6 Similarity

Before

In previous courses and in Chapters 1–5, you learned the following skills, which you'll use in Chapter 6: using properties of parallel lines, using properties of triangles, simplifying expressions, and finding perimeter.

Prerequisite Skills

VOCABULARY CHECK

1. The alternate interior angles formed when a transversal intersects two __?__ lines are congruent.

2. Two triangles are congruent if and only if their corresponding parts are __?__.

SKILLS AND ALGEBRA CHECK

Simplify the expression. *(Review pp. 870, 874 for 6.1.)*

3. $\frac{9 \cdot 20}{15}$ **4.** $\frac{15}{25}$ **5.** $\frac{3+4+5}{6+8+10}$ **6.** $\sqrt{5(5 \cdot 7)}$

Find the perimeter of the rectangle with the given dimensions. *(Review p. 49 for 6.1, 6.2.)*

7. $\ell = 5$ in., $w = 12$ in. **8.** $\ell = 30$ ft, $w = 10$ ft **9.** $A = 56\text{ m}^2$, $\ell = 8$ m

10. Find the slope of a line parallel to the line whose equation is $y - 4 = 7(x + 2)$. *(Review p. 171 for 6.5.)*

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 6, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 417. You will also use the key vocabulary listed below.

Big Ideas

1. **Using ratios and proportions to solve geometry problems**
2. **Showing that triangles are similar**
3. **Using indirect measurement and similarity**

Key Vocabulary

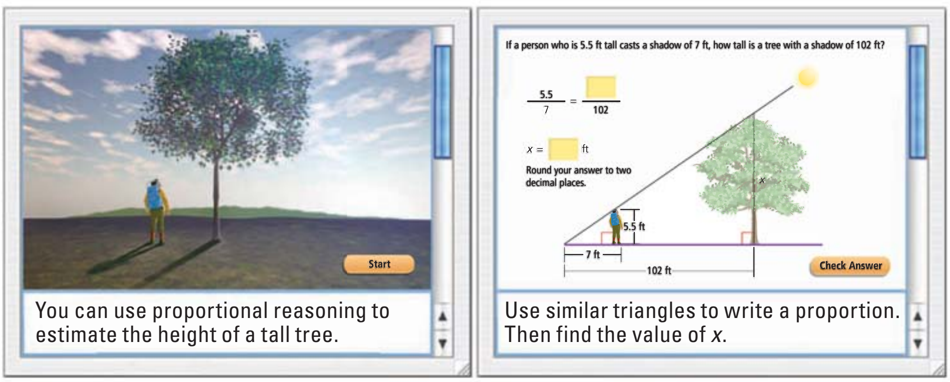
- ratio, *p. 356*
- proportion, *p. 358*
  means, extremes
- geometric mean, *p. 359*
- scale drawing, *p. 365*
- scale, *p. 365*
- similar polygons, *p. 372*
- scale factor of two similar polygons, *p. 373*
- dilation, *p. 409*
- center of dilation, *p. 409*
- scale factor of a dilation, *p. 409*
- reduction, *p. 409*
- enlargement, *p. 409*

Why?

You can use similarity to measure lengths indirectly. For example, you can use similar triangles to find the height of a tree.

Animated Geometry

The animation illustrated below for Exercise 33 on page 394 helps you answer this question: What is the height of the tree?



Geometry at classzone.com

Animated **Geometry** at classzone.com

Other animations for Chapter 6: pages 365, 375, 391, 407, and 414

Other animations for Chapter 1 appear on pages 7, 9, 14, 21, 37, and 50.

6.1 Ratios, Proportions, and the Geometric Mean

Before You solved problems by writing and solving equations.

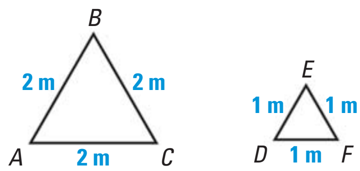
Now You will solve problems by writing and solving proportions.

Why? So you can estimate bird populations, as in Ex. 62.

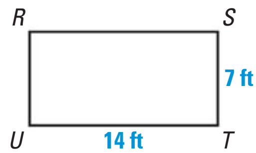
Key Vocabulary
- **ratio**
- **proportion**
  means, extremes
- **geometric mean**

If a and b are two numbers or quantities and $b \neq 0$, then the **ratio of a to b** is $\frac{a}{b}$. The ratio of a to b can also be written as $a:b$.

For example, the ratio of a side length in $\triangle ABC$ to a side length in $\triangle DEF$ can be written as $\frac{2}{1}$ or $2:1$.



Ratios are usually expressed in simplest form. Two ratios that have the same simplified form are called *equivalent ratios*. The ratios 7 : 14 and 1 : 2 in the example below are *equivalent*.



$$\frac{\text{width of } RSTU}{\text{length of } RSTU} = \frac{7 \text{ ft}}{14 \text{ ft}} = \frac{1}{2}$$

EXAMPLE 1 Simplify ratios

Simplify the ratio.

a. 64 m : 6 m

b. $\frac{5 \text{ ft}}{20 \text{ in.}}$

Solution

a. Write 64 m : 6 m as $\frac{64 \text{ m}}{6 \text{ m}}$. Then divide out the units and simplify.

$$\frac{64 \text{ m}}{6 \text{ m}} = \frac{32}{3} = 32:3$$

b. To simplify a ratio with unlike units, multiply by a conversion factor.

$$\frac{5 \text{ ft}}{20 \text{ in.}} = \frac{5 \text{ ft}}{20 \text{ in.}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} = \frac{60}{20} = \frac{3}{1}$$

REVIEW UNIT ANALYSIS

For help with measures and conversion factors, see p. 886 and the Table of Measures on p. 921.

GUIDED PRACTICE for Example 1

Simplify the ratio.

1. 24 yards to 3 yards

2. 150 cm : 6 m

EXAMPLE 2 Use a ratio to find a dimension

PAINTING You are planning to paint a mural on a rectangular wall. You know that the perimeter of the wall is 484 feet and that the ratio of its length to its width is 9 : 2. Find the area of the wall.

Solution

WRITE EXPRESSIONS

Because the ratio in Example 2 is 9 : 2, you can write an equivalent ratio to find expressions for the length and width.

$$\frac{\text{length}}{\text{width}} = \frac{9}{2} = \frac{9}{2} \cdot \frac{x}{x} = \frac{9x}{2x}$$

STEP 1 **Write** expressions for the length and width. Because the ratio of length to width is 9 : 2, you can represent the length by $9x$ and the width by $2x$.

STEP 2 **Solve** an equation to find x.

$2\ell + 2w = P$	Formula for perimeter of rectangle
$2(9x) + 2(2x) = 484$	Substitute for ℓ, w, and P.
$22x = 484$	Multiply and combine like terms.
$x = 22$	Divide each side by 22.

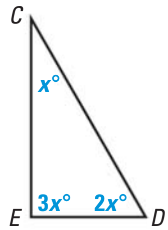
STEP 3 **Evaluate** the expressions for the length and width. Substitute the value of x into each expression.

Length $= 9x = 9(22) = 198$ Width $= 2x = 2(22) = 44$

▶ The wall is 198 feet long and 44 feet wide, so its area is $198 \text{ ft} \cdot 44 \text{ ft} = 8712 \text{ ft}^2$.

EXAMPLE 3 Use extended ratios

xy **ALGEBRA** **The measures of the angles in $\triangle CDE$ are in the *extended ratio* of 1 : 2 : 3. Find the measures of the angles.**



Solution

Begin by sketching the triangle. Then use the extended ratio of 1 : 2 : 3 to label the measures as $x°$, $2x°$, and $3x°$.

$x° + 2x° + 3x° = 180°$	Triangle Sum Theorem
$6x = 180$	Combine like terms.
$x = 30$	Divide each side by 6.

▶ The angle measures are $30°$, $2(30°) = 60°$, and $3(30°) = 90°$.

✓ GUIDED PRACTICE for Examples 2 and 3

3. The perimeter of a room is 48 feet and the ratio of its length to its width is 7 : 5. Find the length and width of the room.

4. A triangle's angle measures are in the extended ratio of 1 : 3 : 5. Find the measures of the angles.

PROPORTIONS An equation that states that two ratios are equal is called a **proportion**.

$$\text{extreme} \rightarrow a, \quad \text{mean} \rightarrow b: \quad \frac{a}{b} = \frac{c}{d} \quad c \leftarrow \text{mean}, \quad d \leftarrow \text{extreme}$$

The numbers b and c are the **means** of the proportion. The numbers a and d are the **extremes** of the proportion.

The property below can be used to solve proportions. To *solve a proportion*, you find the value of any variable in the proportion.

PROPORTIONS
You will learn more properties of proportions on p. 364.

KEY CONCEPT — For Your Notebook

A Property of Proportions

1. **Cross Products Property** In a proportion, the product of the extremes equals the product of the means.

If $\frac{a}{b} = \frac{c}{d}$ where $b \neq 0$ and $d \neq 0$, then $ad = bc$.

$$\frac{2}{3} = \frac{4}{6} \qquad 3 \cdot 4 = 12 \qquad 2 \cdot 6 = 12$$

EXAMPLE 4 Solve proportions

ALGEBRA **Solve the proportion.**

a. $\frac{5}{10} = \frac{x}{16}$

b. $\frac{1}{y+1} = \frac{2}{3y}$

Solution

a.

$\frac{5}{10} = \frac{x}{16}$	Write original proportion.
$5 \cdot 16 = 10 \cdot x$	Cross Products Property
$80 = 10x$	Multiply.
$8 = x$	Divide each side by 10.

b.

$\frac{1}{y+1} = \frac{2}{3y}$	Write original proportion.
$1 \cdot 3y = 2(y + 1)$	Cross Products Property
$3y = 2y + 2$	Distributive Property
$y = 2$	Subtract $2y$ from each side.

ANOTHER WAY
In part (a), you could multiply each side by the denominator, 16. Then $16 \cdot \frac{5}{10} = 16 \cdot \frac{x}{16}$, so $8 = x$.

✓ GUIDED PRACTICE for Example 4

Solve the proportion.

5. $\frac{2}{x} = \frac{5}{8}$

6. $\frac{1}{x-3} = \frac{4}{3x}$

7. $\frac{y-3}{7} = \frac{y}{14}$

EXAMPLE 5 Solve a real-world problem

SCIENCE As part of an environmental study, you need to estimate the number of trees in a 150 acre area. You count 270 trees in a 2 acre area and you notice that the trees seem to be evenly distributed. Estimate the total number of trees.

Solution

Write and solve a proportion involving two ratios that compare the number of trees with the area of the land.

$\frac{270}{2} = \frac{n}{150}$ ← number of trees / ← area in acres	Write proportion.
$270 \cdot 150 = 2 \cdot n$	Cross Products Property
$20{,}250 = n$	Simplify.

▶ There are about 20,250 trees in the 150 acre area.

KEY CONCEPT — *For Your Notebook*

Geometric Mean

The **geometric mean** of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x} = \frac{x}{b}$. So, $x^2 = ab$ and $x = \sqrt{ab}$.

EXAMPLE 6 Find a geometric mean

Find the geometric mean of 24 and 48.

Solution

$x = \sqrt{ab}$	Definition of geometric mean
$= \sqrt{24 \cdot 48}$	Substitute 24 for *a* and 48 for *b*.
$= \sqrt{24 \cdot 24 \cdot 2}$	Factor.
$= 24\sqrt{2}$	Simplify.

▶ The geometric mean of 24 and 48 is $24\sqrt{2} \approx 33.9$.

✓ GUIDED PRACTICE for Examples 5 and 6

8. **WHAT IF?** In Example 5, suppose you count 390 trees in a 3 acre area of the 150 acre area. Make a new estimate of the total number of trees.

Find the geometric mean of the two numbers.

9. 12 and 27 **10.** 18 and 54 **11.** 16 and 18

6.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 27, and 59

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 47, 48, 52, and 63

◆ = **MULTIPLE REPRESENTATIONS** Ex. 66

SKILL PRACTICE

1. **VOCABULARY** Copy the proportion $\frac{m}{n} = \frac{p}{q}$. Identify the means of the proportion and the extremes of the proportion.

2. ★ **WRITING** Write three ratios that are equivalent to the ratio 3 : 4. *Explain* how you found the ratios.

EXAMPLE 1 on p. 356 for Exs. 3–17

SIMPLIFYING RATIOS **Simplify the ratio.**

3. \$20 : \$5

4. $\frac{15 \text{ cm}^2}{12 \text{ cm}^2}$

5. 6 L : 10 mL

6. $\frac{1 \text{ mi}}{20 \text{ ft}}$

7. $\frac{7 \text{ ft}}{12 \text{ in.}}$

8. $\frac{80 \text{ cm}}{2 \text{ m}}$

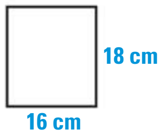
9. $\frac{3 \text{ lb}}{10 \text{ oz}}$

10. $\frac{2 \text{ gallons}}{18 \text{ quarts}}$

WRITING RATIOS **Find the ratio of the width to the length of the rectangle. Then simplify the ratio.**

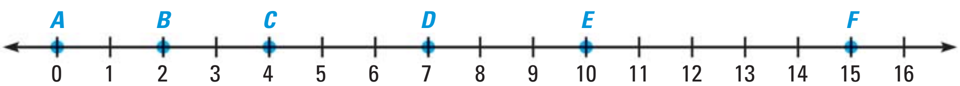
11.

12.



13.

FINDING RATIOS **Use the number line to find the ratio of the distances.**



14. $\frac{AD}{CF}$

15. $\frac{BD}{AB}$

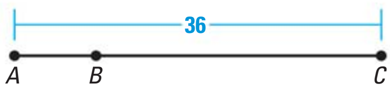
16. $\frac{CE}{EF}$

17. $\frac{BE}{CE}$

EXAMPLE 2 on p. 357 for Exs. 18–19

18. **PERIMETER** The perimeter of a rectangle is 154 feet. The ratio of the length to the width is 10 : 1. Find the length and the width.

19. **SEGMENT LENGTHS** In the diagram, $AB : BC$ is 2 : 7 and $AC = 36$. Find AB and BC.



EXAMPLE 3 on p. 357 for Exs. 20–22

USING EXTENDED RATIOS **The measures of the angles of a triangle are in the extended ratio given. Find the measures of the angles of the triangle.**

20. 3 : 5 : 10

21. 2 : 7 : 9

22. 11 : 12 : 13

EXAMPLE 4 on p. 358 for Exs. 23–30

xy ALGEBRA **Solve the proportion.**

23. $\frac{6}{x} = \frac{3}{2}$

24. $\frac{y}{20} = \frac{3}{10}$

25. $\frac{2}{7} = \frac{12}{z}$

26. $\frac{j+1}{5} = \frac{4}{10}$

27. $\frac{1}{c+5} = \frac{3}{24}$

28. $\frac{4}{a-3} = \frac{2}{5}$

29. $\frac{1+3b}{4} = \frac{5}{2}$

30. $\frac{3}{2p+5} = \frac{1}{9p}$

EXAMPLE 6
on p. 359
for Exs. 31–36

GEOMETRIC MEAN **Find the geometric mean of the two numbers.**

31. 2 and 18 **32.** 4 and 25 **33.** 32 and 8

34. 4 and 16 **35.** 2 and 25 **36.** 6 and 20

37. ERROR ANALYSIS A student incorrectly simplified the ratio. *Describe* and correct the student's error.

$$\frac{8 \text{ in.}}{3 \text{ ft}} = \frac{8 \text{ in.}}{3 \text{ ft}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} = \frac{96 \text{ in.}}{3 \text{ ft}} = \frac{32 \text{ in.}}{1 \text{ ft}}$$

WRITING RATIOS **Let $x = 10$, $y = 3$, and $z = 8$. Write the ratio in simplest form.**

38. $x : z$ **39.** $\frac{8y}{x}$ **40.** $\frac{4}{2x + 2z}$ **41.** $\frac{2x - z}{3y}$

ALGEBRA **Solve the proportion.**

42. $\frac{2x+5}{3} = \frac{x-5}{4}$ **43.** $\frac{2-s}{3} = \frac{2s+1}{5}$ **44.** $\frac{15}{m} = \frac{m}{5}$ **45.** $\frac{7}{q+1} = \frac{q-1}{5}$

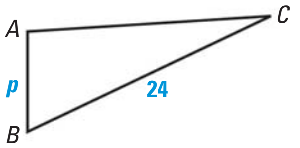
46. ANGLE MEASURES The ratio of the measures of two supplementary angles is 5 : 3. Find the measures of the angles.

47. ★ SHORT RESPONSE The ratio of the measure of an exterior angle of a triangle to the measure of the adjacent interior angle is 1 : 4. Is the triangle *acute* or *obtuse*? *Explain* how you found your answer.

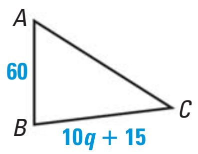
48. ★ SHORT RESPONSE Without knowing its side lengths, can you determine the ratio of the perimeter of a square to the length of one of its sides? *Explain.*

ALGEBRA **In Exercises 49–51, the ratio of two side lengths for the triangle is given. Solve for the variable.**

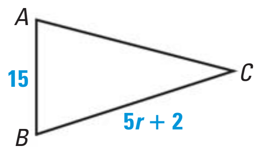
49. $AB : BC$ is 3 : 8.



50. $AB : BC$ is 3 : 4.



51. $AB : BC$ is 5 : 9.



52. ★ MULTIPLE CHOICE What is a value of x that makes $\frac{x}{3} = \frac{4x}{x+3}$ true?

Ⓐ 3 Ⓑ 4 Ⓒ 9 Ⓓ 12

53. AREA The area of a rectangle is 4320 square inches. The ratio of the width to the length is 5 : 6. Find the length and the width.

54. COORDINATE GEOMETRY The points (−3, 2), (1, 1), and (x, 0) are collinear. Use slopes to write a proportion to find the value of x.

55. ALGEBRA Use the proportions $\frac{a+b}{2a-b} = \frac{5}{4}$ and $\frac{b}{a+9} = \frac{5}{9}$ to find a and b.

56. CHALLENGE Find the ratio of x to y given that $\frac{5}{y} + \frac{7}{x} = 24$ and $\frac{12}{y} + \frac{2}{x} = 24$.

PROBLEM SOLVING

EXAMPLE 2 on p. 357 for Ex. 57

57. TILING The perimeter of a room is 66 feet. The ratio of its length to its width is 6 : 5. You want to tile the floor with 12 inch square tiles. Find the length and width of the room, and the area of the floor. How many tiles will you need? The tiles cost $1.98 each. What is the total cost to tile the floor?

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58. GEARS The *gear ratio* of two gears is the ratio of the number of teeth of the larger gear to the number of teeth of the smaller gear. In a set of three gears, the ratio of Gear A to Gear B is equal to the ratio of Gear B to Gear C. Gear A has 36 teeth and Gear C has 16 teeth. How many teeth does Gear B have?

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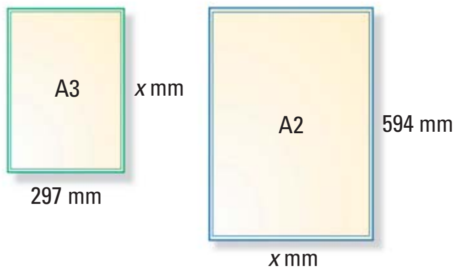
59. TRAIL MIX You need to make 36 one-half cup bags of trail mix for a class trip. The recipe calls for peanuts, chocolate chips, and raisins in the extended ratio 5 : 1 : 4. How many cups of each item do you need?

60. PAPER SIZES International standard paper sizes are commonly used all over the world. The various sizes all have the same width-to-length ratios. Two sizes of paper are shown, called A3 and A2. The distance labeled x is the geometric mean of 297 mm and 594 mm. Find the value of x.



61. BATTING AVERAGE The batting average of a baseball player is the ratio of the number of hits to the number of official at-bats. In 2004, Johnny Damon of the Boston Red Sox had 621 official at-bats and a batting average of .304. Use the proportion to find the number of hits made by Johnny Damon.

$$\frac{\text{Number of hits}}{\text{Number of at-bats}} = \frac{\text{Batting average}}{1.000}$$

EXAMPLE 5 on p. 359 for Ex. 62

62. MULTI-STEP PROBLEM The population of Red-tailed hawks is increasing in many areas of the United States. One long-term survey of bird populations suggests that the Red-tailed hawk population is increasing nationally by 2.7% each year.

a. Write 2.7% as a ratio of hawks in year n to hawks in year $(n - 1)$.

b. In 2004, observers in Corpus Christi, TX, spotted 180 migrating Red-tailed hawks. Assuming this population follows the national trend, about how many Red-tailed hawks can they expect to see in 2005?

c. Observers in Lipan Point, AZ, spotted 951 migrating Red-tailed hawks in 2004. Assuming this population follows the national trend, about how many Red-tailed hawks can they expect to see in 2006?

63. ★ **SHORT RESPONSE** Some common computer screen resolutions are 1024 : 768, 800 : 600, and 640 : 480. *Explain* why these ratios are equivalent.

64. **BIOLOGY** The larvae of the Mother-of-Pearl moth is the fastest moving caterpillar. It can run at a speed of 15 inches per second. When threatened, it can curl itself up and roll away 40 times faster than it can run. How fast can it run in miles per hour? How fast can it roll?

65. **CURRENCY EXCHANGE** Emily took 500 U.S. dollars to the bank to exchange for Canadian dollars. The exchange rate on that day was 1.2 Canadian dollars per U.S. dollar. How many Canadian dollars did she get in exchange for the 500 U.S. dollars?

66. **MULTIPLE REPRESENTATIONS** Let x and y be two positive numbers whose geometric mean is 6.

a. **Making a Table** Make a table of ordered pairs (x, y) such that $\sqrt{xy} = 6$.

b. **Drawing a Graph** Use the ordered pairs to make a scatter plot. Connect the points with a smooth curve.

c. **Analyzing Data** Is the data linear? Why or why not?

67. **ALGEBRA** Use algebra to verify Property 1, the Cross Products Property.

68. **ALGEBRA** Show that the geometric mean of two numbers is equal to the arithmetic mean (or average) of the two numbers only when the numbers are equal. (*Hint*: Solve $\sqrt{xy} = \frac{x + y}{2}$ with $x, y \geq 0$.)

CHALLENGE In Exercises 69–71, use the given information to find the value(s) of x. Assume that the given quantities are nonnegative.

69. The geometric mean of the quantities $(\sqrt{x})$ and $(3\sqrt{x})$ is $(x - 6)$.

70. The geometric mean of the quantities $(x + 1)$ and $(2x + 3)$ is $(x + 3)$.

71. The geometric mean of the quantities $(2x + 1)$ and $(6x + 1)$ is $(4x - 1)$.

MIXED REVIEW

PREVIEW
Prepare for Lesson 6.2 in Exs. 72–75.

Find the reciprocal. *(p. 869)*

72. -6 **73.** $\frac{1}{13}$ **74.** $\frac{-36}{3}$ **75.** -0.2

Solve the quadratic equation. *(p. 882)*

76. $5x^2 = 35$ **77.** $x^2 - 20 = 29$ **78.** $(x - 3)(x + 3) = 27$

Write the equation of the line with the given description. *(p. 180)*

79. Parallel to $y = 3x - 7$, passing through (1, 2)

80. Perpendicular to $y = \frac{1}{4}x + 5$, passing through (0, 24)

6.2 Use Proportions to Solve Geometry Problems

Before You wrote and solved proportions.

Now You will use proportions to solve geometry problems.

Why? So you can calculate building dimensions, as in Ex. 22.

Key Vocabulary
- **scale drawing**
- **scale**

In Lesson 6.1, you learned to use the Cross Products Property to write equations that are equivalent to a given proportion. Three more ways to do this are given by the properties below.

REVIEW RECIPROCALS
For help with reciprocals, see p. 869.

KEY CONCEPT — *For Your Notebook*

Additional Properties of Proportions

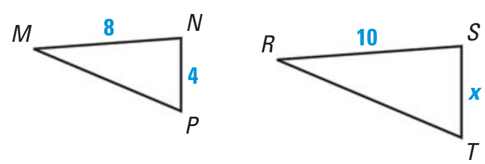
2. **Reciprocal Property** If two ratios are equal, then their reciprocals are also equal. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$.

3. If you interchange the means of a proportion, then you form another true proportion. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.

4. In a proportion, if you add the value of each ratio's denominator to its numerator, then you form another true proportion. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

EXAMPLE 1 Use properties of proportions

In the diagram, $\frac{MN}{RS} = \frac{NP}{ST}$. Write four true proportions.



Solution

Because $\frac{MN}{RS} = \frac{NP}{ST}$, then $\frac{8}{10} = \frac{4}{x}$.

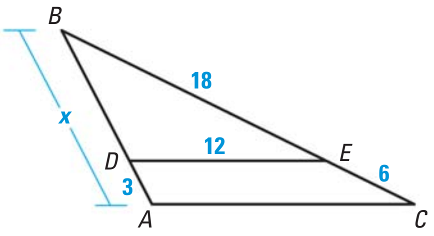
By the Reciprocal Property, the reciprocals are equal, so $\frac{10}{8} = \frac{x}{4}$.

By Property 3, you can interchange the means, so $\frac{8}{4} = \frac{10}{x}$.

By Property 4, you can add the denominators to the numerators, so $\frac{8+10}{10} = \frac{4+x}{x}$, or $\frac{18}{10} = \frac{4+x}{x}$.

EXAMPLE 2 Use proportions with geometric figures

ALGEBRA **In the diagram, $\frac{BD}{DA} = \frac{BE}{EC}$. Find BA and BD.**



Solution

$\frac{BD}{DA} = \frac{BE}{EC}$ — Given

$\frac{BD + DA}{DA} = \frac{BE + EC}{EC}$ — Property of Proportions (Property 4)

$\frac{x}{3} = \frac{18 + 6}{6}$ — Substitution Property of Equality

$6x = 3(18 + 6)$ — Cross Products Property

$x = 12$ — Solve for x.

