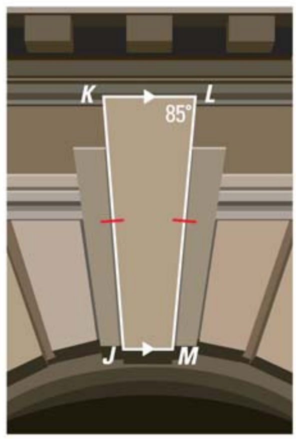
Trapezoid $ABCD$ is isosceles if and only if $\overline{AC} \cong \overline{BD}$.

*Proof:* Exs. 39 and 43, p. 549




## EXAMPLE 2 Use properties of isosceles trapezoids

**ARCH** The stone above the arch in the diagram is an isosceles trapezoid. Find $m\angle K$, $m\angle M$, and $m\angle J$.




### Solution

***STEP 1*** **Find** $m\angle K$. $JKLM$ is an isosceles trapezoid, so $\angle K$ and $\angle L$ are congruent base angles, and $m\angle K = m\angle L = 85°$.

***STEP 2*** **Find** $m\angle M$. Because $\angle L$ and $\angle M$ are consecutive interior angles formed by $\overleftrightarrow{LM}$ intersecting two parallel lines, they are supplementary. So, $m\angle M = 180° - 85° = 95°$.

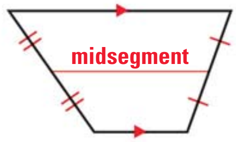
***STEP 3*** **Find** $m\angle J$. Because $\angle J$ and $\angle M$ are a pair of base angles, they are congruent, and $m\angle J = m\angle M = 95°$.

▶ So, $m\angle J = 95°$, $m\angle K = 85°$, and $m\angle M = 95°$.

**READ VOCABULARY**

The midsegment of a trapezoid is sometimes called the *median* of the trapezoid.

**MIDSEGMENTS** Recall that a midsegment of a triangle is a segment that connects the midpoints of two sides of the triangle. The **midsegment of a trapezoid** is the segment that connects the midpoints of its legs.



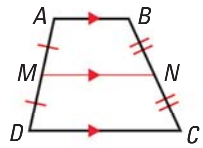

The theorem below is similar to the Midsegment Theorem for Triangles.

## THEOREM — *For Your Notebook*

**THEOREM 8.17 Midsegment Theorem for Trapezoids**

The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

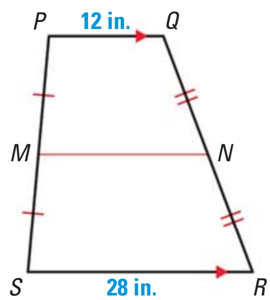
If $\overline{MN}$ is the midsegment of trapezoid $ABCD$, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.




*Justification:* Ex. 40, p. 549
*Proof:* p. 937

## EXAMPLE 3 Use the midsegment of a trapezoid

**In the diagram, $\overline{MN}$ is the midsegment of trapezoid *PQRS*. Find *MN*.**




**Solution**

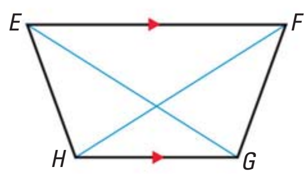
Use Theorem 8.17 to find $MN$.

| | |
|---|---|
| $MN = \frac{1}{2}(PQ + SR)$ | **Apply Theorem 8.17.** |
| $= \frac{1}{2}(12 + 28)$ | **Substitute 12 for *PQ* and 28 for *XU*.** |
| $= 20$ | **Simplify.** |

▶ The length $MN$ is 20 inches.

## ✓ GUIDED PRACTICE for Examples 2 and 3

**In Exercises 3 and 4, use the diagram of trapezoid *EFGH*.**

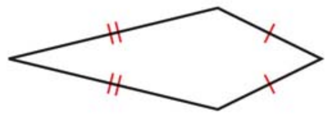



3. If $EG = FH$, is trapezoid $EFGH$ isosceles? *Explain.*

4. If $m\angle HEF = 70°$ and $m\angle FGH = 110°$, is trapezoid $EFGH$ isosceles? *Explain.*

5. In trapezoid $JKLM$, $\angle J$ and $\angle M$ are right angles, and $JK = 9$ cm. The length of the midsegment $\overline{NP}$ of trapezoid $JKLM$ is 12 cm. Sketch trapezoid $JKLM$ and its midsegment. Find $ML$. *Explain* your reasoning.

**KITES** A **kite** is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.



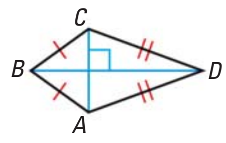
## THEOREMS *For Your Notebook*

### THEOREM 8.18

If a quadrilateral is a kite, then its diagonals are perpendicular.

If quadrilateral $ABCD$ is a kite, then $\overline{AC} \perp \overline{BD}$.

*Proof:* Ex. 41, p. 549

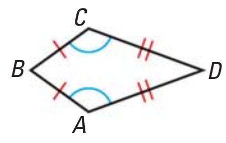



### THEOREM 8.19

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral $ABCD$ is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \not\cong \angle D$.

*Proof:* Ex. 42, p. 549




## EXAMPLE 4 Apply Theorem 8.19

**Find $m\angle D$ in the kite shown at the right.**


**Solution**

By Theorem 8.19, $DEFG$ has exactly one pair of congruent opposite angles. Because $\angle E \not\cong \angle G$, $\angle D$ and $\angle F$ must be congruent. So, $m\angle D = m\angle F$. Write and solve an equation to find $m\angle D$.

$m\angle D + m\angle F + 124° + 80° = 360°$ **Corollary to Theorem 8.1**

$m\angle D + m\angle D + 124° + 80° = 360°$ **Substitute $m\angle D$ for $m\angle F$.**

$2(m\angle D) + 204° = 360°$ **Combine like terms.**

$m\angle D = 78°$ **Solve for $m\angle D$.**

*Animated* **Geometry** at classzone.com

## ✓ GUIDED PRACTICE for Example 4

**6.** In a kite, the measures of the angles are $3x°$, $75°$, $90°$, and $120°$. Find the value of $x$. What are the measures of the angles that are congruent?

# 8.5 EXERCISES

**HOMEWORK KEY**

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 19, and 35

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 28, 31, and 36

## SKILL PRACTICE

1. **VOCABULARY** In trapezoid $PQRS$, $\overline{PQ} \parallel \overline{RS}$. Sketch $PQRS$ and identify its bases and its legs.

2. ★ **WRITING** *Describe* the differences between a kite and a trapezoid.

**EXAMPLES 1 and 2** on pp. 542–543 for Exs. 3–12

**COORDINATE PLANE Points $A$, $B$, $C$, and $D$ are the vertices of a quadrilateral. Determine whether $ABCD$ is a trapezoid.**

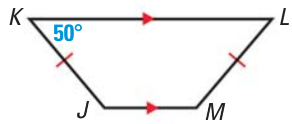
3. $A(0, 4)$, $B(4, 4)$, $C(8, -2)$, $D(2, 1)$

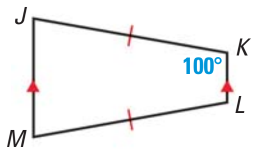
4. $A(-5, 0)$, $B(2, 3)$, $C(3, 1)$, $D(-2, -2)$

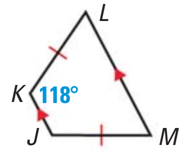
5. $A(2, 1)$, $B(6, 1)$, $C(3, -3)$, $D(-1, -4)$

6. $A(-3, 3)$, $B(-1, 1)$, $C(1, -4)$, $D(-3, 0)$

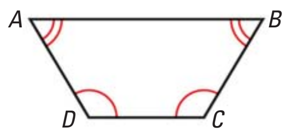
**FINDING ANGLE MEASURES Find $m\angle J$, $m\angle L$, and $m\angle M$.**

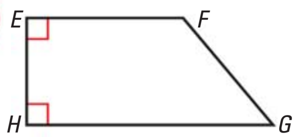
7. 

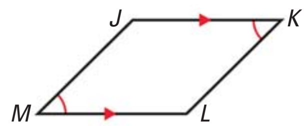

8. 


9. 


**REASONING Determine whether the quadrilateral is a trapezoid. *Explain*.**

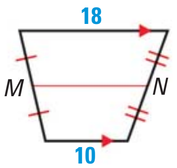
10. 

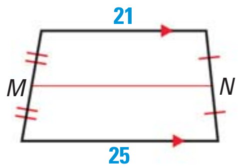

11. 

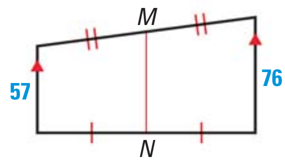

12. 


**EXAMPLE 3** on p. 544 for Exs. 13–16

**FINDING MIDSEGMENTS Find the length of the midsegment of the trapezoid.**

13. 


14. 


15. 


16. ★ **MULTIPLE CHOICE** Which statement is not always true?

Ⓐ The base angles of an isosceles trapezoid are congruent.

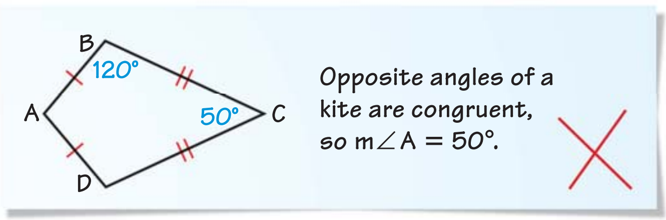
Ⓑ The midsegment of a trapezoid is parallel to the bases.

Ⓒ The bases of a trapezoid are parallel.

Ⓓ The legs of a trapezoid are congruent.

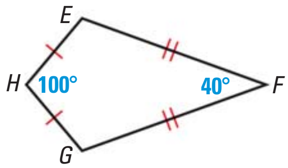
**EXAMPLE 4** on p. 545 for Exs. 17–20

17. **ERROR ANALYSIS** *Describe* and correct the error made in finding $m\angle A$.

