2 Reasoning and Proof

Before

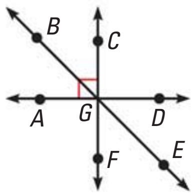
In previous courses and in Chapter 1, you learned the following skills, which you'll use in Chapter 2: naming figures, using notations, drawing diagrams, solving equations, and using postulates.

Prerequisite Skills

VOCABULARY CHECK

Use the diagram to name an example of the described figure.

1. A right angle
2. A pair of vertical angles
3. A pair of supplementary angles
4. A pair of complementary angles



SKILLS AND ALGEBRA CHECK

Describe **what the notation means. Draw the figure.** *(Review p. 2 for 2.4.)*

5. $\overline{AB}$ **6.** $\overleftrightarrow{CD}$ **7.** EF **8.** $\overrightarrow{GH}$

Solve the equation. *(Review p. 875 for 2.5.)*

9. $3x + 5 = 20$ **10.** $4(x - 7) = -12$ **11.** $5(x + 8) = 4x$

Name the postulate used. Draw the figure. *(Review pp. 9, 24 for 2.5.)*

12. $m\angle ABD + m\angle DBC = m\angle ABC$ **13.** $ST + TU = SU$

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 2, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 133. You will also use the key vocabulary listed below.

Big Ideas

1. **Use inductive and deductive reasoning**
2. **Understanding geometric relationships in diagrams**
3. **Writing proofs of geometric relationships**

KEY VOCABULARY

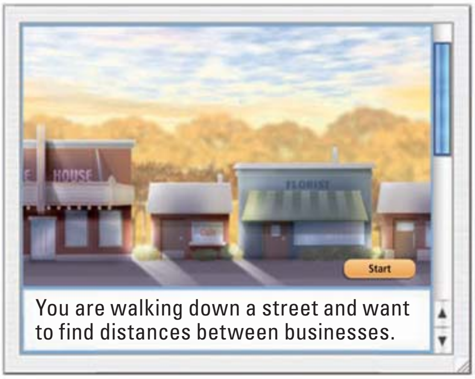
- conjecture, *p. 73*
- inductive reasoning, *p. 73*
- counterexample, *p. 74*
- conditional statement, *p. 79*
  converse, inverse, contrapositive
- if-then form, *p. 79*
  hypothesis, conclusion
- negation, *p. 79*
- equivalent statements, *p. 80*
- perpendicular lines, *p. 81*
- biconditional statement, *p. 82*
- deductive reasoning, *p. 87*
- proof, *p. 112*
- two-column proof, *p. 112*
- theorem, *p. 113*

Why?

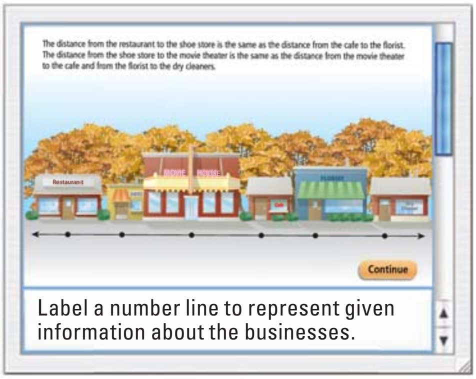
You can use reasoning to draw conclusions. For example, by making logical conclusions from organized information, you can make a layout of a city street.

Animated Geometry

The animation illustrated below for Exercise 29 on page 119 helps you answer this question: Is the distance from the restaurant to the movie theater the same as the distance from the cafe to the dry cleaners?



You are walking down a street and want to find distances between businesses.



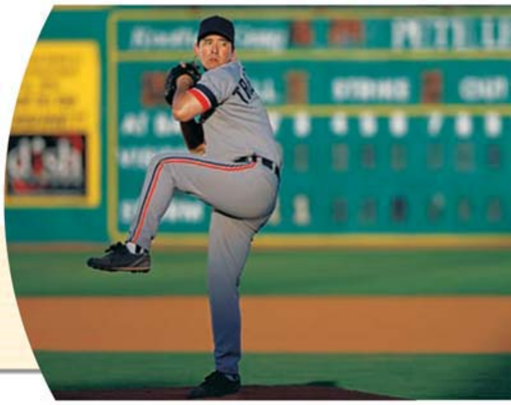
The distance from the restaurant to the shoe store is the same as the distance from the cafe to the florist. The distance from the shoe store to the movie theater is the same as the distance from the movie theater to the cafe and from the florist to the dry cleaners.

Label a number line to represent given information about the businesses.

Animated Geometry at classzone.com

Other animations for Chapter 2: pages 72, 81, 88, 97, 106, and 125

2.1 Use Inductive Reasoning



Before You classified polygons by the number of sides.

Now You will describe patterns and use inductive reasoning.

Why? So you can make predictions about baseball, as in Ex. 32.

Key Vocabulary
- conjecture
- inductive reasoning
- counterexample

Geometry, like much of science and mathematics, was developed partly as a result of people recognizing and describing patterns. In this lesson, you will discover patterns yourself and use them to make predictions.

EXAMPLE 1 Describe a visual pattern

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.

Figure 1

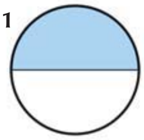


Figure 2

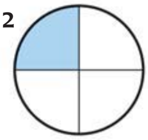
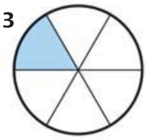


Figure 3



Solution

Each circle is divided into twice as many equal regions as the figure number. Sketch the fourth figure by dividing a circle into eighths. Shade the section just above the horizontal segment at the left.

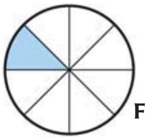


Figure 4

EXAMPLE 2 Describe a number pattern

READ SYMBOLS

The three dots (. . .) tell you that the pattern continues.

Describe the pattern in the numbers −7, −21, −63, −189, . . . and write the next three numbers in the pattern.

Notice that each number in the pattern is three times the previous number.

$$-7, \xrightarrow{\times 3} -21, \xrightarrow{\times 3} -63, \xrightarrow{\times 3} -189, \ldots \xrightarrow{\times 3}$$

▶ Continue the pattern. The next three numbers are −567, −1701, and −5103.

AnimatedGeometry at classzone.com

✓ GUIDED PRACTICE for Examples 1 and 2

1. Sketch the fifth figure in the pattern in Example 1.
2. *Describe* the pattern in the numbers 5.01, 5.03, 5.05, 5.07, ... Write the next three numbers in the pattern.

INDUCTIVE REASONING A **conjecture** is an unproven statement that is based on observations. You use **inductive reasoning** when you find a pattern in specific cases and then write a conjecture for the general case.

EXAMPLE 3 Make a conjecture

Given five collinear points, make a conjecture about the number of ways to connect different pairs of the points.

Solution

Make a table and look for a pattern. Notice the pattern in how the number of connections increases. You can use the pattern to make a conjecture.

Number of points	1	2	3	4	5
Picture					
Number of connections	0	1	3	6	?

+ 1 + 2 + 3 + ?

- **Conjecture** You can connect five collinear points 6 + 4, or 10 different ways.

EXAMPLE 4 Make and test a conjecture

Numbers such as 3, 4, and 5 are called *consecutive numbers.* Make and test a conjecture about the sum of any three consecutive numbers.

Solution

STEP 1 **Find** a pattern using a few groups of small numbers.

$3 + 4 + 5 = 12 = 4 \cdot 3$ $7 + 8 + 9 = 24 = 8 \cdot 3$

$10 + 11 + 12 = 33 = 11 \cdot 3$ $16 + 17 + 18 = 51 = 17 \cdot 3$

- **Conjecture** The sum of any three consecutive integers is three times the second number.

STEP 2 **Test** your conjecture using other numbers. For example, test that it works with the groups −1, 0, 1 and 100, 101, 102.

$-1 + 0 + 1 = 0 = 0 \cdot 3$ ✓ $100 + 101 + 102 = 303 = 101 \cdot 3$ ✓

✓ GUIDED PRACTICE for Examples 3 and 4

3. Suppose you are given seven collinear points. Make a conjecture about the number of ways to connect different pairs of the points.

4. Make and test a conjecture about the sign of the product of any three negative integers.

DISPROVING CONJECTURES To show that a conjecture is true, you must show that it is true for all cases. You can show that a conjecture is false, however, by simply finding one *counterexample*. A **counterexample** is a specific case for which the conjecture is false.

EXAMPLE 5 Find a counterexample

A student makes the following conjecture about the sum of two numbers. Find a counterexample to disprove the student's conjecture.

Conjecture The sum of two numbers is always greater than the larger number.

Solution

To find a counterexample, you need to find a sum that is less than the larger number.

$$-2 + -3 = -5$$
$$-5 \ngtr -3$$

▶ Because a counterexample exists, the conjecture is false.

★ EXAMPLE 6 Standardized Test Practice

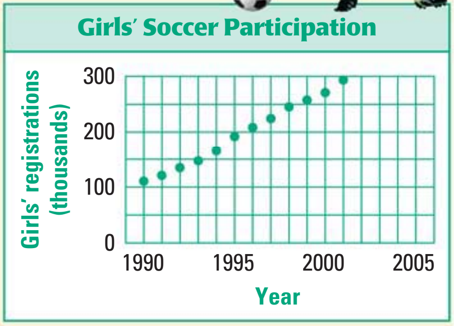
Which conjecture could a high school athletic director make based on the graph at the right?

Ⓐ More boys play soccer than girls.

Ⓑ More girls are playing soccer today than in 1995.

Ⓒ More people are playing soccer today than in the past because the 1994 World Cup games were held in the United States.

Ⓓ The number of girls playing soccer was more in 1995 than in 2001.



ELIMINATE CHOICES
Because the graph does not show data about boys or the World Cup games, you can eliminate choices A and C.

Solution

Choices A and C can be eliminated because they refer to facts not presented by the graph. Choice B is a reasonable conjecture because the graph shows an increase from 1990–2001, but does not give any reasons for that increase.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 5 and 6

5. Find a counterexample to show that the following conjecture is false.

Conjecture The value of x^2 is always greater than the value of x.

6. Use the graph in Example 6 to make a conjecture that *could* be true. Give an explanation that supports your reasoning.

2.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 15, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 5, 19, 22, and 36

◆ = **MULTIPLE REPRESENTATIONS** Ex. 35

SKILL PRACTICE

1. **VOCABULARY** Write a definition of *conjecture* in your own words.

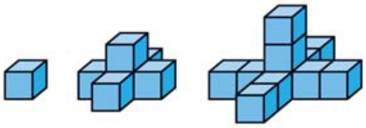
2. ★ **WRITING** The word *counter* has several meanings. Look up the word in a dictionary. Identify which meaning helps you understand the definition of *counterexample*.

EXAMPLE 1
on p. 72
for Exs. 3–5

SKETCHING VISUAL PATTERNS **Sketch the next figure in the pattern.**

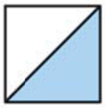
3.

4.



5. ★ **MULTIPLE CHOICE** What is the next figure in the pattern?

Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

EXAMPLE 2
on p. 72
for Exs. 6–11

DESCRIBING NUMBER PATTERNS ***Describe*** **the pattern in the numbers. Write the next number in the pattern.**

6. 1, 5, 9, 13, . . .

7. 3, 12, 48, 192, . . .

8. 10, 5, 2.5, 1.25, . . .

9. 4, 3, 1, −2, . . .

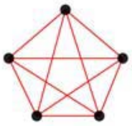
10. $1, \frac{2}{3}, \frac{1}{3}, 0, \ldots$

11. −5, −2, 4, 13, . . .

MAKING CONJECTURES **In Exercises 12 and 13, copy and complete the conjecture based on the pattern you observe in the specific cases.**

EXAMPLE 3
on p. 73
for Ex. 12

12. Given seven noncollinear points, make a conjecture about the number of ways to connect different pairs of the points.

Number of points	3	4	5	6	7
Picture					?
Number of connections	3	6	10	15	?

Conjecture You can connect seven noncollinear points __?__ different ways.

EXAMPLE 4
on p. 73
for Ex. 13

13. Use these sums of odd integers: 3 + 7 = 10, 1 + 7 = 8, 17 + 21 = 38

Conjecture The sum of any two odd integers is __?__.

EXAMPLE 5
on p. 74
for Exs. 14–17

FINDING COUNTEREXAMPLES In Exercises 14–17, show the conjecture is false by finding a counterexample.

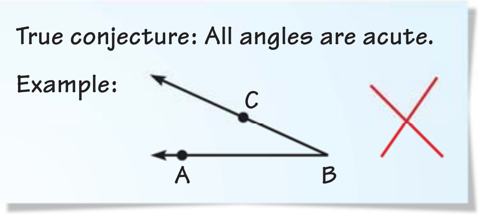
14. If the product of two numbers is positive, then the two numbers must both be positive.

15. The product $(a + b)^2$ is equal to $a^2 + b^2$, for $a \neq 0$ and $b \neq 0$.

16. All prime numbers are odd.

17. If the product of two numbers is even, then the two numbers must both be even.

18. **ERROR ANALYSIS** *Describe* and correct the error in the student's reasoning.


True conjecture: All angles are acute.

Example:

19. ★ **SHORT RESPONSE** *Explain* why only one counterexample is necessary to show that a conjecture is false.

xy **ALGEBRA In Exercises 20 and 21, write a function rule relating x and y.**

20.

x	1	2	3
y	−3	−2	−1

21.

x	1	2	3
y	2	4	6

22. ★ **MULTIPLE CHOICE** What is the first number in the pattern?

?, ?, ?, 81, 243, 729

Ⓐ 1 Ⓑ 3 Ⓒ 9 Ⓓ 27

MAKING PREDICTIONS ***Describe*** **a pattern in the numbers. Write the next number in the pattern. Graph the pattern on a number line.**

23. $2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \ldots$

24. 1, 8, 27, 64, 125, . . .

25. 0.45, 0.7, 0.95, 1.2, . . .

26. 1, 3, 6, 10, 15, . . .

27. 2, 20, 10, 100, 50, . . .

28. $0.4(6), 0.4(6)^2, 0.4(6)^3, \ldots$

29. xy **ALGEBRA** Consider the pattern $5, 5r, 5r^2, 5r^3, \ldots$. For what values of r will the values of the numbers in the pattern be increasing? For what values of r will the values of the numbers be decreasing? *Explain.*

30. **REASONING** A student claims that the next number in the pattern 1, 2, 4, . . . is 8, because each number shown is two times the previous number. Is there another description of the pattern that will give the same first three numbers but will lead to a different pattern? *Explain.*

31. **CHALLENGE** Consider the pattern $1, 1\frac{1}{2}, 1\frac{3}{4}, 1\frac{7}{8}, \ldots$.

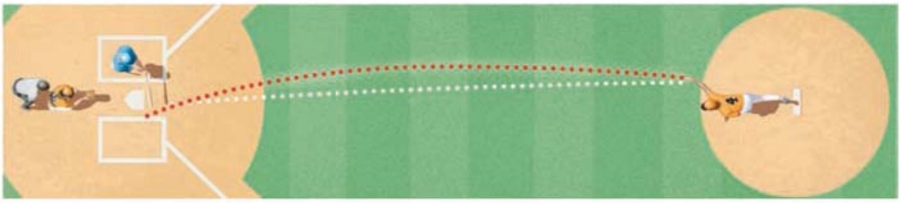
a. *Describe* the pattern. Write the next three numbers in the pattern.

b. What is happening to the values of the numbers?

c. Make a conjecture about later numbers. *Explain* your reasoning.

PROBLEM SOLVING

32. **BASEBALL** You are watching a pitcher who throws two types of pitches, a fastball (F, in white below) and a curveball (C, in red below). You notice that the order of pitches was F, C, F, F, C, C, F, F, F. Assuming that this pattern continues, predict the next five pitches.



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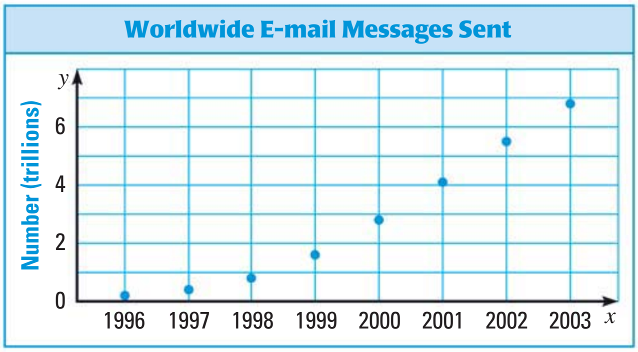
EXAMPLE 6 on p. 74 for Ex. 33

33. **STATISTICS** The scatter plot shows the number of person-to-person e-mail messages sent each year. Make a conjecture that *could* be true. Give an explanation that supports your reasoning.



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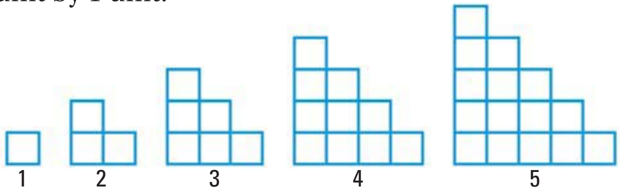
34. **VISUAL REASONING** Use the pattern below. Each figure is made of squares that are 1 unit by 1 unit.



a. Find the distance around each figure. Organize your results in a table.

b. Use your table to *describe* a pattern in the distances.

c. Predict the distance around the 20th figure in this pattern.

35. **MULTIPLE REPRESENTATIONS** Use the given function table relating x and y.

a. Making a Table Copy and complete the table.

b. Drawing a Graph Graph the table of values.

c. Writing an Equation *Describe* the pattern in words and then write an equation relating x and y.

x	y
−3	−5
?	1
5	11
?	15
12	?
15	31

36. ★ **EXTENDED RESPONSE** Your class is selling raffle tickets for \$.25 each.

a. Make a table showing your income if you sold 0, 1, 2, 3, 4, 5, 10, or 20 raffle tickets.

b. Graph your results. *Describe* any pattern you see.

c. Write an equation for your income y if you sold x tickets.

d. If your class paid \$14 for the raffle prize, at least how many tickets does your class need to sell to make a profit? *Explain*.

e. How many tickets does your class need to sell to make a profit of \$50?

37. **FIBONACCI NUMBERS** The *Fibonacci numbers* are shown below. Use the Fibonacci numbers to answer the following questions.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, . . .

a. Copy and complete: After the first two numbers, each number is the __?__ of the __?__ previous numbers.

b. Write the next three numbers in the pattern.

c. **Research** This pattern has been used to describe the growth of the *nautilus shell*. Use an encyclopedia or the Internet to find another real-world example of this pattern.

38. **CHALLENGE** Set A consists of all multiples of 5 greater than 10 and less than 100. Set B consists of all multiples of 8 greater than 16 and less than 100. Show that each conjecture is false by finding a counterexample.

a. Any number in set A is also in set B.

b. Any number less than 100 is either in set A or in set B.

c. No number is in both set A and set B.

MIXED REVIEW

Use the Distributive Property to write the expression without parentheses. *(p. 872)*

39. $4(x - 5)$ **40.** $-2(x - 7)$ **41.** $(-2n + 5)4$ **42.** $x(x + 8)$

PREVIEW
Prepare for Lesson 2.2 in Exs. 43–46.

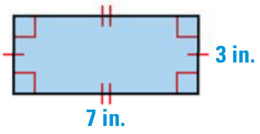
You ask your friends how many pets they have. The results are: 1, 5, 1, 0, 3, 6, 4, 2, 10, and 1. Use these data in Exercises 43–46. *(p. 887)*

43. Find the mean. **44.** Find the median. **45.** Find the mode(s).

46. Tell whether the *mean*, *median*, or *mode(s)* best represent(s) the data.

Find the perimeter and area of the figure. *(p. 49)*

47.



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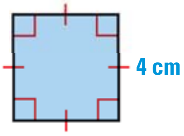

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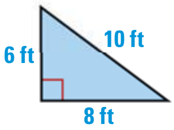


2.2 Analyze Conditional Statements

Before You used definitions.
Now You will write definitions as conditional statements.
Why? So you can verify statements, as in Example 2.

Key Vocabulary
- **conditional statement**
  converse, inverse, contrapositive
- **if-then form**
  hypothesis, conclusion
- **negation**
- **equivalent statements**
- **perpendicular lines**
- **biconditional statement**

A **conditional statement** is a logical statement that has two parts, a *hypothesis* and a *conclusion*. When a conditional statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**. Here is an example:

$$\text{If } \underbrace{\textbf{it is raining}}_{\text{Hypothesis}}\text{, then } \underbrace{\textbf{there are clouds in the sky}}_{\text{Conclusion}}.$$

EXAMPLE 1 Rewrite a statement in if-then form

Rewrite the conditional statement in if-then form.

a. All birds have feathers.

b. Two angles are supplementary if they are a linear pair.

Solution

First, identify the **hypothesis** and the **conclusion**. When you rewrite the statement in if-then form, you may need to reword the hypothesis or conclusion.

a. **All birds** have **feathers**.
If **an animal is a bird**, then **it has feathers**.

b. **Two angles are supplementary** if **they are a linear pair**.
If **two angles are a linear pair**, then **they are supplementary**.

✓ GUIDED PRACTICE for Example 1

Rewrite the conditional statement in if-then form.

1. All 90° angles are right angles.

2. $2x + 7 = 1$, because $x = -3$.

3. When $n = 9$, $n^2 = 81$.

4. Tourists at the Alamo are in Texas.

NEGATION The **negation** of a statement is the *opposite* of the original statement. Notice that Statement 2 is already negative, so its negation is positive.

Statement 1 The ball is red.
Negation 1 The ball is *not* red.

Statement 2 The cat is *not* black.
Negation 2 The cat is black.

VERIFYING STATEMENTS Conditional statements can be true or false. To show that a conditional statement is true, you must prove that the conclusion is true every time the hypothesis is true. To show that a conditional statement is false, you need to give *only one* counterexample.