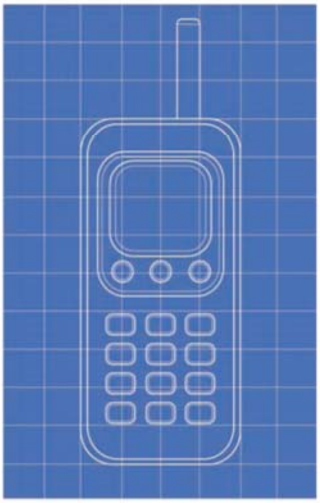
▶ So, $BA = 12$ and $BD = 12 - 3 = 9$.

Animated Geometry at classzone.com

SCALE DRAWING A **scale drawing** is a drawing that is the same shape as the object it represents. The **scale** is a ratio that describes how the dimensions in the drawing are related to the actual dimensions of the object.

EXAMPLE 3 Find the scale of a drawing

BLUEPRINTS The blueprint shows a scale drawing of a cell phone. The length of the antenna on the blueprint is 5 centimeters. The actual length of the antenna is 2 centimeters. What is the scale of the blueprint?



Solution

To find the scale, write the ratio of a length in the drawing to an actual length, then rewrite the ratio so that the denominator is 1.

$$\frac{\text{length on blueprint}}{\text{length of antenna}} = \frac{5 \text{ cm}}{2 \text{ cm}} = \frac{5 \div 2}{2 \div 2} = \frac{2.5}{1}$$

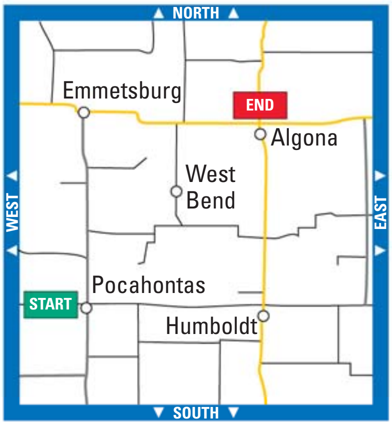
▶ The scale of the blueprint is 2.5 cm : 1 cm.

✓ GUIDED PRACTICE for Examples 1, 2, and 3

1. In Example 1, find the value of x.
2. In Example 2, $\frac{DE}{AC} = \frac{BE}{BC}$. Find AC.
3. **WHAT IF?** In Example 3, suppose the length of the antenna on the blueprint is 10 centimeters. Find the new scale of the blueprint.

EXAMPLE 4 Use a scale drawing

MAPS The scale of the map at the right is 1 inch : 26 miles. Find the actual distance from Pocahontas to Algona.



Solution

Use a ruler. The distance from Pocahontas to Algona on the map is about 1.25 inches. Let x be the actual distance in miles.

$$\frac{1.25 \text{ in.}}{x \text{ mi}} = \frac{1 \text{ in.}}{26 \text{ mi}}$$ ← **distance on map** / ← **actual distance**

$x = 1.25(26)$ **Cross Products Property**

$x = 32.5$ **Simplify.**

▶ The actual distance from Pocahontas to Algona is about 32.5 miles.

EXAMPLE 5 Solve a multi-step problem

SCALE MODEL You buy a 3-D scale model of the Reunion Tower in Dallas, TX. The actual building is 560 feet tall. Your model is 10 inches tall, and the diameter of the dome on your scale model is about 2.1 inches.

a. What is the diameter of the actual dome?

b. About how many times as tall as your model is the actual building?

Solution

a. $$\frac{10 \text{ in.}}{560 \text{ ft}} = \frac{2.1 \text{ in.}}{x \text{ ft}}$$ ← **measurement on model** / ← **measurement on actual building**

$10x = 1176$ **Cross Products Property**

$x = 117.6$ **Solve for *x*.**

▶ The diameter of the actual dome is about 118 feet.

b. To simplify a ratio with unlike units, multiply by a conversion factor.

$$\frac{560 \text{ ft}}{10 \text{ in.}} = \frac{560 \text{ ft}}{10 \text{ in.}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} = 672$$

▶ The actual building is 672 times as tall as the model.

✓ GUIDED PRACTICE for Examples 4 and 5

4. Two cities are 96 miles from each other. The cities are 4 inches apart on a map. Find the scale of the map.

5. **WHAT IF?** Your friend has a model of the Reunion Tower that is 14 inches tall. What is the diameter of the dome on your friend's model?

6.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 13, and 25

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, and 24

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A __?__ is a drawing that has the same shape as the object it represents.

2. ★ **WRITING** Suppose the scale of a model of the Eiffel Tower is 1 inch : 20 feet. *Explain* how to determine how many times taller the actual tower is than the model.

EXAMPLE 1 on p. 364 for Exs. 3–10

REASONING **Copy and complete the statement.**

3. If $\frac{8}{x} = \frac{3}{y}$, then $\frac{8}{3} = \frac{?}{?}$.

4. If $\frac{x}{9} = \frac{y}{20}$, then $\frac{x}{y} = \frac{?}{?}$.

5. If $\frac{x}{6} = \frac{y}{15}$, then $\frac{x+6}{6} = \frac{?}{?}$.

6. If $\frac{14}{3} = \frac{x}{y}$, then $\frac{17}{3} = \frac{?}{?}$.

REASONING **Decide whether the statement is *true* or *false*.**

7. If $\frac{8}{m} = \frac{n}{9}$, then $\frac{8+m}{m} = \frac{n+9}{9}$.

8. If $\frac{5}{7} = \frac{a}{b}$, then $\frac{7}{5} = \frac{a}{b}$.

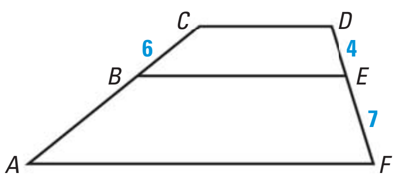
9. If $\frac{d}{2} = \frac{g+10}{11}$, then $\frac{d}{g+10} = \frac{2}{11}$.

10. If $\frac{4+x}{4} = \frac{3+y}{y}$, then $\frac{x}{4} = \frac{3}{y}$.

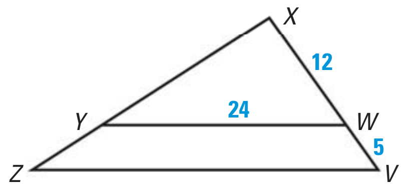
EXAMPLE 2 on p. 365 for Exs. 11–12

PROPERTIES OF PROPORTIONS **Use the diagram and the given information to find the unknown length.**

11. Given $\frac{CB}{BA} = \frac{DE}{EF}$, find BA.

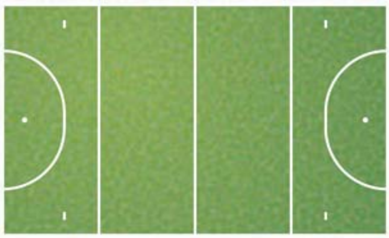


12. Given $\frac{XW}{XV} = \frac{YW}{ZV}$, find ZV.



EXAMPLES 3 and 4 on pp. 365–366 for Exs. 13–14

SCALE DIAGRAMS **In Exercises 13 and 14, use the diagram of the field hockey field in which 1 inch = 50 yards. Use a ruler to approximate the dimension.**



13. Find the actual length of the field.

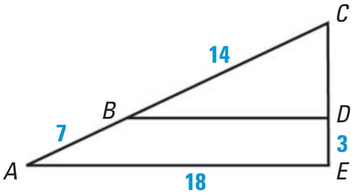
14. Find the actual width of the field.

15. **ERROR ANALYSIS** *Describe* and correct the error made in the reasoning.

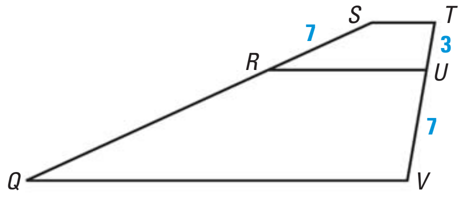
If $\frac{a}{3} = \frac{c}{4}$, then $\frac{a+3}{3} = \frac{c+3}{4}$. ✗

PROPERTIES OF PROPORTIONS **Use the diagram and the given information to find the unknown length.**

16. Given $\frac{CA}{CB} = \frac{AE}{BD}$, find BD.



17. Given $\frac{SQ}{SR} = \frac{TV}{TU}$, find RQ.



18. ★ **MULTIPLE CHOICE** If x, y, z, and q are four different numbers, and the proportion $\frac{x}{y} = \frac{z}{q}$ is true, which of the following is false?

Ⓐ $\frac{y}{x} = \frac{q}{z}$ Ⓑ $\frac{x}{z} = \frac{y}{q}$ Ⓒ $\frac{y}{x} = \frac{z}{q}$ Ⓓ $\frac{x+y}{y} = \frac{z+q}{q}$

CHALLENGE **Two number patterns are *proportional* if there is a nonzero number k such that $(a_1, b_1, c_1, \ldots) = k(a_2, b_2, c_2, \ldots) = ka_2, kb_2, kc_2, \ldots$.**

19. Given the relationship $(8, 16, 20) = k(2, 4, 5)$, find k.

20. Given that $a_1 = ka_2$, $b_1 = kb_2$, and $c_1 = kc_2$, show that $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

21. Given that $a_1 = ka_2$, $b_1 = kb_2$, and $c_1 = kc_2$, show that $\frac{a_1 + b_1 + c_1}{a_2 + b_2 + c_2} = k$.

PROBLEM SOLVING

EXAMPLE 5
on p. 366
for Ex. 22

22. **ARCHITECTURE** A basket manufacturer has headquarters in an office building that has the same shape as a basket they sell.

a. The bottom of the basket is a rectangle with length 15 inches and width 10 inches. The base of the building is a rectangle with length 192 feet. What is the width of the base of the building?

b. About how many times as long as the bottom of the basket is the base of the building?

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Longaberger Company Home Office Newark, Ohio

23. **MAP SCALE** A street on a map is 3 inches long. The actual street is 1 mile long. Find the scale of the map.

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24. ★ **MULTIPLE CHOICE** A model train engine is 12 centimeters long. The actual engine is 18 meters long. What is the scale of the model?

Ⓐ 3 cm : 2 m Ⓑ 1 cm : 1.5 m Ⓒ 1 cm : 3 m Ⓓ 200 cm : 3 m

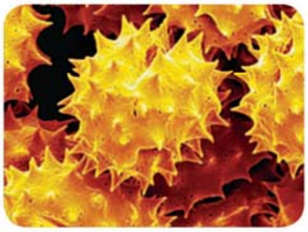
○ = **WORKED-OUT SOLUTIONS** on p. WS1 ★ = **STANDARDIZED TEST PRACTICE**

MAP READING The map of a hiking trail has a scale of 1 inch : 3.2 miles. Use a ruler to approximate the actual distance between the two shelters.

25. Meadow View and Whispering Pines

26. Whispering Pines and Blueberry Hill

27. **POLLEN** The photograph shows a particle of goldenrod pollen that has been magnified under a microscope. The scale of the photograph is 900 : 1. Use a ruler to estimate the width in millimeters of the particle.



RAMP DESIGN Assume that the wheelchair ramps described each have a slope of $\frac{1}{12}$, which is the maximum slope recommended for a wheelchair ramp.

28. A wheelchair ramp has a 21 foot run. What is its rise?

29. A wheelchair ramp rises 4 feet. What is its run?

30. **STATISTICS** Researchers asked 4887 people to pick a number between 1 and 10. The results are shown in the table below.

Answer	1	2	3	4	5
Percent	4.2%	5.1%	11.4%	10.5%	10.7%
Answer	6	7	8	9	10
Percent	10.0%	27.2%	8.8%	6.0%	6.1%

a. Estimate the number of people who picked the number 3.

b. You ask a participant what number she picked. Is the participant more likely to answer 6 or 7? *Explain.*

c. Conduct this experiment with your classmates. Make a table in which you compare the new percentages with the ones given in the original survey. Why might they be different?

xy ALGEBRA Use algebra to verify the property of proportions.

31. Property 2

32. Property 3

33. Property 4

REASONING **Use algebra to *explain* why the property of proportions is true.**

34. If $\frac{a-b}{a+b} = \frac{c-d}{c+d}$, then $\frac{a}{b} = \frac{c}{d}$.

35. If $\frac{a+c}{b+d} = \frac{a-c}{b-d}$, then $\frac{a}{b} = \frac{c}{d}$.

36. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then $\frac{a+c+e}{b+d+f} = \frac{a}{b}$. (*Hint:* Let $\frac{a}{b} = r$.)

37. **CHALLENGE** When fruit is dehydrated, water is removed from the fruit. The water content in fresh apricots is about 86%. In dehydrated apricots, the water content is about 75%. Suppose 5 kilograms of raw apricots are dehydrated. How many kilograms of water are removed from the fruit? What is the approximate weight of the dehydrated apricots?

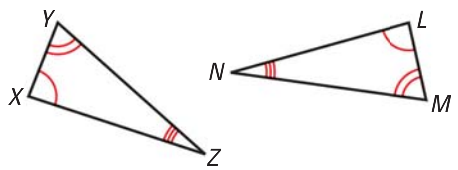
MIXED REVIEW

38. Over the weekend, Claudia drove a total of 405 miles, driving twice as far on Saturday as on Sunday. How far did Claudia travel each day? *(p. 65)*

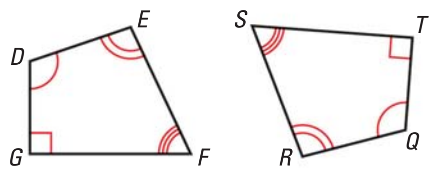
PREVIEW
Prepare for Lesson 6.3 in Exs. 39–40.

Identify all pairs of congruent corresponding parts. Then write another congruence statement for the figures. *(p. 225)*

39. $\triangle XYZ \cong \triangle LMN$



40. $DEFG \cong QRST$



QUIZ *for Lessons 6.1–6.2*

Solve the proportion. *(p. 356)*

1. $\frac{10}{y} = \frac{5}{2}$

2. $\frac{x}{6} = \frac{9}{3}$

3. $\frac{1}{a+3} = \frac{4}{16}$

4. $\frac{6}{d-6} = \frac{4}{8}$

Copy and complete the statement. *(p. 364)*

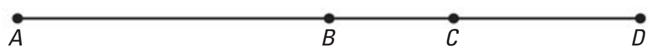
5. If $\frac{9}{x} = \frac{5}{2}$, then $\frac{9}{5} = \frac{?}{?}$.

6. If $\frac{x}{15} = \frac{y}{21}$, then $\frac{x}{y} = \frac{?}{?}$.

7. If $\frac{x}{8} = \frac{y}{12}$, then $\frac{x+8}{8} = \frac{?}{?}$.

8. If $\frac{32}{5} = \frac{x}{y}$, then $\frac{37}{5} = \frac{?}{?}$.

9. In the diagram, $AD = 10$, B is the midpoint of $\overline{AD}$, and AC is the geometric mean of AB and AD. Find AC. *(p. 364)*



6.3 Similar Polygons

MATERIALS • metric ruler • protractor

QUESTION When a figure is reduced, how are the corresponding angles related? How are the corresponding lengths related?

EXPLORE Compare measures of lengths and angles in two photos

STEP 1 ***Measure segments*** Photo 2 is a reduction of Photo 1. In each photo, find AB to the nearest millimeter. Write the ratio of the length of $\overline{AB}$ in Photo 1 to the length of $\overline{AB}$ in Photo 2.

STEP 2 ***Measure angles*** Use a protractor to find the measure of $\angle 1$ in each photo. Write the ratio of $m\angle 1$ in Photo 1 to $m\angle 1$ in Photo 2.

STEP 3 ***Find measurements*** Copy and complete the table. Use the same units for each measurement. Record your results in a table.

Photo 1

Photo 2

Measurement	Photo 1	Photo 2	$\frac{\text{Photo 1}}{\text{Photo 2}}$
AB	?	?	?
AC	?	?	?
DE	?	?	?
$m\angle 1$	?	?	?
$m\angle 2$	?	?	?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Make a conjecture about the relationship between corresponding lengths when a figure is reduced.
2. Make a conjecture about the relationship between corresponding angles when a figure is reduced.
3. Suppose the measure of an angle in Photo 2 is 35°. What is the measure of the corresponding angle in Photo 1?
4. Suppose a segment in Photo 2 is 1 centimeters long. What is the measure of the corresponding segment in Photo 1?
5. Suppose a segment in Photo 1 is 5 centimeters long. What is the measure of the corresponding segment in Photo 2?

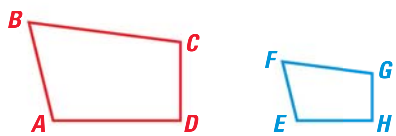
6.3 Use Similar Polygons

Before	You used proportions to solve geometry problems.
Now	You will use proportions to identify similar polygons.
Why?	So you can solve science problems, as in Ex. 34.

Key Vocabulary
- **similar polygons**
- **scale factor**

Two polygons are **similar polygons** if corresponding angles are congruent and corresponding side lengths are proportional.

In the diagram below, *ABCD* is similar to *EFGH*. You can write "*ABCD* is similar to *EFGH*" as $ABCD \sim EFGH$. Notice in the similarity statement that the corresponding vertices are listed in the same order.



$ABCD \sim EFGH$

Corresponding angles

$\angle A \cong \angle E$, $\angle B \cong \angle F$, $\angle C \cong \angle G$, and $\angle D \cong \angle H$

Ratios of corresponding sides

$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$

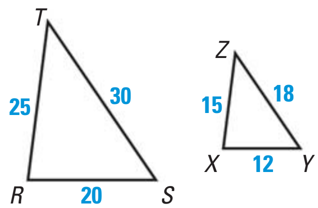
EXAMPLE 1 Use similarity statements

In the diagram, $\triangle RST \sim \triangle XYZ$.

a. List all pairs of congruent angles.

b. Check that the ratios of corresponding side lengths are equal.

c. Write the ratios of the corresponding side lengths in a *statement of proportionality*.



READ VOCABULARY
In a *statement of proportionality*, any pair of ratios forms a true proportion.

Solution

a. $\angle R \cong \angle X$, $\angle S \cong \angle Y$, and $\angle T \cong \angle Z$.

b. $\frac{RS}{XY} = \frac{20}{12} = \frac{5}{3}$ $\qquad \frac{ST}{YZ} = \frac{30}{18} = \frac{5}{3}$ $\qquad \frac{TR}{ZX} = \frac{25}{15} = \frac{5}{3}$

c. Because the ratios in part (b) are equal, $\frac{RS}{XY} = \frac{ST}{YZ} = \frac{TR}{ZX}$.

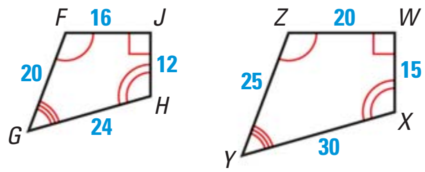
✓ GUIDED PRACTICE for Example 1

1. Given $\triangle JKL \sim \triangle PQR$, list all pairs of congruent angles. Write the ratios of the corresponding side lengths in a statement of proportionality.

SCALE FACTOR If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the **scale factor**. In Example 1, the common ratio of $\frac{5}{3}$ is the scale factor of $\triangle RST$ to $\triangle XYZ$.

EXAMPLE 2 Find the scale factor

Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor of *ZYXW* to *FGHJ*.



Solution

STEP 1 **Identify** pairs of congruent angles. From the diagram, you can see that $\angle Z \cong \angle F$, $\angle Y \cong \angle G$, and $\angle X \cong \angle H$. Angles W and J are right angles, so $\angle W \cong \angle J$. So, the corresponding angles are congruent.

STEP 2 **Show** that corresponding side lengths are proportional.

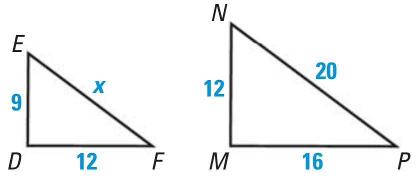
$$\frac{ZY}{FG} = \frac{25}{20} = \frac{5}{4} \qquad \frac{YX}{GH} = \frac{30}{24} = \frac{5}{4} \qquad \frac{XW}{HJ} = \frac{15}{12} = \frac{5}{4} \qquad \frac{WZ}{JF} = \frac{20}{16} = \frac{5}{4}$$

The ratios are equal, so the corresponding side lengths are proportional.

▶ So $ZYXW \sim FGHJ$. The scale factor of $ZYXW$ to $FGHJ$ is $\frac{5}{4}$.

EXAMPLE 3 Use similar polygons

xy ALGEBRA In the diagram, $\triangle DEF \sim \triangle MNP$. Find the value of x.



Solution

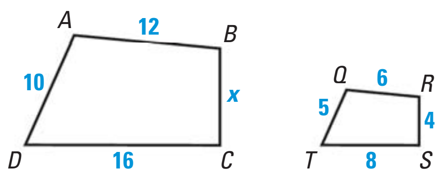
The triangles are similar, so the corresponding side lengths are proportional.

$\frac{MN}{DE} = \frac{NP}{EF}$	**Write proportion.**
$\frac{12}{9} = \frac{20}{x}$	**Substitute.**
$12x = 180$	**Cross Products Property**
$x = 15$	**Solve for *x*.**

ANOTHER WAY
There are several ways to write the proportion. For example, you could write $\frac{DF}{MP} = \frac{EF}{NP}$.

✓ GUIDED PRACTICE for Examples 2 and 3

In the diagram, *ABCD* ~ *QRST*.



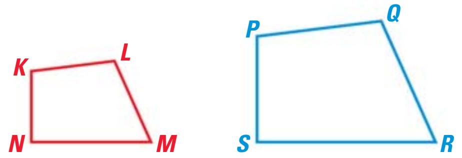
2. What is the scale factor of *QRST* to *ABCD*?
3. Find the value of x.

PERIMETERS The ratios of lengths in similar polygons is the same as the scale factor. Theorem 6.1 shows this is true for the perimeters of the polygons.

THEOREM — *For Your Notebook*

THEOREM 6.1 Perimeters of Similar Polygons

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.



If $KLMN \sim PQRS$, then $\frac{KL + LM + MN + NK}{PQ + QR + RS + SP} = \frac{KL}{PQ} = \frac{LM}{QR} = \frac{MN}{RS} = \frac{NK}{SP}$.

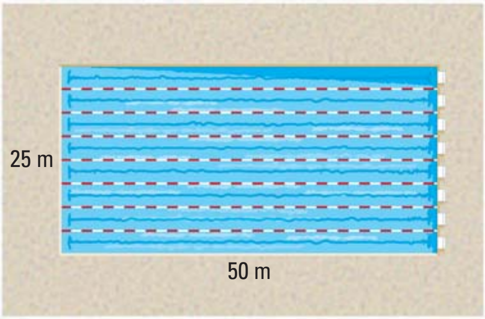
Proof: Ex. 38, p. 379

EXAMPLE 4 Find perimeters of similar figures

SWIMMING A town is building a new swimming pool. An Olympic pool is rectangular with length 50 meters and width 25 meters. The new pool will be similar in shape, but only 40 meters long.

a. Find the scale factor of the new pool to an Olympic pool.

b. Find the perimeter of an Olympic pool and the new pool.



Solution

ANOTHER WAY
Another way to solve Example 4 is to write the scale factor as the decimal 0.8. Then, multiply the perimeter of the Olympic pool by the scale factor to get the perimeter of the new pool:
$0.8(150) = 120$.

a. Because the new pool will be similar to an Olympic pool, the scale factor is the ratio of the lengths, $\frac{40}{50} = \frac{4}{5}$.

b. The perimeter of an Olympic pool is $2(50) + 2(25) = 150$ meters. You can use Theorem 6.1 to find the perimeter x of the new pool.

$\frac{x}{150} = \frac{4}{5}$ Use Theorem 6.1 to write a proportion.

$x = 120$ Multiply each side by 150 and simplify.

▶ The perimeter of the new pool is 120 meters.

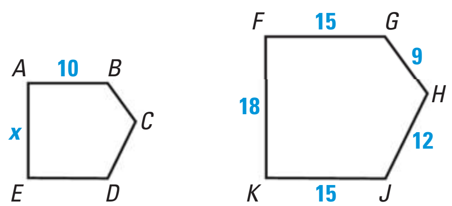
✓ GUIDED PRACTICE for Example 4

In the diagram, *ABCDE* ~ *FGHJK*.

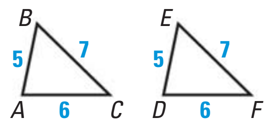
4. Find the scale factor of *FGHJK* to *ABCDE*.

5. Find the value of x.

6. Find the perimeter of *ABCDE*.



SIMILARITY AND CONGRUENCE Notice that any two congruent figures are also similar. Their scale factor is 1 : 1. In $\triangle ABC$ and $\triangle DEF$, the scale factor is $\frac{5}{5} = 1$. You can write $\triangle ABC \sim \triangle DEF$ and $\triangle ABC \cong \triangle DEF$.



READ VOCABULARY
For example, *corresponding lengths* in similar triangles include side lengths, altitudes, medians, midsegments, and so on.

CORRESPONDING LENGTHS You know that perimeters of similar polygons are in the same ratio as corresponding side lengths. You can extend this concept to other segments in polygons.

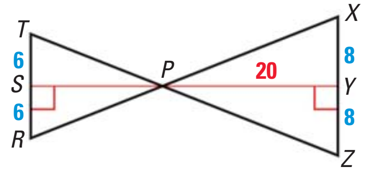
KEY CONCEPT *For Your Notebook*

Corresponding Lengths in Similar Polygons

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

EXAMPLE 5 Use a scale factor

In the diagram, $\triangle TPR \sim \triangle XPZ$. Find the length of the altitude $\overline{PS}$.



Solution

First, find the scale factor of $\triangle TPR$ to $\triangle XPZ$.

$$\frac{TR}{XZ} = \frac{6+6}{8+8} = \frac{12}{16} = \frac{3}{4}$$

Because the ratio of the lengths of the altitudes in similar triangles is equal to the scale factor, you can write the following proportion.

$\frac{PS}{PY} = \frac{3}{4}$ **Write proportion.**

$\frac{PS}{20} = \frac{3}{4}$ **Substitute 20 for *PY*.**

$PS = 15$ **Multiply each side by 20 and simplify.**

▶ The length of the altitude $\overline{PS}$ is 15.