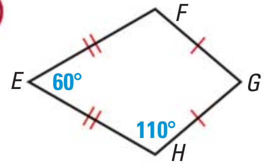


**ANGLES OF KITES** ***EFGH*** **is a kite. Find** $m\angle G$**.**

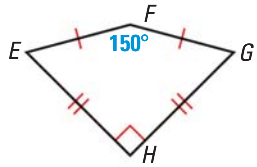
**18.**




**19.**

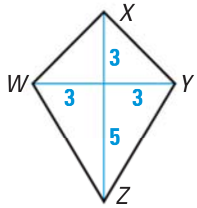



**20.**

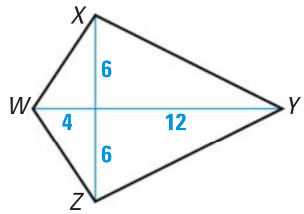



**DIAGONALS OF KITES** **Use Theorem 8.18 and the Pythagorean Theorem to find the side lengths of the kite. Write the lengths in simplest radical form.**

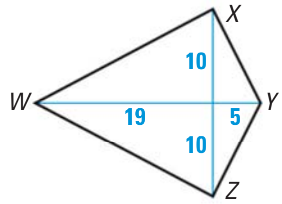
**21.**



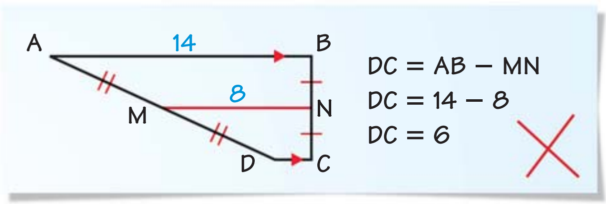

**22.**




**23.**

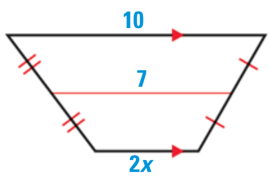



**24.** **ERROR ANALYSIS** In trapezoid $ABCD$, $\overline{MN}$ is the midsegment. *Describe* and correct the error made in finding $DC$.

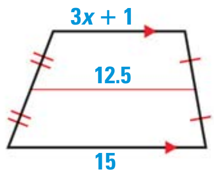



**ALGEBRA** **Find the value of** $x$**.**

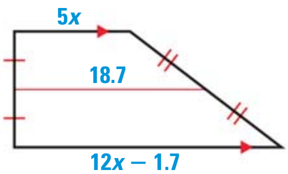
**25.**




**26.**




**27.**




**28.** ★ **SHORT RESPONSE** The points $M(-3, 5)$, $N(-1, 5)$, $P(3, -1)$, and $Q(-5, -1)$ form the vertices of a trapezoid. Draw $MNPQ$ and find $MP$ and $NQ$. What do your results tell you about the trapezoid? *Explain*.

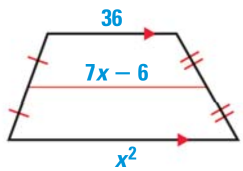
**29.** **DRAWING** In trapezoid $JKLM$, $\overline{JK} \parallel \overline{LM}$ and $JK = 17$. The midsegment of $JKLM$ is $\overline{XY}$, and $XY = 37$. Sketch $JKLM$ and its midsegment. Then find $LM$.

**30.** **RATIOS** The ratio of the lengths of the bases of a trapezoid is $1:3$. The length of the midsegment is 24. Find the lengths of the bases.

**31.** ★ **MULTIPLE CHOICE** In trapezoid $PQRS$, $\overline{PQ} \parallel \overline{RS}$ and $\overline{MN}$ is the midsegment of $PQRS$. If $RS = 5 \cdot PQ$, what is the ratio of $MN$ to $RS$?

(A) $3:5$ (B) $5:3$ (C) $2:1$ (D) $3:1$

**32.** **CHALLENGE** The figure shown at the right is a trapezoid with its midsegment. Find all the possible values of $x$. What is the length of the midsegment? *Explain*. (The figure may not be drawn to scale.)

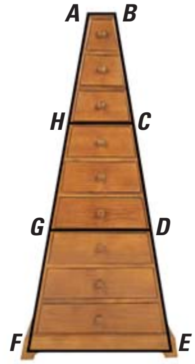



**33.** **REASONING** *Explain* why a kite and a general quadrilateral are the only quadrilaterals that can be concave.

## PROBLEM SOLVING

**EXAMPLES 3 and 4**
on pp. 544–545
for Exs. 34–35

**34. FURNITURE** In the photograph of a chest of drawers, $\overline{HC}$ is the midsegment of trapezoid $ABDG$, $\overline{GD}$ is the midsegment of trapezoid $HCEF$, $AB = 13.9$ centimeters, and $GD = 50.5$ centimeters. Find $HC$. Then find $FE$.

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**35. GRAPHIC DESIGN** You design a logo in the shape of a convex kite. The measure of one angle of the kite is 90°. The measure of another angle is 30°. Sketch a kite that matches this description. Give the measures of all the angles and mark any congruent sides.

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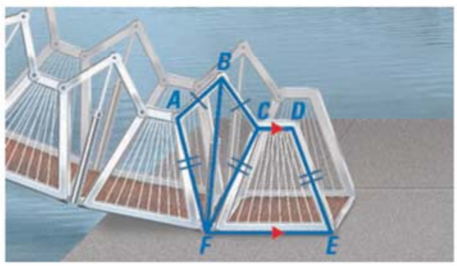
**36. ★ EXTENDED RESPONSE** The bridge below is designed to fold up into an octagon shape. The diagram shows a section of the bridge.

**a.** Classify the quadrilaterals shown in the diagram.

**b.** As the bridge folds up, what happens to the length of $\overline{BF}$? What happens to $m\angle BAF$, $m\angle ABC$, $m\angle BCF$, and $m\angle CFA$?

**c.** Given $m\angle CFE = 65°$, find $m\angle DEF$, $m\angle FCD$, and $m\angle CDE$. *Explain.*

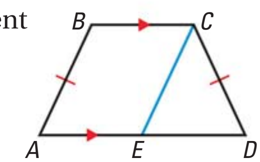



**37. PROVING THEOREM 8.14** Use the diagram and the auxiliary segment to prove Theorem 8.14. In the diagram, $\overline{EC}$ is drawn parallel to $\overline{AB}$.

**GIVEN** ▶ $ABCD$ is an isosceles trapezoid, $\overline{BC} \parallel \overline{AD}$

**PROVE** ▶ $\angle A \cong \angle D$, $\angle B \cong \angle BCD$

*Hint:* Find a way to show that $\triangle ECD$ is an isosceles triangle.

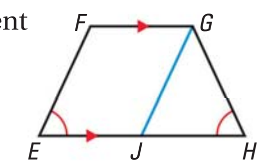



**38. PROVING THEOREM 8.15** Use the diagram and the auxiliary segment to prove Theorem 8.15. In the diagram, $\overline{JG}$ is drawn parallel to $\overline{EF}$.

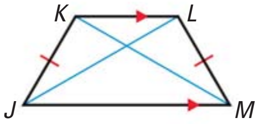
**GIVEN** ▶ $EFGH$ is a trapezoid, $\overline{FG} \parallel \overline{EH}$, $\angle E \cong \angle H$

**PROVE** ▶ $EFGH$ is an isosceles trapezoid.

*Hint:* Find a way to show that $\triangle JGH$ is an isosceles triangle.



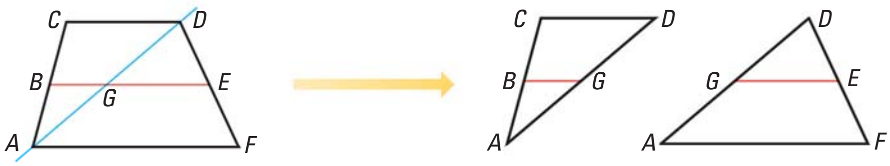
**39.** **PROVING THEOREM 8.16** Prove part of Theorem 8.16.



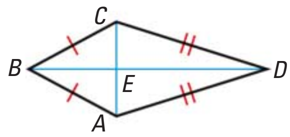

**GIVEN** ▶ $JKLM$ is an isosceles trapezoid.
$\overline{KL} \parallel \overline{JM}$, $\overline{JK} \cong \overline{LM}$

**PROVE** ▶ $\overline{JL} \cong \overline{KM}$

**40.** **REASONING** In the diagram below, $\overline{BG}$ is the midsegment of $\triangle ACD$ and $\overline{GE}$ is the midsegment of $\triangle ADF$. *Explain* why the midsegment of trapezoid $ACDF$ is parallel to each base and why its length is one half the sum of the lengths of the bases.



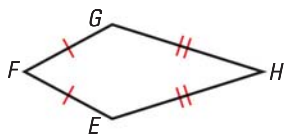

**41.** **PROVING THEOREM 8.18** Prove Theorem 8.18.




**GIVEN** ▶ $ABCD$ is a kite.
$\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$

**PROVE** ▶ $\overline{AC} \perp \overline{BD}$

**42.** **PROVING THEOREM 8.19** Write a paragraph proof of Theorem 8.19.




**GIVEN** ▶ $EFGH$ is a kite.
$\overline{EF} \cong \overline{GF}$, $\overline{EH} \cong \overline{GH}$

**PROVE** ▶ $\angle E \cong \angle G$, $\angle F \ncong \angle H$

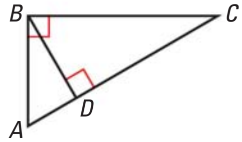
**Plan for Proof** First show that $\angle E \cong \angle G$. Then use an indirect argument to show that $\angle F \ncong \angle H$: If $\angle F \cong \angle H$, then $EFGH$ is a parallelogram. But opposite sides of a parallelogram are congruent. This result contradicts the definition of a kite.