RELATED CONDITIONALS To write the **converse** of a conditional statement, exchange the **hypothesis** and **conclusion**.

READ VOCABULARY
To *negate* part of a conditional statement, you write its negation.

To write the **inverse** of a conditional statement, negate both the hypothesis and the conclusion. To write the **contrapositive**, first write the converse and then negate both the hypothesis and the conclusion.

Conditional statement	If $m\angle A = 99°$, then $\angle A$ is obtuse.
Converse	If $\angle A$ is obtuse, then $m\angle A = 99°$.
Inverse	If $m\angle A \neq 99°$, then $\angle A$ is not obtuse.
Contrapositive	If $\angle A$ is not obtuse, then $m\angle A \neq 99°$.

both false

both true

EXAMPLE 2 Write four related conditional statements

Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement "Guitar players are musicians." Decide whether each statement is *true* or *false*.

Solution

If-then form If you are a guitar player, then you are a musician.
True, guitars players are musicians.

Converse If you are a musician, then you are a guitar player.
False, not all musicians play the guitar.

Inverse If you are not a guitar player, then you are not a musician.
False, even if you don't play a guitar, you can still be a musician.

Contrapositive If you are not a musician, then you are not a guitar player. *True*, a person who is not a musician cannot be a guitar player.

✓ GUIDED PRACTICE for Example 2

Write the converse, the inverse, and the contrapositive of the conditional statement. Tell whether each statement is *true* or *false*.

5. If a dog is a Great Dane, then it is large.

6. If a polygon is equilateral, then the polygon is regular.

EQUIVALENT STATEMENTS A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. Pairs of statements such as these are called *equivalent statements*. In general, when two statements are both true or both false, they are called **equivalent statements**.

DEFINITIONS You can write a definition as a conditional statement in if-then form or as its converse. Both the conditional statement and its converse are true. For example, consider the definition of *perpendicular lines*.

KEY CONCEPT

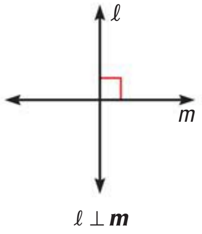
For Your Notebook

Perpendicular Lines

Definition If two lines intersect to form a right angle, then they are **perpendicular lines**.

The definition can also be written using the converse: If two lines are perpendicular lines, then they intersect to form a right angle.

You can write "line ℓ is perpendicular to line m" as $\ell \perp m$.

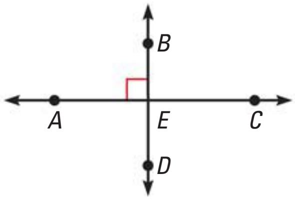


READ DIAGRAMS

In a diagram, a red square may be used to indicate a right angle or that two intersecting lines are perpendicular.

EXAMPLE 3 Use definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.



a. $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

b. $\angle AEB$ and $\angle CEB$ are a linear pair.

c. $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are opposite rays.

Solution

a. This statement is *true*. The right angle symbol in the diagram indicates that the lines intersect to form a right angle. So you can say the lines are perpendicular.

b. This statement is *true*. By definition, if the noncommon sides of adjacent angles are opposite rays, then the angles are a linear pair. Because $\overrightarrow{EA}$ and $\overrightarrow{EC}$ are opposite rays, $\angle AEB$ and $\angle CEB$ are a linear pair.

c. This statement is *false*. Point E does not lie on the same line as A and B, so the rays are not opposite rays.

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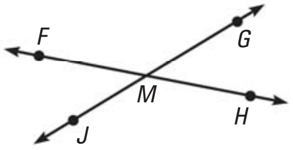
✓ GUIDED PRACTICE for Example 3

Use the diagram shown. Decide whether each statement is true. *Explain* your answer using the definitions you have learned.



7. $\angle JMF$ and $\angle FMG$ are supplementary.

8. Point M is the midpoint of $\overline{FH}$.

9. $\angle JMF$ and $\angle HMG$ are vertical angles.

10. $\overleftrightarrow{FH} \perp \overleftrightarrow{JG}$

READ DEFINITIONS
All definitions can be interpreted forward and backward in this way.

BICONDITIONAL STATEMENTS When a conditional statement and its converse are both true, you can write them as a single *biconditional statement*. A **biconditional statement** is a statement that contains the phrase "if and only if."

Any valid definition can be written as a biconditional statement.

EXAMPLE 4 Write a biconditional

Write the definition of perpendicular lines as a biconditional.

Solution

Definition	If two lines intersect to form a right angle, then they are perpendicular.
Converse	If two lines are perpendicular, then they intersect to form a right angle.
Biconditional	Two lines are perpendicular if and only if they intersect to form a right angle.

✓ GUIDED PRACTICE for Example 4

11. Rewrite the definition of *right angle* as a biconditional statement.

12. Rewrite the statements as a biconditional.

If Mary is in theater class, she will be in the fall play. If Mary is in the fall play, she must be taking theater class.

2.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 17, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 25, 29, 33, 34, and 35

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The __?__ of a conditional statement is found by switching the hypothesis and the conclusion.

2. ★ **WRITING** Write a definition for the term *collinear points*, and show how the definition can be interpreted as a biconditional.

EXAMPLE 1
on p. 79
for Exs. 3–6

REWRITING STATEMENTS **Rewrite the conditional statement in if-then form.**

3. When $x = 6$, $x^2 = 36$.

4. The measure of a straight angle is 180°.

5. Only people who are registered are allowed to vote.

6. **ERROR ANALYSIS** *Describe* and correct the error in writing the if-then statement.

> **Given statement:** All high school students take four English courses.
>
> **If-then statement:** If a high school student takes four courses, then all four are English courses. ✗

EXAMPLE 2
on p. 80
for Exs. 7–15

WRITING RELATED STATEMENTS **For the given statement, write the if-then form, the converse, the inverse, and the contrapositive.**

7. The complementary angles add to 90°.

8. Ants are insects.

9. $3x + 10 = 16$, because $x = 2$.

10. A midpoint bisects a segment.

ANALYZING STATEMENTS **Decide whether the statement is *true* or *false*. If false, provide a counterexample.**

11. If a polygon has five sides, then it is a regular pentagon.

12. If $m\angle A$ is 85°, then the measure of the complement of $\angle A$ is 5°.

13. Supplementary angles are always linear pairs.

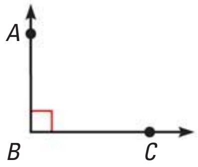
14. If a number is an integer, then it is rational.

15. If a number is a real number, then it is irrational.

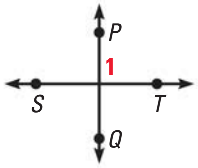
EXAMPLE 3
on p. 81
for Exs. 16–18

USING DEFINITIONS **Decide whether each statement about the diagram is true. *Explain* your answer using the definitions you have learned.**

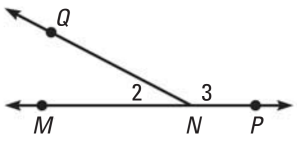
16. $m\angle ABC = 90°$



17. $\overleftrightarrow{PQ} \perp \overleftrightarrow{ST}$



18. $m\angle 2 + m\angle 3 = 180°$



EXAMPLE 4
on p. 82
for Exs. 19–21

REWRITING STATEMENTS **In Exercises 19–21, rewrite the definition as a biconditional statement.**

19. An angle with a measure between 90° and 180° is called *obtuse.*

20. Two angles are a *linear pair* if they are adjacent angles whose noncommon sides are opposite rays.

21. *Coplanar points* are points that lie in the same plane.

DEFINITIONS **Determine whether the statement is a valid definition.**

22. If two rays are *opposite rays*, then they have a common endpoint.

23. If the sides of a triangle are all the same length, then the triangle is *equilateral*.

24. If an angle is a *right angle*, then its measure is greater than that of an acute angle.

25. ★ **MULTIPLE CHOICE** Which statement has the same meaning as the given statement?

GIVEN ▶ You can go to the movie after you do your homework.

Ⓐ If you do your homework, then you can go to the movie afterwards.

Ⓑ If you do not do your homework, then you can go to the movie afterwards.

Ⓒ If you cannot go to the movie afterwards, then do your homework.

Ⓓ If you are going to the movie afterwards, then do not do your homework.

xy ALGEBRA Write the converse of each true statement. Tell whether the converse is true. If false, *explain* why.

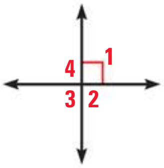
26. If $x > 4$, then $x > 0$.

27. If $x < 6$, then $-x > -6$.

28. If $x \leq -x$, then $x \leq 0$.

29. ★ **OPEN-ENDED MATH** Write a statement that is true but whose converse is false.

30. **CHALLENGE** Write a series of if-then statements that allow you to find the measure of each angle, given that $m\angle 1 = 90°$. Use the definition of linear pairs.



PROBLEM SOLVING

EXAMPLE 1
on p. 82
for Exs. 31–32

In Exercises 31 and 32, use the information about volcanoes to determine whether the biconditional statement is *true* or *false*. If false, provide a counterexample.

VOLCANOES Solid fragments are sometimes ejected from volcanoes during an eruption. The fragments are classified by size, as shown in the table.

31. A fragment is called a *block or bomb* if and only if its diameter is greater than 64 millimeters.

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32. A fragment is called a *lapilli* if and only if its diameter is less than 64 millimeters.

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Type of fragment	Diameter d (millimeters)
Ash	$d < 2$
Lapilli	$2 \leq d \leq 64$
Block or bomb	$d > 64$

33. ★ **SHORT RESPONSE** How can you show that the statement, "If you play a sport, then you wear a helmet." is false? *Explain*.

34. ★ **EXTENDED RESPONSE** You measure the heights of your classmates to get a data set.

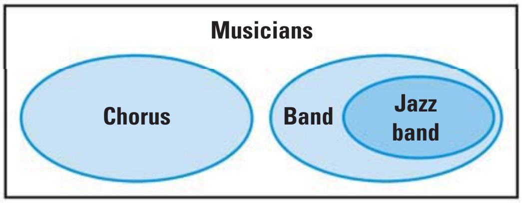
a. Tell whether this statement is true: If x and y are the least and greatest values in your data set, then the mean of the data is between x and y. *Explain* your reasoning.

b. Write the converse of the statement in part (a). Is the converse true? *Explain*.

c. Copy and complete the statement using *mean*, *median*, or *mode* to make a conditional that is true for any data set. *Explain* your reasoning.

Statement If a data set has a mean, a median, and a mode, then the _?_ of the data set will always be one of the measurements.

35. ★ **OPEN-ENDED MATH** The Venn diagram below represents all of the musicians at a high school. Write an if-then statement that describes a relationship between the various groups of musicians.



36. **MULTI-STEP PROBLEM** The statements below describe three ways that rocks are formed. Use these statements in parts (a)–(c).

Igneous rock is formed from the cooling of molten rock.

Sedimentary rock is formed from pieces of other rocks.

Metamorphic rock is formed by changing temperature, pressure, or chemistry.

a. Write each statement in if-then form.

b. Write the converse of each of the statements in part (a). Is the converse of each statement true? *Explain* your reasoning.

c. Write a true if-then statement about rocks. Is the converse of your statement *true* or *false*? *Explain* your reasoning.

37. **ALGEBRA** Can the statement, "If $x^2 - 10 = x + 2$, then $x = 4$," be combined with its converse to form a true biconditional?

38. **REASONING** You are given that the contrapositive of a statement is true. Will that help you determine whether the statement can be written as a true biconditional? *Explain.*

39. **CHALLENGE** Suppose each of the following statements is true. What can you conclude? *Explain* your answer.

If it is Tuesday, then I have art class.

It is Tuesday.

Each school day, I have either an art class or study hall.

If it is Friday, then I have gym class.

Today, I have either music class or study hall.

MIXED REVIEW

PREVIEW
Prepare for Lesson 2.3 in Exs. 40–45.

Find the product of the integers. *(p. 869)*

40. (−2)(10)

41. (15)(−3)

42. (−12)(−4)

43. (−5)(−4)(10)

44. (−3)(6)(−2)

45. (−4)(−2)(−5)

Sketch the figure described. *(p. 2)*

46. $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{CD}$ at point E.

47. $\overleftrightarrow{XY}$ intersects plane P at point Z.

48. $\overleftrightarrow{GH}$ is parallel to $\overleftrightarrow{JK}$.

49. Vertical planes X and Y intersect in $\overleftrightarrow{MN}$.

Find the coordinates of the midpoint of the segment with the given endpoints. *(p. 15)*

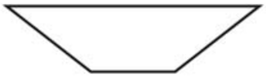
50. $A(10, 5)$ and $B(4, 5)$

51. $P(4, -1)$ and $Q(-2, 3)$

52. $L(2, 2)$ and $N(1, -2)$

Tell whether the figure is a polygon. If it is not, *explain* why. If it is a polygon, tell whether it is *convex* or *concave*. *(p. 42)*

53.



54.

55.

Investigating Geometry ACTIVITY *Use before Lesson 2.3*

2.3 Logic Puzzles

MATERIALS • graph paper • pencils

QUESTION How can reasoning be used to solve a logic puzzle?

EXPLORE Solve a logic puzzle

Using the clues below, you can determine an important mathematical contribution and interesting fact about each of five mathematicians.

Copy the chart onto your graph paper. Use the chart to keep track of the information given in Clues 1–7. Place an X in a box to indicate a definite "no." Place an O in a box to indicate a definite "yes."

Clue 1 Pythagoras had his contribution named after him. He was known to avoid eating beans.

Clue 2 Albert Einstein considered Emmy Noether to be one of the greatest mathematicians and used her work to show the theory of relativity.

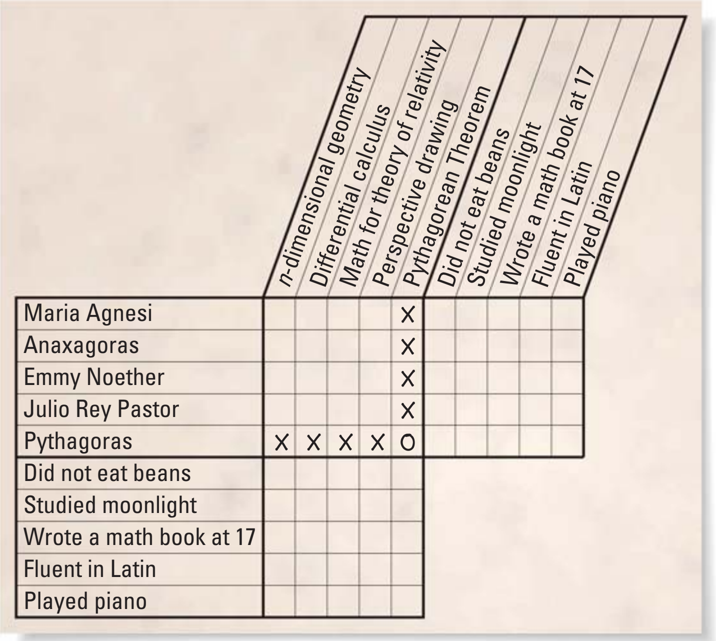
Clue 3 Anaxagoras was the first to theorize that the moon's light is actually the sun's light being reflected.

Clue 4 Julio Rey Pastor wrote a book at age 17.

Clue 5 The mathematician who is fluent in Latin contributed to the study of differential calculus.

Clue 6 The mathematician who did work with n-dimensional geometry was not the piano player.

Clue 7 The person who first used perspective drawing to make scenery for plays was not Maria Agnesi or Julio Rey Pastor.



	n-dimensional geometry	Differential calculus	Math for theory of relativity	Perspective drawing	Pythagorean Theorem	Did not eat beans	Studied moonlight	Wrote a math book at 17	Fluent in Latin	Played piano
Maria Agnesi					X					
Anaxagoras					X					
Emmy Noether					X					
Julio Rey Pastor					X					
Pythagoras	X	X	X	X	O					
Did not eat beans										
Studied moonlight										
Wrote a math book at 17										
Fluent in Latin										
Played piano										

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Write Clue 4 as a conditional statement in if-then form. Then write the contrapositive of the statement. *Explain* why the contrapositive of this statement is a helpful clue.
2. *Explain* how you can use Clue 6 to figure out who played the piano.
3. *Explain* how you can use Clue 7 to figure out who worked with perspective drawing.

2.3 Apply Deductive Reasoning

Before You used inductive reasoning to form a conjecture.

Now You will use deductive reasoning to form a logical argument.

Why So you can reach logical conclusions about locations, as in Ex. 18.

Key Vocabulary
- **deductive reasoning**

Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument. This is different from *inductive reasoning*, which uses specific examples and patterns to form a conjecture.

READ VOCABULARY

The Law of Detachment is also called a *direct argument*. The Law of Syllogism is sometimes called the *chain rule*.

KEY CONCEPT *For Your Notebook*

Laws of Logic

Law of Detachment

If the hypothesis of a true conditional statement is true, then the conclusion is also true.

Law of Syllogism

If **hypothesis** p, then **conclusion** q.
If **hypothesis** q, then **conclusion** r. — If these statements are true,

If **hypothesis** p, then **conclusion** r. — then this statement is true.

EXAMPLE 1 Use the Law of Detachment

Use the Law of Detachment to make a valid conclusion in the true situation.

a. If two segments have the same length, then they are congruent. You know that $BC = XY$.

b. Mary goes to the movies every Friday and Saturday night. Today is Friday.

Solution

a. Because $BC = XY$ satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, $\overline{BC} \cong \overline{XY}$.

b. First, identify the hypothesis and the conclusion of the first statement. The hypothesis is "If it is Friday or Saturday night," and the conclusion is "then Mary goes to the movies."

"Today is Friday" satisfies the hypothesis of the conditional statement, so you can conclude that Mary will go to the movies tonight.

EXAMPLE 2 Use the Law of Syllogism

If possible, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

a. If Rick takes chemistry this year, then Jesse will be Rick's lab partner.

If Jesse is Rick's lab partner, then Rick will get an A in chemistry.

b. If $x^2 > 25$, then $x^2 > 20$.

If $x > 5$, then $x^2 > 25$.

c. If a polygon is regular, then all angles in the interior of the polygon are congruent.

If a polygon is regular, then all of its sides are congruent.

Solution

a. The conclusion of the first statement is the hypothesis of the second statement, so you can write the following new statement.

If Rick takes chemistry this year, then Rick will get an A in chemistry.

AVOID ERRORS

The order in which the statements are given does not affect whether you can use the Law of Syllogism.

b. Notice that the conclusion of the second statement is the hypothesis of the first statement, so you can write the following new statement.

If $x > 5$, then $x^2 > 20$.

c. Neither statement's conclusion is the same as the other statement's hypothesis. You cannot use the Law of Syllogism to write a new conditional statement.

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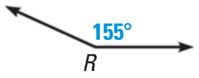
✓ GUIDED PRACTICE for Examples 1 and 2



1. If $90° < m\angle R < 180°$, then $\angle R$ is obtuse. The measure of $\angle R$ is 155°. Using the Law of Detachment, what statement can you make?

2. If Jenelle gets a job, then she can afford a car. If Jenelle can afford a car, then she will drive to school. Using the Law of Syllogism, what statement can you make?

State the law of logic that is illustrated.

3. If you get an A or better on your math test, then you can go to the movies. If you go to the movies, then you can watch your favorite actor.

If you get an A or better on your math test, then you can watch your favorite actor.

4. If $x > 12$, then $x + 9 > 20$. The value of x is 14.

Therefore, $x + 9 > 20$.

ANALYZING REASONING In Geometry, you will frequently use inductive reasoning to make conjectures. You will also be using deductive reasoning to show that conjectures are true or false. You will need to know which type of reasoning is being used.

EXAMPLE 3 Use inductive and deductive reasoning

ALGEBRA What conclusion can you make about the product of an even integer and any other integer?

Solution

STEP 1 **Look** for a pattern in several examples. Use inductive reasoning to make a conjecture.

$(-2)(2) = -4$, $(-1)(2) = -2$, $2(2) = 4$, $3(2) = 6$,

$(-2)(-4) = 8$, $(-1)(-4) = 4$, $2(-4) = -8$, $3(-4) = -12$

Conjecture Even integer • Any integer = Even integer

STEP 2 **Let** n and m each be any integer. Use deductive reasoning to show the conjecture is true.

$2n$ is an even integer because any integer multiplied by 2 is even.

$2nm$ represents the product of an even integer and any integer m.

$2nm$ is the product of 2 and an integer nm. So, $2nm$ is an even integer.

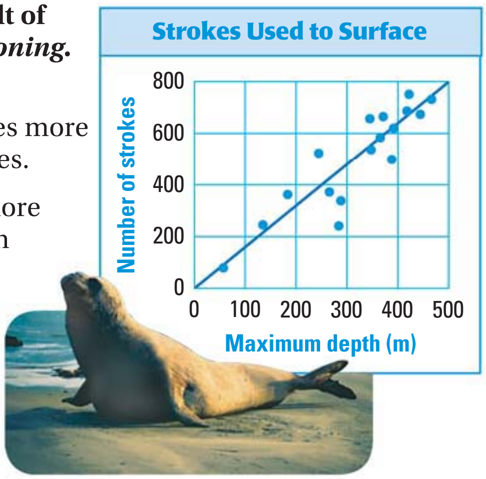
▶ The product of an even integer and any integer is an even integer.

EXAMPLE 4 Reasoning from a graph

Tell whether the statement is the result of *inductive reasoning* or *deductive reasoning*. Explain your choice.

a. The northern elephant seal requires more strokes to surface the deeper it dives.

b. The northern elephant seal uses more strokes to surface from 60 feet than from 250 feet.