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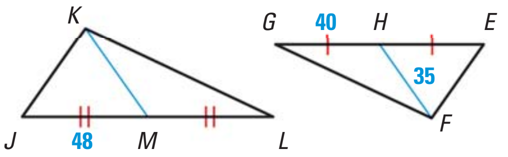
✓ **GUIDED PRACTICE** for Example 5

7. In the diagram, $\triangle JKL \sim \triangle EFG$. Find the length of the median $\overline{KM}$.



6.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 7, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 18, 27, 28, 35, 36, and 37

◆ = **MULTIPLE REPRESENTATIONS** Ex. 33

SKILL PRACTICE

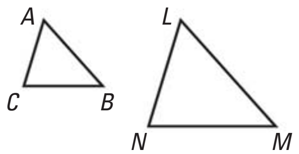
1. **VOCABULARY** Copy and complete: Two polygons are similar if corresponding angles are _?_ and corresponding side lengths are _?_.

2. ★ **WRITING** If two polygons are congruent, must they be similar? If two polygons are similar, must they be congruent? *Explain*.

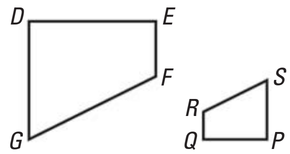
EXAMPLE 1 on p. 372 for Exs. 3–6

USING SIMILARITY **List all pairs of congruent angles for the figures. Then write the ratios of the corresponding sides in a statement of proportionality.**

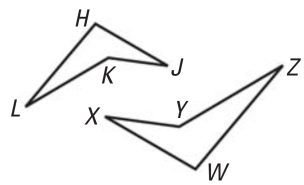
3. $\triangle ABC \sim \triangle LMN$



4. $DEFG \sim PQRS$



5. $HJKL \sim WXYZ$



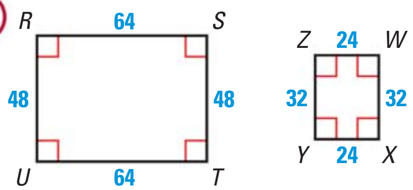
6. ★ **MULTIPLE CHOICE** Triangles *ABC* and *DEF* are similar. Which statement is *not* correct?

(A) $\frac{BC}{EF} = \frac{BC}{EF}$ (B) $\frac{AB}{DE} = \frac{CA}{FD}$ (C) $\frac{CA}{FD} = \frac{BC}{EF}$ (D) $\frac{AB}{EF} = \frac{BC}{DE}$

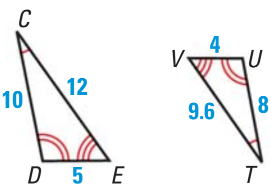
EXAMPLES 2 and 3 on p. 373 for Exs. 7–10

DETERMINING SIMILARITY **Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor.**

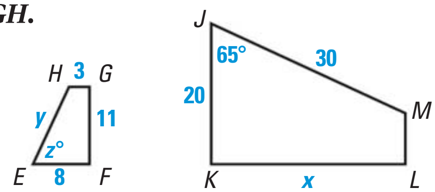
7.



8.



USING SIMILAR POLYGONS **In the diagram, *JKLM* ~ *EFGH*.**



9. Find the scale factor of *JKLM* to *EFGH*.

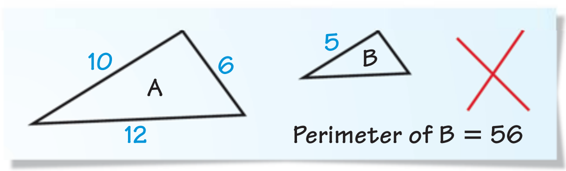
10. Find the values of *x*, *y*, and *z*.

EXAMPLE 4 on p. 374 for Exs. 11–13

11. Find the perimeter of each polygon.

12. **PERIMETER** Two similar FOR SALE signs have a scale factor of 5 : 3. The large sign's perimeter is 60 inches. Find the small sign's perimeter.

13. **ERROR ANALYSIS** The triangles are similar. *Describe* and correct the error in finding the perimeter of Triangle B.



REASONING **Are the polygons *always*, *sometimes*, or *never* similar?**

14. Two isosceles triangles

15. Two equilateral triangles

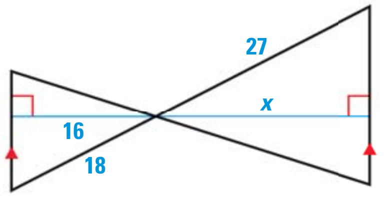
16. A right triangle and an isosceles triangle

17. A scalene triangle and an isosceles triangle

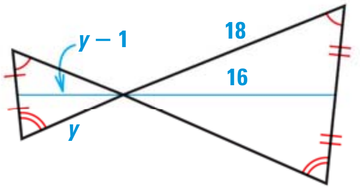
18. ★ **SHORT RESPONSE** The scale factor of Figure A to Figure B is $1:x$. What is the scale factor of Figure B to Figure A? *Explain* your reasoning.

SIMILAR TRIANGLES **Identify the type of special segment shown in blue, and find the value of the variable.**

19.



20.



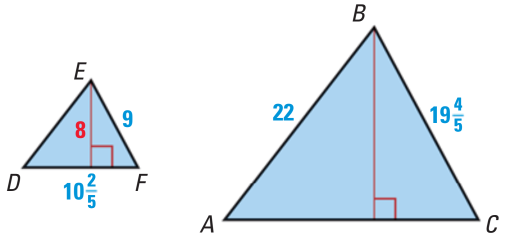
EXAMPLE 5 on p. 375 for Exs. 21–22

USING SCALE FACTOR **Triangles *NPQ* and *RST* are similar. The side lengths of $\triangle NPQ$ are 6 inches, 8 inches, and 10 inches, and the length of an altitude is 4.8 inches. The shortest side of $\triangle RST$ is 8 inches long.**

21. Find the lengths of the other two sides of $\triangle RST$.

22. Find the length of the corresponding altitude in $\triangle RST$.

USING SIMILAR TRIANGLES **In the diagram, $\triangle ABC \sim \triangle DEF$.**



23. Find the scale factor of $\triangle ABC$ to $\triangle DEF$.

24. Find the unknown side lengths in both triangles.

25. Find the length of the altitude shown in $\triangle ABC$.

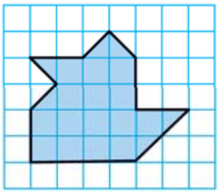
26. Find and compare the areas of both triangles.

27. ★ **SHORT RESPONSE** Suppose you are told that $\triangle PQR \sim \triangle XYZ$ and that the extended ratio of the angle measures in $\triangle PQR$ is $x:x+30:3x$. Do you need to know anything about $\triangle XYZ$ to be able to write its extended ratio of angle measures? *Explain* your reasoning.

28. ★ **MULTIPLE CHOICE** The lengths of the legs of right triangle *ABC* are 3 feet and 4 feet. The shortest side of $\triangle UVW$ is 4.5 feet and $\triangle UVW \sim \triangle ABC$. How long is the hypotenuse of $\triangle UVW$?

Ⓐ 1.5 ft Ⓑ 5 ft Ⓒ 6 ft Ⓓ 7.5 ft

29. **CHALLENGE** Copy the figure at the right and divide it into two similar figures.

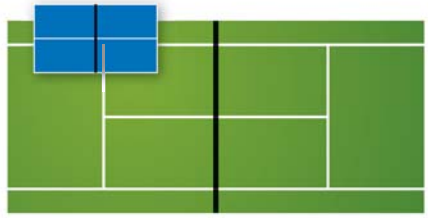


30. **REASONING** Is similarity reflexive? symmetric? transitive? Give examples to support your answers.

PROBLEM SOLVING

EXAMPLE 2
on p. 373 for
Exs. 31–32

31. **TENNIS** In table tennis, the table is a rectangle 9 feet long and 5 feet wide. A tennis court is a rectangle 78 feet long and 36 feet wide. Are the two surfaces similar? *Explain*. If so, find the scale factor of the tennis court to the table.



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32. **DIGITAL PROJECTOR** You are preparing a computer presentation to be digitally projected onto the wall of your classroom. Your computer screen is 13.25 inches wide and 10.6 inches high. The projected image on the wall is 53 inches wide and 42.4 inches high. Are the two shapes similar? If so, find the scale factor of the computer screen to the projected image.

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33. **MULTIPLE REPRESENTATIONS** Use the similar figures shown. The scale factor of Figure 1 to Figure 2 is 7 : 10.

a. Making a Table Copy and complete the table.

	AB	*BC*	*CD*	*DE*	*EA*
Figure 1	3.5	?	?	?	?
Figure 2	5.0	4.0	6.0	8.0	3.0

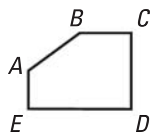


Figure 1

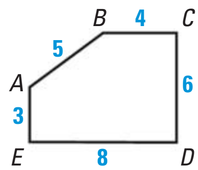
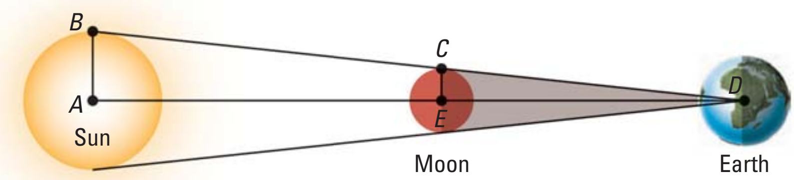


Figure 2

b. Drawing a Graph Graph the data in the table. Let x represent the length of a side in Figure 1 and let y represent the length of the corresponding side in Figure 2. Is the relationship linear?

c. Writing an Equation Write an equation that relates x and y. What is its slope? How is the slope related to the scale factor?

34. **MULTI-STEP PROBLEM** During a total eclipse of the sun, the moon is directly in line with the sun and blocks the sun's rays. The distance ED between Earth and the moon is 240,000 miles, the distance DA between Earth and the sun is 93,000,000 miles, and the radius AB of the sun is 432,500 miles.



Not drawn to scale

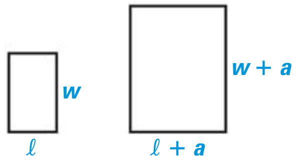
a. Copy the diagram and label the known distances.

b. In the diagram, $\triangle BDA \sim \triangle CDE$. Use this fact to explain a total eclipse of the sun.

c. Estimate the radius CE of the moon.

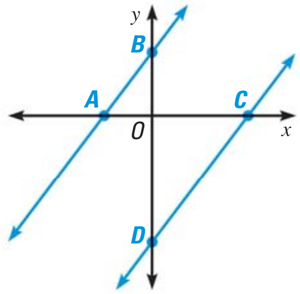
○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE ◆ = MULTIPLE REPRESENTATIONS

35. ★ **SHORT RESPONSE** A rectangular image is enlarged on each side by the same amount. The angles remain unchanged. Can the larger image be similar to the original? *Explain* your reasoning, and give an example to support your answer.



36. ★ **SHORT RESPONSE** How are the areas of similar rectangles related to the scale factor? Use examples to *justify* your reasoning.

37. ★ **EXTENDED RESPONSE** The equations of two lines in the coordinate plane are $y = \frac{4}{3}x + 4$ and $y = \frac{4}{3}x - 8$.



a. *Explain* why the two lines are parallel.

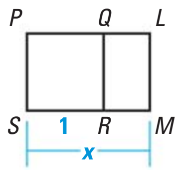
b. Show that $\angle BOA \cong \angle DOC$, $\angle OBA \cong \angle ODC$, and $\angle BAO \cong \angle DCO$.

c. Find the coordinates of points A, B, C, and D. Find the lengths of the sides of $\triangle AOB$ and $\triangle COD$.

d. Show that $\triangle AOB \sim \triangle COD$.

38. **PROVING THEOREM 6.1** Prove the Perimeters of Similar Polygons Theorem for similar rectangles. Include a diagram in your proof.

39. **CHALLENGE** In the diagram, $PQRS$ is a square, and $PLMS \sim LMRQ$. Find the exact value of x. This value is called the *golden ratio*. Golden rectangles have their length and width in this ratio. Show that the similar rectangles in the diagram are golden rectangles.



MIXED REVIEW

PREVIEW
Prepare for Lesson 6.4 in Exs. 40–42.

Given $A(1, 1)$, $B(3, 2)$, $C(2, 4)$, and $D\left(1, \frac{7}{2}\right)$, determine whether the following lines are *parallel*, *perpendicular*, or *neither*. *(p. 171)*

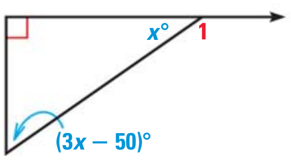
40. $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$

41. $\overleftrightarrow{CD}$ and $\overleftrightarrow{AD}$

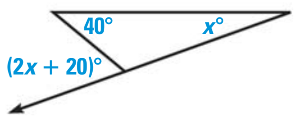
42. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$

Find the measure of the exterior angle shown. *(p. 217)*

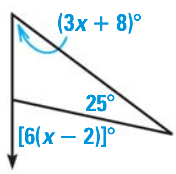
43.



44.

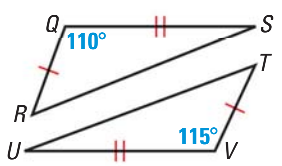


45.

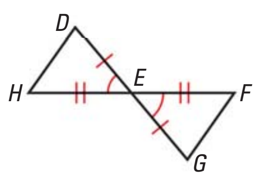


Copy and complete the statement with <, >, or =. *(p. 335)*

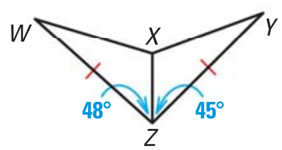
46. RS __?__ TU



47. FG __?__ HD



48. WX __?__ YX



MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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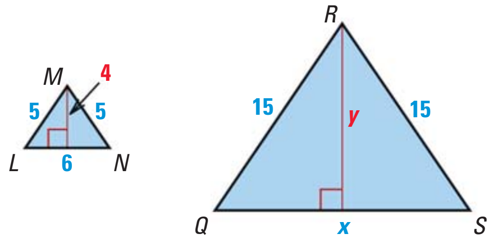
Lessons 6.1–6.3

1. **MULTI-STEP PROBLEM** In the diagram, $\triangle LMN \sim \triangle QRS$.



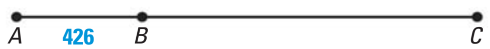
   a. Find the scale factor of $\triangle LMN$ to $\triangle QRS$. Then find the values of x and y.

   b. Find the perimeters of $\triangle LMN$ and $\triangle QRS$.

   c. Find the areas of $\triangle LMN$ and $\triangle QRS$.

   d. *Compare* the ratio of the perimeters to the ratio of the areas of $\triangle LMN$ to $\triangle QRS$. What do you notice?

2. **GRIDDED ANSWER** In the diagram, $AB : BC$ is $3 : 8$. Find AC.



3. **OPEN-ENDED** $\triangle UVW$ is a right triangle with side lengths of 3 cm, 4 cm, and 5 cm. Draw and label $\triangle UVW$. Then draw a triangle similar to $\triangle UVW$ and label its side lengths. What scale factor did you use?

4. **MULTI-STEP PROBLEM** Kelly is going on a trip to England. She takes 600 U.S. dollars with her.

One U.S. Dollar Buys	
EURO	.81
GREAT BRITAIN	.54
CANADA	1.24

   a. In England, she exchanges her U.S. dollars for British pounds. During her stay, Kelly spends 150 pounds. How many British pounds does she have left?

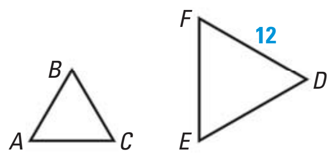
   b. When she returns home, she exchanges her money back to U.S. dollars. How many U.S. dollars does she have at the end of her trip?

5. **SHORT RESPONSE** Kelly bought a 3-D scale model of the Tower Bridge in London, England. The towers of the model are 9 inches tall. The towers of the actual bridge are 206 feet tall, and there are two walkways that are 140 feet high.

   a. Approximate the height of the walkways on the model.

   b. About how many times as tall as the model is the actual structure?

6. **GRIDDED ANSWER** In the diagram, $\triangle ABC \sim \triangle DEF$. The scale factor of $\triangle ABC$ to $\triangle DEF$ is $3 : 5$. Find AC.



7. **EXTENDED RESPONSE** In the United States, 4634 million pounds of apples were consumed in 2002. The population of the United States in that year was 290 million.

   a. Divide the total number of apples consumed by the population to find the per capita consumption.

   b. About how many pounds of apples would a family of four have consumed in one year? in one month?

   c. A medium apple weighs about 5 ounces. Estimate how many apples a family of four would have consumed in one month.

   d. Is it reasonable to assume that a family of four would have eaten that many apples? What other factors could affect the per capita consumption? *Explain*.

6.4 Prove Triangles Similar by AA

Before You used the AAS Congruence Theorem.

Now You will use the AA Similarity Postulate.

Why? So you can use similar triangles to understand aerial photography, as in Ex. 34.

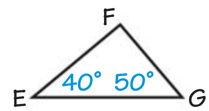
Key Vocabulary
- **similar polygons,** *p. 372*

ACTIVITY ANGLES AND SIMILAR TRIANGLES

QUESTION What can you conclude about two triangles if you know two pairs of corresponding angles are congruent?

Materials:
- protractor
- metric ruler

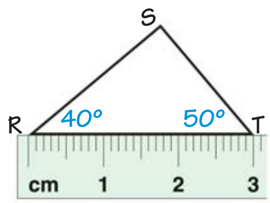
STEP 1 **Draw** $\triangle EFG$ so that $m\angle E = 40°$ and $m\angle G = 50°$.



STEP 2 **Draw** $\triangle RST$ so that $m\angle R = 40°$ and $m\angle T = 50°$, and $\triangle RST$ is not congruent to $\triangle EFG$.

STEP 3 **Calculate** $m\angle F$ and $m\angle S$ using the Triangle Sum Theorem. Use a protractor to check that your results are true.

STEP 4 **Measure** and record the side lengths of both triangles. Use a metric ruler.



DRAW CONCLUSIONS

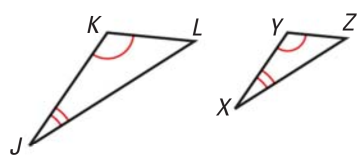
1. Are the triangles similar? Explain your reasoning.
2. Repeat the steps above using different angle measures. Make a conjecture about two triangles with two pairs of congruent corresponding angles.

TRIANGLE SIMILARITY The Activity suggests that two triangles are similar if two pairs of corresponding angles are congruent. In other words, you do not need to know the measures of the sides or the third pair of angles.

POSTULATE *For Your Notebook*

POSTULATE 22 Angle-Angle (AA) Similarity Postulate

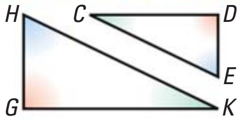
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.



$\triangle JKL \sim \triangle XYZ$

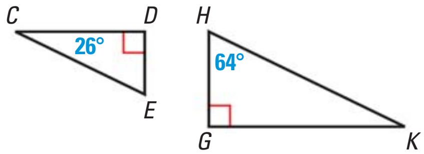
EXAMPLE 1 Use the AA Similarity Postulate

DRAW DIAGRAMS



Use colored pencils to show congruent angles. This will help you write similarity statements.

Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.



Solution

Because they are both right angles, $\angle D$ and $\angle G$ are congruent.

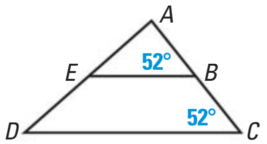
By the Triangle Sum Theorem, $26° + 90° + m\angle E = 180°$, so $m\angle E = 64°$. Therefore, $\angle E$ and $\angle H$ are congruent.

▶ So, $\triangle CDE \sim \triangle KGH$ by the AA Similarity Postulate.

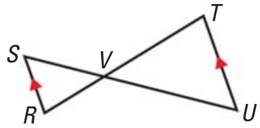
EXAMPLE 2 Show that triangles are similar

Show that the two triangles are similar.

a. $\triangle ABE$ and $\triangle ACD$



b. $\triangle SVR$ and $\triangle UVT$



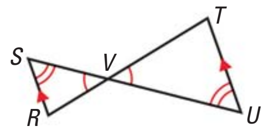
Solution

a. You may find it helpful to redraw the triangles separately.

Because $m\angle ABE$ and $m\angle C$ both equal 52°, $\angle ABE \cong \angle C$. By the Reflexive Property, $\angle A \cong \angle A$.

▶ So, $\triangle ABE \sim \triangle ACD$ by the AA Similarity Postulate.

b. You know $\angle SVR \cong \angle UVT$ by the Vertical Angles Congruence Theorem. The diagram shows $\overline{RS} \parallel \overline{UT}$ so $\angle S \cong \angle U$ by the Alternate Interior Angles Theorem.

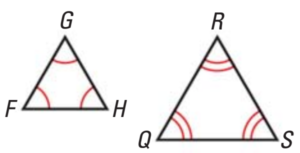


▶ So, $\triangle SVR \sim \triangle UVT$ by the AA Similarity Postulate.

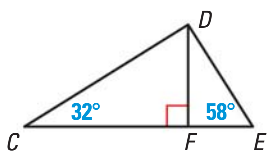
✓ GUIDED PRACTICE for Examples 1 and 2

Show that the triangles are similar. Write a similarity statement.

1. $\triangle FGH$ and $\triangle RQS$



2. $\triangle CDF$ and $\triangle DEF$



3. REASONING Suppose in Example 2, part (b), $\overline{SR} \nparallel \overline{TU}$. Could the triangles still be similar? *Explain.*

INDIRECT MEASUREMENT In Lesson 4.6, you learned a way to use congruent triangles to find measurements indirectly. Another useful way to find measurements indirectly is by using similar triangles.

EXAMPLE 3 Standardized Test Practice

A flagpole casts a shadow that is 50 feet long. At the same time, a woman standing nearby who is five feet four inches tall casts a shadow that is 40 inches long. How tall is the flagpole to the nearest foot?

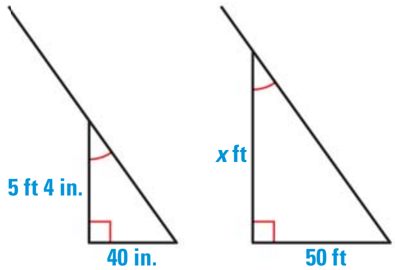
Ⓐ 12 feet   Ⓑ 40 feet

Ⓒ 80 feet   Ⓓ 140 feet

ELIMINATE CHOICES
Notice that the woman's height is greater than her shadow's length. So the flagpole must be taller than its shadow's length. Eliminate choices A and B.

Solution

The flagpole and the woman form sides of two right triangles with the ground, as shown below. The sun's rays hit the flagpole and the woman at the same angle. You have two pairs of congruent angles, so the triangles are similar by the AA Similarity Postulate.



You can use a proportion to find the height x. Write 5 feet 4 inches as 64 inches so that you can form two ratios of feet to inches.

$\frac{x \text{ ft}}{64 \text{ in.}} = \frac{50 \text{ ft}}{40 \text{ in.}}$ **Write proportion of side lengths.**

$40x = 64(50)$ **Cross Products Property**

$x = 80$ **Solve for *x*.**

▶ The flagpole is 80 feet tall. The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Example 3

4. **WHAT IF?** A child who is 58 inches tall is standing next to the woman in Example 3. How long is the child's shadow?

5. You are standing in your backyard, and you measure the lengths of the shadows cast by both you and a tree. Write a proportion showing how you could find the height of the tree.

6.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 13, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 18, 19, 20, 33, and 38

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are _?_.

2. ★ **WRITING** Can you assume that corresponding sides and corresponding angles of any two similar triangles are congruent? *Explain.*

EXAMPLE 1 on p. 382 for Exs. 3–11

REASONING **Use the diagram to complete the statement.**

3. $\triangle ABC \sim$ _?_

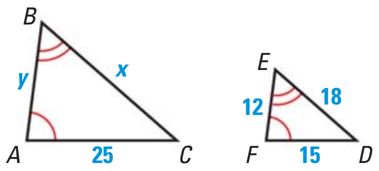
4. $\frac{BA}{?} = \frac{AC}{?} = \frac{CB}{?}$

5. $\frac{25}{?} = \frac{?}{12}$

6. $\frac{?}{25} = \frac{18}{?}$

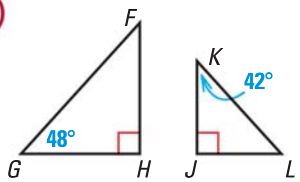
7. $y =$ _?_

8. $x =$ _?_

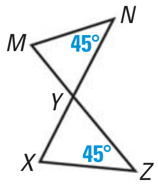


AA SIMILARITY POSTULATE **In Exercises 9–14, determine whether the triangles are similar. If they are, write a similarity statement.**

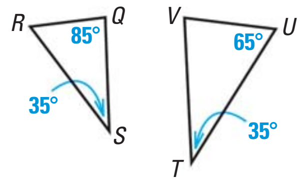
9.



10.

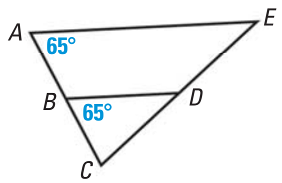


11.

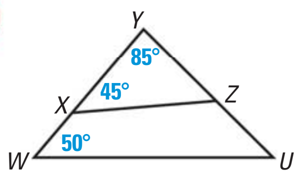


EXAMPLE 2 on p. 382 for Exs. 12–16

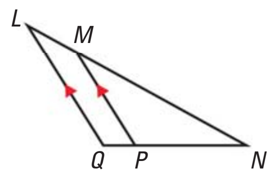
12.



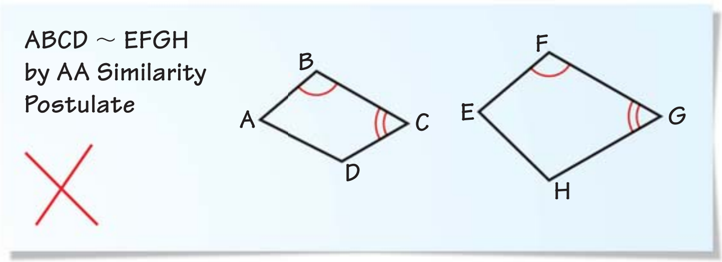
13.