**43.** **CHALLENGE** In Exercise 39, you proved that part of Theorem 8.16 is true. Write the other part of Theorem 8.16 as a conditional statement. Then prove that the statement is true.

## MIXED REVIEW

**44.** Place a right triangle in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex. *(p. 295)*

**Use the diagram to complete the proportion.** *(p. 449)*




**45.** $\dfrac{AB}{AC} = \dfrac{?}{AB}$

**46.** $\dfrac{AB}{BC} = \dfrac{BD}{?}$

**PREVIEW**
Prepare for Lesson 8.6 in Exs. 47–48.

**Three of the vertices of $\square ABCD$ are given. Find the coordinates of point $D$. Show your method.** *(p. 522)*

**47.** $A(-1, -2)$, $B(4, -2)$, $C(6, 2)$, $D(x, y)$

**48.** $A(1, 4)$, $B(0, 1)$, $C(4, 1)$, $D(x, y)$

# Extension

*Use after Lesson 8.5*

# Draw Three-Dimensional Figures

**GOAL** Create isometric drawings and orthographic projections of three-dimensional figures.

**Key Vocabulary**
- **isometric drawing**
- **orthographic projection**

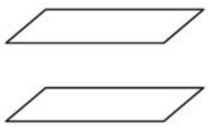
*Technical drawings* are drawings that show different viewpoints of an object. Engineers and architects create technical drawings of products and buildings before actually constructing the actual objects.

## EXAMPLE 1 Draw a rectangular box

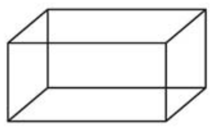
**Draw a rectangular box.**

**Solution**

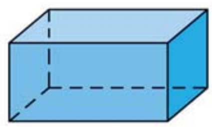
*STEP 1* **Draw** the bases. They are rectangular, but you need to draw them tilted.



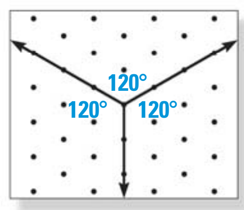
*STEP 2* **Connect** the bases using vertical lines.



*STEP 3* **Erase** parts of the hidden edges so that they are dashed lines.



**ISOMETRIC DRAWINGS** Technical drawings may include **isometric drawings**. These drawings look three-dimensional and can be created on a grid of dots using three axes that intersect to form 120° angles.




## EXAMPLE 2 Create an isometric drawing

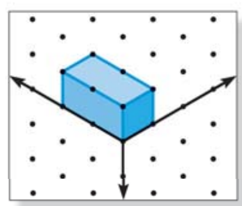
**Create an isometric drawing of the rectangular box in Example 1.**

**Solution**

*STEP 1* **Draw** three axes on isometric dot paper.

*STEP 2* **Draw** the box so that the edges of the box are parallel to the three axes.

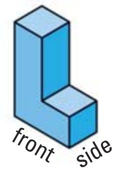
*STEP 3* **Add** depth to the drawing by using different shading for the front, top, and sides.



**ANOTHER VIEW** Technical drawings may also include an *orthographic projection*. An **orthographic projection** is a two-dimensional drawing of the front, top, and side views of an object. The interior lines in these two-dimensional drawings represent edges of the object.

## EXAMPLE 3 Create an orthographic projection

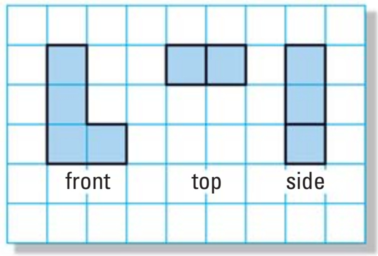
**Create an orthographic projection of the solid.**




**VISUAL REASONING**
In this Extension, you can think of the solids as being constructed from cubes. You can assume there are no cubes hidden from view except those needed to support the visible ones.

### Solution

On graph paper, draw the front, top, and side views of the solid.

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## PRACTICE

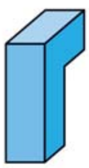
**EXAMPLE 1**
on p. 550
for Exs. 1–3

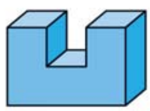
**DRAWING BOXES** **Draw a box with the indicated base.**

1. Equilateral triangle
2. Regular hexagon
3. Square

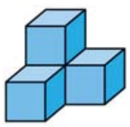
**EXAMPLES 2 and 3**
on pp. 550–551
for Exs. 4–12

**DRAWING SOLIDS** **Create an isometric drawing of the solid. Then create an orthographic projection of the solid.**

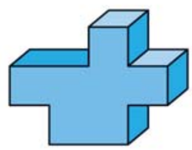
4. 

5. 

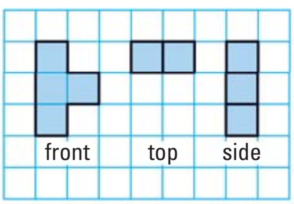
6. 

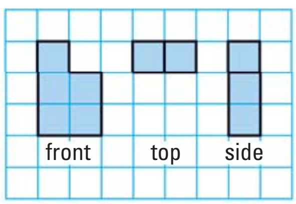
7. 

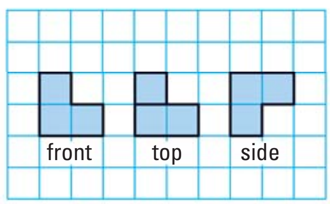
8. 

9. 

**CREATING ISOMETRIC DRAWINGS** **Create an isometric drawing of the orthographic projection.**

10. 


11. 


12. 

# 8.6 Identify Special Quadrilaterals

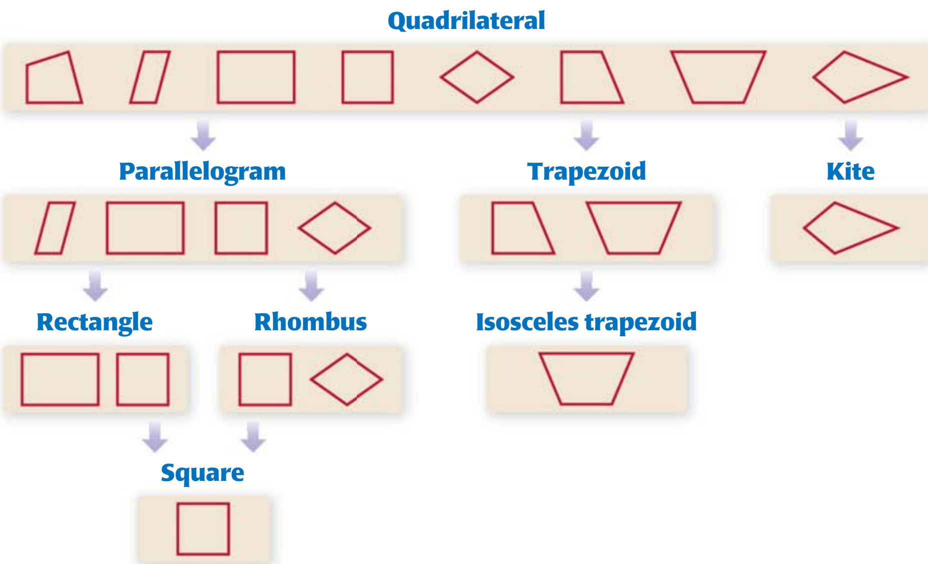
**Before** You identified polygons.

**Now** You will identify special quadrilaterals.

**Why?** So you can describe part of a pyramid, as in Ex. 36.

**Key Vocabulary**
- **parallelogram,** *p. 515*
- **rhombus,** *p. 533*
- **rectangle,** *p. 533*
- **square,** *p. 533*
- **trapezoid,** *p. 542*
- **kite,** *p. 545*

The diagram below shows relationships among the special quadrilaterals you have studied in Chapter 8. Each shape in the diagram has the properties of the shapes linked above it. For example, a rhombus has the properties of a parallelogram and a quadrilateral.

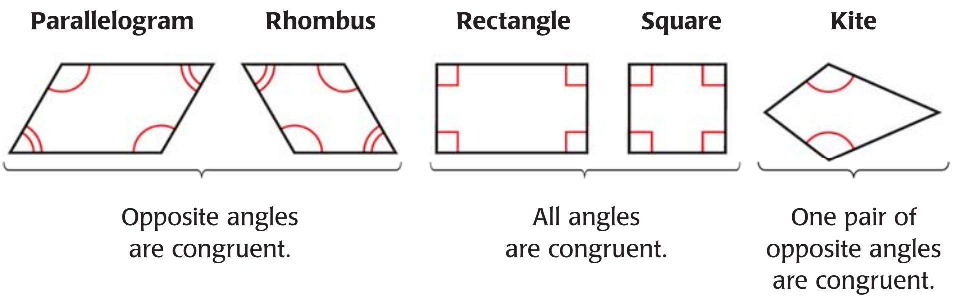



## EXAMPLE 1 Identify quadrilaterals

**Quadrilateral *ABCD* has at least one pair of opposite angles congruent. What types of quadrilaterals meet this condition?**

**Solution**

There are many possibilities.



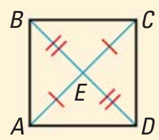
## EXAMPLE 2 Standardized Test Practice

**AVOID ERRORS**
In Example 2, *ABCD* is shaped like a square. But you must rely only on marked information when you interpret a diagram.

**What is the most specific name for quadrilateral *ABCD*?**

Ⓐ Parallelogram  Ⓑ Rhombus

Ⓒ Square  Ⓓ Rectangle




### Solution

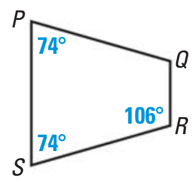
The diagram shows $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$. So, the diagonals bisect each other. By Theorem 8.10, *ABCD* is a parallelogram.

Rectangles, rhombuses and squares are also parallelograms. However, there is no information given about the side lengths or angle measures of *ABCD*. So, you cannot determine whether it is a rectangle, a rhombus, or a square.