Solution

a. Inductive reasoning, because it is based on a pattern in the data

b. Deductive reasoning, because you are comparing values that are given on the graph

✓ GUIDED PRACTICE for Examples 3 and 4

5. Use inductive reasoning to make a conjecture about the sum of a number and itself. Then use deductive reasoning to show the conjecture is true.

6. Use inductive reasoning to write another statement about the graph in Example 4. Then use deductive reasoning to write another statement.

2.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 17, and 21

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 3, 12, 20, and 23

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: If the hypothesis of a true if-then statement is true, then the conclusion is also true by the Law of _?_.

★ **WRITING** **Use deductive reasoning to make a statement about the picture.**

2.

3.

EXAMPLE 1 on p. 87 for Exs. 4–6

LAW OF DETACHMENT **Make a valid conclusion in the situation.**

4. If the measure of an angle is 90°, then it is a right angle. The measure of $\angle A$ is 90°.

5. If $x > 12$, then $-x < -12$. The value of x is 15.

6. If a book is a biography, then it is nonfiction. You are reading a biography.

EXAMPLE 2 on p. 88 for Exs. 7–10

LAW OF SYLLOGISM **In Exercises 7–10, write the statement that follows from the pair of statements that are given.**

7. If a rectangle has four equal side lengths, then it is a square. If a polygon is a square, then it is a regular polygon.

8. If $y > 0$, then $2y > 0$. If $2y > 0$, then $2y - 5 \neq -5$.

9. If you play the clarinet, then you play a woodwind instrument. If you play a woodwind instrument, then you are a musician.

10. If $a = 3$, then $5a = 15$. If $\frac{1}{2}a = 1\frac{1}{2}$, then $a = 3$.

EXAMPLE 3 on p. 89 for Ex. 11

11. **REASONING** What can you say about the sum of an even integer and an even integer? Use inductive reasoning to form a conjecture. Then use deductive reasoning to show that the conjecture is true.

12. ★ **MULTIPLE CHOICE** If two angles are vertical angles, then they have the same measure. You know that $\angle A$ and $\angle B$ are vertical angles. Using the Law of Detachment, which conclusion could you make?

Ⓐ $m\angle A > m\angle B$

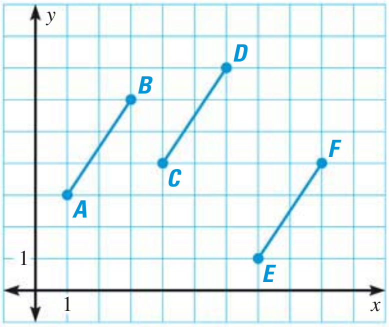
Ⓑ $m\angle A = m\angle B$

Ⓒ $m\angle A + m\angle B = 90°$

Ⓓ $m\angle A + m\angle B = 180°$

13. **ERROR ANALYSIS** *Describe* and correct the error in the argument: "If two angles are a linear pair, then they are supplementary. Angles C and D are supplementary, so the angles are a linear pair."

14. **ALGEBRA** Use the segments in the coordinate plane.



a. Use the distance formula to show that the segments are congruent.

b. Make a conjecture about some segments in the coordinate plane that are congruent to the given segments. Test your conjecture, and *explain* your reasoning.

c. Let one endpoint of a segment be (x, y). Use algebra to show that segments drawn using your conjecture will always be congruent.

d. A student states that the segments described below will each be congruent to the ones shown above. Determine whether the student is correct. *Explain* your reasoning.

$\overline{MN}$, with endpoints $M(3, 5)$ and $N(5, 2)$

$\overline{PQ}$, with endpoints $P(1, -1)$ and $Q(4, -3)$

$\overline{RS}$, with endpoints $R(-2, 2)$ and $S(1, 4)$

15. **CHALLENGE** Make a conjecture about whether the Law of Syllogism works when used with the contrapositives of a pair of statements. Use this pair of statements to *justify* your conjecture.

If a creature is a wombat, then it is a marsupial.

If a creature is a marsupial, then it has a pouch.

PROBLEM SOLVING

EXAMPLES 1 and 2
on pp. 87–88
for Exs. 16–17

USING THE LAWS OF LOGIC In Exercises 16 and 17, what conclusions can you make using the true statement?

16. **CAR COSTS** If you save $2000, then you can buy a car. You have saved $1200.

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17. **PROFIT** The bakery makes a profit if its revenue is greater than its costs. You will get a raise if the bakery makes a profit.

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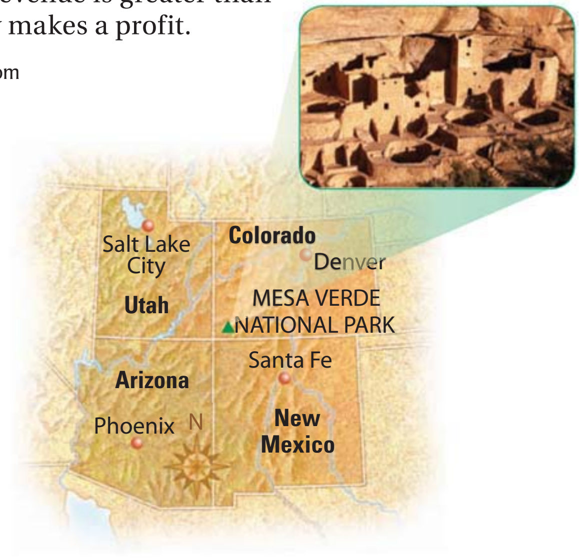
USING DEDUCTIVE REASONING Select the word(s) that make(s) the conclusion true.

18. Mesa Verde National Park is in Colorado. Simone vacationed in Colorado. So, Simone (*must have*, *may have*, or *never*) visited Mesa Verde National Park.

19. The cliff dwellings in Mesa Verde National Park are accessible to visitors only when accompanied by a park ranger. Billy is at a cliff dwelling in Mesa Verde National Park. So, Billy *(is, may be, is not)* with a park ranger.



EXAMPLE 4
on p. 89
for Ex. 20

20. ★ **EXTENDED RESPONSE** Geologists use the Mohs scale to determine a mineral's hardness. Using the scale, a mineral with a higher rating will leave a scratch on a mineral with a lower rating. Geologists use scratch tests to help identify an unknown mineral.

Mineral	 Talc	 Gypsum	 Calcite	 Fluorite
Mohs rating	1	2	3	4

a. Use the table to write three if-then statements such as "If talc is scratched against gypsum, then a scratch mark is left on the talc."

b. You must identify four minerals labeled *A*, *B*, *C*, and *D*. You know that the minerals are the ones shown in the table. The results of your scratch tests are shown below. What can you conclude? *Explain* your reasoning.

Mineral *A* is scratched by Mineral *B*.

Mineral *C* is scratched by all three of the other minerals.

c. What additional test(s) can you use to identify *all* the minerals in part (b)?

REASONING **In Exercises 21 and 22, decide whether *inductive* or *deductive* reasoning is used to reach the conclusion. *Explain* your reasoning.**

21. The rule at your school is that you must attend all of your classes in order to participate in sports after school. You played in a soccer game after school on Monday. Therefore, you went to all of your classes on Monday.

22. For the past 5 years, your neighbor goes on vacation every July 4th and asks you to feed her hamster. You conclude that you will be asked to feed her hamster on the next July 4th.

23. ★ **SHORT RESPONSE** Let an even integer be $2n$ and an odd integer be $2n + 1$. *Explain* why the sum of an even integer and an odd integer is an odd integer.

24. **LITERATURE** George Herbert wrote a poem, *Jacula Prudentum*, that includes the statements shown. Use the Law of Syllogism to write a new conditional statement. *Explain* your reasoning.

> For want of a nail the shoe is lost,
> for want of a shoe the horse is lost,
> for want of a horse the rider is lost.

REASONING **In Exercises 25–28, use the true statements below to determine whether you know the conclusion is *true* or *false*. *Explain* your reasoning.**

If Arlo goes to the baseball game, then he will buy a hot dog.

If the baseball game is not sold out, then Arlo and Mia will go to the game.

If Mia goes to the baseball game, then she will buy popcorn.

The baseball game is not sold out.

25. Arlo bought a hot dog.

26. Arlo and Mia went to the game.

27. Mia bought a hot dog.

28. Arlo had some of Mia's popcorn.

29. **CHALLENGE** Use these statements to answer parts (a)–(c).

Adam says Bob lies.
Bob says Charlie lies.
Charlie says Adam and Bob both lie.

a. If Adam is telling the truth, then Bob is lying. What can you conclude about Charlie's statement?

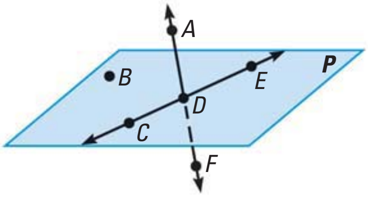
b. Assume Adam is telling the truth. *Explain* how this leads to a contradiction.

c. Who is telling the truth? Who is lying? How do you know?

MIXED REVIEW

PREVIEW
Prepare for Lesson 2.4 in Exs. 30–33.

In Exercises 30–33, use the diagram. *(p. 2)*



30. Name two lines.

31. Name four rays.

32. Name three collinear points.

33. Name four coplanar points.

Plot the given points in a coordinate plane. Then determine whether $\overline{AB}$ and $\overline{CD}$ are congruent. *(p. 9)*

34. $A(1, 4), B(5, 4), C(3, -4), D(3, 0)$

35. $A(-1, 0), B(-1, -5), C(1, 2), D(-5, 2)$

Rewrite the conditional statement in if-then form. *(p. 79)*

36. When $x = -2, x^2 = 4$.

37. The measure of an acute angle is less than 90°.

38. Only people who are members can access the website.

QUIZ *for Lessons 2.1–2.3*

Show the conjecture is false by finding a counterexample. *(p. 72)*

1. If the product of two numbers is positive, then the two numbers must be negative.

2. The sum of two numbers is always greater than the larger number.

In Exercises 3 and 4, write the if-then form and the contrapositive of the statement. *(p. 79)*

3. Points that lie on the same line are called collinear points.

4. $2x - 8 = 2$, because $x = 5$.

5. Make a valid conclusion about the following statements:
If it is above 90°F outside, then I will wear shorts. It is 98°F. *(p. 87)*

6. *Explain* why a number that is divisible by a multiple of 3 is also divisible by 3. *(p. 87)*

Extension

Use after Lesson 2.3

Symbolic Notation and Truth Tables

GOAL Use symbolic notation to represent logical statements.

Key Vocabulary
- **truth value**
- **truth table**

Conditional statements can be written using *symbolic notation*, where letters are used to represent statements. An arrow (→), read "implies," connects the hypothesis and conclusion. To write the negation of a statement p you write the symbol for negation ($\sim$) before the letter. So, "not p" is written $\sim p$.

KEY CONCEPT *For Your Notebook*

Symbolic Notation

Let p be "the angle is a right angle" and let q be "the measure of the angle is 90°."

Conditional If p, then q. $p \rightarrow q$

Example: If an angle is a right angle, then its measure is 90°.

Converse If q, then p. $q \rightarrow p$

Example: If the measure of an angle is 90°, then the angle is a right angle.

Inverse If not p, then not q. $\sim p \rightarrow \sim q$

Example: If an angle is not a right angle, then its measure is not 90°.

Contrapositive If not q, then not p. $\sim q \rightarrow \sim p$

If the measure of an angle is not 90°, then the angle is not a right angle.

Biconditional p if and only if q $p \leftrightarrow q$

Example: An angle is a right angle if and only if its measure is 90°.

EXAMPLE 1 Use symbolic notation

Let p be "the car is running" and let q be "the key is in the ignition."

a. Write the conditional statement $p \rightarrow q$ in words.

b. Write the converse $q \rightarrow p$ in words.

c. Write the inverse $\sim p \rightarrow \sim q$ in words.

d. Write the contrapositive $\sim q \rightarrow \sim p$ in words.

Solution

a. Conditional: If the car is running, then the key is in the ignition.

b. Converse: If the key is in the ignition, then the car is running.

c. Inverse: If the car is not running, then the key is not in the ignition.

d. Contrapositive: If the key is not in the ignition, then the car is not running.

TRUTH TABLES The **truth value** of a statement is either true (T) or false (F). You can determine the conditions under which a conditional statement is true by using a **truth table**. The truth table at the right shows the truth values for hypothesis p and conclusion q. The conditional $p \to q$ is only false when a true hypothesis produces a false conclusion.

Conditional		
p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

EXAMPLE 2 Make a truth table

Use the truth table above to make truth tables for the converse, inverse, and contrapositive of a conditional statement $p \to q$.

Solution

Converse		
p	q	$q \to p$
T	T	T
T	F	T
F	T	F
F	F	T

Inverse				
p	q	$\sim p$	$\sim q$	$\sim p \to \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Contrapositive				
p	q	$\sim q$	$\sim p$	$\sim q \to \sim p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

READ TRUTH TABLES

A conditional statement and its contrapositive are *equivalent statements* because they have the same truth table. The same is true of the converse and the inverse.

PRACTICE

EXAMPLE 1
on p. 94
for Exs. 1–6

1. **WRITING** *Describe* how to use symbolic notation to represent the contrapositive of a conditional statement.

WRITING STATEMENTS **Use p and q to write the symbolic statement in words.**

p: Polygon $ABCDE$ is equiangular and equilateral.

q: Polygon $ABCDE$ is a regular polygon.

2. $p \to q$
3. $\sim p$
4. $\sim q \to \sim p$
5. $p \leftrightarrow q$

6. **LAW OF SYLLOGISM** Use the statements p, q, and r below to write a series of conditionals that would satisfy the Law of Syllogism. How could you write your reasoning using symbolic notation?

p: $x + 5 = 12$

q: $x = 7$

r: $3x = 21$

EXAMPLE 2
on p. 95
for Exs. 7–8

7. **WRITING** Is the truth value of a statement always true (T)? *Explain*.

8. **TRUTH TABLE** Use the statement "If an animal is a poodle, then it is a dog."
   - **a.** Identify the hypothesis p and the conclusion q in the conditional.
   - **b.** Make a truth table for the converse. *Explain* what each row in the table means in terms of the original statement.

2.4 Use Postulates and Diagrams

Before You used postulates involving angle and segment measures.

Now You will use postulates involving points, lines, and planes.

Why? So you can draw the layout of a neighborhood, as in Ex. 39.

Key Vocabulary
- **line perpendicular to a plane**
- **postulate,** *p. 8*

In geometry, rules that are accepted without proof are called *postulates* or *axioms*. Rules that are proved are called *theorems*. Postulates and theorems are often written in conditional form. Unlike the converse of a definition, the converse of a postulate or theorem cannot be assumed to be true.

You learned four postulates in Chapter 1.

POSTULATE 1	Ruler Postulate	page 9
POSTULATE 2	Segment Addition Postulate	page 10
POSTULATE 3	Protractor Postulate	page 24
POSTULATE 4	Angle Addition Postulate	page 25

Here are seven new postulates involving points, lines, and planes.

POSTULATES *For Your Notebook*

Point, Line, and Plane Postulates

POSTULATE 5 Through any two points there exists exactly one line.

POSTULATE 6 A line contains at least two points.

POSTULATE 7 If two lines intersect, then their intersection is exactly one point.

POSTULATE 8 Through any three noncollinear points there exists exactly one plane.

POSTULATE 9 A plane contains at least three noncollinear points.

POSTULATE 10 If two points lie in a plane, then the line containing them lies in the plane.

POSTULATE 11 If two planes intersect, then their intersection is a line.

ALGEBRA CONNECTION You have been using many of Postulates 5–11 in previous courses.

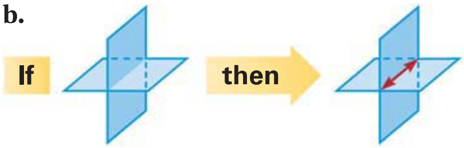
One way to graph a linear equation is to plot two points whose coordinates satisfy the equation and then connect them with a line. Postulate 5 guarantees that there is exactly one such line. A familiar way to find a common solution of two linear equations is to graph the lines and find the coordinates of their intersection. This process is guaranteed to work by Postulate 7.

EXAMPLE 1 Identify a postulate illustrated by a diagram

State the postulate illustrated by the diagram.

a.

b.



Solution

a. **Postulate 7** If two lines intersect, then their intersection is exactly one point.

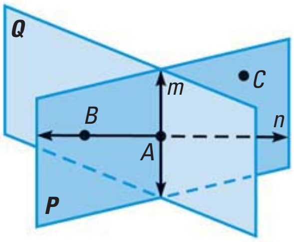
b. **Postulate 11** If two planes intersect, then their intersection is a line.

EXAMPLE 2 Identify postulates from a diagram

Use the diagram to write examples of Postulates 9 and 10.

Postulate 9 Plane *P* contains at least three noncollinear points, *A*, *B*, and *C*.

Postulate 10 Point *A* and point *B* lie in plane *P*, so line *n* containing *A* and *B* also lies in plane *P*.



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✓ GUIDED PRACTICE for Examples 1 and 2

1. Use the diagram in Example 2. Which postulate allows you to say that the intersection of plane *P* and plane *Q* is a line?

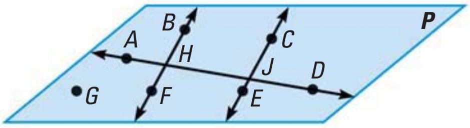
2. Use the diagram in Example 2 to write examples of Postulates 5, 6, and 7.

CONCEPT SUMMARY

For Your Notebook

Interpreting a Diagram

When you interpret a diagram, you can only assume information about size or measure if it is marked.

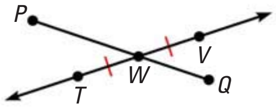


YOU CAN ASSUME	YOU CANNOT ASSUME
All points shown are coplanar.	G, F, and E are collinear.
$\angle AHB$ and $\angle BHD$ are a linear pair.	$\overleftrightarrow{BF}$ and $\overleftrightarrow{CE}$ intersect.
$\angle AHF$ and $\angle BHD$ are vertical angles.	$\overleftrightarrow{BF}$ and $\overleftrightarrow{CE}$ do not intersect.
A, H, J, and D are collinear.	$\angle BHA \cong \angle CJA$
$\overleftrightarrow{AD}$ and $\overleftrightarrow{BF}$ intersect at H.	$\overleftrightarrow{AD} \perp \overleftrightarrow{BF}$ or $m\angle AHB = 90°$

EXAMPLE 3 Use given information to sketch a diagram

Sketch a diagram showing $\overleftrightarrow{TV}$ intersecting $\overline{PQ}$ at point W, so that $\overline{TW} \cong \overline{WV}$.

Solution



STEP 1 **Draw** $\overleftrightarrow{TV}$ and label points T and V.

STEP 2 **Draw** point W at the midpoint of $\overline{TV}$. Mark the congruent segments.

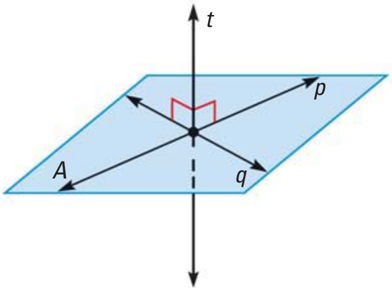
STEP 3 **Draw** $\overline{PQ}$ through W.

AVOID ERRORS

Notice that the picture was drawn so that W does not look like a midpoint of $\overline{PQ}$. Also, it was drawn so that $\overline{PQ}$ is not perpendicular to $\overline{TV}$.

PERPENDICULAR FIGURES A line is a **line perpendicular to a plane** if and only if the line intersects the plane in a point and is perpendicular to every line in the plane that intersects it at that point.

In a diagram, a line perpendicular to a plane must be marked with a right angle symbol.



EXAMPLE 4 Interpret a diagram in three dimensions

Which of the following statements *cannot* be assumed from the diagram?

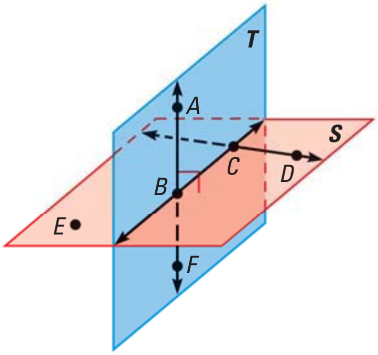
A, B, and F are collinear.

E, B, and D are collinear.

$\overline{AB} \perp$ plane S

$\overline{CD} \perp$ plane T

$\overleftrightarrow{AF}$ intersects $\overleftrightarrow{BC}$ at point B.



Solution

No drawn line connects E, B, and D, so you cannot assume they are collinear. With no right angle marked, you cannot assume $\overline{CD} \perp$ plane T.

✓ **GUIDED PRACTICE** for Examples 3 and 4

In Exercises 3 and 4, refer back to Example 3.

3. If the given information stated $\overline{PW}$ and $\overline{QW}$ are congruent, how would you indicate that in the diagram?

4. Name a pair of supplementary angles in the diagram. *Explain*.

5. In the diagram for Example 4, can you assume plane S intersects plane T at $\overleftrightarrow{BC}$?

6. *Explain* how you know that $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$ in Example 4.

2.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 13, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 10, 24, 25, 33, 39, and 41

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A __?__ is a line that intersects the plane in a point and is perpendicular to every line in the plane that intersects it.

2. ★ **WRITING** *Explain* why you cannot assume $\angle BHA \cong \angle CJA$ in the Concept Summary on page 97.

EXAMPLE 1
on p. 97
for Exs. 3–5

IDENTIFYING POSTULATES **State the postulate illustrated by the diagram.**

3.

4.

5. **CONDITIONAL STATEMENTS** Postulate 8 states that through any three noncollinear points there exists exactly one plane.

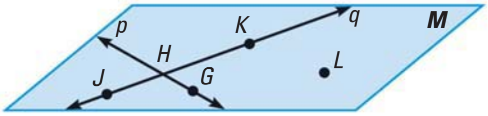
   a. Rewrite Postulate 8 in if-then form.

   b. Write the converse, inverse, and contrapositive of Postulate 8.

   c. Which statements in part (b) are true?