14.



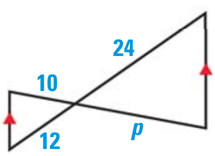
15. **ERROR ANALYSIS** *Explain* why the student's similarity statement is incorrect.



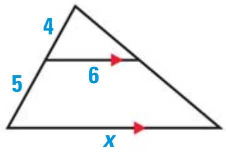
16. ★ **MULTIPLE CHOICE** What is the value of p?

Ⓐ 5　　Ⓑ 20

Ⓒ 28.8　　Ⓓ Cannot be determined



17. **ERROR ANALYSIS** A student uses the proportion $\frac{4}{6} = \frac{5}{x}$ to find the value of x in the figure. *Explain* why this proportion is incorrect and write a correct proportion.



★ **OPEN-ENDED MATH** **In Exercises 18 and 19, make a sketch that can be used to show that the statement is false.**

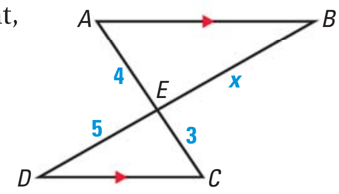
18. If two pairs of sides of two triangles are congruent, then the triangles are similar.

19. If the ratios of two pairs of sides of two triangles are proportional, then the triangles are similar.

20. ★ **MULTIPLE CHOICE** In the figure at the right, find the length of $\overline{BD}$.

Ⓐ $\frac{35}{3}$ Ⓑ $\frac{37}{5}$

Ⓒ $\frac{20}{3}$ Ⓓ $\frac{12}{5}$



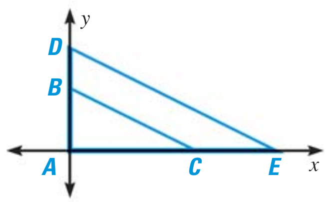
ALGEBRA **Find coordinates for point E so that $\triangle ABC \sim \triangle ADE$.**

21. $A(0, 0)$, $B(0, 4)$, $C(8, 0)$, $D(0, 5)$, $E(x, y)$

22. $A(0, 0)$, $B(0, 3)$, $C(4, 0)$, $D(0, 7)$, $E(x, y)$

23. $A(0, 0)$, $B(0, 1)$, $C(6, 0)$, $D(0, 4)$, $E(x, y)$

24. $A(0, 0)$, $B(0, 6)$, $C(3, 0)$, $D(0, 9)$, $E(x, y)$



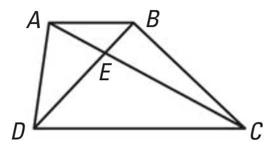
25. **MULTI-STEP PROBLEM** In the diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$, $AE = 6$, $AB = 8$, $CE = 15$, and $DE = 10$.

 a. Copy the diagram and mark all given information.

 b. List two pairs of congruent angles in the diagram.

 c. Name a pair of similar triangles and write a similarity statement.

 d. Find BE and DC.



REASONING **In Exercises 26–29, is it possible for $\triangle JKL$ and $\triangle XYZ$ to be similar? *Explain* why or why not.**

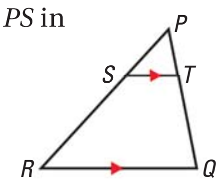
26. $m\angle J = 71°$, $m\angle K = 52°$, $m\angle X = 71°$, and $m\angle Z = 57°$

27. $\triangle JKL$ is a right triangle and $m\angle X + m\angle Y = 150°$.

28. $m\angle J = 87°$ and $m\angle Y = 94°$

29. $m\angle J + m\angle K = 85°$ and $m\angle Y + m\angle Z = 80°$

30. **CHALLENGE** If $PT = x$, $PQ = 3x$, and $SR = \frac{8}{3}x$, find PS in terms of x. *Explain* your reasoning.



PROBLEM SOLVING

EXAMPLE 3 on p. 383 for Exs. 31–32

31. AIR HOCKEY An air hockey player returns the puck to his opponent by bouncing the puck off the wall of the table as shown. From physics, the angles that the path of the puck makes with the wall are congruent. What is the distance d between the puck and the wall when the opponent returns it?

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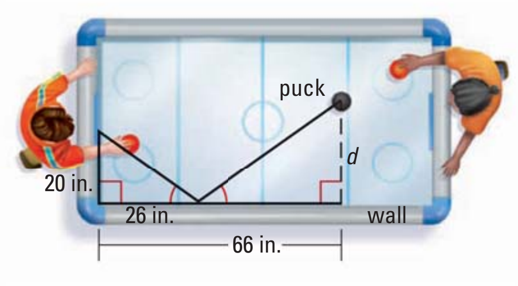
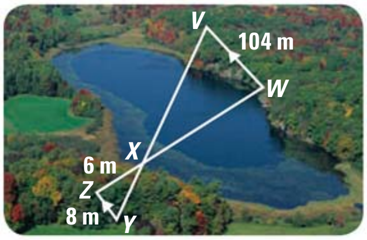


32. LAKES You can measure the width of the lake using a surveying technique, as shown in the diagram.



a. What postulate or theorem can you use to show that the triangles are similar?

b. Find the width of the lake, WX.

c. If $XY = 10$ meters, find VX.

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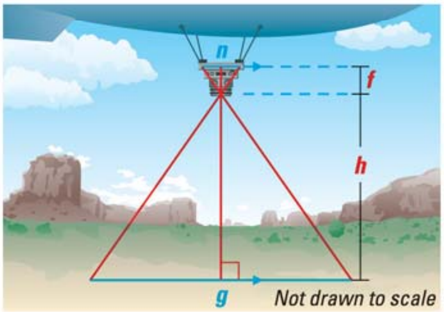
33. ★ **SHORT RESPONSE** *Explain* why all equilateral triangles are similar. Include sketches in your answer.

34. AERIAL PHOTOGRAPHY Low-level aerial photos can be taken using a remote-controlled camera suspended from a blimp. You want to take an aerial photo that covers a ground distance g of 50 meters. Use the proportion $\frac{f}{h} = \frac{n}{g}$ to estimate the altitude h that the blimp should fly at to take the photo. In the proportion, use $f = 8$ centimeters and $n = 3$ centimeters. These two variables are determined by the type of camera used.



35. PROOF Use the given information to draw a sketch. Then write a proof.

GIVEN ▶ $\triangle STU \sim \triangle PQR$
Point V lies on $\overline{TU}$ so that $\overline{SV}$ bisects $\angle TSU$.
Point N lies on $\overline{QR}$ so that $\overline{PN}$ bisects $\angle QPR$.

PROVE ▶ $\frac{SV}{PN} = \frac{ST}{PQ}$

36. PROOF Prove that if an acute angle in one right triangle is congruent to an acute angle in another right triangle, then the triangles are similar.

○ = WORKED-OUT SOLUTIONS on p. WS1   ★ = STANDARDIZED TEST PRACTICE

37. **TECHNOLOGY** Use a graphing calculator or computer.

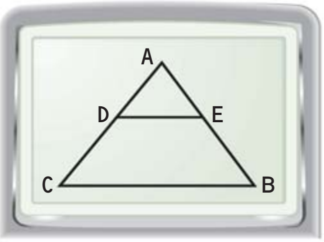
a. Draw $\triangle ABC$. Draw $\overline{DE}$ through two sides of the triangle, parallel to the third side.

b. Measure $\angle ADE$ and $\angle ACB$. Measure $\angle AED$ and $\angle ABC$. What do you notice?

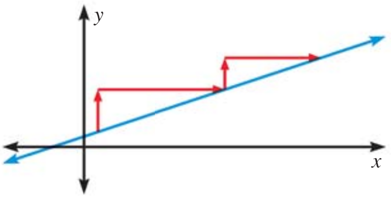
c. What does a postulate in this lesson tell you about $\triangle ADE$ and $\triangle ACB$?

d. Measure all the sides. Show that corresponding side lengths are proportional.

e. Move vertex A to form new triangles. How do your measurements in parts (b) and (d) change? Are the new triangles still similar? *Explain*.



38. ★ **EXTENDED RESPONSE** *Explain* how you could use similar triangles to show that any two points on a line can be used to calculate its slope.



39. **CORRESPONDING LENGTHS** Without using the Corresponding Lengths Property on page 375, prove that the ratio of two corresponding angle bisectors in similar triangles is equal to the scale factor.

40. **CHALLENGE** Prove that if the lengths of two sides of a triangle are a and b respectively, then the lengths of the corresponding altitudes to those sides are in the ratio $\frac{b}{a}$.

MIXED REVIEW

PREVIEW
Prepare for Lesson 6.5 in Exs. 41–44.

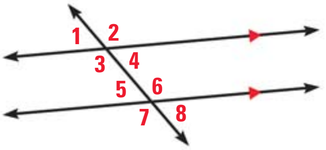
In Exercises 41–44, use the diagram.

41. Name three pairs of corresponding angles. *(p. 147)*

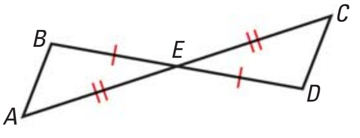
42. Name two pairs of alternate interior angles. *(p. 147)*

43. Name two pairs of alternate exterior angles. *(p. 147)*

44. Find $m\angle 1 + m\angle 7$. *(p. 154)*



45. **CONGRUENCE** Explain why $\triangle ABE \cong \triangle CDE$. *(p. 240)*



Simplify the ratio. *(p. 356)*

46. $\frac{4}{20}$

47. $\frac{36}{18}$

48. 21 : 63

49. 42 : 28

6.5 Prove Triangles Similar by SSS and SAS

Before You used the AA Similarity Postulate to prove triangles similar.

Now You will use the SSS and SAS Similarity Theorems.

Why? So you can show that triangles are similar, as in Ex. 28.

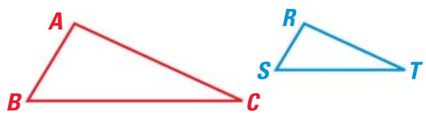
Key Vocabulary
- **ratio,** *p. 356*
- **proportion,** *p. 358*
- **similar polygons,** *p. 372*

In addition to using congruent corresponding angles to show that two triangles are similar, you can use proportional corresponding side lengths.

THEOREM *For Your Notebook*

THEOREM 6.2 Side-Side-Side (SSS) Similarity Theorem

If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

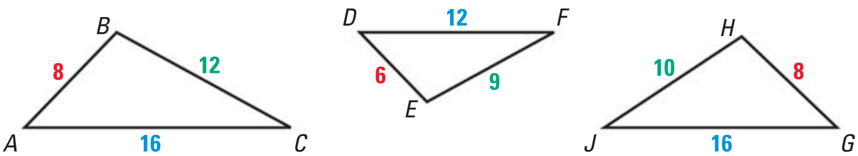


If $\frac{AB}{RS} = \frac{BC}{ST} = \frac{CA}{TR}$, then $\triangle ABC \sim \triangle RST$.

Proof: p. 389

EXAMPLE 1 Use the SSS Similarity Theorem

Is either $\triangle DEF$ or $\triangle GHJ$ similar to $\triangle ABC$?



Solution

APPLY THEOREMS
When using the SSS Similarity Theorem, compare the shortest sides, the longest sides, and then the remaining sides.

Compare $\triangle ABC$ and $\triangle DEF$ by finding ratios of corresponding side lengths.

Shortest sides $\frac{AB}{DE} = \frac{8}{6} = \frac{4}{3}$

Longest sides $\frac{CA}{FD} = \frac{16}{12} = \frac{4}{3}$

Remaining sides $\frac{BC}{EF} = \frac{12}{9} = \frac{4}{3}$

▶ All of the ratios are equal, so $\triangle ABC \sim \triangle DEF$.

Compare $\triangle ABC$ and $\triangle GHJ$ by finding ratios of corresponding side lengths.

Shortest sides $\frac{AB}{GH} = \frac{8}{8} = 1$

Longest sides $\frac{CA}{JG} = \frac{16}{16} = 1$

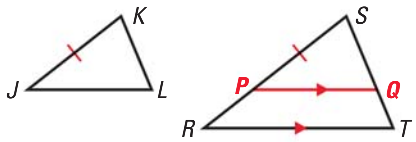
Remaining sides $\frac{BC}{HJ} = \frac{12}{10} = \frac{6}{5}$

▶ The ratios are not all equal, so $\triangle ABC$ and $\triangle GHJ$ are not similar.

PROOF SSS Similarity Theorem

GIVEN ▶ $\frac{RS}{JK} = \frac{ST}{KL} = \frac{TR}{LJ}$

PROVE ▶ $\triangle RST \sim \triangle JKL$



USE AN AUXILIARY LINE

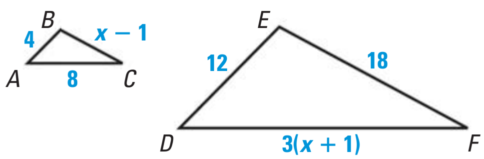
The Parallel Postulate allows you to draw an auxiliary line $\overleftrightarrow{PQ}$ in $\triangle RST$. There is only one line through point P parallel to $\overleftrightarrow{RT}$, so you are able to draw it.

Locate P on $\overline{RS}$ so that $PS = JK$. Draw $\overline{PQ}$ so that $\overline{PQ} \parallel \overline{RT}$. Then $\triangle RST \sim \triangle PSQ$ by the AA Similarity Postulate, and $\frac{RS}{PS} = \frac{ST}{SQ} = \frac{TR}{QP}$.

You can use the given proportion and the fact that $PS = JK$ to deduce that $SQ = KL$ and $QP = LJ$. By the SSS Congruence Postulate, it follows that $\triangle PSQ \cong \triangle JKL$. Finally, use the definition of congruent triangles and the AA Similarity Postulate to conclude that $\triangle RST \sim \triangle JKL$.

EXAMPLE 2 Use the SSS Similarity Theorem

xy ALGEBRA **Find the value of x that makes $\triangle ABC \sim \triangle DEF$.**



Solution

STEP 1 **Find** the value of x that makes corresponding side lengths proportional.

CHOOSE A METHOD

You can use either $\frac{AB}{DE} = \frac{BC}{EF}$ or $\frac{AB}{DE} = \frac{AC}{DF}$ in Step 1.

$\frac{4}{12} = \frac{x-1}{18}$	**Write proportion.**
$4 \cdot 18 = 12(x - 1)$	**Cross Products Property**
$72 = 12x - 12$	**Simplify.**
$7 = x$	**Solve for x.**

STEP 2 **Check** that the side lengths are proportional when $x = 7$.

$BC = x - 1 = 6$ $\qquad$ $DF = 3(x + 1) = 24$

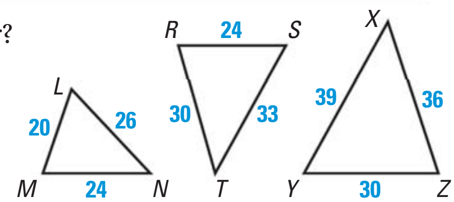
$\frac{AB}{DE} \stackrel{?}{=} \frac{BC}{EF} \Rightarrow \frac{4}{12} = \frac{6}{18}$ ✓ $\qquad$ $\frac{AB}{DE} \stackrel{?}{=} \frac{AC}{DF} \Rightarrow \frac{4}{12} = \frac{8}{24}$ ✓

▶ When $x = 7$, the triangles are similar by the SSS Similarity Theorem.

✓ GUIDED PRACTICE for Examples 1 and 2

1. Which of the three triangles are similar? Write a similarity statement.

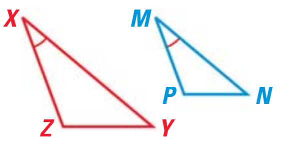
2. The shortest side of a triangle similar to $\triangle RST$ is 12 units long. Find the other side lengths of the triangle.



THEOREM — For Your Notebook

THEOREM 6.3 Side-Angle-Side (SAS) Similarity Theorem

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

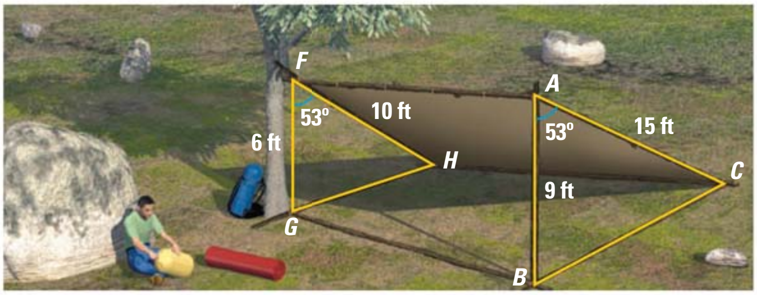


If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

Proof: Ex. 37, p. 395

EXAMPLE 3 Use the SAS Similarity Theorem

LEAN-TO SHELTER You are building a lean-to shelter starting from a tree branch, as shown. Can you construct the right end so it is similar to the left end using the angle measure and lengths shown?



Solution

Both $m\angle A$ and $m\angle F$ equal 53°, so $\angle A \cong \angle F$. Next, compare the ratios of the lengths of the sides that include $\angle A$ and $\angle F$.

Shorter sides $\frac{AB}{FG} = \frac{9}{6} = \frac{3}{2}$ **Longer sides** $\frac{AC}{FH} = \frac{15}{10} = \frac{3}{2}$

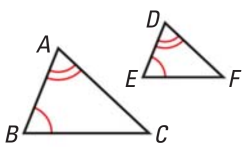
The lengths of the sides that include $\angle A$ and $\angle F$ are proportional.

▸ So, by the SAS Similarity Theorem, $\triangle ABC \sim \triangle FGH$. Yes, you can make the right end similar to the left end of the shelter.

CONCEPT SUMMARY — For Your Notebook

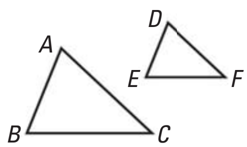
Triangle Similarity Postulate and Theorems

AA Similarity Postulate



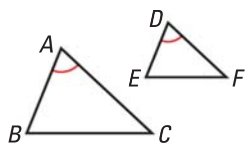
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem



If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

SAS Similarity Theorem

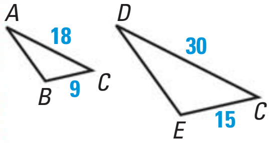


If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

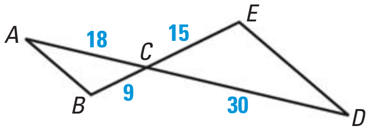
EXAMPLE 4 Choose a method

VISUAL REASONING

To identify corresponding parts, redraw the triangles so that the corresponding parts have the same orientation.



Tell what method you would use to show that the triangles are similar.



Solution

Find the ratios of the lengths of the corresponding sides.

Shorter sides $\frac{BC}{EC} = \frac{9}{15} = \frac{3}{5}$ **Longer sides** $\frac{CA}{CD} = \frac{18}{30} = \frac{3}{5}$

The corresponding side lengths are proportional. The included angles $\angle ACB$ and $\angle DCE$ are congruent because they are vertical angles. So, $\triangle ACB \sim \triangle DCE$ by the SAS Similarity Theorem.

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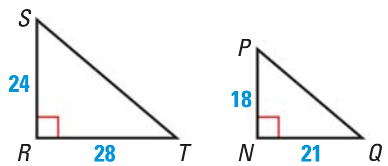
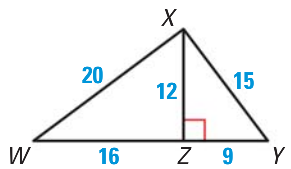
✓ GUIDED PRACTICE for Examples 3 and 4

Explain **how to show that the indicated triangles are similar.**

3. $\triangle SRT \sim \triangle PNQ$



4. $\triangle XZW \sim \triangle YZX$



6.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 7, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 14, 32, 34, and 36

SKILL PRACTICE

1. VOCABULARY You plan to prove that $\triangle ACB$ is similar to $\triangle PXQ$ by the SSS Similarity Theorem. Copy and complete the proportion that is needed to use this theorem: $\frac{AC}{?} = \frac{?}{XQ} = \frac{AB}{?}$.

2. ★ WRITING If you know two triangles are similar by the SAS Similarity Theorem, what additional piece(s) of information would you need to know to show that the triangles are congruent?

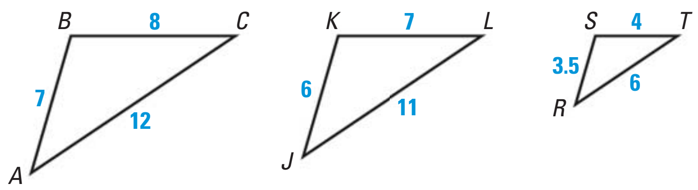
EXAMPLES 1 and 2
on pp. 388–389 for Exs. 3–6

SSS SIMILARITY THEOREM **Verify that $\triangle ABC \sim \triangle DEF$. Find the scale factor of $\triangle ABC$ to $\triangle DEF$.**

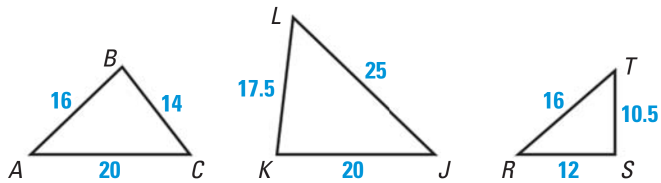
3. $\triangle ABC$: $BC = 18$, $AB = 15$, $AC = 12$
$\triangle DEF$: $EF = 12$, $DE = 10$, $DF = 8$

4. $\triangle ABC$: $AB = 10$, $BC = 16$, $CA = 20$
$\triangle DEF$: $DE = 25$, $EF = 40$, $FD = 50$

5. **SSS SIMILARITY THEOREM** Is either $\triangle JKL$ or $\triangle RST$ similar to $\triangle ABC$?



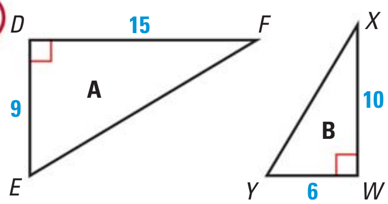
6. **SSS SIMILARITY THEOREM** Is either $\triangle JKL$ or $\triangle RST$ similar to $\triangle ABC$?



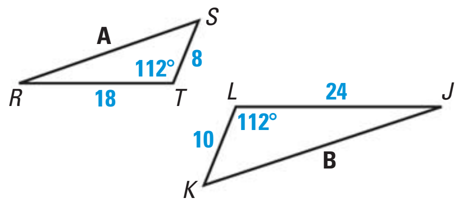
EXAMPLE 3
on p. 390
for Exs. 7–9

SAS SIMILARITY THEOREM **Determine whether the two triangles are similar. If they are similar, write a similarity statement and find the scale factor of Triangle B to Triangle A.**

7.



8.

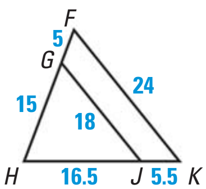


9. **ALGEBRA** Find the value of n that makes $\triangle PQR \sim \triangle XYZ$ when $PQ = 4$, $QR = 5$, $XY = 4(n + 1)$, $YZ = 7n - 1$, and $\angle Q \cong \angle Y$. Include a sketch.

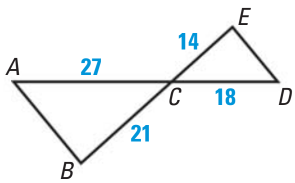
EXAMPLE 4
on p. 391
for Exs. 10–12

SHOWING SIMILARITY **Show that the triangles are similar and write a similarity statement. *Explain* your reasoning.**

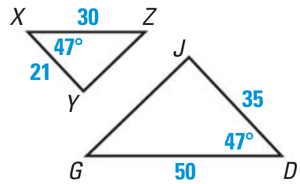
10.



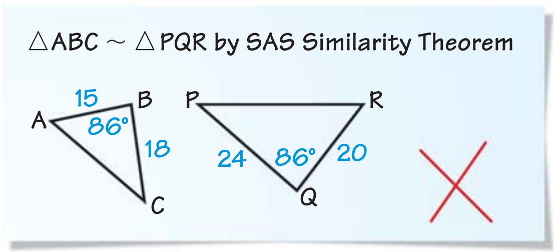
11.



12.



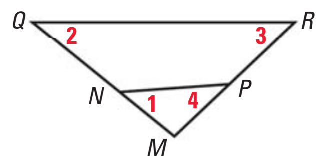
13. **ERROR ANALYSIS** *Describe* and correct the student's error in writing the similarity statement.



14. ★ **MULTIPLE CHOICE** In the diagram, $\frac{MN}{MR} = \frac{MP}{MQ}$. Which of the statements must be true?

Ⓐ $\angle 1 \cong \angle 2$ Ⓑ $\overline{QR} \parallel \overline{NP}$

Ⓒ $\angle 1 \cong \angle 4$ Ⓓ $\triangle MNP \sim \triangle MRQ$



○ = **WORKED-OUT SOLUTIONS** on p. WS1 ★ = **STANDARDIZED TEST PRACTICE**

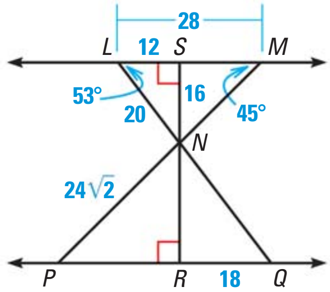
DRAWING TRIANGLES **Sketch the triangles using the given description. *Explain* whether the two triangles can be similar.**

15. In $\triangle XYZ$, $m\angle X = 66°$ and $m\angle Y = 34°$. In $\triangle LMN$, $m\angle M = 34°$ and $m\angle N = 80°$.

16. In $\triangle RST$, $RS = 20$, $ST = 32$, and $m\angle S = 16°$. In $\triangle FGH$, $GH = 30$, $HF = 48$, and $m\angle H = 24°$.

17. The side lengths of $\triangle ABC$ are 24, $8x$, and 54, and the side lengths of $\triangle DEF$ are 15, 25, and $7x$.