▶ The correct answer is A. Ⓐ Ⓑ Ⓒ Ⓓ

## EXAMPLE 3 Identify a quadrilateral

**Is enough information given in the diagram to show that quadrilateral *PQRS* is an isosceles trapezoid? Explain.**




### Solution

**STEP 1** **Show** that *PQRS* is a trapezoid. $\angle R$ and $\angle S$ are supplementary, but $\angle P$ and $\angle S$ are not. So, $\overline{PS} \parallel \overline{QR}$, but $\overline{PQ}$ is not parallel to $\overline{SR}$. By definition, *PQRS* is a trapezoid.

**STEP 2** **Show** that trapezoid *PQRS* is isosceles. $\angle P$ and $\angle S$ are a pair of congruent base angles. So, *PQRS* is an isosceles trapezoid by Theorem 8.15.

▶ Yes, the diagram is sufficient to show that *PQRS* is an isosceles trapezoid.

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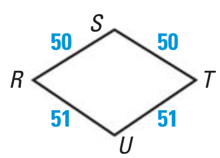
## ✓ GUIDED PRACTICE for Examples 1, 2, and 3

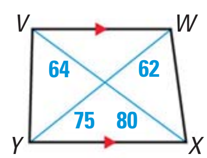
1. Quadrilateral *DEFG* has at least one pair of opposite sides congruent. What types of quadrilaterals meet this condition?

**Give the most specific name for the quadrilateral. *Explain* your reasoning.**

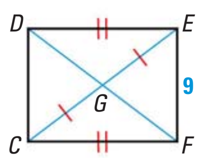
2.




3.




4.




5. **ERROR ANALYSIS** A student knows the following information about quadrilateral *MNPQ*: $\overline{MN} \parallel \overline{PQ}$, $\overline{MP} \cong \overline{NQ}$, and $\angle P \cong \angle Q$. The student concludes that *MNPQ* is an isosceles trapezoid. *Explain* why the student cannot make this conclusion.

# 8.6 EXERCISES

**HOMEWORK KEY**

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 15, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 13, 37, and 38

## SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A quadrilateral that has exactly one pair of parallel sides and diagonals that are congruent is a(n) __?__.

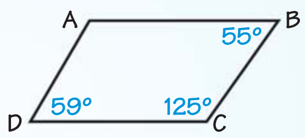
2. ★ **WRITING** *Describe* three methods you could use to prove that a parallelogram is a rhombus.

**EXAMPLE 1** on p. 552 for Exs. 3–12

**PROPERTIES OF QUADRILATERALS** **Copy the chart. Put an X in the box if the shape *always* has the given property.**

| | Property | ▱ | Rectangle | Rhombus | Square | Kite | Trapezoid |
|---|---|---|---|---|---|---|---|
| 3. | All sides are ≅. | ? | ? | ? | ? | ? | ? |
| 4. | Both pairs of opp. sides are ≅. | ? | ? | ? | ? | ? | ? |
| 5. | Both pairs of opp. sides are ∥. | ? | ? | ? | ? | ? | ? |
| 6. | Exactly 1 pair of opp. sides are ∥. | ? | ? | ? | ? | ? | ? |
| 7. | All ∠s are ≅. | ? | ? | ? | ? | ? | ? |
| 8. | Exactly 1 pair of opp. ∠s are ≅. | ? | ? | ? | ? | ? | ? |
| 9. | Diagonals are ⊥. | ? | ? | ? | ? | ? | ? |
| 10. | Diagonals are ≅. | ? | ? | ? | ? | ? | ? |
| 11. | Diagonals bisect each other. | ? | ? | ? | ? | ? | ? |

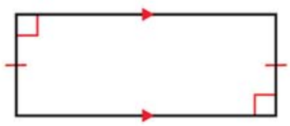
12. **ERROR ANALYSIS** *Describe* and correct the error in classifying the quadrilateral.




∠B and ∠C are supplements, so $\overline{AB} \parallel \overline{CD}$. So, ABCD is a parallelogram.

**EXAMPLE 2** on p. 553 for Exs. 13–17

13. ★ **MULTIPLE CHOICE** What is the most specific name for the quadrilateral shown at the right?

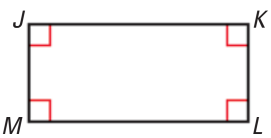


Ⓐ Rectangle  Ⓑ Parallelogram

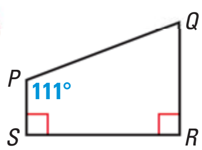
Ⓒ Trapezoid  Ⓓ Isosceles trapezoid

**CLASSIFYING QUADRILATERALS** **Give the most specific name for the quadrilateral. *Explain.***

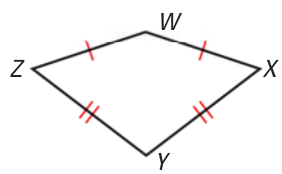
14.




15.




16.

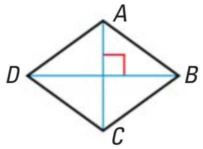


**17.** **DRAWING** Draw a quadrilateral with congruent diagonals and exactly one pair of congruent sides. What is the most specific name for this quadrilateral?

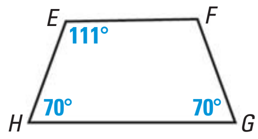
**EXAMPLE 3**
on p. 553
for Exs. 18–20

**IDENTIFYING QUADRILATERALS Tell whether enough information is given in the diagram to classify the quadrilateral by the indicated name. *Explain*.**

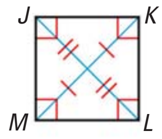
**18.** Rhombus




**19.** Isosceles trapezoid




**20.** Square




**COORDINATE PLANE Points $P$, $Q$, $R$, and $S$ are the vertices of a quadrilateral. Give the most specific name for $PQRS$. *Justify* your answer.**

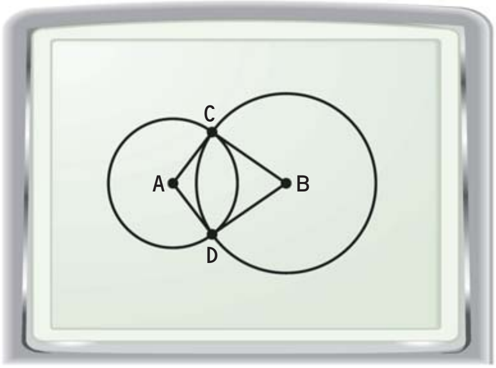
**21.** $P(1, 0)$, $Q(1, 2)$, $R(6, 5)$, $S(3, 0)$

**22.** $P(2, 1)$, $Q(6, 1)$, $R(5, 8)$, $S(3, 8)$

**23.** $P(2, 7)$, $Q(6, 9)$, $R(9, 3)$, $S(5, 1)$

**24.** $P(1, 7)$, $Q(5, 8)$, $R(6, 2)$, $S(2, 1)$

**25.** **TECHNOLOGY** Use geometry drawing software to draw points $A$, $B$, $C$, and segments $AC$ and $BC$. Draw a circle with center $A$ and radius $AC$. Draw a circle with center $B$ and radius $BC$. Label the other intersection of the circles $D$. Draw $\overline{BD}$ and $\overline{AD}$.

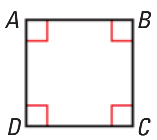



**a.** Drag point $A$, $B$, $C$, or $D$ to change the shape of $ABCD$. What types of quadrilaterals can be formed?

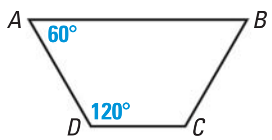
**b.** Are there types of quadrilaterals that cannot be formed? *Explain*.

**DEVELOPING PROOF Which pairs of segments or angles must be congruent so that you can prove that $ABCD$ is the indicated quadrilateral? *Explain*. There may be more than one right answer.**

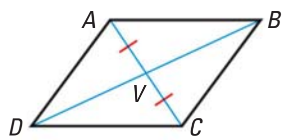
**26.** Square




**27.** Isosceles trapezoid




**28.** Parallelogram

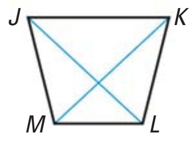



**TRAPEZOIDS In Exercises 29–31, determine whether there is enough information to prove that $JKLM$ is an isosceles trapezoid. *Explain*.**

**29.** **GIVEN** ▶ $\overline{JK} \parallel \overline{LM}$, $\angle JKL \cong \angle KJM$

**30.** **GIVEN** ▶ $\overline{JK} \parallel \overline{LM}$, $\angle JML \cong \angle KLM$, $m\angle KLM \neq 90°$

**31.** **GIVEN** ▶ $\overline{JL} \cong \overline{KM}$, $\overline{JK} \parallel \overline{LM}$, $JK > LM$

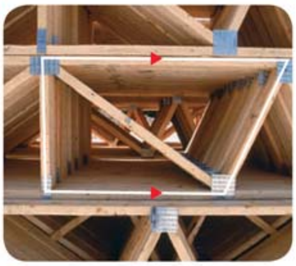



**32.** **CHALLENGE** Draw a rectangle and bisect its angles. What type of quadrilateral is formed by the intersecting bisectors? *Justify* your answer.

## PROBLEM SOLVING

**REAL-WORLD OBJECTS** What type of special quadrilateral is outlined?

**33.**



**34.**


**35.**

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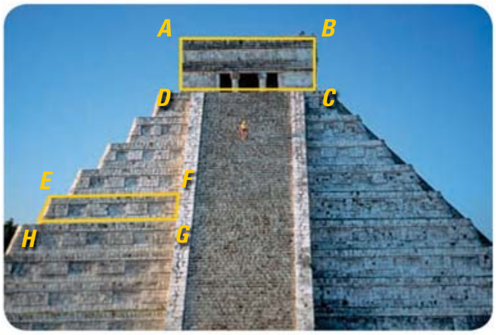
**36. PYRAMID** Use the photo of the Pyramid of Kukulcan in Mexico.