EXAMPLE 2
on p. 97
for Exs. 6–8

USING A DIAGRAM **Use the diagram to write an example of each postulate.**



6. Postulate 6

7. Postulate 7

8. Postulate 8

EXAMPLES 3 and 4
on p. 98
for Exs. 9–10

9. **SKETCHING** Sketch a diagram showing $\overleftrightarrow{XY}$ intersecting $\overleftrightarrow{WV}$ at point T, so $\overleftrightarrow{XY} \perp \overleftrightarrow{WV}$. In your diagram, does $\overline{WT}$ have to be congruent to $\overline{TV}$? *Explain* your reasoning.

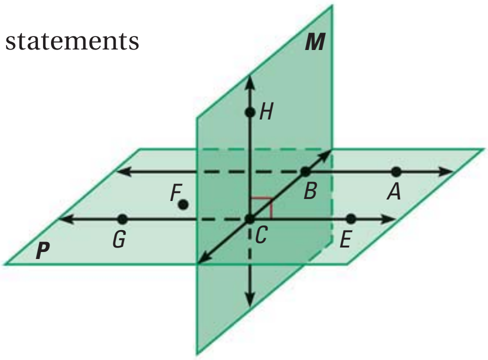
10. ★ **MULTIPLE CHOICE** Which of the following statements *cannot* be assumed from the diagram?

   Ⓐ Points A, B, C, and E are coplanar.

   Ⓑ Points F, B, and G are collinear.

   Ⓒ $\overleftrightarrow{HC} \perp \overleftrightarrow{GE}$

   Ⓓ $\overleftrightarrow{EC}$ intersects plane M at point C.



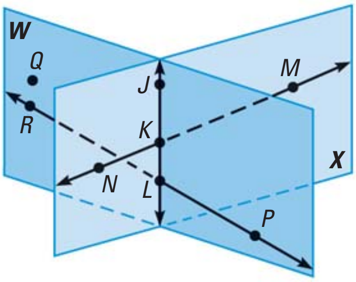
ANALYZING STATEMENTS **Decide whether the statement is true or false. If it is false, give a real-world counterexample.**

11. Through any three points, there exists exactly one line.

12. A point can be in more than one plane.

13. Any two planes intersect.

USING A DIAGRAM **Use the diagram to determine if the statement is *true* or *false*.**



14. Planes W and X intersect at $\overleftrightarrow{KL}$.

15. Points Q, J, and M are collinear.

16. Points K, L, M, and R are coplanar.

17. $\overleftrightarrow{MN}$ and $\overleftrightarrow{RP}$ intersect.

18. $\overleftrightarrow{RP} \perp$ plane W

19. $\overleftrightarrow{JK}$ lies in plane X.

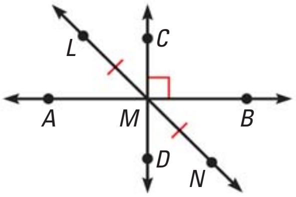
20. $\angle PLK$ is a right angle.

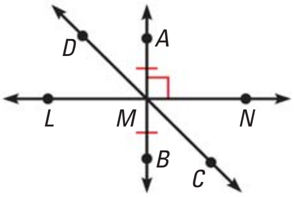
21. $\angle NKL$ and $\angle JKM$ are vertical angles.

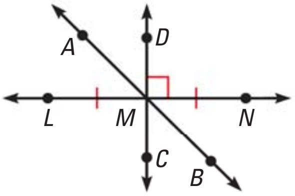
22. $\angle NKJ$ and $\angle JKM$ are supplementary angles.

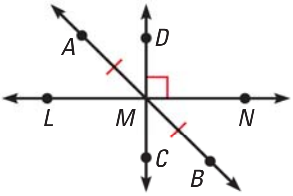
23. $\angle JKM$ and $\angle KLP$ are congruent angles.

24. ★ **MULTIPLE CHOICE** Choose the diagram showing $\overleftrightarrow{LN}$, $\overleftrightarrow{AB}$, and $\overleftrightarrow{DC}$ intersecting at point M, $\overleftrightarrow{AB}$ bisecting $\overline{LN}$, and $\overleftrightarrow{DC} \perp \overleftrightarrow{LN}$.

Ⓐ


Ⓑ


Ⓒ


Ⓓ


25. ★ **OPEN-ENDED MATH** Sketch a diagram of a real-world object illustrating three of the postulates about points, lines, and planes. List the postulates used.

26. **ERROR ANALYSIS** A student made the false statement shown. Change the statement in two different ways to make it true.

Three points are always contained in a line. ✗

27. **REASONING** Use Postulates 5 and 9 to *explain* why every plane contains at least one line.

28. **REASONING** Point X lies in plane M. Use Postulates 6 and 9 to *explain* why there are at least two lines in plane M that contain point X.

29. **CHALLENGE** Sketch a line m and a point C not on line m. Make a conjecture about how many planes can be drawn so that line m and point C lie in the plane. Use postulates to justify your conjecture.

PROBLEM SOLVING

REAL-WORLD SITUATIONS **Which postulate is suggested by the photo?**

30.

31.

32.

33. ★ **SHORT RESPONSE** Give a real-world example of Postulate 6, which states that a line contains at least two points.

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34. **DRAW A DIAGRAM** Sketch two lines that intersect, and another line that does not intersect either one.

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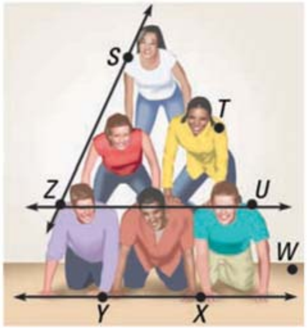
USING A DIAGRAM **Use the pyramid to write examples of the postulate indicated.**



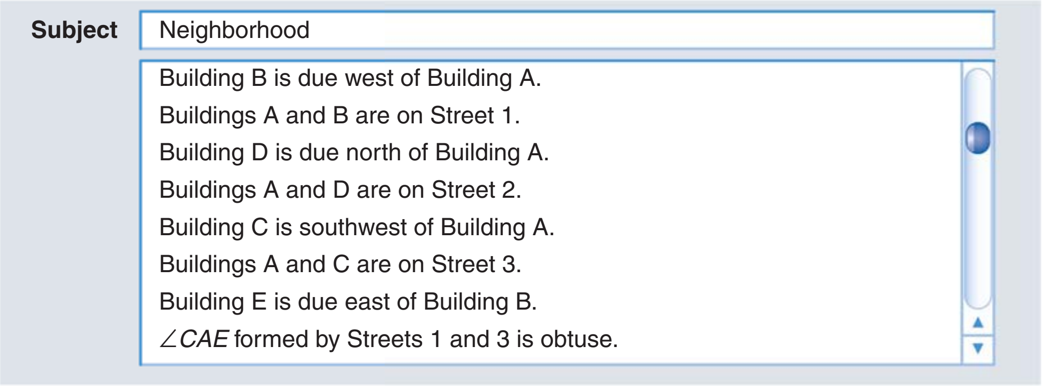
35. Postulate 5

36. Postulate 7

37. Postulate 9

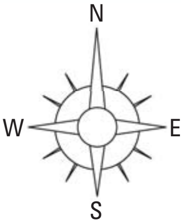
38. Postulate 10

39. ★ **EXTENDED RESPONSE** A friend e-mailed you the following statements about a neighborhood. Use the statements to complete parts (a)–(e).



Subject Neighborhood

Building B is due west of Building A.
Buildings A and B are on Street 1.
Building D is due north of Building A.
Buildings A and D are on Street 2.
Building C is southwest of Building A.
Buildings A and C are on Street 3.
Building E is due east of Building B.
$\angle CAE$ formed by Streets 1 and 3 is obtuse.



a. Draw a diagram of the neighborhood.

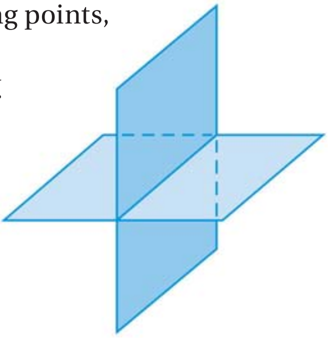
b. Where do Streets 1 and 2 intersect?

c. Classify the angle formed by Streets 1 and 2.

d. Is Building E between Buildings A and B? *Explain*.

e. What street is Building E on?

40. **MULTI-STEP PROBLEM** Copy the figure and label the following points, lines, and planes appropriately.



- **a.** Label the horizontal plane as X and the vertical plane as Y.
- **b.** Draw two points A and B on your diagram so they lie in plane Y, but not in plane X.
- **c.** Illustrate Postulate 5 on your diagram.
- **d.** If point C lies in both plane X and plane Y, where would it lie? Draw point C on your diagram.
- **e.** Illustrate Postulate 9 for plane X on your diagram.

41. ★ **SHORT RESPONSE** Points E, F, and G all lie in plane P and in plane Q. What must be true about points E, F, and G if P and Q are different planes? What must be true about points E, F, and G to force P and Q to be the same plane? Make sketches to support your answers.

DRAWING DIAGRAMS **$\overleftrightarrow{AC}$ and $\overleftrightarrow{DB}$ intersect at point E. Draw one diagram that meets the additional condition(s) and another diagram that does not.**

42. $\angle AED$ and $\angle AEB$ are right angles.

43. Point E is the midpoint of $\overline{AC}$.

44. $\overrightarrow{EA}$ and $\overrightarrow{EC}$ are opposite rays. $\overrightarrow{EB}$ and $\overrightarrow{ED}$ are not opposite rays.

45. **CHALLENGE** Suppose none of the four legs of a chair are the same length. What is the maximum number of planes determined by the lower ends of the legs? Suppose exactly three of the legs of a second chair have the same length. What is the maximum number of planes determined by the lower ends of the legs of the second chair? *Explain* your reasoning.

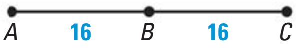
MIXED REVIEW

PREVIEW
Prepare for Lesson 2.5 in Exs. 46–48.

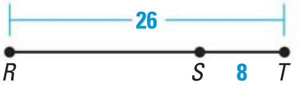
Draw an example of the type of angle described. *(p. 9)*

46. Find MP.

47. Find AC.

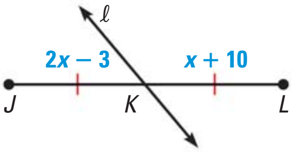


48. Find RS.

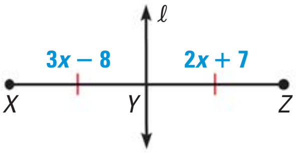


Line ℓ bisects the segment. Find the indicated length. *(p. 15)*

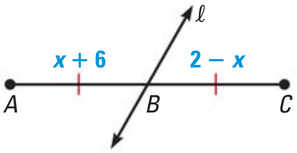
49. Find JK.



50. Find XZ.



51. Find BC.



Draw an example of the type of angle described. *(p. 24)*

52. Right angle **53.** Acute angle **54.** Obtuse angle **55.** Straight angle

56. Two angles form a linear pair. The measure of one angle is 9 times the measure of the other angle. Find the measure of each angle. *(p. 35)*

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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Lessons 2.1–2.4

1. **MULTI-STEP PROBLEM** The table below shows the time of the sunrise on different days in Galveston, Texas.

Date in 2006	Time of sunrise (Central Standard Time)
Jan. 1	7:14 A.M.
Feb. 1	7:08 A.M.
Mar. 1	6:45 A.M.
Apr. 1	6:09 A.M.
May 1	5:37 A.M.
June 1	5:20 A.M.
July 1	5:23 A.M.
Aug. 1	5:40 A.M.

   a. *Describe* the pattern, if any, in the times shown in the table.

   b. Use the times in the table to make a reasonable prediction about the time of the sunrise on September 1, 2006.

2. **SHORT RESPONSE** As shown in the table below, hurricanes are categorized by the speed of the wind in the storm. Use the table to determine whether the statement is *true* or *false*. If false, provide a counterexample.

Hurricane category	Wind speed w (mi/h)
1	$74 \le w \le 95$
2	$96 \le w \le 110$
3	$111 \le w \le 130$
4	$131 \le w \le 155$
5	$w > 155$

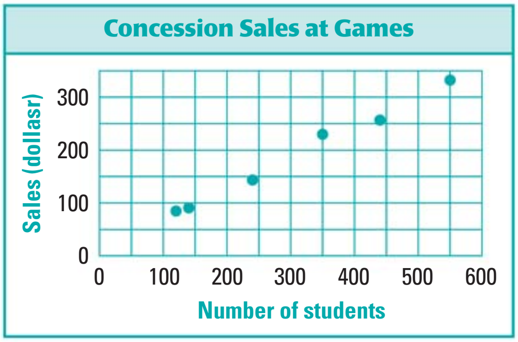
   a. A hurricane is a category 5 hurricane if and only if its wind speed is greater than 155 miles per hour.

   b. A hurricane is a category 3 hurricane if and only if its wind speed is less than 130 miles per hour.

3. **GRIDDED ANSWER** Write the next number in the pattern.

   1, 2, 5, 10, 17, 26, . . .

4. **EXTENDED RESPONSE** The graph shows concession sales at six high school football games. Tell whether each statement is the result of *inductive reasoning* or *deductive reasoning*. *Explain* your thinking.



   a. If 500 students attend a football game, the high school can expect concession sales to reach \$300.

   b. Concession sales were highest at the game attended by 550 students.

   c. The average number of students who come to a game is about 300.

5. **SHORT RESPONSE** Select the phrase that makes the conclusion true. *Explain* your reasoning.

   a. A person needs a library card to check out books at the public library. You checked out a book at the public library. You (*must have, may have,* or *do not have*) a library card.

   b. The islands of Hawaii are volcanoes. Bob has never been to the Hawaiian Islands. Bob (*has visited, may have visited,* or *has never visited*) volcanoes.

6. **SHORT RESPONSE** Sketch a diagram showing $\overleftrightarrow{PQ}$ intersecting $\overleftrightarrow{RS}$ at point N. In your diagram, $\angle PNS$ should be an obtuse angle. Identify two acute angles in your diagram. *Explain* how you know that these angles are acute.

Investigating Geometry ACTIVITY *Use before Lesson 2.5*

2.5 Justify a Number Trick

MATERIALS • paper • pencil

QUESTION **How can you use algebra to justify a number trick?**

Number tricks can allow you to guess the result of a series of calculations.

EXPLORE **Play the number trick**

STEP 1 ***Pick a number*** Follow the directions below.

a. Pick any number between 11 and 98 that does not end in a zero.	23
b. Double the number.	23 • 2
c. Add 4 to your answer.	46 + 4
d. Multiply your answer by 5.	50 • 5
e. Add 12 to your answer.	250 + 12
f. Multiply your answer by 10.	262 • 10
g. Subtract 320 from your answer.	2620 − 320
h. Cross out the zeros in your answer.	$23\cancel{00}$

STEP 2 ***Repeat the trick*** Repeat the trick three times using three different numbers. What do you notice?

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Let *x* represent the number you chose in the Explore. Write algebraic expressions for each step. Remember to use the Order of Operations.
2. *Justify* each expression you wrote in Exercise 1.
3. Another number trick is as follows:

   Pick any number.
   Multiply your number by 2.
   Add 18 to your answer.
   Divide your answer by 2.
   Subtract your original number from your answer.

   What is your answer? Does your answer depend on the number you chose? How can you change the trick so your answer is always 15? *Explain*.

4. **REASONING** Write your own number trick.

2.5 Reason Using Properties from Algebra

Before	You used deductive reasoning to form logical arguments.
Now	You will use algebraic properties in logical arguments too.
Why	So you can apply a heart rate formula, as in Example 3.

Key Vocabulary
- **equation,** *p. 875*
- **solve an equation,** *p. 875*

When you *solve an equation*, you use properties of real numbers. Segment lengths and angle measures are real numbers, so you can also use these properties to write logical arguments about geometric figures.

KEY CONCEPT *For Your Notebook*

Algebraic Properties of Equality

Let a, b, and c be real numbers.

Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $ac = bc$.
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	If $a = b$, then a can be substituted for b in any equation or expression.

EXAMPLE 1 Write reasons for each step

Solve $2x + 5 = 20 - 3x$. Write a reason for each step.

Equation	Explanation	Reason
$2x + 5 = 20 - 3x$	Write original equation.	**Given**
$2x + 5 + 3x = 20 - 3x + 3x$	Add $3x$ to each side.	**Addition Property of Equality**
$5x + 5 = 20$	Combine like terms.	**Simplify.**
$5x = 15$	Subtract 5 from each side.	**Subtraction Property of Equality**
$x = 3$	Divide each side by 5.	**Division Property of Equality**

▶ The value of x is 3.

KEY CONCEPT *For Your Notebook*

Distributive Property

$a(b + c) = ab + ac$, where a, b, and c are real numbers.

EXAMPLE 2 Use the Distributive Property

Solve $-4(11x + 2) = 80$. Write a reason for each step.

Solution

Equation	Explanation	Reason
$-4(11x + 2) = 80$	Write original equation.	**Given**
$-44x - 8 = 80$	Multiply.	**Distributive Property**
$-44x = 88$	Add 8 to each side.	**Addition Property of Equality**
$x = -2$	Divide each side by -44.	**Division Property of Equality**

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EXAMPLE 3 Use properties in the real world

HEART RATE When you exercise, your target heart rate should be between 50% to 70% of your maximum heart rate. Your target heart rate r at 70% can be determined by the formula $r = 0.70(220 - a)$ where a represents your age in years. Solve the formula for a.

Solution

Equation	Explanation	Reason
$r = 0.70(220 - a)$	Write original equation.	**Given**
$r = 154 - 0.70a$	Multiply.	**Distributive Property**
$r - 154 = -0.70a$	Subtract 154 from each side.	**Subtraction Property of Equality**
$\frac{r - 154}{-0.70} = a$	Divide each side by -0.70.	**Division Property of Equality**

GUIDED PRACTICE for Examples 1, 2, and 3

In Exercises 1 and 2, solve the equation and write a reason for each step.

1. $4x + 9 = -3x + 2$

2. $14x + 3(7 - x) = -1$

3. Solve the formula $A = \frac{1}{2}bh$ for b.

PROPERTIES The following properties of equality are true for all real numbers. Segment lengths and angle measures are real numbers, so these properties of equality are true for segment lengths and angle measures.

KEY CONCEPT

For Your Notebook

Reflexive Property of Equality

Real Numbers	For any real number a, $a = a$.
Segment Length	For any segment $\overline{AB}$, $AB = AB$.
Angle Measure	For any angle $\angle A$, $m\angle A = m\angle A$.

Symmetric Property of Equality

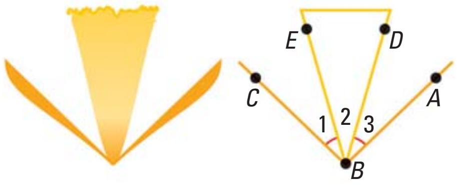
Real Numbers	For any real numbers a and b, if $a = b$, then $b = a$.
Segment Length	For any segments $\overline{AB}$ and $\overline{CD}$, if $AB = CD$, then $CD = AB$.
Angle Measure	For any angles $\angle A$ and $\angle B$, if $m\angle A = m\angle B$, then $m\angle B = m\angle A$.

Transitive Property of Equality

Real Numbers	For any real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.
Segment Length	For any segments $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$, if $AB = CD$ and $CD = EF$, then $AB = EF$.
Angle Measure	For any angles $\angle A$, $\angle B$, and $\angle C$, if $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

EXAMPLE 4 Use properties of equality

LOGO You are designing a logo to sell daffodils. Use the information given. Determine whether $m\angle EBA = m\angle DBC$.

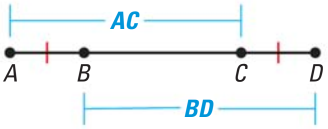


Solution

Equation	Explanation	Reason
$m\angle 1 = m\angle 3$	Marked in diagram.	**Given**
$m\angle EBA = m\angle 3 + m\angle 2$	Add measures of adjacent angles.	**Angle Addition Postulate**
$m\angle EBA = m\angle 1 + m\angle 2$	Substitute $m\angle 1$ for $m\angle 3$.	**Substitution Property of Equality**
$m\angle 1 + m\angle 2 = m\angle DBC$	Add measures of adjacent angles.	**Angle Addition Postulate**
$m\angle EBA = m\angle DBC$	Both measures are equal to the sum of $m\angle 1 + m\angle 2$.	**Transitive Property of Equality**

EXAMPLE 5 Use properties of equality

In the diagram, $AB = CD$. Show that $AC = BD$.



Solution

Equation	Explanation	Reason
$AB = CD$	Marked in diagram.	**Given**
$AC = AB + BC$	Add lengths of adjacent segments.	**Segment Addition Postulate**
$BD = BC + CD$	Add lengths of adjacent segments.	**Segment Addition Postulate**
$AB + BC = CD + BC$	Add BC to each side of $AB = CD$.	**Addition Property of Equality**
$AC = BD$	Substitute AC for $AB + BC$ and BD for $BC + CD$.	**Substitution Property of Equality**

✓ GUIDED PRACTICE for Examples 4 and 5

Name the property of equality the statement illustrates.

4. If $m\angle 6 = m\angle 7$, then $m\angle 7 = m\angle 6$.

5. If $JK = KL$ and $KL = 12$, then $JK = 12$.

6. $m\angle W = m\angle W$

2.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 21, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 5, 27, and 35

◆ = **MULTIPLE REPRESENTATIONS** Ex. 36

SKILL PRACTICE

1. VOCABULARY The following statement is true because of what property? The measure of an angle is equal to itself.

2. ★ WRITING *Explain* how to check the answer to Example 3 on page 106.