FINDING MEASURES **In Exercises 18–23, use the diagram to copy and complete the statements.**



18. $m\angle NQP =$ __?__

19. $m\angle QPN =$ __?__

20. $m\angle PNQ =$ __?__

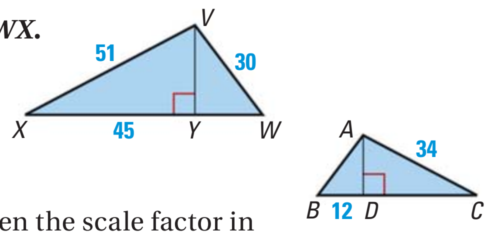
21. $RN =$ __?__

22. $PQ =$ __?__

23. $NM =$ __?__

24. **SIMILAR TRIANGLES** In the diagram at the right, name the three pairs of triangles that are similar.

CHALLENGE **In the figure at the right, $\triangle ABC \sim \triangle VWX$.**



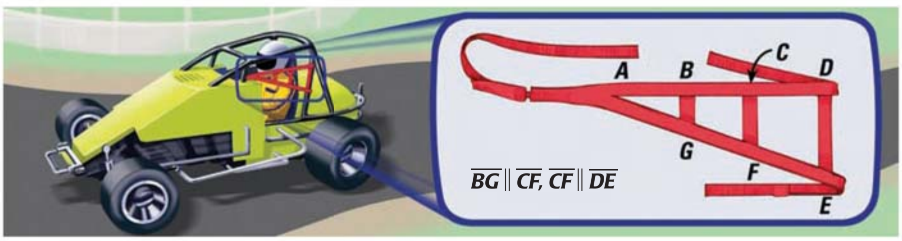
25. Find the scale factor of $\triangle VWX$ to $\triangle ABC$.

26. Find the ratio of the area of $\triangle VWX$ to the area of $\triangle ABC$.

27. Make a conjecture about the relationship between the scale factor in Exercise 25 and the ratio in Exercise 26. *Justify* your conjecture.

PROBLEM SOLVING

28. **RACECAR NET** Which postulate or theorem could you use to show that the three triangles that make up the racecar window net are similar? *Explain*.



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EXAMPLE 1 on p. 388 for Ex. 29

29. **STAINED GLASS** Certain sections of stained glass are sold in triangular *beveled* pieces. Which of the three beveled pieces, if any, are similar?

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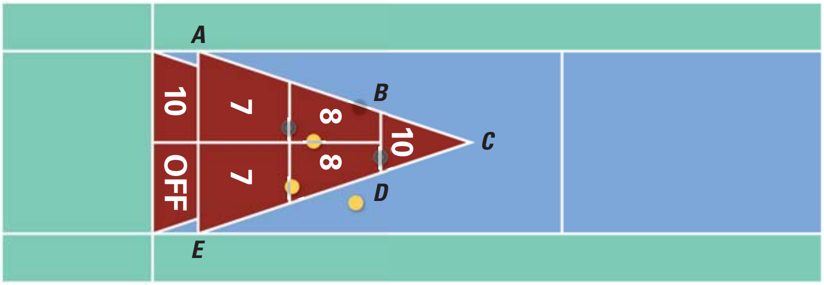
SHUFFLEBOARD In the portion of the shuffleboard court shown, $\frac{BC}{AC} = \frac{BD}{AE}$.



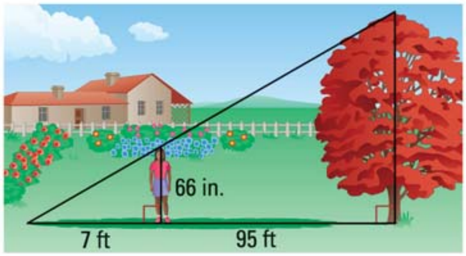
30. What additional piece of information do you need in order to show that $\triangle BCD \sim \triangle ACE$ using the SSS Similarity Theorem?

31. What additional piece of information do you need in order to show that $\triangle BCD \sim \triangle ACE$ using the SAS Similarity Theorem?

32. ★ **OPEN-ENDED MATH** Use a diagram to show why there is no Side-Side-Angle Similarity Postulate.

EXAMPLE 4
on p. 391
for Ex. 33

33. **MULTI-STEP PROBLEM** Ruby is standing in her back yard and she decides to estimate the height of a tree. She stands so that the tip of her shadow coincides with the tip of the tree's shadow, as shown. Ruby is 66 inches tall. The distance from the tree to Ruby is 95 feet and the distance between the tip of the shadows and Ruby is 7 feet.



a. What postulate or theorem can you use to show that the triangles in the diagram are similar?

b. About how tall is the tree, to the nearest foot?

c. **What If?** Curtis is 75 inches tall. At a different time of day, he stands so that the tip of his shadow and the tip of the tree's shadow coincide, as described above. His shadow is 6 feet long. How far is Curtis from the tree?

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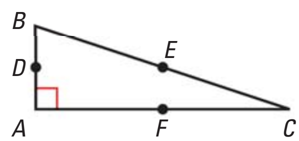
34. ★ **EXTENDED RESPONSE** Suppose you are given two right triangles with one pair of corresponding legs and the pair of corresponding hypotenuses having the same length ratios.

a. The lengths of the given pair of corresponding legs are 6 and 18, and the lengths of the hypotenuses are 10 and 30. Use the Pythagorean Theorem to solve for the lengths of the other pair of corresponding legs. Draw a diagram.

b. Write the ratio of the lengths of the second pair of corresponding legs.

c. Are these triangles similar? Does this suggest a Hypotenuse-Leg Similarity Theorem for right triangles?

35. **PROOF** Given that $\triangle ABC$ is a right triangle and D, E, and F are midpoints, prove that $m\angle DEF = 90°$.



36. ★ **WRITING** Can two triangles have all pairs of corresponding angles in proportion? *Explain.*

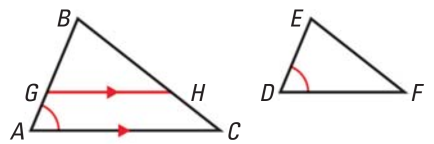
○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

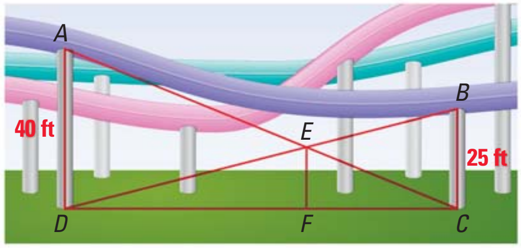
37. **PROVING THEOREM 6.3** Write a paragraph proof of the SAS Similarity Theorem.

GIVEN ▶ $\angle A \cong \angle D, \frac{AB}{DE} = \frac{AC}{DF}$

PROVE ▶ $\triangle ABC \sim \triangle DEF$



38. **CHALLENGE** A portion of a water slide in an amusement park is shown. Find the length of $\overline{EF}$. (*Note:* The posts form right angles with the ground.)

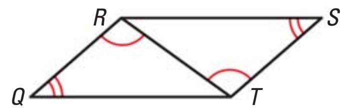


MIXED REVIEW

Find the slope of the line that passes through the given points. *(p. 171)*

39. (0, −8), (4, 16)
40. (−2, −9), (1, −3)
41. (−3, 9), (7, 2)

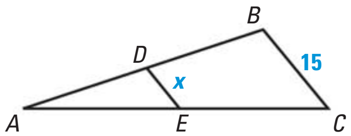
42. State the postulate or theorem you would use to prove the triangles congruent. Then write a congruence statement. *(p. 249)*



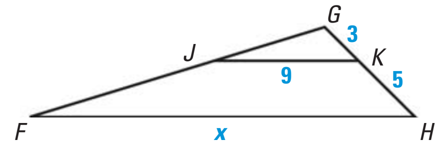
PREVIEW
Prepare for Lesson 6.6 in Exs. 43–44.

Find the value of *x*.

43. $\overline{DE}$ is a midsegment of $\triangle ABC$. *(p. 295)*



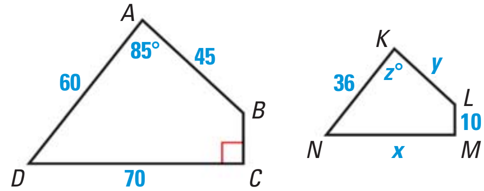
44. $\frac{GK}{GH} = \frac{JK}{FH}$ *(p. 364)*



QUIZ *for Lessons 6.3–6.5*

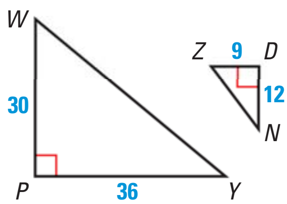
In the diagram, *ABCD* ~ *KLMN*. *(p. 372)*

1. Find the scale factor of *ABCD* to *KLMN*.
2. Find the values of *x*, *y*, and *z*.
3. Find the perimeter of each polygon.

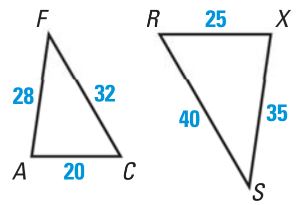


Determine whether the triangles are similar. If they are similar, write a similarity statement. *(pp. 381, 388)*

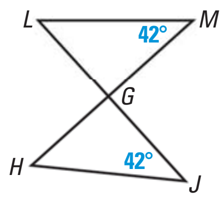
4.



5.



6.



@HomeTutor classzone.com Keystrokes

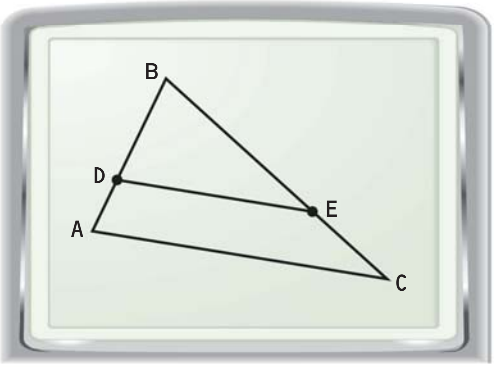
6.6 Investigate Proportionality

MATERIALS • graphing calculator or computer

QUESTION How can you use geometry drawing software to compare segment lengths in triangles?

EXPLORE 1 Construct a line parallel to a triangle's third side

STEP 1 ***Draw a triangle*** Draw a triangle. Label the vertices A, B, and C. Draw a point on $\overline{AB}$. Label the point D.



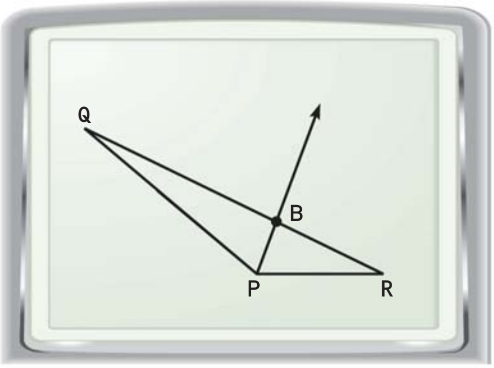
STEP 2 ***Draw a parallel line*** Draw a line through D that is parallel to $\overline{AC}$. Label the intersection of the line and $\overline{BC}$ as point E.

STEP 3 ***Measure segments*** Measure $\overline{BD}$, $\overline{DA}$, $\overline{BE}$, and $\overline{EC}$. Calculate the ratios $\frac{BD}{DA}$ and $\frac{BE}{EC}$.

STEP 4 ***Compare ratios*** Move one or more of the triangle's vertices to change its shape. *Compare* the ratios from Step 3 as the shape changes. Save as "EXPLORE1."

EXPLORE 2 Construct an angle bisector of a triangle

STEP 1 ***Draw a triangle*** Draw a triangle. Label the vertices P, Q, and R. Draw the angle bisector of $\angle QPR$. Label the intersection of the angle bisector and $\overline{QR}$ as point B.



STEP 2 ***Measure segments*** Measure $\overline{BR}$, $\overline{RP}$, $\overline{BQ}$, and $\overline{QP}$. Calculate the ratios $\frac{BR}{BQ}$ and $\frac{RP}{QP}$.

STEP 3 ***Compare ratios*** Move one or more of the triangle's vertices to change its shape. *Compare* the ratios from Step 3. Save as "EXPLORE2."

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Make a conjecture about the ratios of the lengths of the segments formed when two sides of a triangle are cut by a line parallel to the triangle's third side.
2. Make a conjecture about how the ratio of the lengths of two sides of a triangle is related to the ratio of the lengths of the segments formed when an angle bisector is drawn to the third side.

6.6 Use Proportionality Theorems

Before You used proportions with similar triangles.

Now You will use proportions with a triangle or parallel lines.

Why? So you can use perspective drawings, as in Ex. 28.

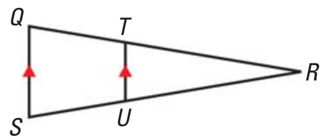
Key Vocabulary
- **corresponding angles,** *p. 147*
- **ratio,** *p. 356*
- **proportion,** *p. 358*

The Midsegment Theorem, which you learned on page 295, is a special case of the Triangle Proportionality Theorem and its converse.

THEOREMS *For Your Notebook*

THEOREM 6.4 Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

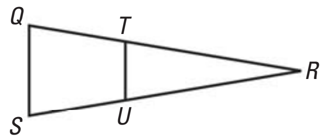


If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.

Proof: Ex. 22, p. 402

THEOREM 6.5 Converse of the Triangle Proportionality Theorem

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

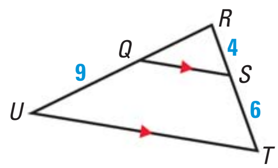


If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.

Proof: Ex. 26, p. 402

EXAMPLE 1 Find the length of a segment

In the diagram, $\overline{QS} \parallel \overline{UT}$, $RS = 4$, $ST = 6$, and $QU = 9$. What is the length of $\overline{RQ}$?



Solution

$\frac{RQ}{QU} = \frac{RS}{ST}$ Triangle Proportionality Theorem

$\frac{RQ}{9} = \frac{4}{6}$ Substitute.

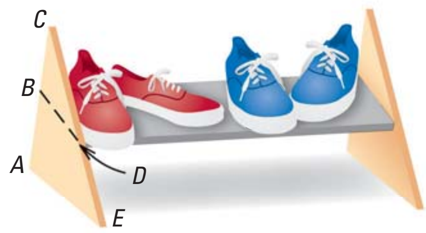
$RQ = 6$ Multiply each side by 9 and simplify.

REASONING Theorems 6.4 and 6.5 also tell you that if the lines are *not* parallel, then the proportion is *not* true, and vice-versa.

So if $\overline{TU} \nparallel \overline{QS}$, then $\frac{RT}{TQ} \neq \frac{RU}{US}$. Also, if $\frac{RT}{TQ} \neq \frac{RU}{US}$, then $\overline{TU} \nparallel \overline{QS}$.

EXAMPLE 2 Solve a real-world problem

SHOERACK On the shoerack shown, $AB = 33$ cm, $BC = 27$ cm, $CD = 44$ cm, and $DE = 25$ cm. *Explain* why the gray shelf is not parallel to the floor.



Solution

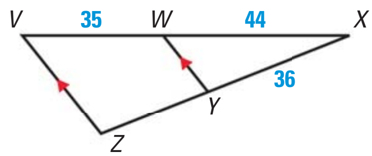
Find and simplify the ratios of lengths determined by the shoerack.

$\frac{CD}{DE} = \frac{44}{25}$ $\qquad$ $\frac{CB}{BA} = \frac{27}{33} = \frac{9}{11}$

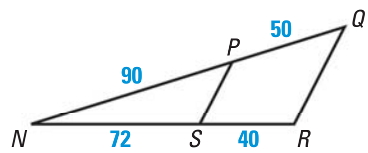
▶ Because $\frac{44}{25} \neq \frac{9}{11}$, $\overline{BD}$ is not parallel to $\overline{AE}$. So, the shelf is not parallel to the floor.

✓ GUIDED PRACTICE for Examples 1 and 2

1. Find the length of $\overline{YZ}$.



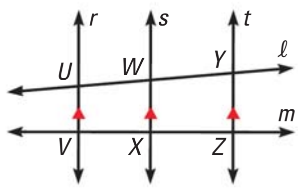
2. Determine whether $\overline{PS} \parallel \overline{QR}$.



THEOREMS *For Your Notebook*

THEOREM 6.6

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

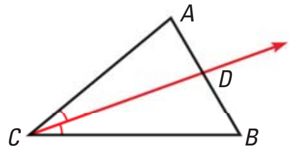


$\frac{UW}{WY} = \frac{VX}{XZ}$

Proof: Ex. 23, p. 402

THEOREM 6.7

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

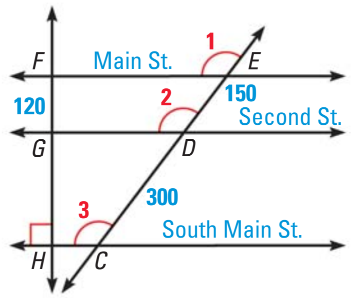


$\frac{AD}{DB} = \frac{CA}{CB}$

Proof: Ex. 27, p. 403

EXAMPLE 3 Use Theorem 6.6

CITY TRAVEL In the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are all congruent and $GF = 120$ yards, $DE = 150$ yards, and $CD = 300$ yards. Find the distance HF between Main Street and South Main Street.



ANOTHER WAY

For alternative methods for solving the problem in Example 3, turn to page 404 for the **Problem Solving Workshop**.

Solution

Corresponding angles are congruent, so $\overleftrightarrow{FE}$, $\overleftrightarrow{GD}$, and $\overleftrightarrow{HC}$ are parallel. Use Theorem 6.6.

$\frac{HG}{GF} = \frac{CD}{DE}$ **Parallel lines divide transversals proportionally.**

$\frac{HG + GF}{GF} = \frac{CD + DE}{DE}$ **Property of proportions (Property 4)**

$\frac{HF}{120} = \frac{300 + 150}{150}$ **Substitute.**

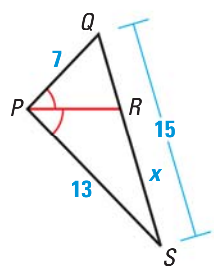
$\frac{HF}{120} = \frac{450}{150}$ **Simplify.**

$HF = 360$ **Multiply each side by 120 and simplify.**

▶ The distance between Main Street and South Main Street is 360 yards.

EXAMPLE 4 Use Theorem 6.7

In the diagram, $\angle QPR \cong \angle RPS$. Use the given side lengths to find the length of $\overline{RS}$.



Solution

Because $\overrightarrow{PR}$ is an angle bisector of $\angle QPS$, you can apply Theorem 6.7. Let $RS = x$. Then $RQ = 15 - x$.

$\frac{RQ}{RS} = \frac{PQ}{PS}$ **Angle bisector divides opposite side proportionally.**

$\frac{15 - x}{x} = \frac{7}{13}$ **Substitute.**

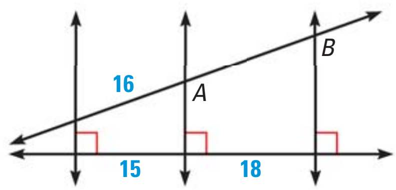
$7x = 195 - 13x$ **Cross Products Property**

$x = 9.75$ **Solve for *x*.**

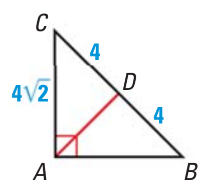
✓ GUIDED PRACTICE for Examples 3 and 4

Find the length of $\overline{AB}$.

3.



4.



6.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 9, and 21

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 8, 13, 25, and 28

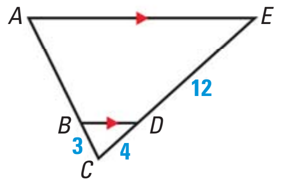
SKILL PRACTICE

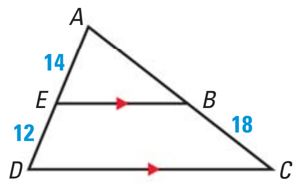
1. **VOCABULARY** State the Triangle Proportionality Theorem. Draw a diagram.

2. ★ **WRITING** *Compare* the Midsegment Theorem (see page 295) and the Triangle Proportionality Theorem. How are they related?

EXAMPLE 1 on p. 397 for Exs. 3–4

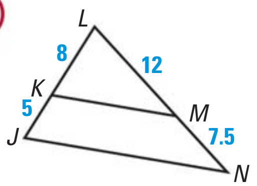
FINDING THE LENGTH OF A SEGMENT **Find the length of $\overline{AB}$.**

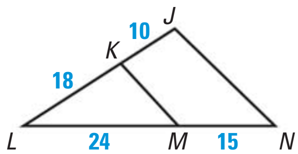
3. 

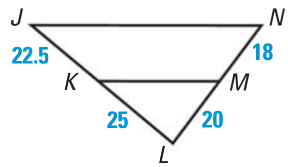
4. 

EXAMPLE 2 on p. 398 for Exs. 5–7

REASONING **Use the given information to determine whether $\overline{KM} \parallel \overline{JN}$. *Explain* your reasoning.**

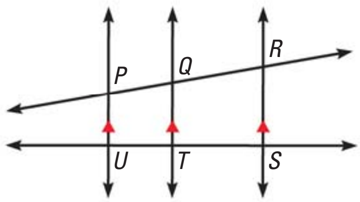
5. 

6. 

7. 

EXAMPLE 3 on p. 399 for Ex. 8

8. ★ **MULTIPLE CHOICE** For the figure at the right, which statement is *not* necessarily true?

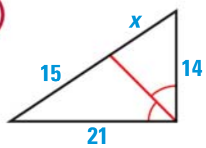


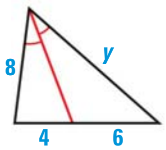
Ⓐ $\frac{PQ}{QR} = \frac{UT}{TS}$ Ⓑ $\frac{TS}{UT} = \frac{QR}{PQ}$

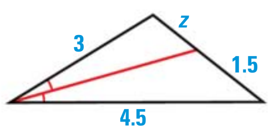
Ⓒ $\frac{QR}{RS} = \frac{TS}{RS}$ Ⓓ $\frac{PQ}{PR} = \frac{UT}{US}$

EXAMPLE 4 on p. 399 for Exs. 9–12

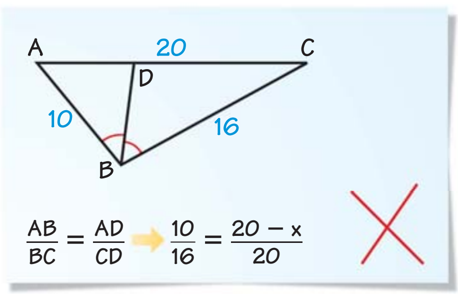
ALGEBRA **Find the value of the variable.**

9. 

10. 

11. 

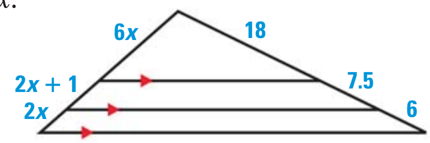
12. **ERROR ANALYSIS** A student begins to solve for the length of $\overline{AD}$ as shown. *Describe* and correct the student's error.



13. ★ **MULTIPLE CHOICE** Find the value of x.

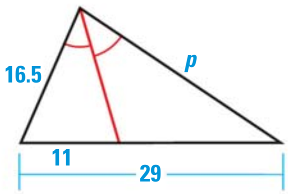
Ⓐ $\frac{1}{2}$ Ⓑ 1

Ⓒ 2 Ⓓ 3

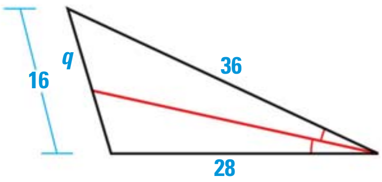


ALGEBRA **Find the value of the variable.**

14.

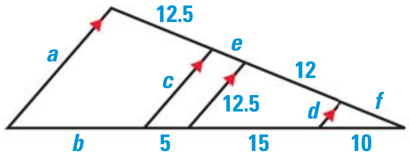


15.

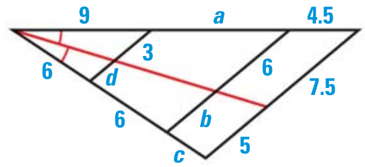


FINDING SEGMENT LENGTHS **Use the diagram to find the value of each variable.**

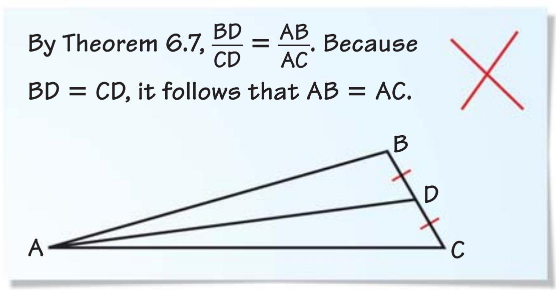
16.



17.

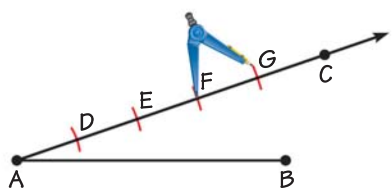


18. **ERROR ANALYSIS** A student claims that $AB = AC$ using the method shown. *Describe* and correct the student's error.



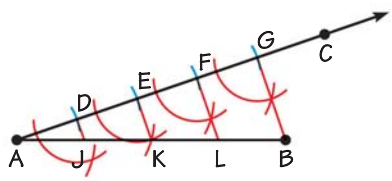
19. **CONSTRUCTION** Follow the instructions for constructing a line segment that is divided into four equal parts.

a. Draw a line segment that is about 3 inches long, and label its endpoints A and B. Choose any point C not on $\overline{AB}$. Draw $\overrightarrow{AC}$.

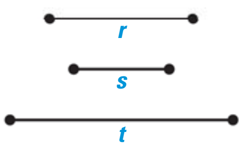


b. Using any length, place the compass point at A and make an arc intersecting $\overrightarrow{AC}$ at D. Using the same compass setting, make additional arcs on $\overrightarrow{AC}$. Label the points E, F, and G so that $AD = DE = EF = FG$.

c. Draw $\overline{GB}$. Construct a line parallel to $\overline{GB}$ through D. Continue constructing parallel lines and label the points as shown. *Explain* why $AJ = JK = KL = LB$.