**a.** $\overline{EF} \parallel \overline{HG}$, and $\overline{EH}$ and $\overline{FG}$ are not parallel. What shape is this part of the pyramid?

**b.** $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$, and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are all congruent to each other. What shape is this part of the pyramid?

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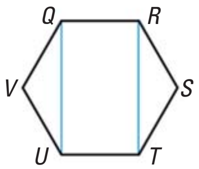
**37.** ★ **SHORT RESPONSE** *Explain* why a parallelogram with one right angle must be a rectangle.

**38.** ★ **EXTENDED RESPONSE** Segments $AC$ and $BD$ bisect each other.

**a.** Suppose that $\overline{AC}$ and $\overline{BD}$ are congruent, but not perpendicular. Draw quadrilateral $ABCD$ and classify it. *Justify* your answer.

**b.** Suppose that $\overline{AC}$ and $\overline{BD}$ are perpendicular, but not congruent. Draw quadrilateral $ABCD$ and classify it. *Justify* your answer.

**39. MULTI-STEP PROBLEM** Polygon $QRSTUV$ shown at the right is a regular hexagon, and $\overline{QU}$ and $\overline{RT}$ are diagonals. Follow the steps below to classify quadrilateral $QRTU$. *Explain* your reasoning in each step.




**a.** Show that $\triangle QVU$ and $\triangle RST$ are congruent isosceles triangles.

**b.** Show that $\overline{QR} \cong \overline{UT}$ and that $\overline{QU} \cong \overline{RT}$.

**c.** Show that $\angle UQR \cong \angle QRT \cong \angle RTU \cong \angle TUQ$. Find the measure of each of these angles.

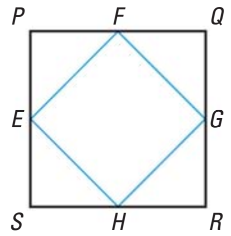
**d.** Classify quadrilateral $QRTU$.

**40. REASONING** In quadrilateral $WXYZ$, $\overline{WY}$ and $\overline{XZ}$ intersect each other at point $V$. $\overline{WV} \cong \overline{XV}$ and $\overline{YV} \cong \overline{ZV}$, but $\overline{WY}$ and $\overline{XZ}$ do not bisect each other. Draw $\overline{WY}$, $\overline{XY}$, and $WXYZ$. What special type of quadrilateral is $WXYZ$? Write a plan for a proof of your answer.

**CHALLENGE** **What special type of quadrilateral is *EFGH*? Write a paragraph proof to show that your answer is correct.**

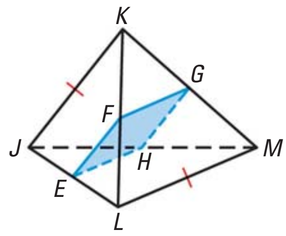
**41.** **GIVEN ▶** *PQRS* is a square. *E, F, G,* and *H* are midpoints of the sides of the square.

**PROVE ▶** *EFGH* is a __?__.



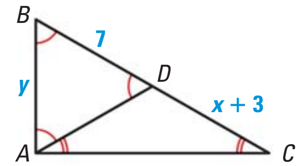

**42.** **GIVEN ▶** In the three-dimensional figure, $\overline{JK} \cong \overline{LM}$; *E, F, G,* and *H* are the midpoints of $\overline{JL}$, $\overline{KL}$, $\overline{KM}$, and $\overline{JM}$.

**PROVE ▶** *EFGH* is a __?__.




## MIXED REVIEW

**In Exercises 43 and 44, use the diagram.** *(p. 264)*




**43.** Find the values of *x* and *y*. *Explain* your reasoning.

**44.** Find $m\angle ADC$, $m\angle DAC$, and $m\angle DCA$. *Explain* your reasoning.

**PREVIEW**
Prepare for Lesson 9.1 in Exs. 45–46.

**The vertices of quadrilateral *ABCD* are *A*(−2, 1), *B*(2, 5), *C*(3, 2), and *D*(1, −1). Draw *ABCD* in a coordinate plane. Then draw its image after the indicated translation.** *(p. 272)*

**45.** $(x, y) \rightarrow (x + 1, y - 3)$

**46.** $(x, y) \rightarrow (x - 2, y - 2)$

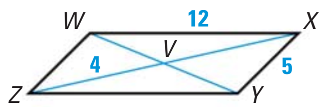
**Use the diagram of ▱*WXYZ* to find the indicated length.** *(p. 515)*

**47.** *YZ*

**48.** *WZ*

**49.** *XV*

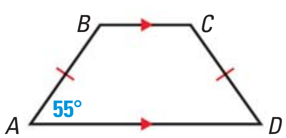
**50.** *XZ*



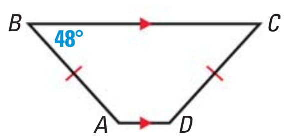

## QUIZ for Lessons 8.5–8.6

**Find the unknown angle measures.** *(p. 542)*

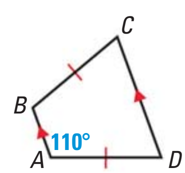
**1.**




**2.**




**3.**




**4.** The diagonals of quadrilateral *ABCD* are congruent and bisect each other. What types of quadrilaterals match this description? *(p. 552)*

**5.** In quadrilateral *EFGH*, $\angle E \cong \angle G$, $\angle F \cong \angle H$, and $\overline{EF} \cong \overline{EH}$. What is the most specific name for quadrilateral *EFGH*? *(p. 552)*

# MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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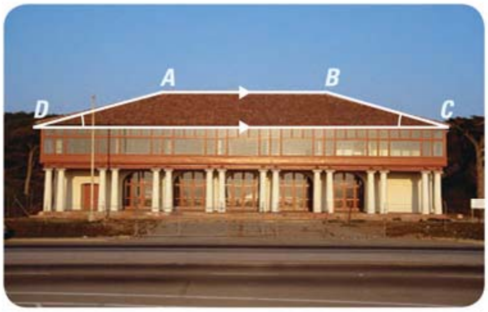
## Lessons 8.4–8.6

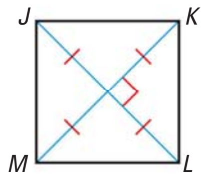
**1. MULTI-STEP PROBLEM** In the photograph shown below, quadrilateral *ABCD* represents the front view of the roof.



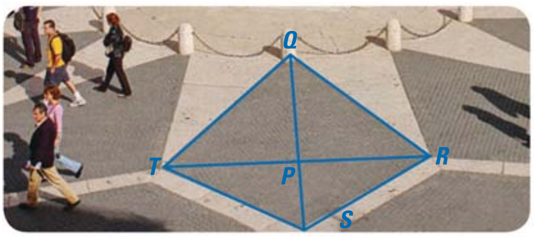

**a.** *Explain* how you know that the shape of the roof is a trapezoid.

**b.** Do you have enough information to determine that the roof is an isosceles trapezoid? *Explain* your reasoning.

**2. SHORT RESPONSE** Is enough information given in the diagram to show that quadrilateral *JKLM* is a square? *Explain* your reasoning.



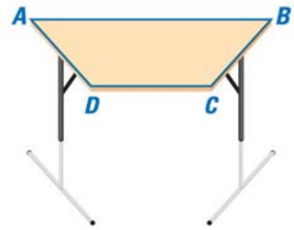

**3. EXTENDED RESPONSE** In the photograph, quadrilateral *QRST* is a kite.



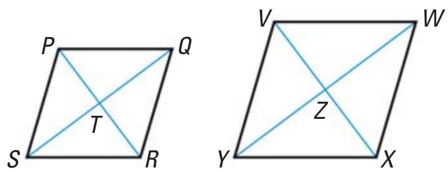

**a.** If $m\angle TQR = 102°$ and $m\angle RST = 125°$, find $m\angle QTS$. *Explain* your reasoning.

**b.** If $QS = 11$ ft, $TR = 14$ ft, and $\overline{TP} \cong \overline{QP} \cong \overline{RP}$, find *QR*, *RS*, *ST*, and *TQ*. Round your answers to the nearest foot. Show your work.

**4. GRIDDED ANSWER** The top of the table shown is shaped like an isosceles trapezoid. In *ABCD*, $AB = 48$ inches, $BC = 19$ inches, $CD = 24$ inches, and $DA = 19$ inches. Find the length (in inches) of the midsegment of *ABCD*.




**5. SHORT RESPONSE** Rhombus *PQRS* is similar to rhombus *VWXY*. In the diagram below, $QS = 32$, $QR = 20$, and $WZ = 20$. Find *WX*. *Explain* your reasoning.




**6. OPEN-ENDED** In quadrilateral *MNPQ*, $\overline{MP} \cong \overline{NQ}$.

**a.** What types of quadrilaterals could *MNPQ* be? Use the most specific names. *Explain*.

**b.** For each of your answers in part (a), tell what additional information would allow you to conclude that *MNPQ* is that type of quadrilateral. *Explain* your reasoning. (There may be more than one correct answer.)

**7. EXTENDED RESPONSE** Three of the vertices of quadrilateral *EFGH* are $E(0, 4)$, $F(2, 2)$, and $G(4, 4)$.

**a.** Suppose that *EFGH* is a rhombus. Find the coordinates of vertex *H*. *Explain* why there is only one possible location for *H*.

**b.** Suppose that *EFGH* is a convex kite. Show that there is more than one possible set of coordinates for vertex *H*. *Describe* what all the possible sets of coordinates have in common.