EXAMPLES 1 and 2 on pp. 105–106 for Exs. 3–14

WRITING REASONS **Copy the logical argument. Write a reason for each step.**

3.

$3x - 12 = 7x + 8$ Given

$-4x - 12 = 8$?

$-4x = 20$?

$x = -5$?

4.

$5(x - 1) = 4x + 13$ Given

$5x - 5 = 4x + 13$?

$x - 5 = 13$?

$x = 18$?

5. ★ **MULTIPLE CHOICE** Name the property of equality the statement illustrates: If $XY = AB$ and $AB = GH$, then $XY = GH$.

Ⓐ Substitution Ⓑ Reflexive Ⓒ Symmetric Ⓓ Transitive

WRITING REASONS Solve the equation. Write a reason for each step.

6. $5x - 10 = -40$

7. $4x + 9 = 16 - 3x$

8. $5(3x - 20) = -10$

9. $3(2x + 11) = 9$

10. $2(-x - 5) = 12$

11. $44 - 2(3x + 4) = -18x$

12. $4(5x - 9) = -2(x + 7)$

13. $2x - 15 - x = 21 + 10x$

14. $3(7x - 9) - 19x = -15$

EXAMPLE 3
on p. 106
for Exs. 15–20

xy ALGEBRA Solve the equation for *y*. Write a reason for each step.

15. $5x + y = 18$

16. $-4x + 2y = 8$

17. $12 - 3y = 30x$

18. $3x + 9y = -7$

19. $2y + 0.5x = 16$

20. $\frac{1}{2}x - \frac{3}{4}y = -2$

EXAMPLES 4 and 5
on pp. 107–108
for Exs. 21–25

COMPLETING STATEMENTS In Exercises 21–25, use the property to copy and complete the statement.

21. Substitution Property of Equality: If $AB = 20$, then $AB + CD = \underline{\ ?\ }$.

22. Symmetric Property of Equality: If $m\angle 1 = m\angle 2$, then $\underline{\ ?\ }$.

23. Addition Property of Equality: If $AB = CD$, then $\underline{\ ?\ } + EF = \underline{\ ?\ } + EF$.

24. Distributive Property: If $5(x + 8) = 2$, then $\underline{\ ?\ }x + \underline{\ ?\ } = 2$.

25. Transitive Property of Equality: If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $\underline{\ ?\ }$.

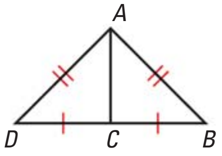
26. **ERROR ANALYSIS** *Describe* and correct the error in solving the equation for *x*.

$7x = x + 24$	Given
$8x = 24$	Addition Property of Equality
$x = 3$	Division Property of Equality

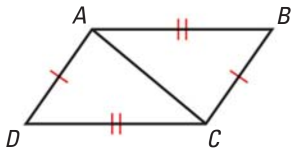
27. ★ **OPEN-ENDED MATH** Write examples from your everyday life that could help you remember the *Reflexive*, *Symmetric*, and *Transitive* Properties of Equality.

PERIMETER In Exercises 28 and 29, show that the perimeter of triangle *ABC* is equal to the perimeter of triangle *ADC*.

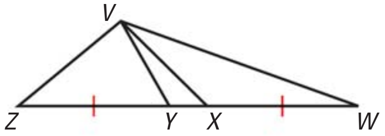
28.



29.



30. **CHALLENGE** In the figure at the right, $\overline{ZY} \cong \overline{XW}$, $ZX = 5x + 17$, $YW = 10 - 2x$, and $YX = 3$. Find ZY and XW.



PROBLEM SOLVING

EXAMPLE 3
on p. 106
for Exs. 31–32

31. **PERIMETER** The formula for the perimeter P of a rectangle is $P = 2\ell + 2w$ where ℓ is the length and w is the width. Solve the formula for ℓ and write a reason for each step. Then find the length of a rectangular lawn whose perimeter is 55 meters and whose width is 11 meters.

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32. **AREA** The formula for the area A of a triangle is $A = \frac{1}{2}bh$ where b is the base and h is the height. Solve the formula for h and write a reason for each step. Then find the height of a triangle whose area is 1768 square inches and whose base is 52 inches.

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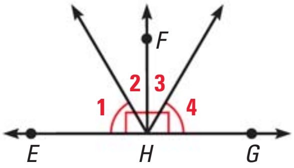
33. **PROPERTIES OF EQUALITY** Copy and complete the table to show $m\angle 2 = m\angle 3$.



Equation	Explanation	Reason
$m\angle 1 = m\angle 4$, $m\angle EHF = 90°$, $m\angle GHF = 90°$	?	Given
$m\angle EHF = m\angle GHF$	?	Substitution Property of Equality
$m\angle EHF = m\angle 1 + m\angle 2$ $m\angle GHF = m\angle 3 + m\angle 4$	Add measures of adjacent angles.	?
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	Write expressions equal to the angle measures.	?
?	Substitute $m\angle 1$ for $m\angle 4$.	?
$m\angle 2 = m\angle 3$	?	Subtraction Property of Equality

34. **MULTI-STEP PROBLEM** Points A, B, C, and D represent stops, in order, along a subway route. The distance between Stops A and C is the same as the distance between Stops B and D.

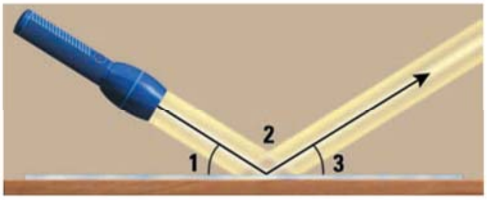
a. Draw a diagram to represent the situation.

b. Use the Segment Addition Postulate to show that the distance between Stops A and B is the same as the distance between Stops C and D.

c. *Justify* part (b) using the Properties of Equality.

EXAMPLE 4
on p. 107
for Ex. 35

35. ★ **SHORT RESPONSE** A flashlight beam is reflected off a mirror lying flat on the ground. Use the information given below to find $m\angle 2$.



$m\angle 1 + m\angle 2 + m\angle 3 = 180°$

$m\angle 1 + m\angle 2 = 148°$

$m\angle 1 = m\angle 3$

○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

◆ = **MULTIPLE REPRESENTATIONS**

36. **MULTIPLE REPRESENTATIONS** The formula to convert a temperature in degrees Fahrenheit (°F) to degrees Celsius (°C) is $C = \frac{5}{9}(F - 32)$.

a. **Writing an Equation** Solve the formula for F. Write a reason for each step.

b. **Making a Table** Make a table that shows the conversion to Fahrenheit for each temperature: 0°C, 20°C, 32°C, and 41°C.

c. **Drawing a Graph** Use your table to graph the temperature in degrees Celsius (°C) as a function of the temperature in degrees Fahrenheit (°F). Is this a linear function?

CHALLENGE **In Exercises 37 and 38, decide whether the relationship is *reflexive*, *symmetric*, or *transitive*.**

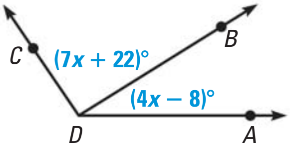
37. **Group:** two employees in a grocery store
Relationship: "worked the same hours as"
Example: Yen worked the same hours as Jim.

38. **Group:** negative numbers on a number line
Relationship: "is less than"
Example: −4 is less than −1.

MIXED REVIEW

PREVIEW Prepare for Lesson 2.6 in Exs. 39–40.

In the diagram, $m\angle ADC = 124°$. *(p. 24)*



39. Find $m\angle ADB$.

40. Find $m\angle BDC$.

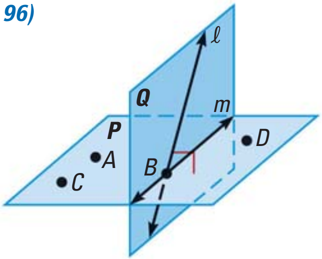
41. Find a counterexample to show the conjecture is false.

Conjecture All polygons have five sides. *(p. 72)*

42. Select the word(s) that make(s) the conclusion true. If $m\angle X = m\angle Y$ and $m\angle Y = m\angle Z$, then $m\angle X$ (*is*, *may be*, or *is not*) equal to $m\angle Z$. *(p. 87)*

QUIZ *for Lessons 2.4–2.5*

Use the diagram to determine if the statement is *true* or *false*. *(p. 96)*



1. Points B, C, and D are coplanar.

2. Point A is on line ℓ.

3. Plane P and plane Q are perpendicular.

Solve the equation. Write a reason for each step. *(p. 105)*

4. $x + 20 = 35$

5. $5x - 14 = 16 + 3x$

Use the property to copy and complete the statement. *(p. 105)*

6. Subtraction Property of Equality: If $AB = CD$, then _?_ $- EF =$ _?_ $- EF$.

7. Transitive Property of Equality: If $a = b$ and $b = c$, then _?_ = _?_.

2.6 Prove Statements about Segments and Angles

Before You used deductive reasoning.

Now You will write proofs using geometric theorems.

Why? So you can prove angles are congruent, as in Ex. 21.

Key Vocabulary
- **proof**
- **two-column proof**
- **theorem**

A **proof** is a logical argument that shows a statement is true. There are several formats for proofs. A **two-column proof** has numbered statements and corresponding reasons that show an argument in a logical order.

In a two-column proof, each statement in the left-hand column is either given information or the result of applying a known property or fact to statements already made. Each reason in the right-hand column is the explanation for the corresponding statement.

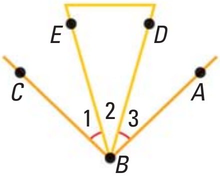
EXAMPLE 1 Write a two-column proof

WRITE PROOFS
Writing a two-column proof is a formal way of organizing your reasons to show a statement is true.

Write a two-column proof for the situation in Example 4 on page 107.

GIVEN ▶ $m\angle 1 = m\angle 3$

PROVE ▶ $m\angle EBA = m\angle DBC$



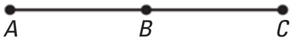
STATEMENTS	REASONS
1. $m\angle 1 = m\angle 3$	**1.** Given
2. $m\angle EBA = m\angle 3 + m\angle 2$	**2.** Angle Addition Postulate
3. $m\angle EBA = m\angle 1 + m\angle 2$	**3.** Substitution Property of Equality
4. $m\angle 1 + m\angle 2 = m\angle DBC$	**4.** Angle Addition Postulate
5. $m\angle EBA = m\angle DBC$	**5.** Transitive Property of Equality

✓ GUIDED PRACTICE for Example 1

1. Four steps of a proof are shown. Give the reasons for the last two steps.

GIVEN ▶ $AC = AB + AB$

PROVE ▶ $AB = BC$



STATEMENTS	REASONS
1. $AC = AB + AB$	**1.** Given
2. $AB + BC = AC$	**2.** Segment Addition Postulate
3. $AB + AB = AB + BC$	**3.** ?
4. $AB = BC$	**4.** ?

THEOREMS The reasons used in a proof can include definitions, properties, postulates, and *theorems*. A **theorem** is a statement that can be proven. Once you have proven a theorem, you can use the theorem as a reason in other proofs.

TAKE NOTES
Be sure to copy all new theorems in your notebook. Notice that the theorem box tells you where to find the proof(s).

THEOREMS *For Your Notebook*

THEOREM 2.1 Congruence of Segments

Segment congruence is reflexive, symmetric, and transitive.

Reflexive For any segment AB, $\overline{AB} \cong \overline{AB}$.

Symmetric If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Proofs: p. 137; Ex. 5, p. 121; Ex. 26, p. 118

THEOREM 2.2 Congruence of Angles

Angle congruence is reflexive, symmetric, and transitive.

Reflexive For any angle A, $\angle A \cong \angle A$.

Symmetric If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

Proofs: Ex. 25, p. 118; Concept Summary, p. 114; Ex. 21, p. 137

EXAMPLE 2 Name the property shown

Name the property illustrated by the statement.

a. If $\angle R \cong \angle T$ and $\angle T \cong \angle P$, then $\angle R \cong \angle P$.

b. If $\overline{NK} \cong \overline{BD}$, then $\overline{BD} \cong \overline{NK}$.

Solution

a. Transitive Property of Angle Congruence

b. Symmetric Property of Segment Congruence

✓ GUIDED PRACTICE for Example 2

Name the property illustrated by the statement.

2. $\overline{CD} \cong \overline{CD}$

3. If $\angle Q \cong \angle V$, then $\angle V \cong \angle Q$.

In this lesson, most of the proofs involve showing that congruence and equality are equivalent. You may find that what you are asked to prove seems to be obviously true. It is important to practice writing these proofs so that you will be prepared to write more complicated proofs in later chapters.

EXAMPLE 3 Use properties of equality

Prove this property of midpoints: If you know that M is the midpoint of $\overline{AB}$, prove that AB is two times AM and AM is one half of AB.

WRITE PROOFS
Before writing a proof, organize your reasoning by copying or drawing a diagram for the situation described. Then identify the GIVEN and PROVE statements.

GIVEN ▶ M is the midpoint of $\overline{AB}$.

PROVE ▶ **a.** $AB = 2 \cdot AM$

b. $AM = \frac{1}{2}AB$

STATEMENTS	REASONS
1. M is the midpoint of $\overline{AB}$.	**1.** Given
2. $\overline{AM} \cong \overline{MB}$	**2.** Definition of midpoint
3. $AM = MB$	**3.** Definition of congruent segments
4. $AM + MB = AB$	**4.** Segment Addition Postulate
5. $AM + AM = AB$	**5.** Substitution Property of Equality
a. 6. $2AM = AB$	**6.** Distributive Property
b. 7. $AM = \frac{1}{2}AB$	**7.** Division Property of Equality

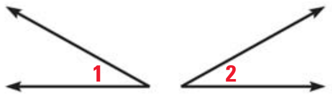
✓ GUIDED PRACTICE for Example 3

4. WHAT IF? Look back at Example 3. What would be different if you were proving that $AB = 2 \cdot MB$ and that $MB = \frac{1}{2}AB$ instead?

CONCEPT SUMMARY *For Your Notebook*

Writing a Two-Column Proof

In a proof, you make one statement at a time, until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.



Copy or draw diagrams and label given information to help develop proofs.

Proof of the Symmetric Property of Angle Congruence

GIVEN ▶ $\angle 1 \cong \angle 2$

PROVE ▶ $\angle 2 \cong \angle 1$

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	**1.** Given
2. $m\angle 1 = m\angle 2$	**2.** Definition of congruent angles
3. $m\angle 2 = m\angle 1$	**3.** Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	**4.** Definition of congruent angles

Statements based on facts that you know or on conclusions from deductive reasoning →

← Definitions, postulates, or proven theorems that allow you to state the corresponding statement

↑ The number of statements will vary.

↑ Remember to give a reason for the last statement.

EXAMPLE 4 Solve a multi-step problem

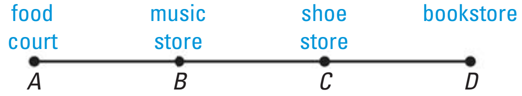
SHOPPING MALL Walking down a hallway at the mall, you notice the music store is halfway between the food court and the shoe store. The shoe store is halfway between the music store and the bookstore. Prove that the distance between the entrances of the food court and music store is the same as the distance between the entrances of the shoe store and bookstore.

ANOTHER WAY

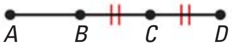
For an alternative method for solving the problem in Example 4, turn to page 120 for the **Problem Solving Workshop**.

Solution

STEP 1 **Draw** and label a diagram.



STEP 2 **Draw** separate diagrams to show mathematical relationships.



STEP 3 **State** what is given and what is to be proved for the situation. Then write a proof.

GIVEN ▶ B is the midpoint of $\overline{AC}$.
C is the midpoint of $\overline{BD}$.

PROVE ▶ $AB = CD$

STATEMENTS	REASONS
1. B is the midpoint of $\overline{AC}$. C is the midpoint of $\overline{BD}$.	**1.** Given
2. $\overline{AB} \cong \overline{BC}$	**2.** Definition of midpoint
3. $\overline{BC} \cong \overline{CD}$	**3.** Definition of midpoint
4. $\overline{AB} \cong \overline{CD}$	**4.** Transitive Property of Congruence
5. $AB = CD$	**5.** Definition of congruent segments

✓ GUIDED PRACTICE for Example 4

5. In Example 4, does it matter what the actual distances are in order to prove the relationship between AB and CD? *Explain.*

6. In Example 4, there is a clothing store halfway between the music store and the shoe store. What other two store entrances are the same distance from the entrance of the clothing store?

2.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 15, and 21

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 4, 12, 19, 27, and 28

SKILL PRACTICE

1. **VOCABULARY** What is a *theorem*? How is it different from a *postulate*?

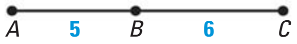
2. ★ **WRITING** You can use theorems as reasons in a two-column proof. What other types of statements can you use as reasons in a two-column proof? Give examples.

EXAMPLE 1 on p. 112 for Exs. 3–4

3. **DEVELOPING PROOF** Copy and complete the proof.

GIVEN ▶ $AB = 5, BC = 6$

PROVE ▶ $AC = 11$



STATEMENTS	REASONS
1. $AB = 5, BC = 6$	1. Given
2. $AC = AB + BC$	2. Segment Addition Postulate
3. $AC = 5 + 6$	3. $\underline{\ ?\ }$
4. $\underline{\ ?\ }$	4. Simplify.

4. ★ **MULTIPLE CHOICE** Which property listed is the reason for the last step in the proof?

GIVEN ▶ $m\angle 1 = 59°, m\angle 2 = 59°$

PROVE ▶ $m\angle 1 = m\angle 2$

STATEMENTS	REASONS
1. $m\angle 1 = 59°, m\angle 2 = 59°$	1. Given
2. $59° = m\angle 2$	2. Symmetric Property of Equality
3. $m\angle 1 = m\angle 2$	3. $\underline{\ ?\ }$

Ⓐ Transitive Property of Equality
Ⓑ Reflexive Property of Equality
Ⓒ Symmetric Property of Equality
Ⓓ Distributive Property

EXAMPLES 2 and 3 on pp. 113–114 for Exs. 5–13

USING PROPERTIES **Use the property to copy and complete the statement.**

5. Reflexive Property of Congruence: $\underline{\ ?\ } \cong \overline{SE}$

6. Symmetric Property of Congruence: If $\underline{\ ?\ } \cong \underline{\ ?\ }$, then $\angle RST \cong \angle JKL$.

7. Transitive Property of Congruence: If $\angle F \cong \angle J$ and $\underline{\ ?\ } \cong \underline{\ ?\ }$, then $\angle F \cong \angle L$.

NAMING PROPERTIES **Name the property illustrated by the statement.**

8. If $\overline{DG} \cong \overline{CT}$, then $\overline{CT} \cong \overline{DG}$.

9. $\angle VWX \cong \angle VWX$

10. If $\overline{JK} \cong \overline{MN}$ and $\overline{MN} \cong \overline{XY}$, then $\overline{JK} \cong \overline{XY}$.

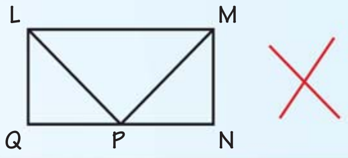
11. $YZ = ZY$

12. ★ **MULTIPLE CHOICE** Name the property illustrated by the statement "If $\overline{CD} \cong \overline{MN}$, then $\overline{MN} \cong \overline{CD}$."

Ⓐ Reflexive Property of Equality
Ⓑ Symmetric Property of Equality
Ⓒ Symmetric Property of Congruence
Ⓓ Transitive Property of Congruence

13. **ERROR ANALYSIS** In the diagram below, $\overline{MN} \cong \overline{LQ}$ and $\overline{LQ} \cong \overline{PN}$. *Describe* and correct the error in the reasoning.

Because $\overline{MN} \cong \overline{LQ}$ and $\overline{LQ} \cong \overline{PN}$, then $\overline{MN} \cong \overline{PN}$ by the Reflexive Property of Segment Congruence.



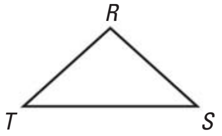
EXAMPLE 4
on p. 115
for Exs. 14–15

MAKING A SKETCH **In Exercises 14 and 15, sketch a diagram that represents the given information.**

14. **CRYSTALS** The shape of a crystal can be represented by intersecting lines and planes. Suppose a crystal is *cubic*, which means it can be represented by six planes that intersect at right angles.

15. **BEACH VACATION** You are on vacation at the beach. Along the boardwalk, the bike rentals are halfway between your cottage and the kite shop. The snack shop is halfway between your cottage and the bike rentals. The arcade is halfway between the bike rentals and the kite shop.

16. **DEVELOPING PROOF** Copy and complete the proof.



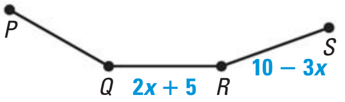
GIVEN ▶ $RT = 5$, $RS = 5$, $\overline{RT} \cong \overline{TS}$

PROVE ▶ $\overline{RS} \cong \overline{TS}$

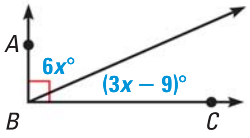
STATEMENTS	REASONS
1. $RT = 5$, $RS = 5$, $\overline{RT} \cong \overline{TS}$	**1.** ?
2. $RS = RT$	**2.** Transitive Property of Equality
3. $RT = TS$	**3.** Definition of congruent segments
4. $RS = TS$	**4.** Transitive Property of Equality
5. $\overline{RS} \cong \overline{TS}$	**5.** ?

xy **ALGEBRA** **Solve for x using the given information. *Explain* your steps.**

17. **GIVEN** ▶ $\overline{QR} \cong \overline{PQ}$, $\overline{RS} \cong \overline{PQ}$



18. **GIVEN** ▶ $m\angle ABC = 90°$



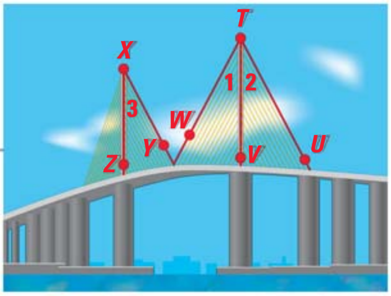
19. ★ **SHORT RESPONSE** *Explain* why writing a proof is an example of deductive reasoning, not inductive reasoning.