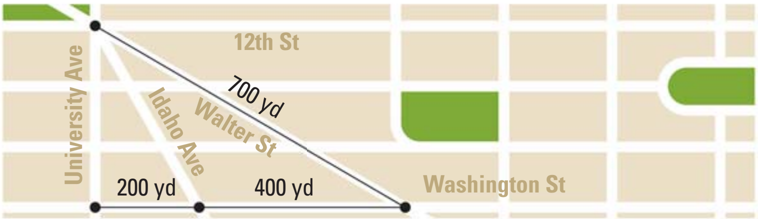


20. **CHALLENGE** Given segments with lengths r, s, and t, construct a segment of length x, such that $\frac{r}{s} = \frac{t}{x}$.



PROBLEM SOLVING

21. **CITY MAP** On the map below, Idaho Avenue bisects the angle between University Avenue and Walter Street. To the nearest yard, what is the distance along University Avenue from 12th Street to Washington Street?



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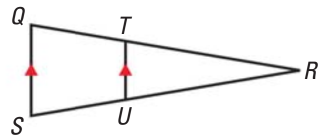
22. **PROVING THEOREM 6.4** Prove the Triangle Proportionality Theorem.

GIVEN ▶ $\overline{QS} \parallel \overline{TU}$

PROVE ▶ $\frac{QT}{TR} = \frac{SU}{UR}$



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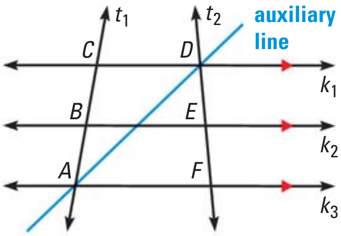
23. **PROVING THEOREM 6.6** Use the diagram with the auxiliary line drawn to write a paragraph proof of Theorem 6.6.

GIVEN ▶ $k_1 \parallel k_2$, $k_2 \parallel k_3$

PROVE ▶ $\frac{CB}{BA} = \frac{DE}{EF}$

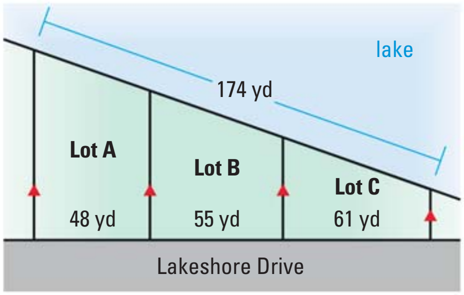


24. **MULTI-STEP PROBLEM** The real estate term *lake frontage* refers to the distance along the edge of a piece of property that touches a lake.

a. Find the lake frontage (to the nearest tenth of a yard) for each lot shown.

b. In general, the more lake frontage a lot has, the higher its selling price. Which of the lots should be listed for the highest price?

c. Suppose that lot prices are in the same ratio as lake frontages. If the least expensive lot is $100,000, what are the prices of the other lots? *Explain* your reasoning.



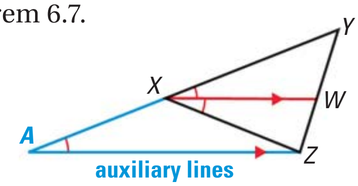
25. ★ **SHORT RESPONSE** Sketch an isosceles triangle. Draw a ray that bisects the angle opposite the base. This ray divides the base into two segments. By Theorem 6.7, the ratio of the legs is proportional to the ratio of these two segments. *Explain* why this ratio is 1 : 1 for an isosceles triangle.

26. **PLAN FOR PROOF** Use the diagram given for the proof of Theorem 6.4 in Exercise 22 to write a plan for proving Theorem 6.5, the Triangle Proportionality Converse.

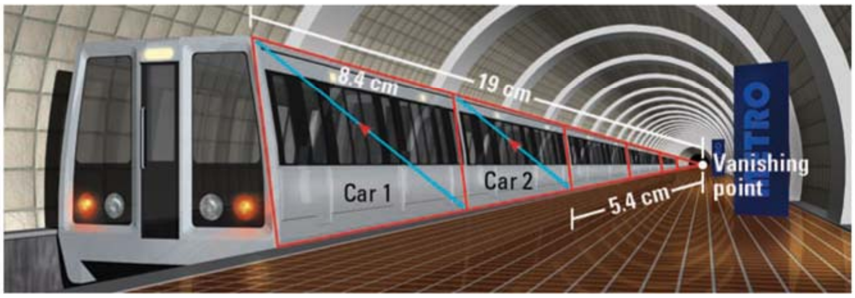
27. PROVING THEOREM 6.7 Use the diagram with the auxiliary lines drawn to write a paragraph proof of Theorem 6.7.

GIVEN ▶ $\angle YXW \cong \angle WXZ$

PROVE ▶ $\frac{YW}{WZ} = \frac{XY}{XZ}$



28. ★ EXTENDED RESPONSE In *perspective drawing*, lines that are parallel in real life must meet at a vanishing point on the horizon. To make the train cars in the drawing appear equal in length, they are drawn so that the lines connecting the opposite corners of each car are parallel.

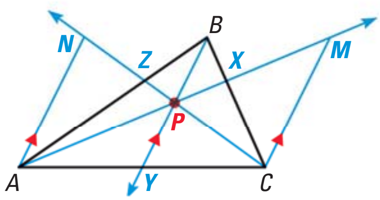


a. Use the dimensions given and the red parallel lines to find the length of the bottom edge of the drawing of Car 2.

b. What other set of parallel lines exist in the figure? *Explain* how these can be used to form a set of similar triangles.

c. Find the length of the top edge of the drawing of Car 2.

29. CHALLENGE Prove *Ceva's Theorem:* If P is any point inside $\triangle ABC$, then $\frac{AY}{YC} \cdot \frac{CX}{XB} \cdot \frac{BZ}{ZA} = 1$. (*Hint:* Draw lines parallel to $\overline{BY}$ through A and C. Apply Theorem 6.4 to $\triangle ACM$. Show that $\triangle APN \sim \triangle MPC$, $\triangle CXM \sim \triangle BXP$, and $\triangle BZP \sim \triangle AZN$.)



MIXED REVIEW

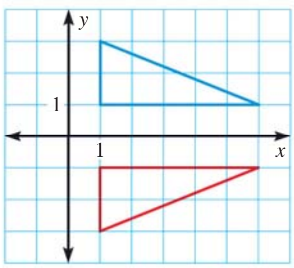
PREVIEW
Prepare for Lesson 6.7 in Exs. 30–36.

Perform the following operations. Then simplify.

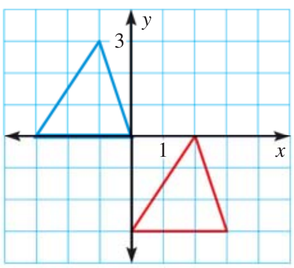
30. $(-3) \cdot \frac{7}{2}$ *(p. 869)*   **31.** $\frac{4}{3} \cdot \frac{1}{2}$ *(p. 869)*   **32.** $5\left(\frac{1}{2}\right)^2$ *(p. 871)*   **33.** $\left(\frac{5}{4}\right)^3$ *(p. 871)*

Describe **the translation in words and write the coordinate rule for the translation.** *(p. 272)*

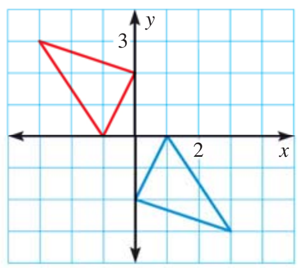
34.



35.



36.



PROBLEM SOLVING WORKSHOP
LESSON 6.6

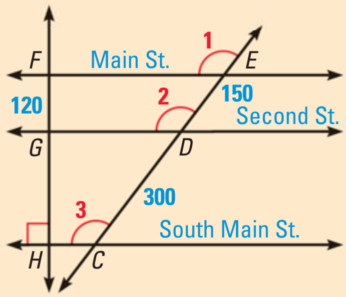
Using ALTERNATIVE METHODS

Another Way to Solve Example 3, page 399

MULTIPLE REPRESENTATIONS In Lesson 6.6, you used proportionality theorems to find lengths of segments formed when transversals intersect two or more parallel lines. Now, you will learn two different ways to solve Example 3 on page 399.

PROBLEM

CITY TRAVEL In the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are all congruent and $GF = 120$ yards, $DE = 150$ yards, and $CD = 300$ yards. Find the distance HF between Main Street and South Main Street.



METHOD 1

Applying a Ratio One alternative approach is to look for ratios in the diagram.

STEP 1 **Read** the problem. Because Main Street, Second Street, and South Main Street are all parallel, the lengths of the segments of the cross streets will be in proportion, so they have the same ratio.

STEP 2 **Apply** a ratio. Notice that on $\overleftrightarrow{CE}$, the distance CD between South Main Street and Second Street is twice the distance DE between Second Street and Main Street. So the same will be true for the distances HG and GF.

$HG = 2 \cdot GF$	**Write equation.**
$= 2 \cdot 120$	**Substitute.**
$= 240$	**Simplify.**

STEP 3 **Calculate** the distance. Line HF is perpendicular to both Main Street and South Main Street, so the distance between Main Street and South Main Street is this perpendicular distance, HF.

$HF = HG + GF$	**Segment Addition Postulate**
$= 120 + 240$	**Substitute.**
$= 360$	**Simplify.**

STEP 4 **Check** page 399 to verify your answer, and confirm that it is the same.

METHOD 2 **Writing a Proportion** Another alternative approach is to use a graphic organizer to set up a proportion.

STEP 1 **Make** a table to compare the distances.

	$\overleftrightarrow{CE}$	$\overleftrightarrow{HF}$
Total distance	300 + 150, or 450	x
Partial distance	150	120

STEP 2 **Write** and solve a proportion.

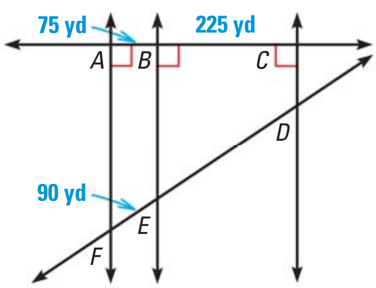
$\frac{450}{150} = \frac{x}{120}$ **Write proportion.**

$360 = x$ **Multiply each side by 12 and simplify.**

▸ The distance is 360 yards.

PRACTICE

1. **MAPS** Use the information on the map.



a. Find DE.

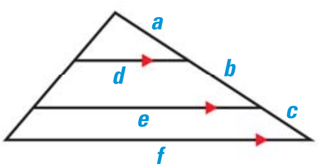
b. **What If?** Suppose there is an alley one fourth of the way from $\overline{BE}$ to $\overline{CD}$ and parallel to $\overline{BE}$. What is the distance from E to the alley along $\overleftrightarrow{FD}$?

2. **REASONING** Given the diagram below, *explain* why the three given proportions are true.

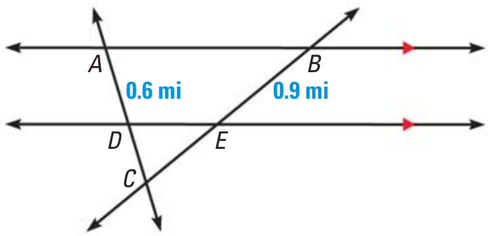
$\frac{a}{a+b} = \frac{d}{e}$

$\frac{a}{a+b+c} = \frac{d}{f}$

$\frac{a+b}{a+b+c} = \frac{d}{f}$

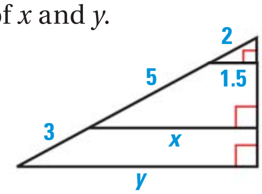


3. **WALKING** Two people leave points A and B at the same time. They intend to meet at point C at the same time. The person who leaves point A walks at a speed of 3 miles per hour. How fast must the person who leaves point B walk?



4. **ERROR ANALYSIS** A student who attempted to solve the problem in Exercise 3 claims that you need to know the length of $\overline{AC}$ to solve the problem. *Describe* and correct the error that the student made.

5. **xy ALGEBRA** Use the diagram to find the values of x and y.



Extension

Use after Lesson 6.6

Fractals

GOAL Explore the properties of fractals.

Key Vocabulary
- fractal
- self-similarity
- iteration

A **fractal** is an object that is *self-similar*. An object is **self-similar** if one part of the object can be enlarged to look like the whole object. In nature, fractals can be found in ferns and branches of a river. Scientists use fractals to map out clouds in order to predict rain.

A Mandelbrot fractal

Many fractals are formed by a repetition of a sequence of the steps called **iteration**. The first stage of drawing a fractal is considered Stage 0. Helge van Koch (1870–1924) described a fractal known as the *Koch snowflake*, shown in Example 1.

HISTORY NOTE

Computers made it easier to study mathematical iteration by reducing the time needed to perform calculations. Using fractals, mathematicians have been able to create better models of coastlines, clouds, and other natural objects.

EXAMPLE 1 Draw a fractal

Use the directions below to draw a Koch snowflake.

Starting with an equilateral triangle, at each stage each side is divided into thirds and a new equilateral triangle is formed using the middle third as the triangle side length.

Solution

STAGE 0 **Draw** an equilateral triangle with a side length of one unit.

STAGE 1 **Replace** the middle third of each side with an equilateral triangle.

STAGE 2 **Repeat** Stage 1 with the six smaller equilateral triangles.

STAGE 3 **Repeat** Stage 1 with the eighteen smaller equilateral triangles.

MEASUREMENT Benoit Mandelbrot (b. 1924) was the first mathematician to formalize the idea of fractals when he observed methods used to measure the lengths of coastlines. Coastlines cannot be measured as straight lines because of the inlets and rocks. Mandelbrot used fractals to model coastlines.

EXAMPLE 2 Find lengths in a fractal

Make a table to study the lengths of the sides of a Koch snowflake at different stages.

Stage number	Edge length	Number of edges	Perimeter
0	1	3	3
1	$\frac{1}{3}$	$3 \cdot 4 = 12$	4
2	$\frac{1}{9}$	$12 \cdot 4 = 48$	$\frac{48}{9} = 5\frac{1}{3}$
3	$\frac{1}{27}$	$48 \cdot 4 = 192$	$\frac{192}{27} = 7\frac{1}{9}$
n	$\frac{1}{3^n}$	$3 \cdot 4^n$	$\frac{4^n}{3^{n-1}}$

AnimatedGeometry at classzone.com

PRACTICE

EXAMPLES 1 and 2 for Exs. 1–3

1. **PERIMETER** Find the ratio of the edge length of the triangle in Stage 0 of a Koch snowflake to the edge length of the triangle in Stage 1. How is the perimeter of the triangle in Stage 0 related to the perimeter of the triangle in Stage 1? *Explain*.

2. **MULTI-STEP PROBLEM** Use the *Cantor set*, which is a fractal whose iteration consists of dividing a segment into thirds and erasing the middle third.
   - **a.** Draw Stage 0 through Stage 5 of the Cantor set. Stage 0 has a length of one unit.
   - **b.** Make a table showing the stage number, number of segments, segment length, and total length of the Cantor set.
   - **c.** What is the total length of the Cantor set at Stage 10? Stage 20? Stage n?

3. **EXTENDED RESPONSE** A *Sierpinski carpet* starts with a square with side length one unit. At each stage, divide the square into nine equal squares with the middle square shaded a different color.
   - **a.** Draw Stage 0 through Stage 3 of a Sierpinski Carpet.
   - **b.** *Explain* why the carpet is said to be *self-similar* by comparing the upper left hand square to the whole square.
   - **c.** Make a table to find the total area of the colored squares at Stage 3.

Investigating Geometry ACTIVITY *Use before Lesson 6.7*

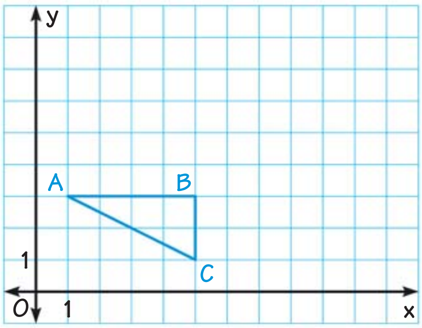
6.7 Dilations

MATERIALS • graph paper • straightedge • compass • ruler

QUESTION How can you construct a similar figure?

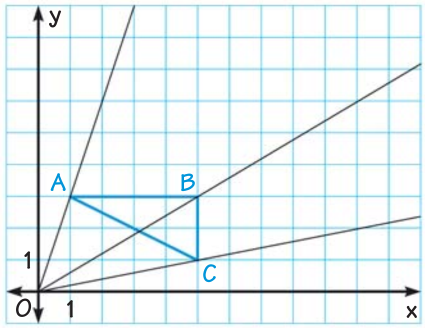
EXPLORE Construct a similar triangle

STEP 1



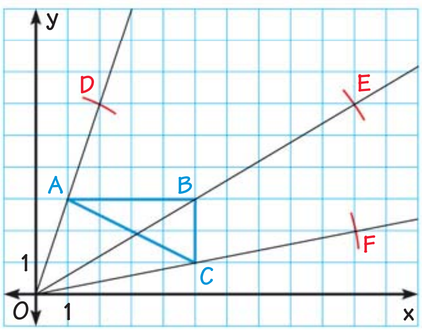
Draw a triangle Plot the points $A(1, 3)$, $B(5, 3)$, and $C(5, 1)$ in a coordinate plane. Draw $\triangle ABC$.

STEP 2



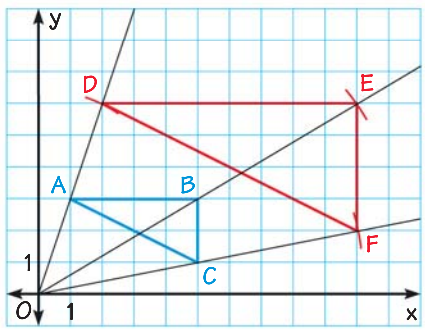
Draw rays Using the origin as an endpoint O, draw $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$.

STEP 3



Draw equal segments Use a compass to mark a point D on $\overrightarrow{OA}$ so $OA = AD$. Mark a point E on $\overrightarrow{OB}$ so $OB = BE$. Mark a point F on $\overrightarrow{OC}$ so $OC = CF$.

STEP 4



Draw the image Connect points D, E, and F to form a right triangle.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Measure $\overline{AB}$, $\overline{BC}$, $\overline{DE}$, and $\overline{EF}$. Calculate the ratios $\frac{DE}{AB}$ and $\frac{EF}{BC}$. Using this information, show that the two triangles are similar.

2. Repeat the steps in the Explore to construct $\triangle GHJ$ so that $3 \cdot OA = AG$, $3 \cdot OB = BH$, and $3 \cdot OC = CJ$.

6.7 Perform Similarity Transformations

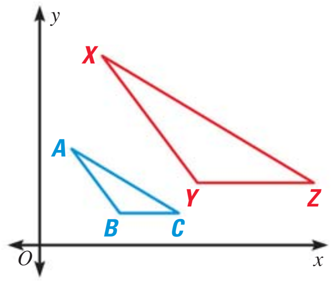
Before	You performed congruence transformations.
Now	You will perform dilations.
Why?	So you can solve problems in art, as in Ex. 26.

Key Vocabulary
- **dilation**
- **center of dilation**
- **scale factor of a dilation**
- **reduction**
- **enlargement**
- **transformation,** *p. 272*

A **dilation** is a transformation that stretches or shrinks a figure to create a similar figure. A dilation is a type of *similarity transformation*.

In a dilation, a figure is enlarged or reduced with respect to a fixed point called the **center of dilation**.

The **scale factor of a dilation** is the ratio of a side length of the image to the corresponding side length of the original figure. In the figure shown, $\triangle XYZ$ is the image of $\triangle ABC$. The center of dilation is (0, 0) and the scale factor is $\frac{XY}{AB}$.



KEY CONCEPT — For Your Notebook

Coordinate Notation for a Dilation

You can describe a dilation with respect to the origin with the notation $(x, y) \rightarrow (kx, ky)$, where k is the scale factor.

If $0 < k < 1$, the dilation is a **reduction**. If $k > 1$, the dilation is an **enlargement**.

EXAMPLE 1 Draw a dilation with a scale factor greater than 1

READ DIAGRAMS
All of the dilations in this lesson are in the coordinate plane and each center of dilation is the origin.

Draw a dilation of quadrilateral *ABCD* with vertices *A*(2, 1), *B*(4, 1), *C*(4, −1), and *D*(1, −1). Use a scale factor of 2.

Solution

First draw *ABCD*. Find the dilation of each vertex by multiplying its coordinates by 2. Then draw the dilation.

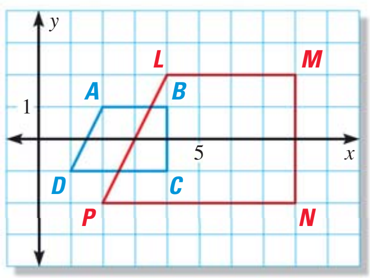
$$(x, y) \rightarrow (2x, 2y)$$

$A(2, 1) \rightarrow L(4, 2)$

$B(4, 1) \rightarrow M(8, 2)$

$C(4, -1) \rightarrow N(8, -2)$

$D(1, -1) \rightarrow P(2, -2)$



EXAMPLE 2 Verify that a figure is similar to its dilation

A triangle has the vertices $A(4, -4)$, $B(8, 2)$, and $C(8, -4)$. The image of $\triangle ABC$ after a dilation with a scale factor of $\frac{1}{2}$ is $\triangle DEF$.

a. Sketch $\triangle ABC$ and $\triangle DEF$.

b. Verify that $\triangle ABC$ and $\triangle DEF$ are similar.

Solution

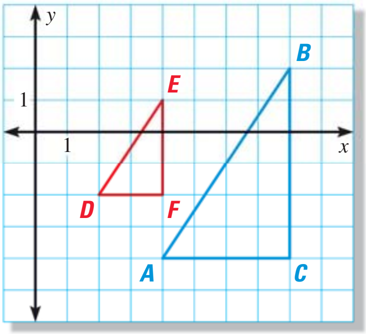
a. The scale factor is less than one, so the dilation is a reduction.

$$(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$$

$$A(4, -4) \rightarrow D(2, -2)$$

$$B(8, 2) \rightarrow E(4, 1)$$

$$C(8, -4) \rightarrow F(4, -2)$$



b. Because $\angle C$ and $\angle F$ are both right angles, $\angle C \cong \angle F$. Show that the lengths of the sides that include $\angle C$ and $\angle F$ are proportional. Find the horizontal and vertical lengths from the coordinate plane.

$$\frac{AC}{DF} \stackrel{?}{=} \frac{BC}{EF} \rightarrow \frac{4}{2} = \frac{6}{3} \checkmark$$

So, the lengths of the sides that include $\angle C$ and $\angle F$ are proportional.

▸ Therefore, $\triangle ABC \sim \triangle DEF$ by the SAS Similarity Theorem.

✓ GUIDED PRACTICE for Examples 1 and 2

Find the coordinates of L, M, and N so that $\triangle LMN$ is a dilation of $\triangle PQR$ with a scale factor of k. Sketch $\triangle PQR$ and $\triangle LMN$.

1. $P(-2, -1)$, $Q(-1, 0)$, $R(0, -1)$; $k = 4$

2. $P(5, -5)$, $Q(10, -5)$, $R(10, 5)$; $k = 0.4$

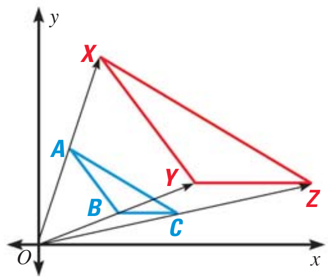
EXAMPLE 3 Find a scale factor

PHOTO STICKERS You are making your own photo stickers. Your photo is 4 inches by 4 inches. The image on the stickers is 1.1 inches by 1.1 inches. What is the scale factor of the reduction?

Solution

The scale factor is the ratio of a side length of the sticker image to a side length of the original photo, or $\frac{1.1 \text{ in.}}{4 \text{ in.}}$. In simplest form, the scale factor is $\frac{11}{40}$.

READING DIAGRAMS Generally, for a center of dilation at the origin, a point of the figure and its image lie on the same ray from the origin. However, if a point of the figure *is* the origin, its image is also the origin.



EXAMPLE 4 Standardized Test Practice

You want to create a quadrilateral *EFGH* that is similar to quadrilateral *PQRS*. What are the coordinates of *H*?

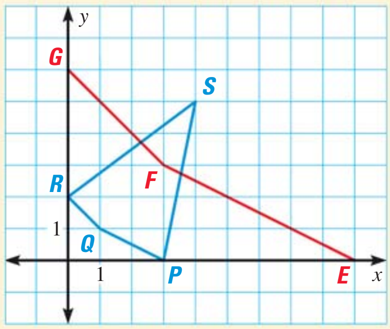
ELIMINATE CHOICES
You can eliminate choice A, because you can tell by looking at the graph that *H* is in Quadrant I. The point (12, −15) is in Quadrant II.

Ⓐ (12, −15)

Ⓑ (7, 8)

Ⓒ (12, 15)

Ⓓ (15, 18)



Solution

Determine if *EFGH* is a dilation of *PQRS* by checking whether the same scale factor can be used to obtain *E*, *F*, and *G* from *P*, *Q*, and *R*.

$(x, y) \rightarrow (kx, ky)$

$P(3, 0) \rightarrow E(9, 0)$ $k = 3$

$Q(1, 1) \rightarrow F(3, 3)$ $k = 3$

$R(0, 2) \rightarrow G(0, 6)$ $k = 3$

Because k is the same in each case, the image is a dilation with a scale factor of 3. So, you can use the scale factor to find the image *H* of point *S*.

$S(4, 5) \rightarrow H(3 \cdot 4, 3 \cdot 5) = H(12, 15)$

▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

CHECK Draw rays from the origin through each point and its image.

GUIDED PRACTICE for Examples 3 and 4

3. **WHAT IF?** In Example 3, what is the scale factor of the reduction if your photo is 5.5 inches by 5.5 inches?

4. Suppose a figure containing the origin is dilated. *Explain* why the corresponding point in the image of the figure is also the origin.

6.7 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 11, and 27

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 13, 21, 22, 28, 30, and 31

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: In a dilation, the image is _?_ to the original figure.