# 8 CHAPTER SUMMARY

## BIG IDEAS — For Your Notebook

Big Idea 1

### Using Angle Relationships in Polygons

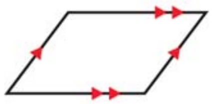
You can use theorems about the interior and exterior angles of convex polygons to solve problems.

| Polygon Interior Angles Theorem | Polygon Exterior Angles Theorem |
|---|---|
| The sum of the interior angle measures of a convex $n$-gon is $(n - 2) \cdot 180°$. | The sum of the exterior angle measures of a convex $n$-gon is $360°$. |

Big Idea 2

### Using Properties of Parallelograms

By definition, a parallelogram is a quadrilateral with both pairs of opposite sides parallel. Other properties of parallelograms:



- Opposite sides are congruent.
- Opposite angles are congruent.
- Diagonals bisect each other.
- Consecutive angles are supplementary.

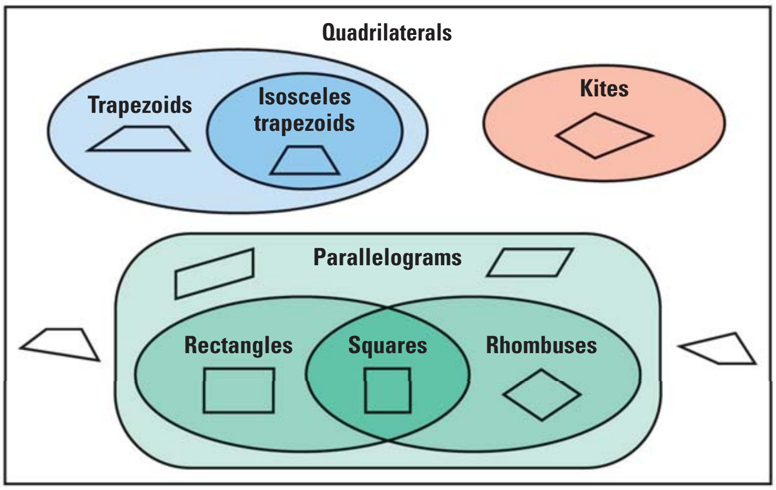
**Ways to show that a quadrilateral is a parallelogram:**

- Show both pairs of opposite sides are parallel.
- Show both pairs of opposite sides or opposite angles are congruent.
- Show one pair of opposite sides are congruent and parallel.
- Show the diagonals bisect each other.

Big Idea 3

### Classifying Quadrilaterals by Their Properties

Special quadrilaterals can be classified by their properties. In a parallelogram, both pairs of opposite sides are parallel. In a trapezoid, only one pair of sides are parallel. A kite has two pairs of consecutive congruent sides, but opposite sides are not congruent.



# 8 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

## REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- diagonal, *p. 507*
- parallelogram, *p. 515*
- rhombus, *p. 533*
- rectangle, *p. 533*
- square, *p. 533*
- trapezoid, *p. 542*
- bases of a trapezoid, *p. 542*
- base angles of a trapezoid, *p. 542*
- legs of a trapezoid, *p. 542*
- isosceles trapezoid, *p. 543*
- midsegment of a trapezoid, *p. 544*
- kite, *p. 545*

### VOCABULARY EXERCISES

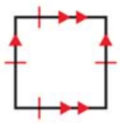
**In Exercises 1 and 2, copy and complete the statement.**

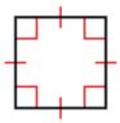
**1.** The __?__ of a trapezoid is parallel to the bases.

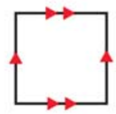
**2.** A(n) __?__ of a polygon is a segment whose endpoints are nonconsecutive vertices.

**3. WRITING** *Describe* the different ways you can show that a trapezoid is an isosceles trapezoid.

**In Exercises 4–6, match the figure with the most specific name.**

**4.**


**5.**


**6.**


**A.** Square **B.** Parallelogram **C.** Rhombus

## REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 8.

### 8.1 Find Angle Measures in Polygons *pp. 507–513*

**EXAMPLE**

**The sum of the measures of the interior angles of a convex regular polygon is 1080°. Classify the polygon by the number of sides. What is the measure of each interior angle?**

Write and solve an equation for the number of sides $n$.

$(n - 2) \cdot 180° = 1080°$ **Polygon Interior Angles Theorem**

$n = 8$ **Solve for *n*.**

The polygon has 8 sides, so it is an octagon.

A regular octagon has 8 congruent interior angles, so divide to find the measure of each angle: $1080° \div 8 = 135°$. The measure of each interior angle is 135°.

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Chapter Review Practice

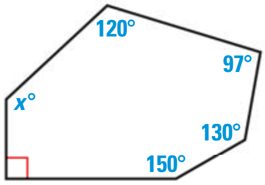
**EXAMPLES 2, 3, 4, and 5** on pp. 508–510 for Exs. 7–11

## EXERCISES

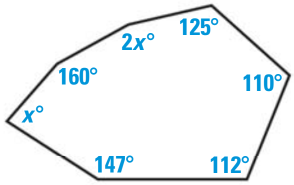
**7.** The sum of the measures of the interior angles of a convex regular polygon is 3960°. Classify the polygon by the number of sides. What is the measure of each interior angle?

**In Exercises 8–10, find the value of *x*.**

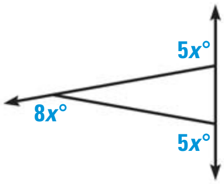
**8.**




**9.**




**10.**



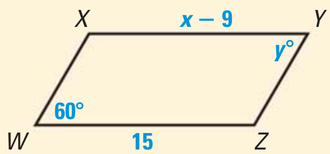

**11.** In a regular nonagon, the exterior angles are all congruent. What is the measure of one of the exterior angles? *Explain*.

## 8.2 Use Properties of Parallelograms

*pp. 515–521*

### EXAMPLE

**Quadrilateral *WXYZ* is a parallelogram. Find the values of *x* and *y*.**




To find the value of $x$, apply Theorem 8.3.

| | |
|---|---|
| $XY = WZ$ | **Opposite sides of a ▱ are ≅.** |
| $x - 9 = 15$ | **Substitute.** |
| $x = 24$ | **Add 9 to each side.** |

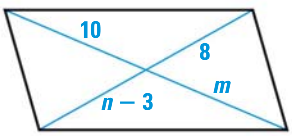
By Theorem 8.4, $\angle W \cong \angle Y$, or $m\angle W = m\angle Y$. So, $y = 60$.

**EXAMPLES 1, 2, and 3** on pp. 515, 517 for Exs. 12–17

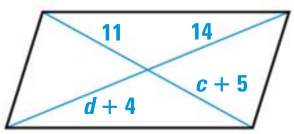
## EXERCISES

**Find the value of each variable in the parallelogram.**

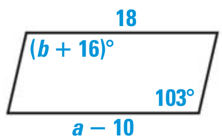
**12.**




**13.**




**14.**




**15.** In ▱$PQRS$, $PQ = 5$ centimeters, $QR = 10$ centimeters, and $m\angle PQR = 36°$. Sketch $PQRS$. Find and label all of its side lengths and interior angle measures.

**16.** The perimeter of ▱$EFGH$ is 16 inches. If $EF$ is 5 inches, find the lengths of all the other sides of $EFGH$. *Explain* your reasoning.

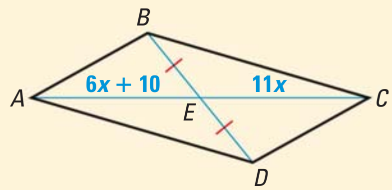
**17.** In ▱$JKLM$, the ratio of the measure of $\angle J$ to the measure of $\angle M$ is 5 : 4. Find $m\angle J$ and $m\angle M$. *Explain* your reasoning.

# 8 CHAPTER REVIEW

## 8.3 Show that a Quadrilateral is a Parallelogram
pp. 522–529

**EXAMPLE**

**For what value of $x$ is quadrilateral $ABCD$ a parallelogram?**




If the diagonals bisect each other, then $ABCD$ is a parallelogram. The diagram shows that $\overline{BE} \cong \overline{DE}$. You need to find the value of $x$ that makes $\overline{AE} \cong \overline{CE}$.

$AE = CE$ — **Set the segment lengths equal.**

$6x + 10 = 11x$ — **Substitute expressions for the lengths.**

$x = 2$ — **Solve for $x$.**

When $x = 2$, $AE = 6(2) + 10 = 22$ and $CE = 11(2) = 22$. So, $\overline{AE} \cong \overline{CE}$.

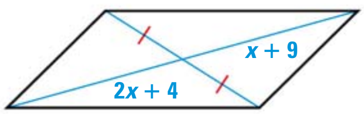
Quadrilateral $ABCD$ is a parallelogram when $x = 2$.

### EXERCISES

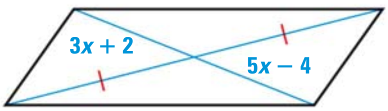
**EXAMPLE 3** on p. 524 for Exs. 18–19

**For what value of $x$ is the quadrilateral a parallelogram?**

**18.**



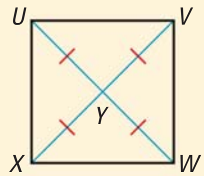

**19.**




## 8.4 Properties of Rhombuses, Rectangles, and Squares
pp. 533–540

**EXAMPLE**

**Classify the special quadrilateral.**



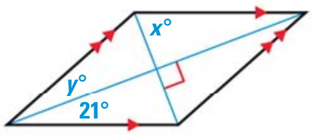

In quadrilateral $UVWX$, the diagonals bisect each other. So, $UVWX$ is a parallelogram. Also, $\overline{UY} \cong \overline{VY} \cong \overline{WY} \cong \overline{XY}$. So, $UY + YW = VY + XY$. Because $UY + YW = UW$, and $VY + XY = VX$, you can conclude that $\overline{UW} \cong \overline{VX}$. By Theorem 8.13, $UVWX$ is a rectangle.