20. **CHALLENGE** Point P is the midpoint of $\overline{MN}$ and point Q is the midpoint of $\overline{MP}$. Suppose $\overline{AB}$ is congruent to $\overline{MP}$, and $\overline{PN}$ has length x. Write the length of the segments in terms of x. *Explain.*

a. $\overline{AB}$ **b.** $\overline{MN}$ **c.** $\overline{MQ}$ **d.** $\overline{NQ}$

PROBLEM SOLVING

21. **BRIDGE** In the bridge in the illustration, it is known that $\angle 2 \cong \angle 3$ and $\overrightarrow{TV}$ bisects $\angle UTW$. Copy and complete the proof to show that $\angle 1 \cong \angle 3$.



STATEMENTS	REASONS
1. $\overrightarrow{TV}$ bisects $\angle UTW$.	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** ___?___
3. $\angle 2 \cong \angle 3$	**3.** Given
4. $\angle 1 \cong \angle 3$	**4.** ___?___

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EXAMPLE 3
on p. 114
for Ex. 22

22. **DEVELOPING PROOF** Write a complete proof by matching each statement with its corresponding reason.

GIVEN ▶ $\overrightarrow{QS}$ is an angle bisector of $\angle PQR$.

PROVE ▶ $m\angle PQS = \frac{1}{2} m\angle PQR$

STATEMENTS	REASONS
1. $\overrightarrow{QS}$ is an angle bisector of $\angle PQR$.	**A.** Definition of angle bisector
2. $\angle PQS \cong \angle SQR$	**B.** Distributive Property
3. $m\angle PQS = m\angle SQR$	**C.** Angle Addition Postulate
4. $m\angle PQS + m\angle SQR = m\angle PQR$	**D.** Given
5. $m\angle PQS + m\angle PQS = m\angle PQR$	**E.** Division Property of Equality
6. $2 \cdot m\angle PQS = m\angle PQR$	**F.** Definition of congruent angles
7. $m\angle PQS = \frac{1}{2} m\angle PQR$	**G.** Substitution Property of Equality

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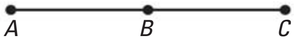
PROOF **Use the given information and the diagram to prove the statement.**

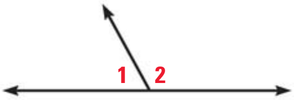
23. **GIVEN** ▶ $2AB = AC$

PROVE ▶ $AB = BC$



24. **GIVEN** ▶ $m\angle 1 + m\angle 2 = 180°$
$m\angle 1 = 62°$

PROVE ▶ $m\angle 2 = 118°$

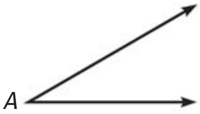


PROVING PROPERTIES **Prove the indicated property of congruence.**

25. Reflexive Property of Angle Congruence

GIVEN ▶ A is an angle.

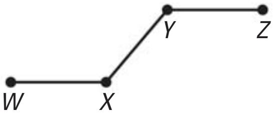
PROVE ▶ $\angle A \cong \angle A$



26. Transitive Property of Segment Congruence

GIVEN ▶ $\overline{WX} \cong \overline{XY}$ and $\overline{XY} \cong \overline{YZ}$

PROVE ▶ $\overline{WX} \cong \overline{YZ}$



27. ★ **SHORT RESPONSE** In the sculpture shown, $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$. Classify the triangle and *justify* your reasoning.

28. ★ **SHORT RESPONSE** You use a computer drawing program to create a line segment. You copy the segment and paste it. You copy the pasted segment and then paste it, and so on. How do you know all the line segments are congruent?

EXAMPLE 4
on p. 115
for Ex. 29

29. **MULTI-STEP PROBLEM** The distance from the restaurant to the shoe store is the same as the distance from the cafe to the florist. The distance from the shoe store to the movie theater is the same as the distance from the movie theater to the cafe, and from the florist to the dry cleaners.

Use the steps below to prove that the distance from the restaurant to the movie theater is the same as the distance from the cafe to the dry cleaners.

a. Draw and label a diagram to show the mathematical relationships.

b. State what is given and what is to be proved for the situation.

c. Write a two-column proof.

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30. **CHALLENGE** The distance from Springfield to Lakewood City is equal to the distance from Springfield to Bettsville. Janisburg is 50 miles farther from Springfield than Bettsville is. Moon Valley is 50 miles farther from Springfield than Lakewood City is.

a. Assume all five cities lie in a straight line. Draw a diagram that represents this situation.

b. Suppose you do not know that all five cities lie in a straight line. Draw a diagram that is different from the one in part (a) to represent the situation.

c. *Explain* the differences in the two diagrams.

MIXED REVIEW

PREVIEW
Prepare for Lesson 2.7 in Exs. 31–33.

Given $m\angle 1$, find the measure of an angle that is complementary to $\angle 1$ and the measure of an angle that is supplementary to $\angle 1$. *(p. 35)*

31. $m\angle 1 = 47°$ **32.** $m\angle 1 = 29°$ **33.** $m\angle 1 = 89°$

Solve the equation. Write a reason for each step. *(p. 105)*

34. $5x + 14 = -16$ **35.** $2x - 9 = 15 - 4x$ **36.** $x + 28 = -11 - 3x - 17$

PROBLEM SOLVING WORKSHOP
LESSON 2.6

Using ALTERNATIVE METHODS

Another Way to Solve Example 4, page 115

MULTIPLE REPRESENTATIONS The first step in writing any proof is to make a plan. A diagram or *visual organizer* can help you plan your proof. The steps of a proof must be in a logical order, but there may be more than one correct order.

PROBLEM

SHOPPING MALL Walking down a hallway at the mall, you notice the music store is halfway between the food court and the shoe store. The shoe store is halfway between the music store and the bookstore. Prove that the distance between the entrances of the food court and music store is the same as the distance between the entrances of the shoe store and bookstore.

METHOD

Using a Visual Organizer

STEP 1 **Use** a visual organizer to map out your proof.

The music store is halfway between the food court and the shoe store. The shoe store is halfway between the music store and the bookstore.

Given information	M is halfway between F and S.	S is halfway between M and B.
Deductions from given information	M is the midpoint of $\overline{FS}$. So, $FM = MS$.	S is the midpoint of $\overline{MB}$. So, $MS = SB$.
Statement to prove	$FM = SB$	

STEP 2 **Write** a proof using the lengths of the segments.

GIVEN ▶ M is halfway between F and S.
S is halfway between M and B.

PROVE ▶ $FM = SB$

STATEMENTS	REASONS
1. M is halfway between F and S.	**1.** Given
2. S is halfway between M and B.	**2.** Given
3. M is the midpoint of $\overline{FS}$.	**3.** Definition of midpoint
4. S is the midpoint of $\overline{MB}$.	**4.** Definition of midpoint
5. $FM = MS$ and $MS = SB$	**5.** Definition of midpoint
6. $MS = MS$	**6.** Reflexive Property of Equality
7. $FM = SB$	**7.** Substitution Property of Equality

PRACTICE

1. **COMPARE PROOFS** *Compare* the proof on the previous page and the proof in Example 4 on page 115.
   - **a.** How are the proofs the same? How are they different?
   - **b.** Which proof is easier for you to understand? *Explain*.

2. **REASONING** Below is a proof of the Transitive Property of Angle Congruence. What is another reason you could give for Statement 3? *Explain*.

GIVEN ▶ $\angle A \cong \angle B$ and $\angle B \cong \angle C$

PROVE ▶ $\angle A \cong \angle C$

STATEMENTS	REASONS
1. $\angle A \cong \angle B, \angle B \cong \angle C$	**1.** Given
2. $m\angle A = m\angle B, m\angle B = m\angle C$	**2.** Definition of congruent angles
3. $m\angle A = m\angle C$	**3.** Transitive Property of Equality
4. $\angle A \cong \angle C$	**4.** Definition of congruent angles

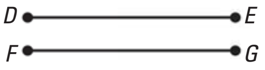
3. **SHOPPING MALL** You are at the same mall as on page 120 and you notice that the bookstore is halfway between the shoe store and the toy store. Draw a diagram or make a visual organizer, then write a proof to show that the distance from the entrances of the food court and music store is the same as the distance from the entrances of the book store and toy store.

4. **WINDOW DESIGN** The entrance to the mall has a decorative window above the main doors as shown. The colored dividers form congruent angles. Draw a diagram or make a visual organizer, then write a proof to show that the angle measure between the red dividers is half the measure of the angle between the blue dividers.

5. **COMPARE PROOFS** Below is a proof of the Symmetric Property of Segment Congruence.

GIVEN ▶ $\overline{DE} \cong \overline{FG}$

PROVE ▶ $\overline{FG} \cong \overline{DE}$



STATEMENTS	REASONS
1. $\overline{DE} \cong \overline{FG}$	**1.** Given
2. $DE = FG$	**2.** Definition of congruent segments
3. $FG = DE$	**3.** Symmetric Property of Equality
4. $\overline{FG} \cong \overline{DE}$	**4.** Definition of congruent segments

   - **a.** *Compare* this proof to the proof of the Symmetric Property of Angle Congruence in the Concept Summary on page 114. What makes the proofs different? *Explain*.
   - **b.** *Explain* why Statement 2 above cannot be $\overline{FG} \cong \overline{DE}$.

Investigating Geometry ACTIVITY *Use before Lesson 2.7*

2.7 Angles and Intersecting Lines

MATERIALS • graphing calculator or computer

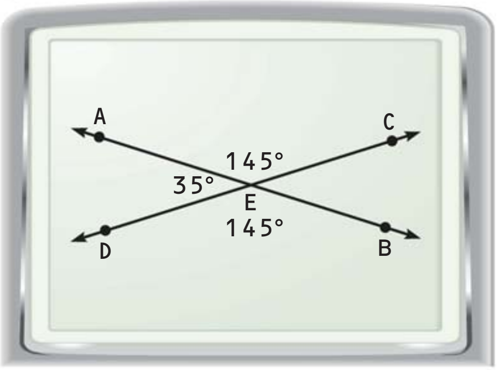
QUESTION **What is the relationship between the measures of the angles formed by intersecting lines?**

You can use geometry drawing software to investigate the measures of angles formed when lines intersect.

EXPLORE 1 Measure linear pairs formed by intersecting lines

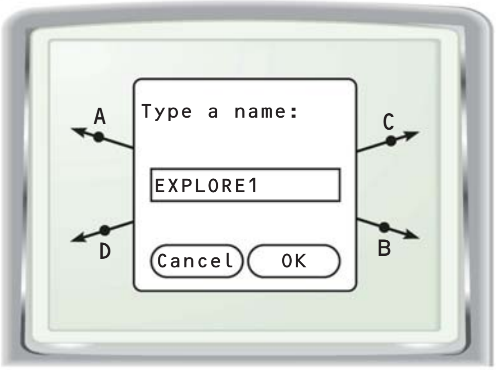
STEP 1 ***Draw two intersecting lines*** Draw and label $\overleftrightarrow{AB}$. Draw and label $\overleftrightarrow{CD}$ so that it intersects $\overleftrightarrow{AB}$. Draw and label the point of intersection E.

STEP 2



Measure angles Measure $\angle AEC$, $\angle AED$, and $\angle DEB$. Move point C to change the angles.

STEP 3



Type a name:

EXPLORE1

Save Save as “EXPLORE1” by choosing Save from the F1 menu and typing the name.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. *Describe* the relationship between $\angle AEC$ and $\angle AED$.
2. *Describe* the relationship between $\angle AED$ and $\angle DEB$.
3. What do you notice about $\angle AEC$ and $\angle DEB$?
4. In Explore 1, what happens when you move C to a different position? Do the angle relationships stay the same? Make a conjecture about two angles supplementary to the same angle.
5. Do you think your conjecture will be true for supplementary angles that are not adjacent? *Explain*.

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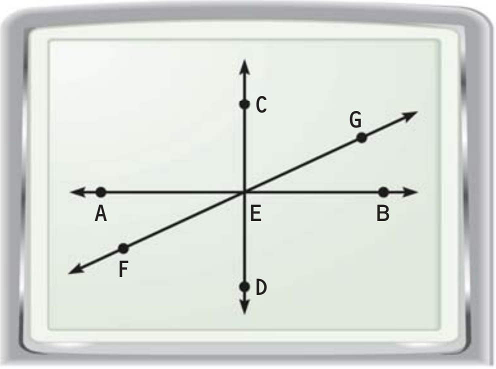
Keystrokes

EXPLORE 2 Measure complementary angles

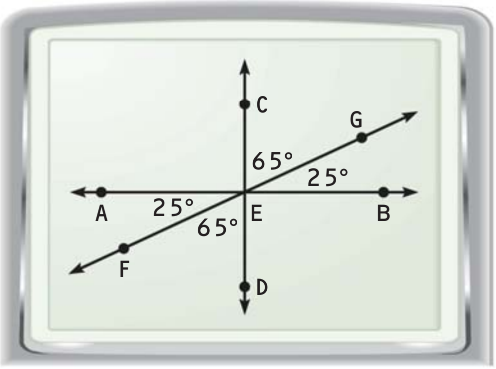
STEP 1 ***Draw two perpendicular lines*** Draw and label $\overleftrightarrow{AB}$. Draw point E on $\overleftrightarrow{AB}$. Draw and label $\overleftrightarrow{EC} \perp \overleftrightarrow{AB}$. Draw and label point D on $\overleftrightarrow{EC}$ so that E is between C and D as shown in Step 2.

STEP 2



Draw another line Draw and label $\overleftrightarrow{EG}$ so that G is in the interior of $\angle CEB$. Draw point F on $\overleftrightarrow{EG}$ as shown. Save as "EXPLORE2".

STEP 3



Measure angles Measure $\angle AEF$, $\angle FED$, $\angle CEG$, and $\angle GEB$. Move point G to change the angles.

EXPLORE 3 Measure vertical angles formed by intersecting lines

STEP 1 ***Draw two intersecting lines*** Draw and label $\overleftrightarrow{AB}$. Draw and label $\overleftrightarrow{CD}$ so that it intersects $\overleftrightarrow{AB}$. Draw and label the point of intersection E.

STEP 2 ***Measure angles*** Measure $\angle AEC$, $\angle AED$, $\angle BEC$, and $\angle DEB$. Move point C to change the angles. Save as "EXPLORE3".

DRAW CONCLUSIONS Use your observations to complete these exercises

6. In Explore 2, does the angle relationship stay the same as you move G?

7. In Explore 2, make a conjecture about the relationship between $\angle CEG$ and $\angle GEB$. Write your conjecture in if-then form.

8. In Explore 3, the intersecting lines form two pairs of vertical angles. Make a conjecture about the relationship between any two vertical angles. Write your conjecture in if-then form.

9. Name the pairs of vertical angles in Explore 2. Use this drawing to test your conjecture from Exercise 8.

2.7 Prove Angle Pair Relationships

Before You identified relationships between pairs of angles.

Now You will use properties of special pairs of angles.

Why? So you can describe angles found in a home, as in Ex. 44.

Key Vocabulary
- **complementary angles,** *p. 35*
- **supplementary angles,** *p. 35*
- **linear pair,** *p. 37*
- **vertical angles,** *p. 37*

Sometimes, a new theorem describes a relationship that is useful in writing proofs. For example, using the *Right Angles Congruence Theorem* will reduce the number of steps you need to include in a proof involving right angles.

THEOREM

For Your Notebook

THEOREM 2.3 Right Angles Congruence Theorem

All right angles are congruent.

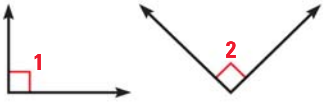
Proof: below

PROOF Right Angles Congruence Theorem

WRITE PROOFS
When you prove a theorem, write the hypothesis of the theorem as the GIVEN statement. The conclusion is what you must PROVE.

GIVEN ▶ $\angle 1$ and $\angle 2$ are right angles.

PROVE ▶ $\angle 1 \cong \angle 2$



STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are right angles.	**1.** Given
2. $m\angle 1 = 90°, m\angle 2 = 90°$	**2.** Definition of right angle
3. $m\angle 1 = m\angle 2$	**3.** Transitive Property of Equality
4. $\angle 1 \cong \angle 2$	**4.** Definition of congruent angles

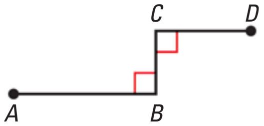
EXAMPLE 1 Use right angle congruence

Write a proof.

AVOID ERRORS
The given information in Example 1 is about perpendicular lines. You must then use deductive reasoning to show the angles are right angles.

GIVEN ▶ $\overline{AB} \perp \overline{BC}, \overline{DC} \perp \overline{BC}$

PROVE ▶ $\angle B \cong \angle C$

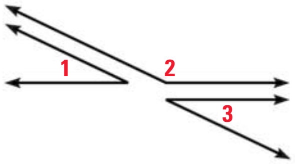


STATEMENTS	REASONS
1. $\overline{AB} \perp \overline{BC}, \overline{DC} \perp \overline{BC}$	**1.** Given
2. $\angle B$ and $\angle C$ are right angles.	**2.** Definition of perpendicular lines
3. $\angle B \cong \angle C$	**3.** Right Angles Congruence Theorem

THEOREMS *For Your Notebook*

THEOREM 2.4 Congruent Supplements Theorem

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

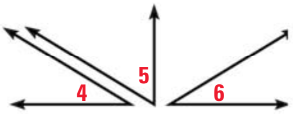


If $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 2$ are supplementary, then $\angle 1 \cong \angle 3$.

Proof: Example 2, below; Ex. 36, p. 129

THEOREM 2.5 Congruent Complements Theorem

If two angles are complementary to the same angle (or to congruent angles), then they are congruent.



If $\angle 4$ and $\angle 5$ are complementary and $\angle 6$ and $\angle 5$ are complementary, then $\angle 4 \cong \angle 6$.

Proof: Ex. 37, p. 129; Ex. 41, p. 130

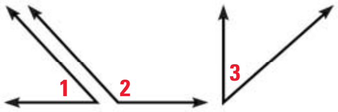
To prove Theorem 2.4, you must prove two cases: one with angles supplementary to the same angle and one with angles supplementary to congruent angles. The proof of Theorem 2.5 also requires two cases.

EXAMPLE 2 Prove a case of Congruent Supplements Theorem

Prove that two angles supplementary to the same angle are congruent.

GIVEN ▶ $\angle 1$ and $\angle 2$ are supplements.
$\angle 3$ and $\angle 2$ are supplements.

PROVE ▶ $\angle 1 \cong \angle 3$



STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are supplements. $\angle 3$ and $\angle 2$ are supplements.	**1.** Given
2. $m\angle 1 + m\angle 2 = 180°$ $m\angle 3 + m\angle 2 = 180°$	**2.** Definition of supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	**3.** Transitive Property of Equality
4. $m\angle 1 = m\angle 3$	**4.** Subtraction Property of Equality
5. $\angle 1 \cong \angle 3$	**5.** Definition of congruent angles

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✓ GUIDED PRACTICE for Examples 1 and 2

1. How many steps do you save in the proof in Example 1 by using the *Right Angles Congruence Theorem*?

2. Draw a diagram and write GIVEN and PROVE statements for a proof of each case of the *Congruent Complements Theorem*.

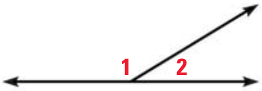
INTERSECTING LINES When two lines intersect, pairs of vertical angles and linear pairs are formed. The relationship that you used in Lesson 1.5 for linear pairs is formally stated below as the *Linear Pair Postulate*. This postulate is used in the proof of the *Vertical Angles Congruence Theorem*.

POSTULATE For Your Notebook

POSTULATE 12 Linear Pair Postulate

If two angles form a linear pair, then they are supplementary.

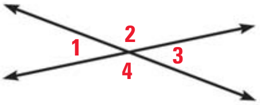
$\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 = 180°$.



THEOREM For Your Notebook

THEOREM 2.6 Vertical Angles Congruence Theorem

Vertical angles are congruent.



$\angle 1 \cong \angle 3, \angle 2 \cong \angle 4$

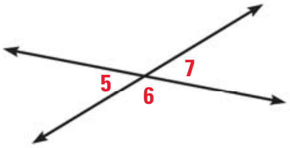
Proof: Example 3, below

EXAMPLE 3 Prove the Vertical Angles Congruence Theorem

Prove vertical angles are congruent.

GIVEN ▶ $\angle 5$ and $\angle 7$ are vertical angles.

PROVE ▶ $\angle 5 \cong \angle 7$

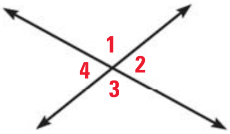


USE A DIAGRAM
You can use information labeled in a diagram in your proof.

STATEMENTS	REASONS
1. $\angle 5$ and $\angle 7$ are vertical angles.	**1.** Given
2. $\angle 5$ and $\angle 6$ are a linear pair. $\angle 6$ and $\angle 7$ are a linear pair.	**2.** Definition of linear pair, as shown in the diagram
3. $\angle 5$ and $\angle 6$ are supplementary. $\angle 6$ and $\angle 7$ are supplementary.	**3.** Linear Pair Postulate
4. $\angle 5 \cong \angle 7$	**4.** Congruent Supplements Theorem

✓ GUIDED PRACTICE for Example 3

In Exercises 3–5, use the diagram.