2. ★ **WRITING** *Explain* how to find the scale factor of a dilation. How do you know whether a dilation is an enlargement or a reduction?

EXAMPLES 1 and 2 on pp. 409–410 for Exs. 3–8

DRAWING DILATIONS **Draw a dilation of the polygon with the given vertices using the given scale factor k.**

3. $A(-2, 1)$, $B(-4, 1)$, $C(-2, 4)$; $k = 2$

4. $A(-5, 5)$, $B(-5, -10)$, $C(10, 0)$; $k = \frac{3}{5}$

5. $A(1, 1)$, $B(6, 1)$, $C(6, 3)$; $k = 1.5$

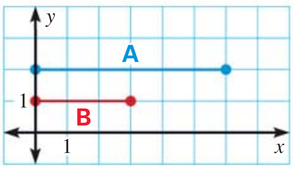
6. $A(2, 8)$, $B(8, 8)$, $C(16, 4)$; $k = 0.25$

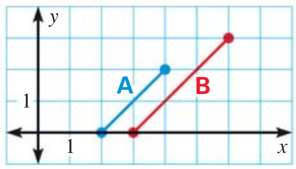
7. $A(-8, 0)$, $B(0, 8)$, $C(4, 0)$, $D(0, -4)$; $k = \frac{3}{8}$

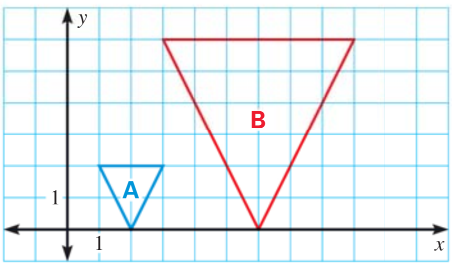
8. $A(0, 0)$, $B(0, 3)$, $C(2, 4)$, $D(2, -1)$; $k = \frac{13}{2}$

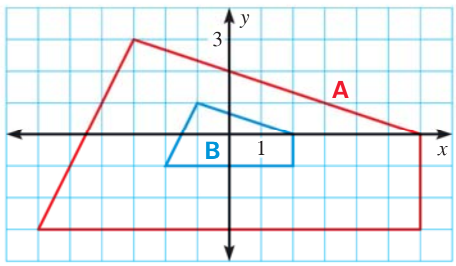
EXAMPLE 3 on p. 410 for Exs. 9–12

IDENTIFYING DILATIONS **Determine whether the dilation from Figure A to Figure B is a *reduction* or an *enlargement*. Then find its scale factor.**

9.


10.


11.


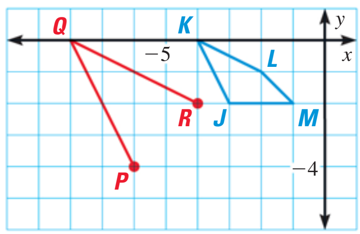
12.


EXAMPLE 4 on p. 411 for Ex. 13

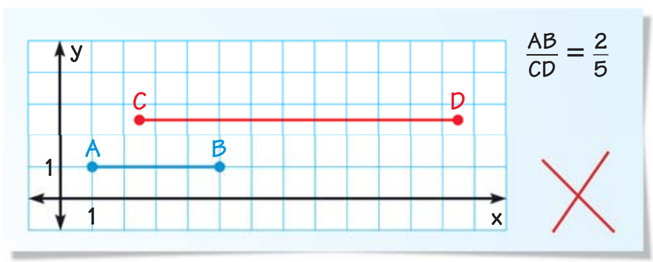
13. ★ **MULTIPLE CHOICE** You want to create a quadrilateral $PQRS$ that is similar to quadrilateral $JKLM$. What are the coordinates of S?

Ⓐ $(2, 4)$ Ⓑ $(4, -2)$

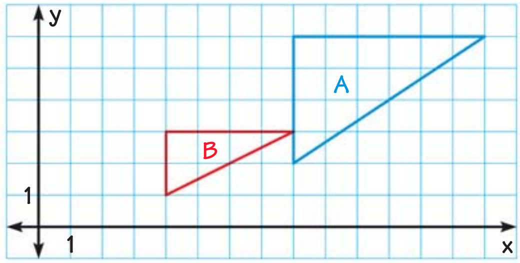
Ⓒ $(-2, -4)$ Ⓓ $(-4, -2)$



14. **ERROR ANALYSIS** A student found the scale factor of the dilation from $\overline{AB}$ to $\overline{CD}$ to be $\frac{2}{5}$. *Describe* and correct the student's error.

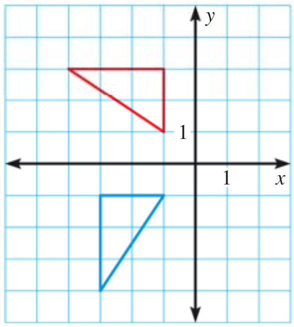


15. ERROR ANALYSIS A student says that the figure shown represents a dilation. What is wrong with this statement?

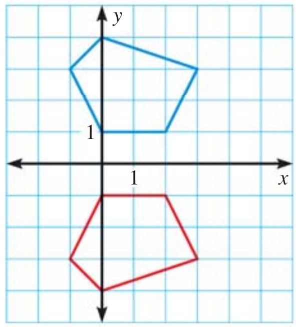


IDENTIFYING TRANSFORMATIONS **Determine whether the transformation shown is a *translation*, *reflection*, *rotation*, or *dilation*.**

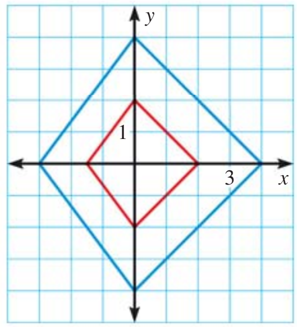
16.



17.

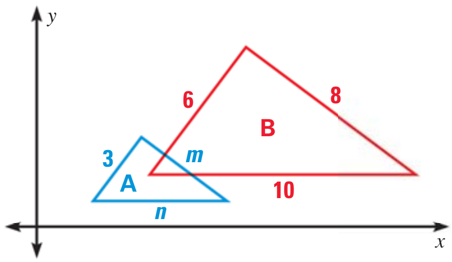


18.

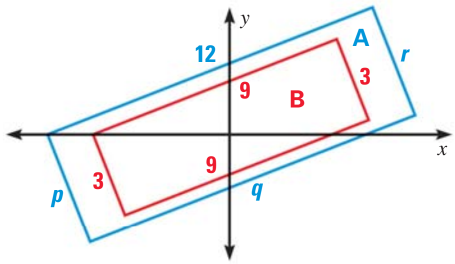


FINDING SCALE FACTORS **Find the scale factor of the dilation of Figure A to Figure B. Then give the unknown lengths of Figure A.**

19.



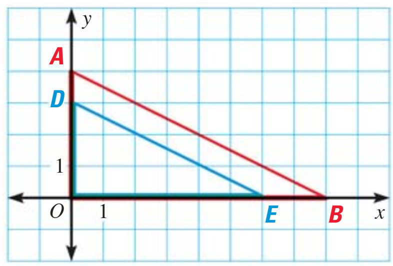
20.



21. ★ MULTIPLE CHOICE In the diagram shown, $\triangle ABO$ is a dilation of $\triangle DEO$. The length of a median of $\triangle ABO$ is what percent of the length of the corresponding median of $\triangle DEO$?

Ⓐ 50% Ⓑ 75%

Ⓒ $133\frac{1}{3}\%$ Ⓓ 200%



22. ★ SHORT RESPONSE Suppose you dilate a figure using a scale factor of 2. Then, you dilate the image using a scale factor of $\frac{1}{2}$. *Describe* the size and shape of this new image.

CHALLENGE ***Describe*** **the two transformations, the first followed by the second, that combined will transform $\triangle ABC$ into $\triangle DEF$.**

23. $A(-3, 3)$, $B(-3, 1)$, $C(0, 1)$
$D(6, 6)$, $E(6, 2)$, $F(0, 2)$

24. $A(6, 0)$, $B(9, 6)$, $C(12, 6)$
$D(0, 3)$, $E(1, 5)$, $F(2, 5)$

PROBLEM SOLVING

EXAMPLE 3 on p. 410 for Exs. 25–27

25. BILLBOARD ADVERTISEMENT A billboard advertising agency requires each advertisement to be drawn so that it fits in a 12-inch by 6-inch rectangle. The agency uses a scale factor of 24 to enlarge the advertisement to create the billboard. What are the dimensions of a billboard, in feet?

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26. POTTERY Your pottery is used on a poster for a student art show. You want to make postcards using the same image. On the poster, the image is 8 inches in width and 6 inches in height. If the image on the postcard can be 5 inches wide, what scale should you use for the image on the postcard?

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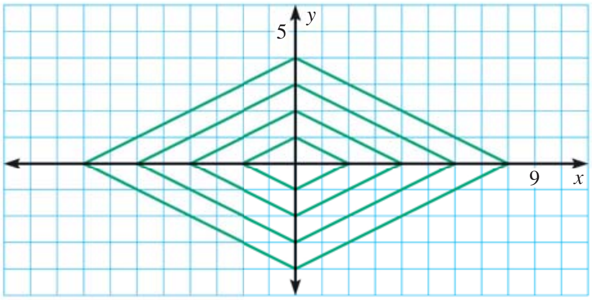
27. SHADOWS You and your friend are walking at night. You point a flashlight at your friend, and your friend's shadow is cast on the building behind him. The shadow is an enlargement, and is 15 feet tall. Your friend is 6 feet tall. What is the scale factor of the enlargement?

28. ★ **OPEN-ENDED MATH** *Describe* how you can use dilations to create the figure shown below.



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29. MULTI-STEP PROBLEM $\triangle ABC$ has vertices $A(3, -3)$, $B(3, 6)$, and $C(15, 6)$.

a. Draw a dilation of $\triangle ABC$ using a scale factor of $\frac{2}{3}$.

b. Find the ratio of the perimeter of the image to the perimeter of the original figure. How does this ratio compare to the scale factor?

c. Find the ratio of the area of the image to the area of the original figure. How does this ratio compare to the scale factor?

30. ★ **EXTENDED RESPONSE** Look at the coordinate notation for a dilation on page 409. Suppose the definition of dilation allowed $k < 0$.

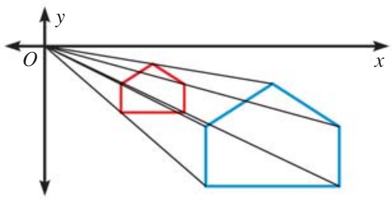
a. *Describe* the dilation if $-1 < k < 0$.

b. *Describe* the dilation if $k < -1$.

c. Use a rotation to describe a dilation with $k = -1$.

○ = WORKED-OUT SOLUTIONS on p. WS1   ★ = STANDARDIZED TEST PRACTICE

31. ★ **SHORT RESPONSE** *Explain* how you can use dilations to make a perspective drawing with the center of dilation as a vanishing point. Draw a diagram.



32. **MIDPOINTS** Let $\overline{XY}$ be a dilation of $\overline{PQ}$ with scale factor k. Show that the image of the midpoint of $\overline{PQ}$ is the midpoint of $\overline{XY}$.

33. **REASONING** In Exercise 32, show that $\overline{XY} \parallel \overline{PQ}$.

34. **CHALLENGE** A rectangle has vertices $A(0, 0)$, $B(0, 6)$, $C(9, 6)$, and $D(9, 0)$. *Explain* how to dilate the rectangle to produce an image whose area is twice the area of the original rectangle. Make a conjecture about how to dilate any polygon to produce an image whose area is n times the area of the original polygon.

MIXED REVIEW

Simplify the expression. *(p. 873)*

35. $(3x + 2)^2 + (x - 5)^2$

36. $4\left(\frac{1}{2}ab\right) + (b - a)^2$

37. $(a + b)^2 - (a - b)^2$

Find the distance between each pair of points. *(p. 15)*

38. (0, 5) and (4, 3)

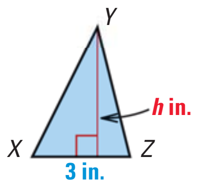
39. (−3, 0) and (2, 4)

40. (−2, −4) and (3, −2)

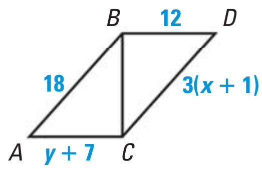
PREVIEW
Prepare for Lesson 7.1 in Exs. 41–43.

Find the value(s) of the variable(s).

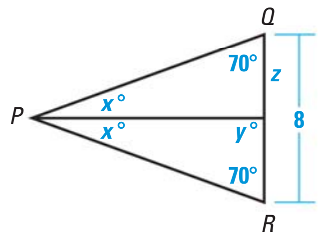
41. Area = 6 in.^{2} *(p. 49)*



42. $\triangle ABC \cong \triangle DCB$ *(p. 256)*



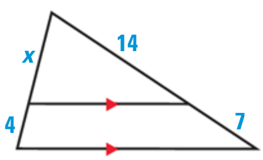
43. $\triangle PQR$ is isosceles. *(p. 303)*



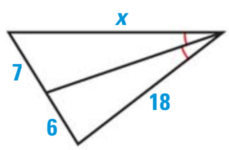
QUIZ for Lessons 6.6–6.7

Find the value of x. *(p. 397)*

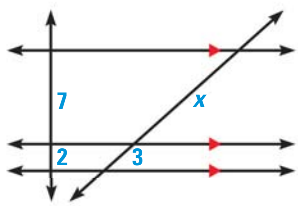
1.



2.



3.



Draw a dilation of $\triangle ABC$ with the given vertices and scale factor k. *(p. 409)*

4. $A(-5, 5)$, $B(-5, -10)$, $C(10, 0)$; $k = 0.4$

5. $A(-2, 1)$, $B(-4, 1)$, $C(-2, 4)$; $k = 2.5$

STATE TEST PRACTICE
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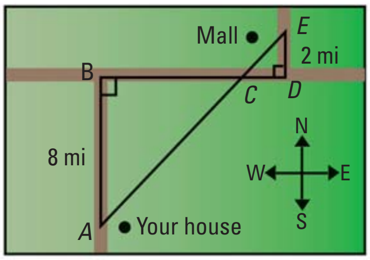
Lessons 6.4–6.7

1. **OPEN-ENDED** The diagram shows the front of a house. What information would you need in order to show that $\triangle WXY \sim \triangle VXZ$ using the SAS Similarity Theorem?

2. **EXTENDED RESPONSE** You leave your house to go to the mall. You drive due north 8 miles, due east 7.5 miles, and due north again 2 miles.

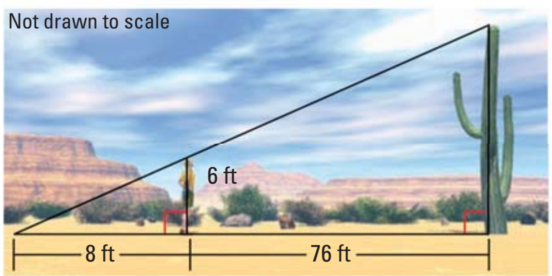


   a. *Explain* how to prove that $\triangle ABC \sim \triangle EDC$.

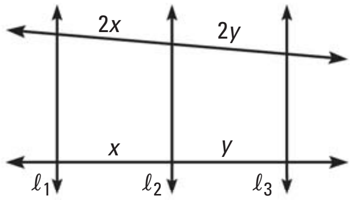
   b. Find CD.

   c. Find AE, the distance between your house and the mall.

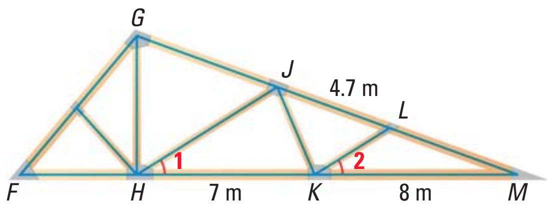
3. **SHORT RESPONSE** The Cardon cactus found in the Sonoran Desert in Mexico is the tallest type of cactus in the world. Marco stands 76 feet from the cactus so that his shadow coincides with the cactus' shadow. Marco is 6 feet tall and his shadow is 8 feet long. How tall is the Cardon cactus? *Explain.*



4. **SHORT RESPONSE** In the diagram, is it *always*, *sometimes*, or *never* true that $l_1 \parallel l_2 \parallel l_3$? *Explain.*



5. **GRIDDED ANSWER** In the diagram of the roof truss, $HK = 7$ meters, $KM = 8$ meters, $JL = 4.7$ meters, and $\angle 1 \cong \angle 2$. Find LM to the nearest tenth of a meter.



6. **GRIDDED ANSWER** You are designing a catalog for a greeting card company. The catalog features a $2\frac{4}{5}$ inch by 2 inch photograph of each card. The actual dimensions of a greeting card are 7 inches by 5 inches. What is the scale factor of the reduction?

7. **MULTI-STEP PROBLEM** Rectangle $ABCD$ has vertices $A(2, 2)$, $B(4, 2)$, $C(4, -4)$, and $D(2, -4)$.

   a. Draw rectangle $ABCD$. Then draw a dilation of rectangle $ABCD$ using a scale factor of $\frac{5}{4}$. Label the image $PQRS$.

   b. Find the ratio of the perimeter of the image to the perimeter of the original figure. How does this ratio compare to the scale factor?

   c. Find the ratio of the area of the image to the area of the original figure. How does this ratio compare to the scale factor?

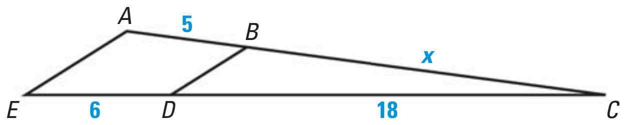
6 CHAPTER SUMMARY

BIG IDEAS *For Your Notebook*

Big Idea 1

Using Ratios and Proportions to Solve Geometry Problems

You can use properties of proportions to solve a variety of algebraic and geometric problems.



For example, in the diagram above, suppose you know that $\frac{AB}{BC} = \frac{ED}{DC}$. Then you can write any of the following relationships.

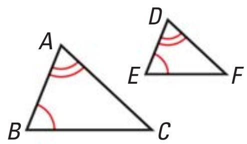
$\frac{5}{x} = \frac{6}{18}$ $\quad 5 \cdot 18 = 6x \quad$ $\frac{x}{5} = \frac{18}{6}$ $\quad$ $\frac{5}{6} = \frac{x}{18}$ $\quad$ $\frac{5 + x}{x} = \frac{6 + 18}{18}$

Big Idea 2

Showing that Triangles are Similar

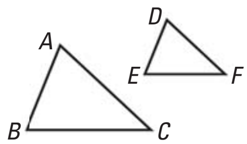
You learned three ways to prove two triangles are similar.

AA Similarity Postulate



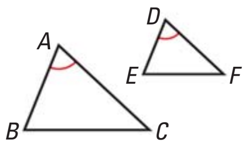
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem



If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

SAS Similarity Theorem

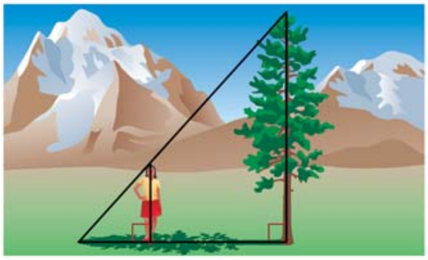


If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

Big Idea 3

Using Indirect Measurement and Similarity

You can use triangle similarity theorems to apply indirect measurement in order to find lengths that would be inconvenient or impossible to measure directly.



Consider the diagram shown. Because the two triangles formed by the person and the tree are similar by the AA Similarity Postulate, you can write the following proportion to find the height of the tree.

$$\frac{\text{height of person}}{\text{length of person's shadow}} = \frac{\text{height of tree}}{\text{length of tree's shadow}}$$

You also learned about dilations, a type of similarity transformation. In a dilation, a figure is either enlarged or reduced in size.

6 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- ratio, *p. 356*
- proportion, *p. 358*
  means, extremes
- geometric mean, *p. 359*
- scale drawing, *p. 365*
- scale, *p. 365*
- similar polygons, *p. 372*
- scale factor of two similar polygons, *p. 373*
- dilation, *p. 409*
- center of dilation, *p. 409*
- scale factor of a dilation, *p. 409*
- reduction, *p. 409*
- enlargement, *p. 409*

VOCABULARY EXERCISES

Copy and complete the statement.

1. A __?__ is a transformation in which the original figure and its image are similar.

2. If $\triangle PQR \sim \triangle XYZ$, then $\frac{PQ}{XY} = \frac{?}{YZ} = \frac{?}{?}$.

3. **WRITING** *Describe* the relationship between a ratio and a proportion. Give an example of each.

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 6.

6.1 Ratios, Proportions, and the Geometric Mean *pp. 356–363*

EXAMPLE

The measures of the angles in $\triangle ABC$ are in the extended ratio of 3 : 4 : 5. Find the measures of the angles.

Use the extended ratio of 3 : 4 : 5 to label the angle measures as $3x°$, $4x°$, and $5x°$.

$3x° + 4x° + 5x° = 180°$ **Triangle Sum Theorem**

$12x = 180$ **Combine like terms.**

$x = 15$ **Divide each side by 12.**

So, the angle measures are $3(15°) = 45°$, $4(15°) = 60°$, and $5(15°) = 75°$.

EXERCISES

EXAMPLES 1, 3, and 6 on pp. 356–359 for Exs. 4–6

4. The length of a rectangle is 20 meters and the width is 15 meters. Find the ratio of the width to the length of the rectangle. Then simplify the ratio.

5. The measures of the angles in $\triangle UVW$ are in the extended ratio of 1 : 1 : 2. Find the measures of the angles.

6. Find the geometric mean of 8 and 12.

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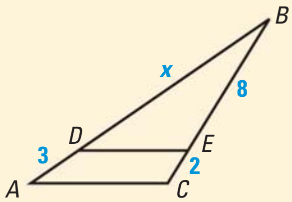
Chapter Review Practice

6.2 Use Proportions to Solve Geometry Problems *pp. 364–370*

EXAMPLE

In the diagram, $\frac{BA}{DA} = \frac{BC}{EC}$. Find BD.



$\frac{x+3}{3} = \frac{8+2}{2}$ Substitution Property of Equality

$2x + 6 = 30$ Cross Products Property

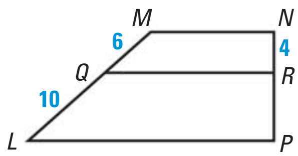
$x = 12$ Solve for *x*.

EXERCISES

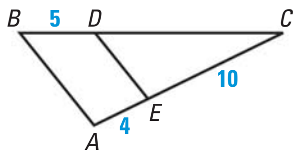
EXAMPLE 2 on p. 365 for Exs. 7–8

Use the diagram and the given information to find the unknown length.

7. Given $\frac{RN}{RP} = \frac{QM}{QL}$, find RP.



8. Given $\frac{CD}{DB} = \frac{CE}{EA}$, find CD.

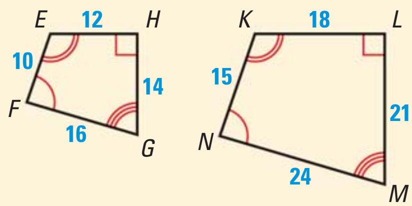


6.3 Use Similar Polygons *pp. 372–379*

EXAMPLE

In the diagram, $EHGF \sim KLMN$. Find the scale factor.

From the diagram, you can see that $\overline{EH}$ and $\overline{KL}$ correspond. So, the scale factor of $EHGF$ to $KLMN$ is $\frac{EH}{KL} = \frac{12}{18} = \frac{2}{3}$.

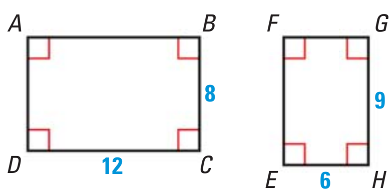


EXERCISES

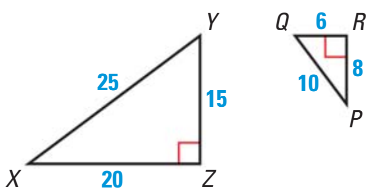
EXAMPLES 2 and 4 on pp. 373–374 for Exs. 9–11

In Exercises 9 and 10, determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor.

9.



10.



11. POSTERS Two similar posters have a scale factor of 4 : 5. The large poster's perimeter is 85 inches. Find the small poster's perimeter.

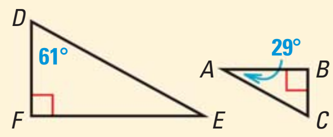
6 CHAPTER REVIEW

6.4 Prove Triangles Similar by AA

pp. 381–387

EXAMPLE

Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.



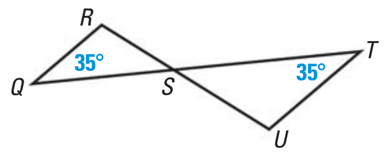
Because they are right angles, $\angle F \cong \angle B$. By the Triangle Sum Theorem, $61° + 90° + m\angle E = 180°$, so $m\angle E = 29°$ and $\angle E \cong \angle A$. Then, two angles of $\triangle DFE$ are congruent to two angles of $\triangle CBA$. So, $\triangle DFE \sim \triangle CBA$.

EXERCISES

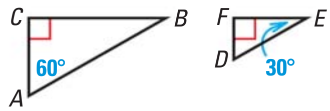
EXAMPLES 2 and 3 on pp. 382–383 for Exs. 12–14

Use the AA Similarity Postulate to show that the triangles are similar.

12.



13.



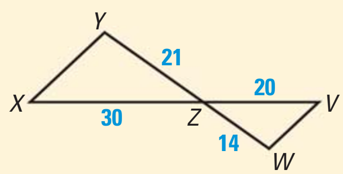
14. CELL TOWER A cellular telephone tower casts a shadow that is 72 feet long, while a tree nearby that is 27 feet tall casts a shadow that is 6 feet long. How tall is the tower?

6.5 Prove Triangles Similar by SSS and SAS

pp. 388–395

EXAMPLE

Show that the triangles are similar.