### EXERCISES

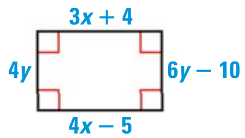
**EXAMPLES 2 and 3** on pp. 534–535 for Exs. 20–22

**Classify the special quadrilateral. Then find the values of $x$ and $y$.**

**20.**




**21.**




**22.** The diagonals of a rhombus are 10 centimeters and 24 centimeters. Find the length of a side. *Explain.*

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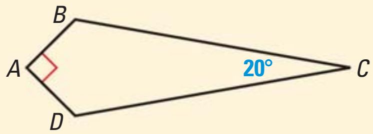
Chapter Review Practice

## 8.5 Use Properties of Trapezoids and Kites *pp. 542–549*

### EXAMPLE

**Quadrilateral *ABCD* is a kite. Find $m\angle B$ and $m\angle D$.**




A kite has exactly one pair of congruent opposite angles. Because $\angle A \ncong \angle C$, $\angle B$ and $\angle D$ must be congruent. Write and solve an equation.

$$90° + 20° + m\angle B + m\angle D = 360° \quad \textbf{Corollary to Theorem 8.1}$$

$$110° + m\angle B° + m\angle D = 360° \quad \textbf{Combine like terms.}$$

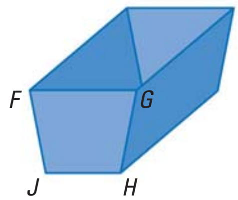
$$m\angle B + m\angle D = 250° \quad \textbf{Subtract 110° from each side.}$$

Because $\angle B \cong \angle D$, you can substitute $m\angle B$ for $m\angle D$ in the last equation. Then $m\angle B + m\angle B = 250°$, and $m\angle B = m\angle D = 125°$.

### EXERCISES

**EXAMPLES 2 and 3** on pp. 543–544 for Exs. 20–22

**In Exercises 23 and 24, use the diagram of a recycling container. One end of the container is an isosceles trapezoid with $\overline{FG} \parallel \overline{JH}$ and $m\angle F = 79°$.**



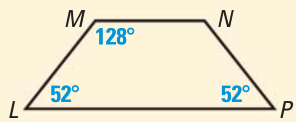

**23.** Find $m\angle G$, $m\angle H$, and $m\angle J$.

**24.** Copy trapezoid *FGHJ* and sketch its midsegment. If the midsegment is 16.5 inches long and $\overline{FG}$ is 19 inches long, find *JH*.

## 8.6 Identify Special Quadrilaterals *pp. 552–557*

### EXAMPLE

**Give the most specific name for quadrilateral *LMNP*.**




In *LMNP*, $\angle L$ and $\angle M$ are supplementary, but $\angle L$ and $\angle P$ are not. So, $\overline{MN} \parallel \overline{LP}$, but $\overline{LM}$ is not parallel to $\overline{NP}$. By definition, *LMNP* is a trapezoid.

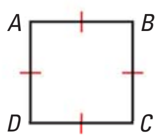
Also, $\angle L$ and $\angle P$ are a pair of base angles and $\angle L \cong \angle P$. So, *LMNP* is an isosceles trapezoid by Theorem 8.15.

### EXERCISES

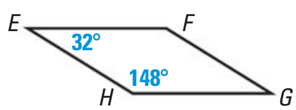
**EXAMPLE 2** on p. 553 for Exs. 25–28

**Give the most specific name for the quadrilateral. *Explain* your reasoning.**

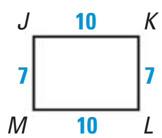
**25.**




**26.**



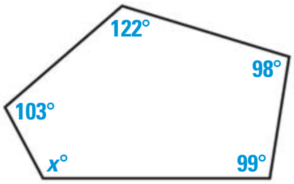

**27.**

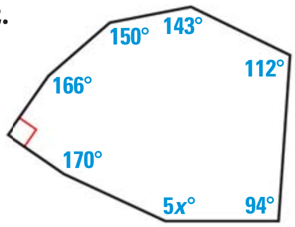


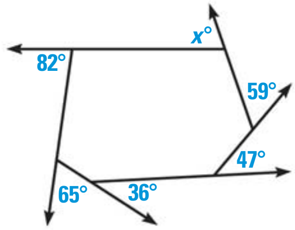

**28.** In quadrilateral *RSTU*, $\angle R$, $\angle T$, and $\angle U$ are right angles, and $RS = ST$. What is the most specific name for quadrilateral *RSTU*? *Explain.*

# 8 CHAPTER TEST

**Find the value of *x*.**

**1.** 

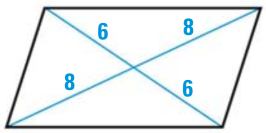

**2.** 

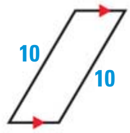

**3.** 


**4.** In ▱*EFGH*, $m\angle F$ is 40° greater than $m\angle G$. Sketch ▱*EFGH* and label each angle with its correct angle measure. *Explain* your reasoning.

**Are you given enough information to determine whether the quadrilateral is a parallelogram? *Explain* your reasoning.**

**5.** 


**6.** 


**7.** 


**In Exercises 8–11, list each type of quadrilateral—*parallelogram*, *rectangle*, *rhombus*, and *square*—for which the statement is always true.**

**8.** It is equilateral.

**9.** Its interior angles are all right angles.

**10.** The diagonals are congruent.

**11.** Opposite sides are parallel.

**12.** The vertices of quadrilateral *PQRS* are *P*(−2, 0), *Q*(0, 3), *R*(6, −1), and *S*(1, −2). Draw *PQRS* in a coordinate plane. Show that it is a trapezoid.

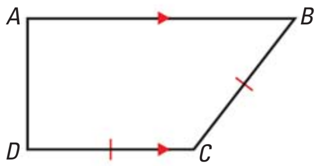
**13.** One side of a quadrilateral *JKLM* is longer than another side.

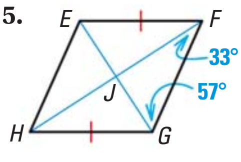
**a.** Suppose *JKLM* is an isosceles trapezoid. In a coordinate plane, find possible coordinates for the vertices of *JKLM*. *Justify* your answer.

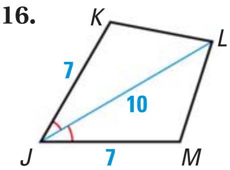
**b.** Suppose *JKLM* is a kite. In a coordinate plane, find possible coordinates for the vertices of *JKLM*. *Justify* your answer.

**c.** Name other special quadrilaterals that *JKLM* could be.

**Give the most specific name for the quadrilateral. *Explain* your reasoning.**

**14.** 


**15.** 


**16.** 


**17.** In trapezoid *WXYZ*, $\overline{WX} \parallel \overline{YZ}$, and $YZ = 4.25$ centimeters. The midsegment of trapezoid *WXYZ* is 2.75 centimeters long. Find *WX*.

**18.** In ▱*RSTU*, $\overline{RS}$ is 3 centimeters shorter than $\overline{ST}$. The perimeter of ▱*RSTU* is 42 centimeters. Find *RS* and *ST*.

# GRAPH NONLINEAR FUNCTIONS

## EXAMPLE 1 Graph a quadratic function in vertex form

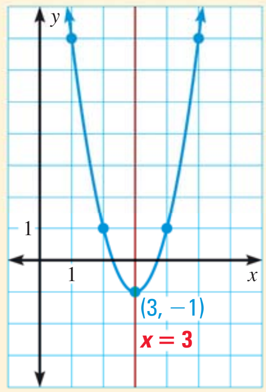
**Graph $y = 2(x - 3)^2 - 1$.**

The *vertex form* of a quadratic function is $y = a(x - h)^2 + k$. Its graph is a parabola with vertex at $(h, k)$ and axis of symmetry $x = h$.

The given function is in vertex form. So, $a = 2$, $h = 3$, and $k = -1$. Because $a > 0$, the parabola opens up.

Graph the vertex at $(3, -1)$. Sketch the axis of symmetry, $x = 3$. Use a table of values to find points on each side of the axis of symmetry. Draw a parabola through the points.

| x | 3 | 1 | 2 | 4 | 5 |
|---|---|---|---|---|---|
| y | −1 | 7 | 1 | 1 | 7 |

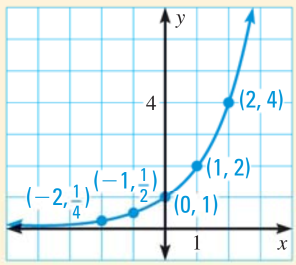



## EXAMPLE 2 Graph an exponential function

**Graph $y = 2^x$.**

Make a table by choosing a few values for $x$ and finding the values for $y$. Plot the points and connect them with a smooth curve.

| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | $\frac{1}{4}$ | $\frac{1}{2}$ | 1 | 2 | 4 |




## EXERCISES

**EXAMPLE 1** for Exs. 1–6

**Graph the quadratic function. Label the vertex and sketch the axis of symmetry.**

1. $y = 3x^2 + 5$
2. $y = -2x^2 + 4$
3. $y = 0.5x^2 - 3$
4. $y = 3(x + 3)^2 - 3$
5. $y = -2(x - 4)^2 - 1$
6. $y = \frac{1}{2}(x - 4)^2 + 3$

**EXAMPLE 2** for Exs. 7–10

**Graph the exponential function.**

7. $y = 3^x$
8. $y = 8^x$
9. $y = 2.2^x$
10. $y = \left(\frac{1}{3}\right)^x$

**Use a table of values to graph the cubic or absolute value function.**

11. $y = x^3$
12. $y = x^3 - 2$
13. $y = 3x^3 - 1$
14. $y = 2|x|$
15. $y = 2|x| - 4$
16. $y = -|x| - 1$

# 8 ★ Standardized TEST PREPARATION

## CONTEXT-BASED MULTIPLE CHOICE QUESTIONS

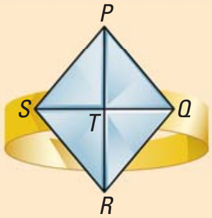
Some of the information you need to solve a context-based multiple choice question may appear in a table, a diagram, or a graph.

**PROBLEM 1**

Which of the statements about the rhombus-shaped ring is not always true?