3. If $m\angle 1 = 112°$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

4. If $m\angle 2 = 67°$, find $m\angle 1$, $m\angle 3$, and $m\angle 4$.

5. If $m\angle 4 = 71°$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$.

6. Which previously proven theorem is used in Example 3 as a reason?

★ EXAMPLE 4 Standardized Test Practice

ELIMINATE CHOICES
Look for angle pair relationships in the diagram. The angles in the diagram are supplementary, not complementary or congruent, so eliminate choices A and C.

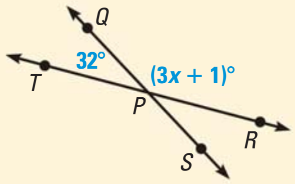
Which equation can be used to find x?

Ⓐ $32 + (3x + 1) = 90$

Ⓑ $32 + (3x + 1) = 180$

Ⓒ $32 = 3x + 1$

Ⓓ $3x + 1 = 212$



Solution

Because $\angle TPQ$ and $\angle QPR$ form a linear pair, the sum of their measures is 180°.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Example 4

Use the diagram in Example 4.

7. Solve for x.

8. Find $m\angle TPS$.

2.7 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 13, and 39
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 16, 30, and 45

SKILL PRACTICE

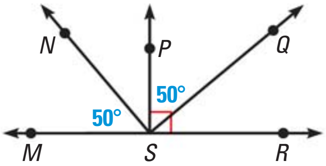
1. VOCABULARY Copy and complete: If two lines intersect at a point, then the _?_ angles formed by the intersecting lines are congruent.

2. ★ **WRITING** *Describe* the relationship between the angle measures of complementary angles, supplementary angles, vertical angles, and linear pairs.

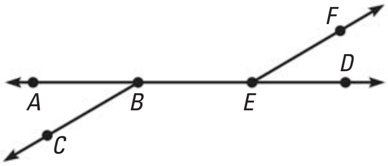
EXAMPLES 1 and 2
on pp. 124–125 for Exs. 3–7

IDENTIFY ANGLES **Identify the pair(s) of congruent angles in the figures below. *Explain* how you know they are congruent.**

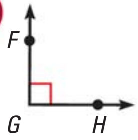
3.

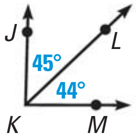


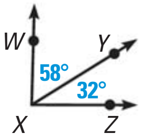
4. $\angle ABC$ is supplementary to $\angle CBD$. $\angle CBD$ is supplementary to $\angle DEF$.



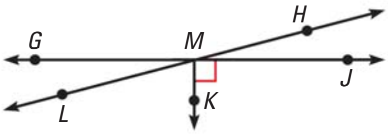
5.







6.



7. ★ **SHORT RESPONSE** The x-axis and y-axis in a coordinate plane are perpendicular to each other. The axes form four angles. Are the four angles congruent right angles? *Explain.*

EXAMPLE 3
on p. 126
for Exs. 8–11

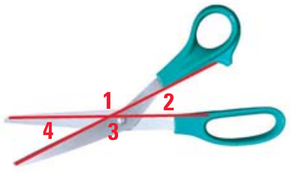
FINDING ANGLE MEASURES **In Exercises 8–11, use the diagram at the right.**

8. If $m\angle 1 = 145^\circ$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

9. If $m\angle 3 = 168^\circ$, find $m\angle 1$, $m\angle 2$, and $m\angle 4$.

10. If $m\angle 4 = 37^\circ$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$.

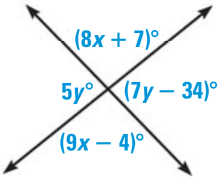
11. If $m\angle 2 = 62^\circ$, find $m\angle 1$, $m\angle 3$, and $m\angle 4$.



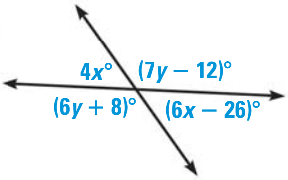
EXAMPLE 4
on p. 127
for Exs. 12–14

xy ALGEBRA **Find the values of x and y.**

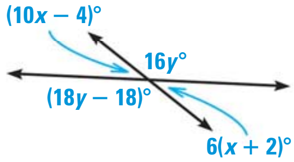
12.



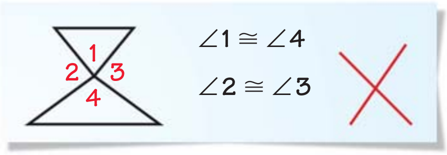
13.



14.



15. **ERROR ANALYSIS** *Describe* the error in stating that $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$.



16. ★ **MULTIPLE CHOICE** In a figure, $\angle A$ and $\angle D$ are complementary angles and $m\angle A = 4x^\circ$. Which expression can be used to find $m\angle D$?

Ⓐ $(4x + 90)^\circ$ Ⓑ $(180 - 4x)^\circ$ Ⓒ $(180 + 4x)^\circ$ Ⓓ $(90 - 4x)^\circ$

FINDING ANGLE MEASURES **In Exercises 17–21, copy and complete the statement given that $m\angle FHE = m\angle BHG = m\angle AHF = 90^\circ$.**

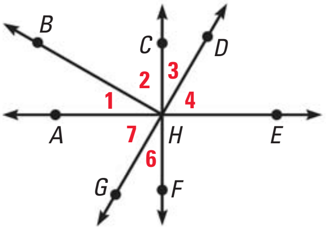
17. If $m\angle 3 = 30^\circ$, then $m\angle 6 =$ _?_.

18. If $m\angle BHF = 115^\circ$, then $m\angle 3 =$ _?_.

19. If $m\angle 6 = 27^\circ$, then $m\angle 1 =$ _?_.

20. If $m\angle DHF = 133^\circ$, then $m\angle CHG =$ _?_.

21. If $m\angle 3 = 32^\circ$, then $m\angle 2 =$ _?_.



ANALYZING STATEMENTS **Two lines that are not perpendicular intersect such that $\angle 1$ and $\angle 2$ are a linear pair, $\angle 1$ and $\angle 4$ are a linear pair, and $\angle 1$ and $\angle 3$ are vertical angles. Tell whether the statement is true.**

22. $\angle 1 \cong \angle 2$

23. $\angle 1 \cong \angle 3$

24. $\angle 1 \cong \angle 4$

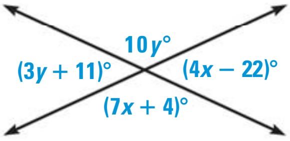
25. $\angle 3 \cong \angle 2$

26. $\angle 2 \cong \angle 4$

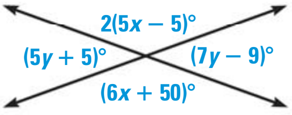
27. $m\angle 3 + m\angle 4 = 180^\circ$

xy ALGEBRA **Find the measure of each angle in the diagram.**

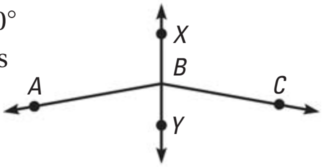
28.



29.



30. ★ **OPEN-ENDED MATH** In the diagram, $m\angle CBY = 80°$ and $\overleftrightarrow{XY}$ bisects $\angle ABC$. Give two more true statements about the diagram.



DRAWING CONCLUSIONS **In Exercises 31–34, use the given statement to name two congruent angles. Then give a reason that justifies your conclusion.**

31. In triangle GFE, $\overrightarrow{GH}$ bisects $\angle EGF$.

32. $\angle 1$ is a supplement of $\angle 6$, and $\angle 9$ is a supplement of $\angle 6$.

33. $\overline{AB}$ is perpendicular to $\overline{CD}$, and $\overline{AB}$ and $\overline{CD}$ intersect at E.

34. $\angle 5$ is complementary to $\angle 12$, and $\angle 1$ is complementary to $\angle 12$.

35. **CHALLENGE** Sketch two intersecting lines j and k. Sketch another pair of lines ℓ and m that intersect at the same point as j and k and that bisect the angles formed by j and k. Line ℓ is perpendicular to line m. *Explain* why this is true.

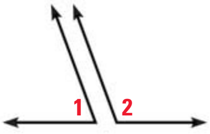
PROBLEM SOLVING

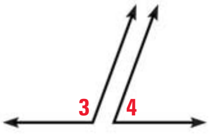
EXAMPLE 2
on p. 125
for Ex. 36

36. **PROVING THEOREM 2.4** Prove the second case of the Congruent Supplements Theorem where two angles are supplementary to congruent angles.

GIVEN ▶ $\angle 1$ and $\angle 2$ are supplements.
$\angle 3$ and $\angle 4$ are supplements.
$\angle 1 \cong \angle 4$

PROVE ▶ $\angle 2 \cong \angle 3$





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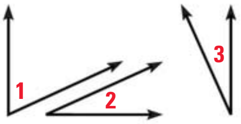
37. **PROVING THEOREM 2.5** Copy and complete the proof of the first case of the Congruent Complements Theorem where two angles are complementary to the same angles.

GIVEN ▶ $\angle 1$ and $\angle 2$ are complements.
$\angle 1$ and $\angle 3$ are complements.

PROVE ▶ $\angle 2 \cong \angle 3$



STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are complements. $\angle 1$ and $\angle 3$ are complements.	**1.** ?
2. $m\angle 1 + m\angle 2 = 90°$ $m\angle 1 + m\angle 3 = 90°$	**2.** ?
3. ?	**3.** Transitive Property of Equality
4. ?	**4.** Subtraction Property of Equality
5. $\angle 2 \cong \angle 3$	**5.** ?

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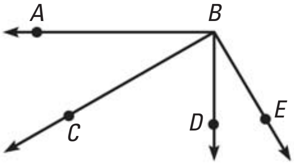
PROOF **Use the given information and the diagram to prove the statement.**

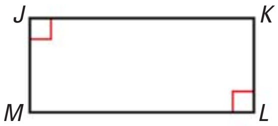
38. **GIVEN ▶** $\angle ABD$ is a right angle.
$\angle CBE$ is a right angle.

PROVE ▶ $\angle ABC \cong \angle DBE$



39. **GIVEN ▶** $\overline{JK} \perp \overline{JM}$, $\overline{KL} \perp \overline{ML}$,
$\angle J \cong \angle M$, $\angle K \cong \angle L$

PROVE ▶ $\overline{JM} \perp \overline{ML}$ and $\overline{JK} \perp \overline{KL}$



40. **MULTI-STEP PROBLEM** Use the photo of the folding table.

a. If $m\angle 1 = x°$, write expressions for the other three angle measures.

b. Estimate the value of x. What are the measures of the other angles?

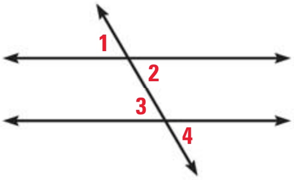
c. As the table is folded up, $\angle 4$ gets smaller. What happens to the other three angles? *Explain* your reasoning.

41. **PROVING THEOREM 2.5** Write a two-column proof for the second case of Theorem 2.5 where two angles are complementary to congruent angles.

WRITING PROOFS **Write a two-column proof.**

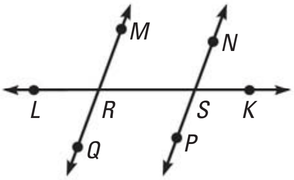
42. **GIVEN ▶** $\angle 1 \cong \angle 3$

PROVE ▶ $\angle 2 \cong \angle 4$



43. **GIVEN ▶** $\angle QRS$ and $\angle PSR$ are supplementary.

PROVE ▶ $\angle QRL \cong \angle PSR$



44. **STAIRCASE** Use the photo and the given information to prove the statement.

GIVEN ▶ $\angle 1$ is complementary to $\angle 3$.
$\angle 2$ is complementary to $\angle 4$.

PROVE ▶ $\angle 1 \cong \angle 4$

45. ★ **EXTENDED RESPONSE** $\angle STV$ is bisected by $\overrightarrow{TW}$, and $\overrightarrow{TX}$ and $\overrightarrow{TW}$ are opposite rays. You want to show $\angle STX \cong \angle VTX$.

a. Draw a diagram.

b. Identify the GIVEN and PROVE statements for the situation.

c. Write a two-column proof.

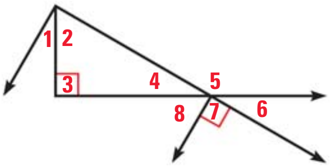
46. **USING DIAGRAMS** Copy and complete the statement with <, >, or =.

a. $m\angle 3 \underline{\ ?\ } m\angle 7$

b. $m\angle 4 \underline{\ ?\ } m\angle 6$

c. $m\angle 8 + m\angle 6 \underline{\ ?\ } 150°$

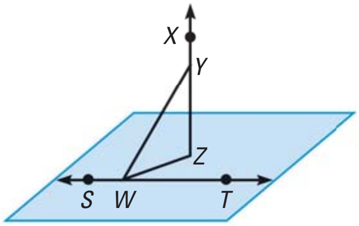
d. If $m\angle 4 = 30°$, then $m\angle 5 \underline{\ ?\ } m\angle 4$



CHALLENGE **In Exercises 47 and 48, write a two-column proof.**

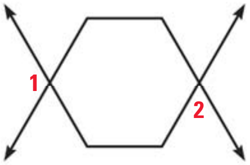
47. **GIVEN** ▶ $m\angle WYZ = m\angle TWZ = 45°$

PROVE ▶ $\angle SWZ \cong \angle XYW$



48. **GIVEN** ▶ The hexagon is regular.

PROVE ▶ $m\angle 1 + m\angle 2 = 180°$



MIXED REVIEW

PREVIEW
Prepare for Lesson 3.1 in Exs. 49–52.

In Exercises 49–52, sketch a plane. Then sketch the described situation. *(p. 2)*

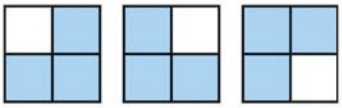
49. Three noncollinear points that lie in the plane

50. A line that intersects the plane at one point

51. Two perpendicular lines that lie in the plane

52. A plane perpendicular to the given plane

53. Sketch the next figure in the pattern. *(p. 72)*



QUIZ *for Lessons 2.6–2.7*

Match the statement with the property that it illustrates. *(p. 112)*

1. If $\overline{HJ} \cong \overline{LM}$, then $\overline{LM} \cong \overline{HJ}$.

2. If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 4$, then $\angle 1 \cong \angle 4$.

3. $\angle XYZ \cong \angle XYZ$

A. Reflexive Property of Congruence

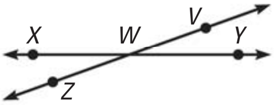
B. Symmetric Property of Congruence

C. Transitive Property of Congruence

4. Write a two-column proof. *(p. 124)*

GIVEN ▶ $\angle XWY$ is a straight angle.
$\angle ZWV$ is a straight angle.

PROVE ▶ $\angle XWV \cong \angle ZWY$



MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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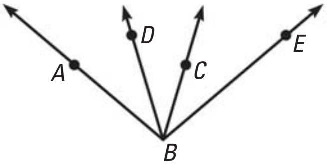
Lessons 2.5–2.7

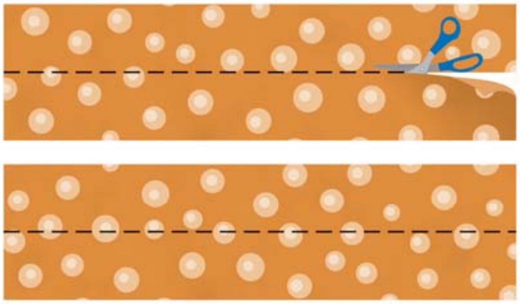
1. **MULTI-STEP PROBLEM** In the diagram below, $\overrightarrow{BD}$ bisects $\angle ABC$ and $\overrightarrow{BC}$ bisects $\angle DBE$.



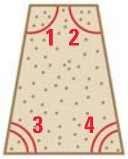
   a. Prove $m\angle ABD = m\angle CBE$.

   b. If $m\angle ABE = 99°$, what is $m\angle DBC$? *Explain.*

2. **SHORT RESPONSE** You are cutting a rectangular piece of fabric into strips that you will weave together to make a placemat. As shown, you cut the fabric in half lengthwise to create two congruent pieces. You then cut each of these pieces in half lengthwise. Do all of the strips have the same width? *Explain* your reasoning.



3. **GRIDDED ANSWER** The cross section of a concrete retaining wall is shown below. Use the given information to find the measure of $\angle 1$ in degrees.



$m\angle 1 = m\angle 2$

$m\angle 3 = m\angle 4$

$m\angle 3 = 80°$

$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$

4. **EXTENDED RESPONSE** *Explain* how the Congruent Supplements Theorem and the Transitive Property of Angle Congruence can both be used to show how angles that are supplementary to the same angle are congruent.

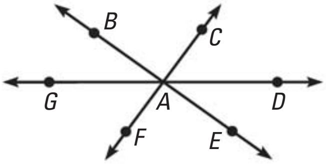
5. **EXTENDED RESPONSE** A formula you can use to calculate the total cost of an item including sales tax is $T = c(1 + s)$, where T is the total cost including sales tax, c is the cost not including sales tax, and s is the sales tax rate written as a decimal.

   a. Solve the formula for s. Give a reason for each step.

   b. Use your formula to find the sales tax rate on a purchase that was \$26.75 with tax and \$25 without tax.

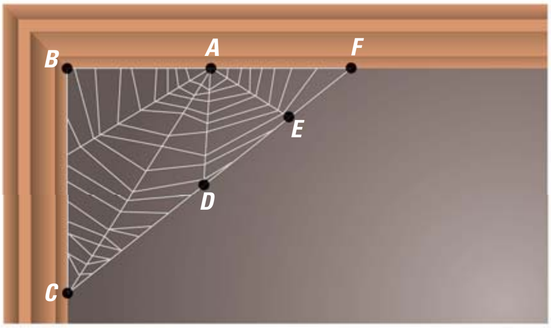
   c. Look back at the steps you used to solve the formula for s. Could you have solved for s in a different way? *Explain.*

6. **OPEN-ENDED** In the diagram below, $m\angle GAB = 36°$. What additional information do you need to find $m\angle BAC$ and $m\angle CAD$? *Explain* your reasoning.



7. **SHORT RESPONSE** Two lines intersect to form $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$. The measure of $\angle 3$ is three times the measure of $\angle 1$ and $m\angle 1 = m\angle 2$. Find all four angle measures. *Explain* your reasoning.

8. **SHORT RESPONSE** Part of a spider web is shown below. If you know that $\angle CAD$ and $\angle DAE$ are complements and that $\overrightarrow{AB}$ and $\overrightarrow{AF}$ are opposite rays, what can you conclude about $\angle BAC$ and $\angle EAF$? *Explain* your reasoning.



2 CHAPTER SUMMARY

BIG IDEAS

For Your Notebook

Big Idea 1

Using Inductive and Deductive Reasoning

When you make a conjecture based on a pattern, you use inductive reasoning. You use deductive reasoning to show whether the conjecture is true or false by using facts, definitions, postulates, or proven theorems. If you can find one counterexample to the conjecture, then you know the conjecture is false.

Big Idea 2

Understanding Geometric Relationships in Diagrams

The following can be assumed from the diagram:

A, B, and C are coplanar.

$\angle ABH$ and $\angle HBF$ are a linear pair.

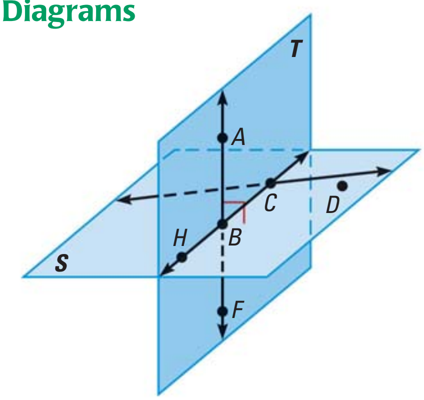
Plane T and plane S intersect in $\overleftrightarrow{BC}$.

$\overleftrightarrow{CD}$ lies in plane S.

$\angle ABC$ and $\angle HBF$ are vertical angles.

$\overleftrightarrow{AB} \perp$ plane S.

Diagram assumptions are reviewed on page 97.



Big Idea 3

Writing Proofs of Geometric Relationships

You can write a logical argument to show a geometric relationship is true. In a two-column proof, you use deductive reasoning to work from GIVEN information to reach a conjecture you want to PROVE.

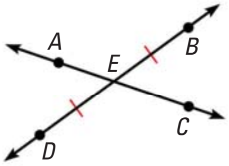


Diagram of geometric relationship with given information labeled to help you write the proof

GIVEN ▶ The hypothesis of an if-then statement

PROVE ▶ The conclusion of an if-then statement

STATEMENTS	REASONS
1. Hypothesis	1. Given
n. Conclusion	*n.* ______

Statements based on facts that you know or conclusions from deductive reasoning

Use postulates, proven theorems, definitions, and properties of numbers and congruence as reasons.

Proof summary is on page 114.

2 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

See pp. 926–931 for a list of postulates and theorems.

- conjecture, *p. 73*
- inductive reasoning, *p. 73*
- counterexample, *p. 74*
- conditional statement, *p. 79*
  converse, inverse, contrapositive
- if-then form, *p. 79*
  hypothesis, conclusion
- negation, *p. 79*
- equivalent statements, *p. 80*
- perpendicular lines, *p. 81*
- biconditional statement, *p. 82*
- deductive reasoning, *p. 87*
- line perpendicular to a plane, *p. 98*
- proof, *p. 112*
- two-column proof, *p. 112*
- theorem, *p. 113*

VOCABULARY EXERCISES

1. Copy and complete: A statement that can be proven is called a(n) __?__.

2. **WRITING** *Compare* the inverse of a conditional statement to the converse of the conditional statement.

3. You know $m\angle A = m\angle B$ and $m\angle B = m\angle C$. What does the Transitive Property of Equality tell you about the measures of the angles?