Notice that the lengths of two pairs of corresponding sides are proportional.

$$\frac{WZ}{YZ} = \frac{14}{21} = \frac{2}{3} \qquad \frac{VZ}{XZ} = \frac{20}{30} = \frac{2}{3}$$

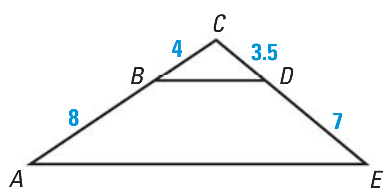
The included angles for these sides, $\angle XZY$ and $\angle VZW$, are vertical angles, so $\angle XZY \cong \angle VZW$. Then $\triangle XYZ \sim \triangle VWZ$ by the SAS Similarity Theorem.

EXERCISES

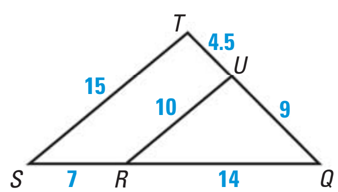
EXAMPLE 4 on p. 391 for Exs. 15–16

Use the SSS Similarity Theorem or SAS Similarity Theorem to show that the triangles are similar.

15.



16.



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Chapter Review Practice

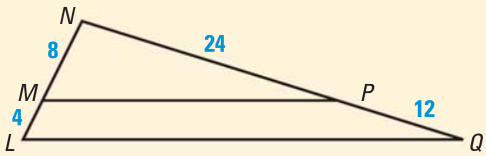
6.6 Use Proportionality Theorems

pp. 397–403

EXAMPLE

Determine whether $\overline{MP} \parallel \overline{LQ}$.

Begin by finding and simplifying ratios of lengths determined by $\overline{MP}$.



$\frac{NM}{ML} = \frac{8}{4} = \frac{2}{1}$ $\frac{NP}{PQ} = \frac{24}{12} = \frac{2}{1}$

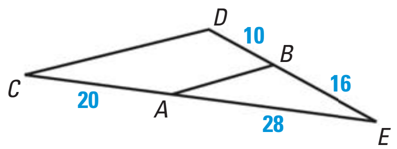
Because $\frac{NM}{ML} = \frac{NP}{PQ}$, $\overline{MP}$ is parallel to $\overline{LQ}$ by Theorem 6.5, the Triangle Proportionality Converse.

EXERCISES

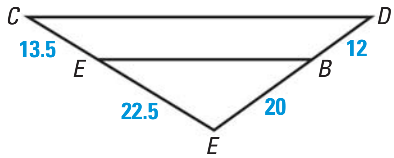
EXAMPLE 2
on p. 398
for Exs. 17–18

Use the given information to determine whether $\overline{AB} \parallel \overline{CD}$.

17.



18.



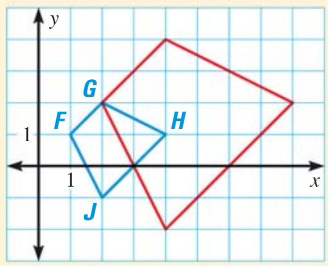
6.7 Perform Similarity Transformations

pp. 409–415

EXAMPLE

Draw a dilation of quadrilateral *FGHJ* with vertices *F*(1, 1), *G*(2, 2), *H*(4, 1), and *J*(2, −1). Use a scale factor of 2.

First draw *FGHJ*. Find the dilation of each vertex by multiplying its coordinates by 2. Then draw the dilation.



$(x, y) \rightarrow (2x, 2y)$

$F(1, 1) \rightarrow (2, 2)$

$G(2, 2) \rightarrow (4, 4)$

$H(4, 1) \rightarrow (8, 2)$

$J(2, -1) \rightarrow (4, -2)$

EXERCISES

EXAMPLES 1 and 2
on pp. 409–410
for Exs. 19–21

Draw a dilation of the polygon with the given vertices using the given scale factor *k*.

19. $T(0, 8)$, $U(6, 0)$, $V(0, 0)$; $k = \frac{3}{2}$

20. $A(6, 0)$, $B(3, 9)$, $C(0, 0)$, $D(3, 1)$; $k = 4$

21. $P(8, 2)$, $Q(4, 0)$, $R(3, 1)$, $S(6, 4)$; $k = 0.5$

6 CHAPTER TEST

Solve the proportion.

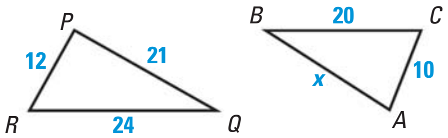
1. $\frac{6}{x} = \frac{9}{24}$ **2.** $\frac{5}{4} = \frac{y-5}{12}$ **3.** $\frac{3-2b}{4} = \frac{3}{2}$ **4.** $\frac{7}{2a+8} = \frac{1}{a-1}$

In Exercises 5–7, use the diagram where $\triangle PQR \sim \triangle ABC$.

5. List all pairs of congruent angles.

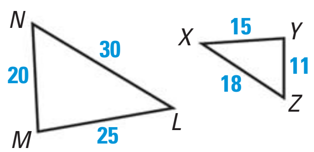
6. Write the ratios of the corresponding sides in a statement of proportionality.

7. Find the value of x.

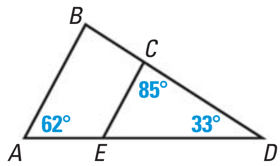


Determine whether the triangles are similar. If so, write a similarity statement and the postulate or theorem that justifies your answer.

8.



9.



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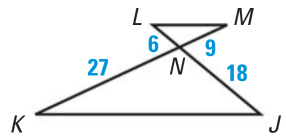
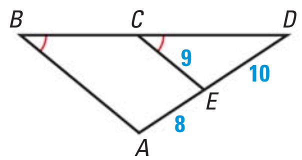
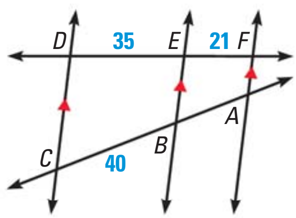


In Exercises 11–13, find the length of $\overline{AB}$.

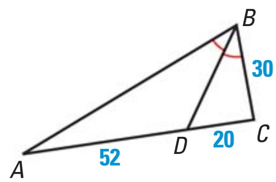
11.



12.

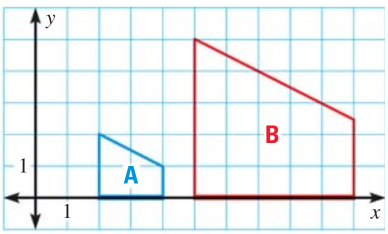


13.

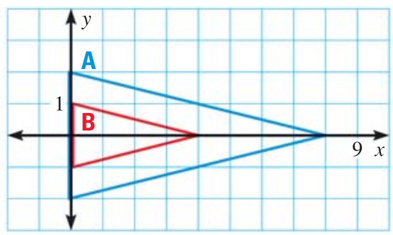


Determine whether the dilation from Figure A to Figure B is a *reduction* or an *enlargement*. Then find its scale factor.

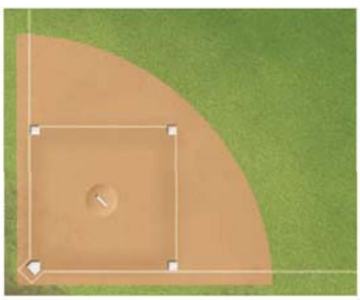
14.



15.



16. SCALE MODEL You are making a scale model of your school's baseball diamond as part of an art project. The distance between two consecutive bases is 90 feet. If you use a scale factor of $\frac{1}{180}$ to build your model, what will be the distance around the bases on your model?



SOLVE QUADRATIC EQUATIONS AND SIMPLIFY RADICALS

A radical expression is *simplified* when the radicand has no perfect square factor except 1, there is no fraction in the radicand, and there is no radical in a denominator.

EXAMPLE 1 Solve quadratic equations by finding square roots

Solve the equation $4x^2 - 3 = 109$.

$4x^2 - 3 = 109$	Write original equation.
$4x^2 = 112$	Add 3 to each side.
$x^2 = 28$	Divide each side by 4.
$x = \pm\sqrt{28}$	$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, so $\sqrt{28} = \pm\sqrt{4} \cdot \sqrt{7}$.
$x = \pm 2\sqrt{7}$	Simplify.

EXAMPLE 2 Simplify quotients with radicals

Simplify the expression.

a. $\sqrt{\frac{10}{8}}$ **b.** $\sqrt{\frac{1}{5}}$

Solution

a.

$\sqrt{\frac{10}{8}} = \sqrt{\frac{5}{4}}$	Simplify fraction.
$= \frac{\sqrt{5}}{\sqrt{4}}$	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.
$= \frac{\sqrt{5}}{2}$	Simplify.

b.

$\sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$. $\sqrt{1} = 1$.
$= \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$	Multiply numerator and denominator by $\sqrt{5}$.
$= \frac{\sqrt{5}}{5}$	Multiply fractions. $\sqrt{a} \cdot \sqrt{a} = a$.

EXERCISES

EXAMPLE 1 for Exs. 1–9

Solve the equation or write *no solution*.

1. $x^2 + 8 = 108$
2. $2x^2 - 1 = 49$
3. $x^2 - 9 = 8$
4. $5x^2 + 11 = 1$
5. $2(x^2 - 7) = 6$
6. $9 = 21 + 3x^2$
7. $3x^2 - 17 = 43$
8. $56 - x^2 = 20$
9. $-3(-x^2 + 5) = 39$

EXAMPLE 2 for Exs. 10–17

Simplify the expression.

10. $\sqrt{\frac{7}{81}}$
11. $\sqrt{\frac{3}{5}}$
12. $\sqrt{\frac{24}{27}}$
13. $\frac{3\sqrt{7}}{\sqrt{12}}$
14. $\sqrt{\frac{75}{64}}$
15. $\frac{\sqrt{2}}{\sqrt{200}}$
16. $\frac{9}{\sqrt{27}}$
17. $\sqrt{\frac{21}{42}}$

Scoring Rubric

Full Credit
- solution is complete and correct

Partial Credit
- solution is complete but has errors,

or
- solution is without error but is incomplete

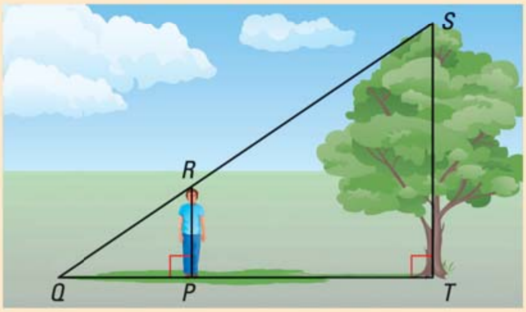
No Credit
- no solution is given,

or
- solution makes no sense

EXTENDED RESPONSE QUESTIONS

PROBLEM

To find the height of a tree, a student 63 inches in height measures the length of the tree's shadow and the length of his own shadow, as shown. The student casts a shadow 81 inches in length and the tree casts a shadow 477 inches in length.



a. Explain why $\triangle PQR \sim \triangle TQS$.

b. Find the height of the tree.

c. Suppose the sun is a little lower in the sky. Can you still use this method to measure the height of the tree? *Explain.*

Below are sample solutions to the problem. Read each solution and the comments in blue to see why the sample represents full credit, partial credit, or no credit.

SAMPLE 1: Full credit solution

The reasoning is complete.

a. Because they are both right angles, $\angle QPR \cong \angle QTS$. Also, $\angle Q \cong \angle Q$ by the Reflexive Property. So, $\triangle PQR \sim \triangle TQS$ by the AA Similarity Postulate.

The proportion and calculations are correct.

b.

$$\frac{PR}{PQ} = \frac{TS}{TQ}$$

$$\frac{63}{81} = \frac{TS}{477}$$

$$63(477) = 81 \cdot TS$$

$$371 = TS$$

The height of the tree is 371 inches.

In part (b), the question is answered correctly.

c. As long as the sun creates two shadows, I can use this method. Angles P and T will always be right angles. The measure of $\angle Q$ will change as the sun's position changes, but the angle will still be congruent to itself. So, $\triangle PQR$ and $\triangle TQS$ will still be similar, and I can write a proportion.

In part (c), the reasoning is complete and correct.

Sample 2: Partial credit solution

In part (a), there is no explanation of why the postulate can be applied.

In part (b), the proportion is incorrect, which leads to an incorrect solution.

In part (c), a partial explanation is given.

a. $\triangle PQR \sim \triangle TQS$ by the Angle-Angle Similarity Postulate.

b. $\frac{PR}{PQ} = \frac{TS}{TP}$

$\frac{63}{81} = \frac{TS}{396}$

$308 = TS$

The height of the tree is 308 inches.

c. As long as the sun creates two shadows, I can use this method because the triangles will always be similar.

Sample 3: No credit solution

The reasoning in part (a) is incomplete.

In part (b), no work is shown.

The answer in part (c) is incorrect.

a. The triangles are similar because the lines are parallel and the angles are congruent.

b. $TS = 371$ inches

c. No. The angles in the triangle will change, so you can't write a proportion.

PRACTICE Apply the Scoring Rubric

1. A student's solution to the problem on the previous page is given below. Score the solution as *full credit*, *partial credit*, or *no credit*. *Explain* your reasoning. If you choose *partial credit* or *no credit*, *explain* how you would change the solution so that it earns a score of full credit.

a. $\angle QPR \cong \angle PTS$, and $\angle Q$ is in both triangles. So, $\triangle PQR \sim \triangle TQS$.

b. $\frac{PR}{PQ} = \frac{QT}{ST}$

$\frac{63}{81} = \frac{477}{x}$

$63x = 81(477)$

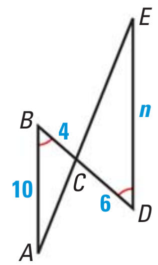
$x \approx 613.3$

The tree is about 613.3 inches tall.

c. The method will still work because the triangles will still be similar if the sun changes position. The right angles will stay right angles, and $\angle Q$ is in both triangles, so it does not matter if its measure changes.

EXTENDED RESPONSE

1. Use the diagram.

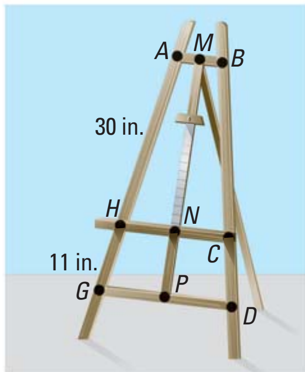


a. *Explain* how you know that $\triangle ABC \sim \triangle EDC$.

b. Find the value of n.

c. The perimeter of $\triangle ABC$ is 22. What is the perimeter of $\triangle EDC$? *Justify* your answer.

2. On the easel shown at the right, $\overline{AB} \parallel \overline{HC} \parallel \overline{GD}$, and $\overline{AG} \cong \overline{BD}$.



a. Find BD, BC, and CD. *Justify* your answer.

b. On the easel, $\overline{MP}$ is a support bar attached to $\overline{AB}$, $\overline{HC}$, and $\overline{GD}$. On this support bar, $NP = 10$ inches. Find the length of $\overline{MP}$ to the nearest inch. *Justify* your answer.

c. The support bar $\overline{MP}$ bisects $\overline{AB}$, $\overline{HC}$, and $\overline{GD}$. Does this mean that polygons $AMNH$ and $AMPG$ are similar? *Explain*.

3. A handmade rectangular rug is available in two sizes at a rug store. A small rug is 24 inches long and 16 inches wide. A large rug is 36 inches long and 24 inches wide.

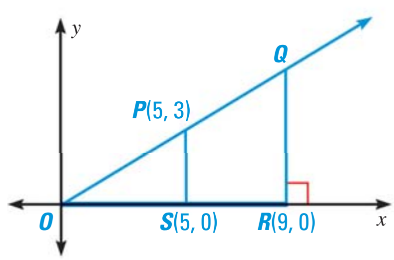
a. Are the rugs similar? If so, what is the ratio of their corresponding sides? *Explain*.

b. Find the perimeter and area of each rug. Then find the ratio of the perimeters (large rug to small rug) and the ratio of the areas (large rug to small rug).

c. It takes 250 feet of wool yarn to make 1 square foot of either rug. How many inches of yarn are used for each rug? *Explain*.

d. The price of a large rug is 1.5 times the price of a small rug. The store owner wants to change the prices for the rugs, so that the price for each rug is based on the amount of yarn used to make the rug. If the owner changes the prices, about how many times as much will the price of a large rug be than the price of a small rug? *Explain*.

4. In the diagram shown at the right, $\overleftrightarrow{OQ}$ passes through the origin.



a. Explain how you know that $\triangle OPS \sim \triangle OQR$.

b. Find the coordinates of point Q. *Justify* your answer.

c. The x-coordinate of a point on $\overleftrightarrow{OQ}$ is a. Write the y-coordinate of this point in terms of a. *Justify* your answer.

MULTIPLE CHOICE

5. If $\triangle PQR \sim \triangle STU$, which proportion is not necessarily true?

Ⓐ $\dfrac{PQ}{QR} = \dfrac{ST}{TU}$ Ⓑ $\dfrac{PQ}{SU} = \dfrac{PR}{TU}$

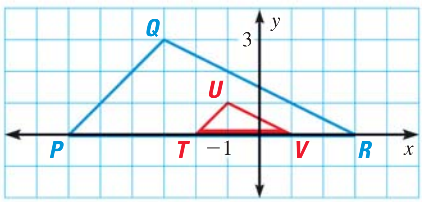
Ⓒ $\dfrac{PR}{SU} = \dfrac{QR}{TU}$ Ⓓ $\dfrac{PQ}{PR} = \dfrac{ST}{SU}$

6. On a map, the distance between two cities is $2\frac{3}{4}$ inches. The scale on the map is 1 in.:80 mi. What is the actual distance between the two cities?

Ⓐ 160 mi Ⓑ 180 mi

Ⓒ 200 mi Ⓓ 220 mi

7. In the diagram, what is the scale factor of the dilation from $\triangle PQR$ to $\triangle TUV$?

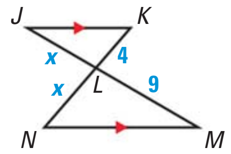


Ⓐ $\frac{1}{2}$ Ⓑ $\frac{1}{3}$

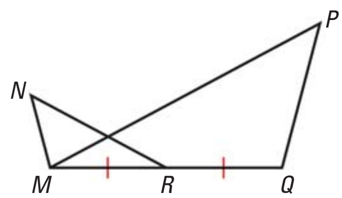
Ⓒ 2 Ⓓ 3

GRIDDED ANSWER

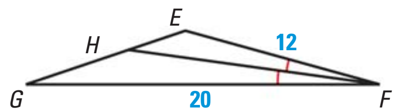
8. Find the value of x.



9. In the diagram below, $\triangle PQM \sim \triangle NMR$, and $\overline{MR} \cong \overline{QR}$. If $NR = 12$, find PM.



10. Given $GE = 10$, find HE.



11. In an acute isosceles triangle, the measures of two of the angles are in the ratio 4 : 1. Find the measure of a base angle in the triangle.

SHORT RESPONSE

12. On a school campus, the gym is 400 feet from the art studio.

a. Suppose you draw a map of the school campus using a scale of $\frac{1}{4}$ inch: 100 feet. How far will the gym be from the art studio on your map?

b. Suppose you draw a map of the school campus using a scale of $\frac{1}{2}$ inch : 100 feet. Will the distance from the gym to the art studio on this map be *greater than* or *less than* the distance on the map in part (a)? *Explain.*

13. Rectangles *ABCD* and *EFGH* are similar, and the ratio of *AB* to *EF* is 1 : 3. In each rectangle, the length is twice the width. The area of *ABCD* is 32 square inches. Find the length, width, and area of *EFGH*. *Explain.*

Find $m\angle 2$ if $\angle 1$ and $\angle 2$ are (a) complementary angles and (b) supplementary angles. *(p. 24)*

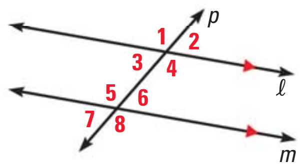
1. $m\angle 1 = 57°$
2. $m\angle 1 = 23°$
3. $m\angle 1 = 88°$
4. $m\angle 1 = 46°$

Solve the equation and write a reason for each step. *(p. 105)*

5. $3x - 19 = 47$
6. $30 - 4(x - 3) = -x + 18$
7. $-5(x + 2) = 25$

State the postulate or theorem that justifies the statement. *(pp. 147, 154)*

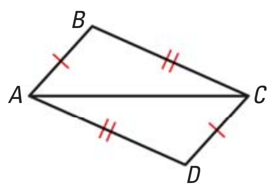
8. $\angle 1 \cong \angle 8$
9. $\angle 3 \cong \angle 6$
10. $m\angle 3 + m\angle 5 = 180°$
11. $\angle 3 \cong \angle 7$
12. $\angle 2 \cong \angle 3$
13. $m\angle 7 + m\angle 8 = 180°$

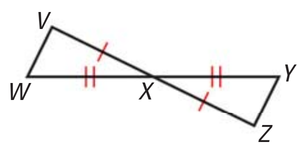


The variable expressions represent the angle measures of a triangle. Find the measure of each angle. Then classify the triangle by its angles. *(p. 217)*

14. $m\angle A = x°$
 $m\angle B = 3x°$
 $m\angle C = 4x°$
15. $m\angle A = 2x°$
 $m\angle B = 2x°$
 $m\angle C = (x - 15)°$
16. $m\angle A = (3x - 15)°$
 $m\angle B = (x + 5)°$
 $m\angle C = (x - 20)°$

Determine whether the triangles are congruent. If so, write a congruence statement and state the postulate or theorem you used. *(pp. 234, 240, 249)*

17.


18.


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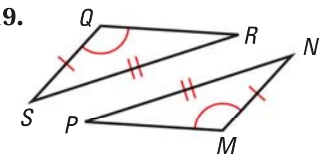
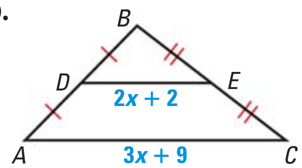
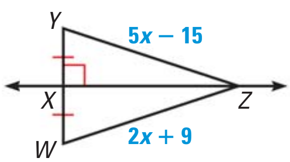
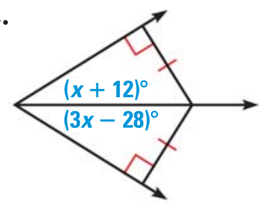


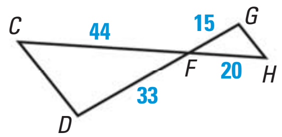
Find the value of *x*. *(pp. 295, 303, 310)*

20.


21.


22.


Determine whether the triangles are similar. If they are, write a similarity statement and state the postulate or theorem you used. *(pp. 381, 388)*

23.


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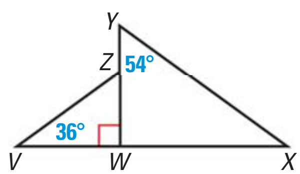
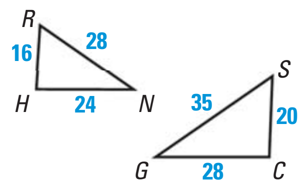


25.


26. PROFITS A company's profits for two years are shown in the table. Plot and connect the points (x, y). Use the Midpoint Formula to estimate the company's profits in 2003. (Assume that profits followed a linear pattern.) *(p. 15)*

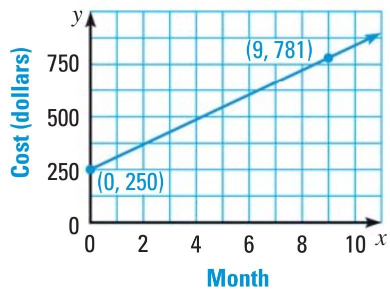
Years since 2000, x	1	5
Profit, y (in dollars)	21,000	36,250

27. TENNIS MEMBERSHIP The graph at the right models the accumulated cost for an individual adult tennis club membership for several months. *(p. 180)*

a. Write an equation of the line.

b. Tell what the slope and y-intercept mean in this situation.

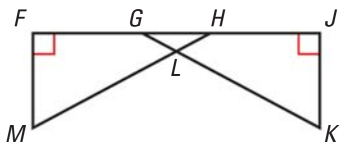
c. Find the accumulated cost for one year.



PROOF **Write a two-column proof or a paragraph proof.** *(pp. 234, 240, 249)*

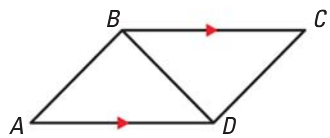
28. GIVEN ▶ $\overline{FG} \cong \overline{HJ}$, $\overline{MH} \cong \overline{KG}$, $\overline{MF} \perp \overline{FJ}$, $\overline{KJ} \perp \overline{FJ}$

PROVE ▶ $\triangle FHM \cong \triangle JGK$

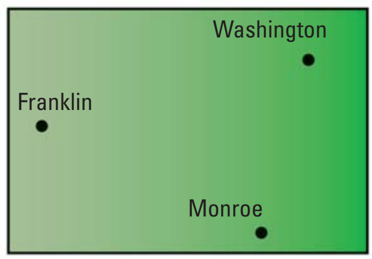


29. GIVEN ▶ $\overline{BC} \parallel \overline{AD}$, $\overline{BC} \cong \overline{AD}$

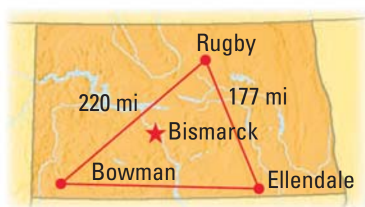
PROVE ▶ $\triangle BCD \cong \triangle DAB$



30. COMMUNITY CENTER A building committee needs to choose a site for a new community center. The committee decides that the new center should be located so that it is the same distance from each of the three local schools. Use the diagram to make a sketch of the triangle formed by the three schools. *Explain* how you can use this triangle to locate the site for the new community center. *(p. 303)*



31. GEOGRAPHY The map shows the distances between three cities in North Dakota. *Describe* the range of possible distances from Bowman to Ellendale. *(p. 328)*



32. CALENDAR You send 12 photos to a company that makes personalized wall calendars. The company enlarges the photos and inserts one for each month on the calendar. Each photo is 4 inches by 6 inches. The image for each photo on the calendar is 10 inches by 15 inches. What is the scale factor of the enlargement? *(p. 409)*