Ⓐ $m\angle SPT = m\angle TPQ$ Ⓑ $PT = TR$

Ⓒ $m\angle STR = 90°$ Ⓓ $PR = SQ$




### Plan

**INTERPRET THE DIAGRAM** The diagram shows rhombus *PQRS* with its diagonals intersecting at point *T*. Use properties of rhombuses to figure out which statement is not always true.

### Solution

**STEP 1** Evaluate choice A.

Consider choice A: $m\angle SPT = m\angle TPQ$.

Each diagonal of a rhombus bisects each of a pair of opposite angles. The diagonal $\overline{PR}$ bisects $\angle SPQ$, so $m\angle SPT = m\angle TPQ$. Choice A is true.

**STEP 2** Evaluate choice B.

Consider choice B: $PT = TR$.

The diagonals of a parallelogram bisect each other. A rhombus is also a parallelogram, so the diagonals of *PQRS* bisect each other. So, $PT = TR$. Choice B is true.

**STEP 3** Evaluate choice C.

Consider choice C: $m\angle STR = 90°$.

The diagonals of a rhombus are perpendicular. *PQRS* is a rhombus, so its diagonals are perpendicular. Therefore, $m\angle STR = 90°$. Choice C is true.

**STEP 3** Evaluate choice D.

Consider choice D: $PR = SQ$.

If the diagonals of a parallelogram are congruent, then it is a rectangle. But *PQRS* is a rhombus. Only in the special case where it is also a square (a type of rhombus that is also a rectangle), would choice D be true. So, choice D is not always true.

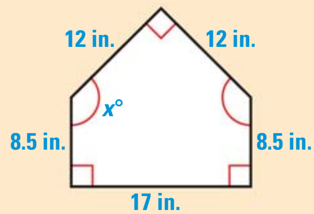
The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

## PROBLEM 2

The official dimensions of home plate in professional baseball are shown on the diagram. What is the value of $x$?

Ⓐ 90 Ⓑ 108

Ⓒ 135 Ⓓ 150




### Plan

**INTERPRET THE DIAGRAM** From the diagram, you can see that home plate is a pentagon. Use what you know about the interior angles of a polygon and the markings given on the diagram to find the value of $x$.

### Solution

***STEP 1*** Find the sum of the measures of the interior angles.

Home plate has 5 sides. Use the Polygon Interior Angles Theorem to find the sum of the measures of the interior angles.

$(n - 2) \cdot 180° = (5 - 2) \cdot 180°$ **Substitute 5 for *n*.**

$= 3 \cdot 180°$ **Subtract.**

$= 540°$ **Multiply.**

***STEP 2*** Write and solve an equation.

From the diagram, you know that three interior angles are right angles. The two other angles are congruent, including the one whose measure is $x°$. Use this information to write an equation. Then solve the equation.

$3 \cdot 90° + 2 \cdot x° = 540°$ **Write equation.**

$270 + 2x = 540$ **Multiply.**

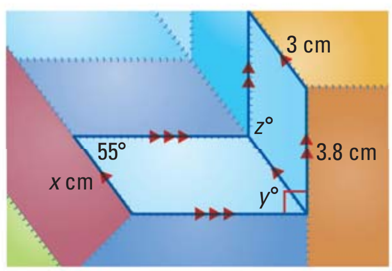
$2x = 270$ **Subtract 270 from each side.**

$x = 135$ **Divide each side by 2.**

The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

## PRACTICE

**In Exercises 1 and 2, use the part of the quilt shown.**




1. What is the value of $x$?

   Ⓐ 3 Ⓑ 3.4

   Ⓒ 3.8 Ⓓ 5.5

2. What is the value of $z$?

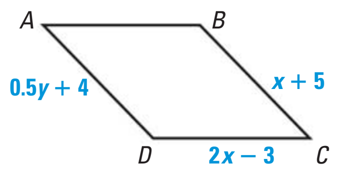
   Ⓐ 35 Ⓑ 55

   Ⓒ 125 Ⓓ 145

# 8 ★ Standardized TEST PRACTICE

## MULTIPLE CHOICE

**In Exercises 1 and 2, use the diagram of rhombus *ABCD* below.**




1. What is the value of $x$?

Ⓐ 2 Ⓑ 4.6

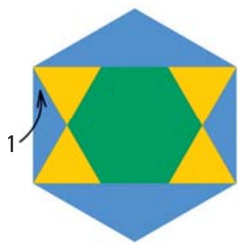
Ⓒ 8 Ⓓ 13

2. What is the value of $y$?

Ⓐ 1.8 Ⓑ 2

Ⓒ 8 Ⓓ 18

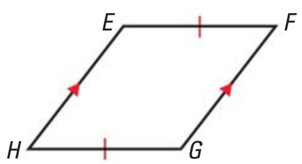
3. In the design shown below, a green regular hexagon is surrounded by yellow equilateral triangles and blue isosceles triangles. What is the measure of $\angle 1$?




Ⓐ 30° Ⓑ 40°

Ⓒ 50° Ⓓ 60°

4. Which statement about *EFGH* can be concluded from the given information?




Ⓐ It is not a kite.

Ⓑ It is not an isosceles trapezoid.

Ⓒ It is not a square.

Ⓓ It is not a rhombus.

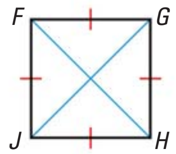
5. What is the most specific name for quadrilateral *FGHJ*?

Ⓐ Parallelogram

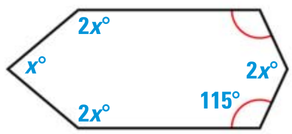
Ⓑ Rhombus

Ⓒ Rectangle

Ⓓ Square



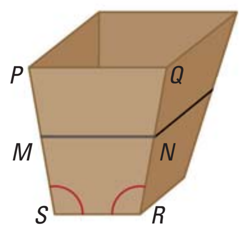

6. What is the measure of the smallest interior angle of the hexagon shown?




Ⓐ 50° Ⓑ 60°

Ⓒ 70° Ⓓ 80°

**In Exercises 7 and 8, use the diagram of a cardboard container. In the diagram, $\angle S \cong \angle R$, $\overline{PQ} \parallel \overline{SR}$, and $\overline{PS}$ and $\overline{QR}$ are not parallel.**




7. Which statement is true?

Ⓐ $PR = SQ$

Ⓑ $m\angle S + m\angle R = 180°$

Ⓒ $PQ = 2 \cdot SR$

Ⓓ $PQ = QR$

8. The bases of trapezoid *PQRS* are $\overline{PQ}$ and $\overline{SR}$, and the midsegment is $\overline{MN}$. Given $PQ = 9$ centimeters, and $MN = 7.2$ centimeters, what is $SR$?

Ⓐ 5.4 cm Ⓑ 8.1 cm

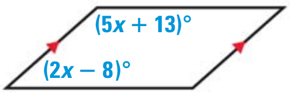
Ⓒ 10.8 cm Ⓓ 12.6 cm

## GRIDDED ANSWER

**9.** How many degrees greater is the measure of an interior angle of a regular octagon than the measure of an interior angle of a regular pentagon?

**10.** Parallelogram $ABCD$ has vertices $A(-3, -1)$, $B(-1, 3)$, $C(4, 3)$, and $D(2, -1)$. What is the sum of the $x$- and $y$-coordinates of the point of intersection of the diagonals of $ABCD$?

**11.** For what value of $x$ is the quadrilateral shown below a parallelogram?

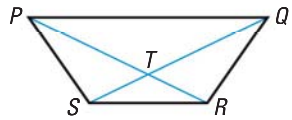



**12.** In kite $JKLM$, the ratio of $JK$ to $KL$ is $3:2$. The perimeter of $JKLM$ is 30 inches. Find the length (in inches) of $\overline{JK}$.

## SHORT RESPONSE

**13.** The vertices of quadrilateral $EFGH$ are $E(-1, -2)$, $F(-1, 3)$, $G(2, 4)$, and $H(3, 1)$. What type of quadrilateral is $EFGH$? *Explain*.

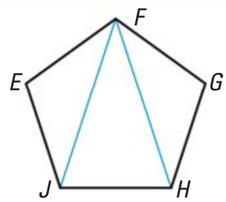
**14.** In the diagram below, $PQRS$ is an isosceles trapezoid with $\overline{PQ} \parallel \overline{RS}$. *Explain* how to show that $\triangle PTS \cong \triangle QTR$.




**15.** In trapezoid $ABCD$, $\overline{AB} \parallel \overline{CD}$, $\overline{XY}$ is the midsegment of $ABCD$, and $\overline{CD}$ is twice as long as $\overline{AB}$. Find the ratio of $XY$ to $AB$. *Justify* your answer.

## EXTENDED RESPONSE

**16.** The diagram shows a regular pentagon and diagonals drawn from vertex $F$.




- **a.** The diagonals divide the pentagon into three triangles. Classify the triangles by their angles and side measures. *Explain* your reasoning.
- **b.** Which triangles are congruent? *Explain* how you know.
- **c.** For each triangle, find the interior angle measures. *Explain* your reasoning.

**17.** In parts (a)–(c), you are given information about a quadrilateral with vertices $A$, $B$, $C$, $D$. In each case, $ABCD$ is a different quadrilateral.

- **a.** Suppose that $\overline{AB} \parallel \overline{CD}$, $AB = DC$, and $\angle C$ is a right angle. Draw quadrilateral $ABCD$ and give the most specific name for $ABCD$. *Justify* your answer.
- **b.** Suppose that $\overline{AB} \parallel \overline{CD}$ and $ABCD$ has *exactly* two right angles, one of which is $\angle C$. Draw quadrilateral $ABCD$ and give the most specific name for $ABCD$. *Justify* your answer.
- **c.** Suppose you are given only that $\overline{AB} \parallel \overline{CD}$. What additional information would you need to know about $\overline{AC}$ and $\overline{BD}$ to conclude that $ABCD$ is a rhombus? *Explain*.