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 2.

2.1 Use Inductive Reasoning

pp. 72–78

EXAMPLE

Describe the pattern in the numbers 3, 21, 147, 1029, ..., and write the next three numbers in the pattern.

Each number is seven times the previous number.

So, the next three numbers are 7203, 50,421, and 352,947.

EXERCISES

EXAMPLES 2 and 5 on pp. 72–74 for Exs. 4–5

4. *Describe* the pattern in the numbers −20,480, −5120, −1280, −320, ... Write the next three numbers.

5. Find a counterexample to disprove the conjecture:

If the quotient of two numbers is positive, then the two numbers must both be positive.

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Chapter Review Practice

2.2 Analyze Conditional Statements pp. 79–85

EXAMPLE

Write the if-then form, the converse, the inverse, and the contrapositive of the statement “Black bears live in North America.”

a. If-then form: If a bear is a black bear, then it lives in North America.

b. Converse: If a bear lives in North America, then it is a black bear.

c. Inverse: If a bear is not a black bear, then it does not live in North America.

d. Contrapositive: If a bear does not live in North America, then it is not a black bear.

EXERCISES

EXAMPLES 2, 3, and 4 on pp. 80–82 for Exs. 6–8

6. Write the if-then form, the converse, the inverse, and the contrapositive of the statement “An angle whose measure is 34° is an acute angle.”

7. Is this a valid definition? *Explain* why or why not.

“If the sum of the measures of two angles is 90°, then the angles are complementary.”

8. Write the definition of *equiangular* as a biconditional statement.

2.3 Apply Deductive Reasoning pp. 87–93

EXAMPLE

Use the Law of Detachment to make a valid conclusion in the true situation.

If two angles have the same measure, then they are congruent. You know that $m\angle A = m\angle B$.

▶ Because $m\angle A = m\angle B$ satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, $\angle A \cong \angle B$.

EXERCISES

EXAMPLES 1, 2, and 4 on pp. 87–89 for Exs. 9–11

9. Use the Law of Detachment to make a valid conclusion.

If an angle is a right angle, then the angle measures 90°. $\angle B$ is a right angle.

10. Use the Law of Syllogism to write the statement that follows from the pair of true statements.

If $x = 3$, then $2x = 6$.

If $4x = 12$, then $x = 3$.

11. What can you say about the sum of any two odd integers? Use inductive reasoning to form a conjecture. Then use deductive reasoning to show that the conjecture is true.

2 CHAPTER REVIEW

2.4 Use Postulates and Diagrams

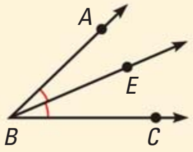
pp. 96–102

EXAMPLE

$\angle ABC$, an acute angle, is bisected by $\overrightarrow{BE}$. Sketch a diagram that represents the given information.

1. Draw $\angle ABC$, an acute angle, and label points A, B, and C.

2. Draw angle bisector $\overrightarrow{BE}$. Mark congruent angles.



EXERCISES

EXAMPLES 3 and 4 on p. 98 for Exs. 12–13

12. Straight angle CDE is bisected by $\overrightarrow{DK}$. Sketch a diagram that represents the given information.

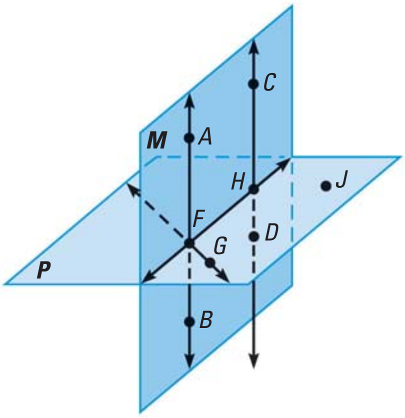
13. Which of the following statements *cannot* be assumed from the diagram?

Ⓐ A, B, and C are coplanar.

Ⓑ $\overleftrightarrow{CD} \perp$ plane P

Ⓒ A, F, and B are collinear.

Ⓓ Plane M intersects plane P in $\overleftrightarrow{FH}$.



2.5 Reason Using Properties from Algebra

pp. 105–111

EXAMPLE

Solve $3x + 2(2x + 9) = -10$. Write a reason for each step.

$3x + 2(2x + 9) = -10$ **Write original equation.**

$3x + 4x + 18 = -10$ **Distributive Property**

$7x + 18 = -10$ **Simplify.**

$7x = -28$ **Subtraction Property of Equality**

$x = -4$ **Division Property of Equality**

EXERCISES

EXAMPLES 1 and 2 on pp. 105–106 for Exs. 14–17

Solve the equation. Write a reason for each step.

14. $-9x - 21 = -20x - 87$

15. $15x + 22 = 7x + 62$

16. $3(2x + 9) = 30$

17. $5x + 2(2x - 23) = -154$

2.6 Prove Statements about Segments and Angles

pp. 112–119

EXAMPLE

Prove the Reflexive Property of Segment Congruence.

GIVEN ▶ $\overline{AB}$ is a line segment.

PROVE ▶ $\overline{AB} \cong \overline{AB}$

STATEMENTS	REASONS
1. $\overline{AB}$ is a line segment.	**1.** Given
2. AB is the length of $\overline{AB}$.	**2.** Ruler Postulate
3. $AB = AB$	**3.** Reflexive Property of Equality
4. $\overline{AB} \cong \overline{AB}$	**4.** Definition of congruent segments

EXAMPLES 2 and 3 on pp. 113–114 for Exs. 18–21

EXERCISES

Name the property illustrated by the statement.

18. If $\angle DEF \cong \angle JKL$, then $\angle JKL \cong \angle DEF$.

19. $\angle C \cong \angle C$

20. If $MN = PQ$ and $PQ = RS$, then $MN = RS$.

21. Prove the Transitive Property of Angle Congruence.

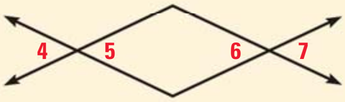
2.7 Prove Angle Pair Relationships

pp. 124–131

EXAMPLE

GIVEN ▶ $\angle 5 \cong \angle 6$

PROVE ▶ $\angle 4 \cong \angle 7$

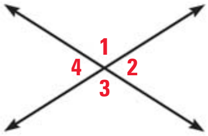


STATEMENTS	REASONS
1. $\angle 5 \cong \angle 6$	**1.** Given
2. $\angle 4 \cong \angle 5$	**2.** Vertical Angles Congruence Theorem
3. $\angle 4 \cong \angle 6$	**3.** Transitive Property of Congruence
4. $\angle 6 \cong \angle 7$	**4.** Vertical Angles Congruence Theorem
5. $\angle 4 \cong \angle 7$	**5.** Transitive Property of Congruence

EXAMPLES 2 and 3 on pp. 125–126 for Exs. 22–24

EXERCISES

In Exercises 22 and 23, use the diagram at the right.



22. If $m\angle 1 = 114°$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

23. If $m\angle 4 = 57°$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$.

24. Write a two-column proof.

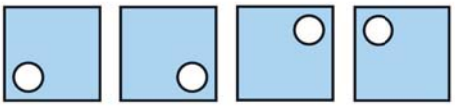
GIVEN ▶ $\angle 3$ and $\angle 2$ are complementary.
$m\angle 1 + m\angle 2 = 90°$

PROVE ▶ $\angle 3 \cong \angle 1$

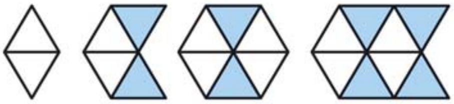
2 CHAPTER TEST

Sketch the next figure in the pattern.

1.



2.



Describe **the pattern in the numbers. Write the next number.**

3. $-6, -1, 4, 9, \ldots$

4. $100, -50, 25, -12.5, \ldots$

In Exercises 5–8, write the if-then form, the converse, the inverse, and the contrapositive for the given statement.

5. All right angles are congruent.

6. Frogs are amphibians.

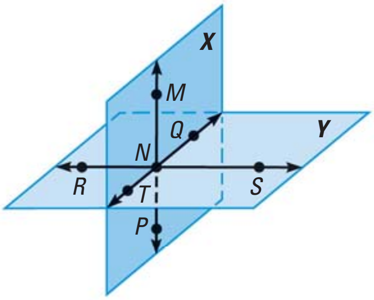
7. $5x + 4 = -6$, because $x = -2$.

8. A regular polygon is equilateral.

9. If you decide to go to the football game, then you will miss band practice. Tonight, you are going the football game. Using the Law of Detachment, what statement can you make?

10. If Margot goes to college, then she will major in Chemistry. If Margot majors in Chemistry, then she will need to buy a lab manual. Using the Law of Syllogism, what statement can you make?

Use the diagram to write examples of the stated postulate.



11. A line contains at least two points.

12. A plane contains at least three noncollinear points.

13. If two planes intersect, then their intersection is a line.

Solve the equation. Write a reason for each step.

14. $9x + 31 = -23$

15. $-7(-x + 2) = 42$

16. $26 + 2(3x + 11) = -18x$

In Exercises 17–19, match the statement with the property that it illustrates.

17. If $\angle RST \cong \angle XYZ$, then $\angle XYZ \cong \angle RST$.

18. $\overline{PQ} \cong \overline{PQ}$

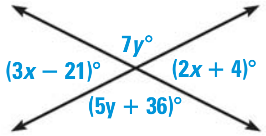
19. If $\overline{FG} \cong \overline{JK}$ and $\overline{JK} \cong \overline{LM}$, then $\overline{FG} \cong \overline{LM}$.

A. Reflexive Property of Congruence

B. Symmetric Property of Congruence

C. Transitive Property of Congruence

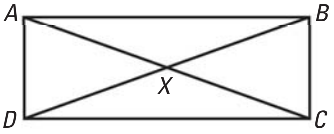
20. Use the Vertical Angles Congruence Theorem to find the measure of each angle in the diagram at the right.



21. Write a two-column proof.

GIVEN ▶ $\overline{AX} \cong \overline{DX}$, $\overline{XB} \cong \overline{XC}$

PROVE ▶ $\overline{AC} \cong \overline{BD}$



2 xy ALGEBRA REVIEW

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SIMPLIFY RATIONAL AND RADICAL EXPRESSIONS

xy EXAMPLE 1 *Simplify rational expressions*

a. $\frac{2x^2}{4xy}$ **b.** $\frac{3x^2 + 2x}{9x + 6}$

Solution

To simplify a rational expression, factor the numerator and denominator. Then divide out any common factors.

a. $\frac{2x^2}{4xy} = \frac{\cancel{2} \cdot \cancel{x} \cdot x}{\cancel{2} \cdot 2 \cdot \cancel{x} \cdot y} = \frac{x}{2y}$ **b.** $\frac{3x^2 + 2x}{9x + 6} = \frac{x\cancel{(3x + 2)}}{3\cancel{(3x + 2)}} = \frac{x}{3}$

xy EXAMPLE 2 *Simplify radical expressions*

a. $\sqrt{54}$ **b.** $2\sqrt{5} - 5\sqrt{2} - 3\sqrt{5}$ **c.** $(3\sqrt{2})(-6\sqrt{6})$

Solution

a. $\sqrt{54} = \sqrt{9} \cdot \sqrt{6}$ **Use product property of radicals.**

$= 3\sqrt{6}$ **Simplify.**

b. $2\sqrt{5} - 5\sqrt{2} - 3\sqrt{5} = -\sqrt{5} - 5\sqrt{2}$ **Combine like terms.**

c. $(3\sqrt{2})(-6\sqrt{6}) = -18\sqrt{12}$ **Use product property and associative property.**

$= -18 \cdot 2\sqrt{3}$ **Simplify $\sqrt{12}$.**

$= -36\sqrt{3}$ **Simplify.**

EXERCISES

EXAMPLE 1
for Exs. 1–9

Simplify the expression, if possible.

1. $\frac{5x^4}{20x^2}$
2. $\frac{-12ab^3}{9a^2b}$
3. $\frac{5m + 35}{5}$
4. $\frac{36m - 48m}{6m}$
5. $\frac{k + 3}{-2k + 3}$
6. $\frac{m + 4}{m^2 + 4m}$
7. $\frac{12x + 16}{8 + 6x}$
8. $\frac{3x^3}{5x + 8x^2}$
9. $\frac{3x^2 - 6x}{6x^2 - 3x}$

EXAMPLE 2
for Exs. 10–24

Simplify the expression, if possible. All variables are positive.

10. $\sqrt{75}$
11. $-\sqrt{180}$
12. $\pm\sqrt{128}$
13. $\sqrt{2} - \sqrt{18} + \sqrt{6}$
14. $\sqrt{28} - \sqrt{63} - \sqrt{35}$
15. $4\sqrt{8} + 3\sqrt{32}$
16. $(6\sqrt{5})(2\sqrt{2})$
17. $(-4\sqrt{10})(-5\sqrt{5})$
18. $(2\sqrt{6})^2$
19. $\sqrt{(25)^2}$
20. $\sqrt{x^2}$
21. $\sqrt{-(a)^2}$
22. $\sqrt{(3y)^2}$
23. $\sqrt{3^2 + 2^2}$
24. $\sqrt{h^2 + k^2}$

2 ★ Standardized TEST PREPARATION

Scoring Rubric

Full Credit
- solution is complete and correct

Partial Credit
- solution is complete but has errors,

or
- solution is without error but incomplete

No Credit
- no solution is given,

or
- solution makes no sense

EXTENDED RESPONSE QUESTIONS

PROBLEM

Seven members of the student government (Frank, Gina, Henry, Isabelle, Jack, Katie, and Leah) are posing for a picture for the school yearbook. For the picture, the photographer will arrange the students in a row according to the following restrictions:

Henry must stand in the middle spot.

Katie must stand in the right-most spot.

There must be exactly two spots between Gina and Frank.

Isabelle cannot stand next to Henry.

Frank must stand next to Katie.

a. *Describe* one possible ordering of the students.

b. Which student(s) can stand in the second spot from the left?

c. If the condition that Leah must stand in the left-most spot is added, will there be exactly one ordering of the students? *Justify* your answer.

Below are sample solutions to the problem. Read each solution and the comments in blue to see why the sample represents full credit, partial credit, or no credit.

SAMPLE 1: Full credit solution

The method of representation is clearly explained.

a. Using the first letters of the students' names, here is one possible ordering of the students:

I L G H J F K

The conclusion is correct and shows understanding of the problem.

b. The only students without fixed positions are Isabelle, Leah, and Jack. There are no restrictions on placement in the second spot from the left, so any of these three students can occupy that location.

c. Henry, Frank, Katie, and Gina have fixed positions according to the restrictions. If Leah must stand in the left-most spot, the ordering looks like:

L _ G H _ F K

The reasoning behind the answer is explained clearly.

Because Isabelle cannot stand next to Henry, she must occupy the spot next to Leah. Therefore, Jack stands next to Henry and the only possible order would have to be:

L I G H J F K.

Yes, there would be exactly one ordering of the students.

SAMPLE 2: Partial credit solution

The answer to part (a) is correct.

a. One possible ordering of the students is:
Jack, Isabelle, Gina, Henry, Leah, Frank, and Katie.

Part (b) is correct but not explained.

b. There are three students who could stand in the second spot from the left. They are Isabelle, Leah, and Jack.

The student did not recall that Isabelle cannot stand next to Henry; therefore, the conclusion is incorrect.

c. No, there would be two possible orderings of the students. With Leah in the left-most spot, the ordering looks like:

Leah, ____, Gina, Henry, ____, Frank, and Katie

Therefore, the two possible orderings are
Leah, Isabelle, Gina, Henry, Jack, Frank, and Katie
or
Leah, Jack, Gina, Henry, Isabelle, Frank, and Katie.

SAMPLE 3: No credit solution

The answer to part (a) is incorrect because Isabelle is next to Henry.

a. One possible ordering of the students is **L G J H I F K**.

b. There are four students who can stand in the second spot from the left. Those students are Leah, Gina, Isabelle, and Jack.

Parts (b) and (c) are based on the incorrect conclusion in part (a).

c. The two possible orderings are **L G J H I F K** and **L J G H I F K**.

PRACTICE Apply the Scoring Rubric

1. A student's solution to the problem on the previous page is given below. Score the solution as *full credit*, *partial credit*, or *no credit*. *Explain* your reasoning. If you choose *partial credit* or *no credit*, explain how you would change the solution so that it earns a score of full credit.

a. A possible ordering of the students is I - J - G - H - L - F - K.

b. There are no restrictions on the second spot from the left. Leah, Isabelle, and Jack could all potentially stand in this location.

c. The positions of Gina, Henry, Frank, and Katie are fixed.

_ - _ - G - H - _ - F - K.

Because Isabelle cannot stand next to Henry, she must occupy the left-most spot or the second spot from the left. There are no restrictions on Leah or Jack. That leaves four possible orderings:

I - J - G - H - L - F - K
L - I - G - H - J - F - K
I - L - G - H - J - F - K
J - I - G - H - L - F - K.

If the restriction is added that Leah must occupy the left-most spot, there is exactly one ordering that would satisfy all conditions:

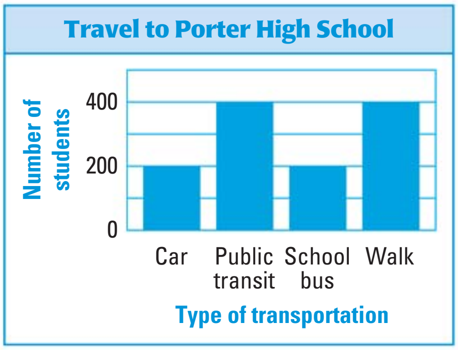
L - I - G - H - J - F - K.

2 ★ Standardized TEST PRACTICE

EXTENDED RESPONSE

1. In some bowling leagues, the handicap H of a bowler with an average score A is found using the formula $H = \frac{4}{5}(200 - A)$. The handicap is then added to the bowler's score.
   a. Solve the formula for A. Write a reason for each step.
   b. Use your formula to find a bowler's average score with a handicap of 12.
   c. Using this formula, is it possible to calculate a handicap for a bowler with an average score above 200? *Explain* your reasoning.

2. A survey was conducted at Porter High School asking students what form of transportation they use to go to school. All students in the high school were surveyed. The results are shown in the bar graph.
   a. Does the statement "About 1500 students attend Porter High School" follow from the data? *Explain*.
   b. Does the statement "About one third of all students at Porter take public transit to school" follow from the data? *Explain*.
   c. John makes the conclusion that Porter High School is located in a city or a city suburb. *Explain* his reasoning and tell if his conclusion is the result of *inductive reasoning* or *deductive reasoning*.
   d. Betty makes the conclusion that there are twice as many students who walk as take a car to school. *Explain* her reasoning and tell if her conclusion is the result of *inductive reasoning* or *deductive reasoning*.



3. The senior class officers are planning a meeting with the principal and some class officers from the other grades. The senior class president, vice president, treasurer, and secretary will all be present. The junior class president and treasurer will attend. The sophomore class president and vice president, and freshmen treasurer will attend. The secretary makes a seating chart for the meeting using the following conditions.

   The principal will sit in chair 10. The senior class treasurer will sit at the other end.

   The senior class president will sit to the left of the principal, next to the junior class president, and across from the sophomore class president.

   All three treasurers will sit together. The two sophomores will sit next to each other.

   The two vice presidents and the freshman treasurer will sit on the same side of the table.

   a. Draw a diagram to show where everyone will sit.
   b. *Explain* why the senior class secretary must sit between the junior class president and junior class treasurer.
   c. Can the senior class vice-president sit across from the junior class president? *Justify* your answer.

MULTIPLE CHOICE

4. If d represents an odd integer, which of the expressions represents an even integer?

Ⓐ $d + 2$

Ⓑ $2d - 1$

Ⓒ $3d + 1$

Ⓓ $3d + 2$

5. In the repeating decimal 0.23142314. . . , where the digits 2314 repeat, which digit is in the 300th place to the right of the decimal point?

Ⓐ 1

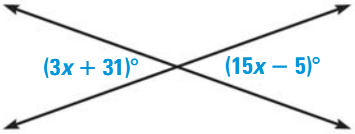
Ⓑ 2

Ⓒ 3

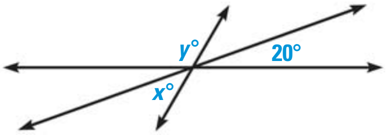
Ⓓ 4

GRIDDED ANSWER

6. Use the diagram to find the value of x.



7. Three lines intersect in the figure shown. What is the value of $x + y$?



8. R is the midpoint of $\overline{PQ}$, and S and T are the midpoints of $\overline{PR}$ and $\overline{RQ}$, respectively. If $ST = 20$, what is PT?

SHORT RESPONSE

9. Is this a correct conclusion from the given information? If so, *explain* why. If not, *explain* the error in the reasoning.

> If you are a soccer player, then you wear shin guards. Your friend is wearing shin guards. Therefore, she is a soccer player.

10. *Describe* the pattern in the numbers. Write the next number in the pattern.

> 192, −48, 12, −3, . . .

11. Points A, B, C, D, E, and F are coplanar. Points A, B, and F are collinear. The line through A and B is perpendicular to the line through C and D, and the line through C and D is perpendicular to the line through E and F. Which four points must lie on the same line? *Justify* your answer.

12. Westville High School offers after-school tutoring with five student volunteer tutors for this program: Jen, Kim, Lou, Mike, and Nina. On any given weekday, three tutors are scheduled to work. Due to the students' other commitments after school, the tutoring work schedule must meet the following conditions.

> Jen can work any day except every other Monday and Wednesday.
>
> Kim can only work on Thursdays and Fridays.
>
> Lou can work on Tuesdays and Wednesdays.
>
> Mike cannot work on Fridays.
>
> Nina cannot work on Tuesdays.

Name three tutors who can work on *any* Wednesday. *Justify* your answer.