12 Surface Area and Volume of Solids

Before

In previous chapters, you learned the following skills, which you'll use in Chapter 12: properties of similar polygons, areas and perimeters of two-dimensional figures, and right triangle trigonometry.

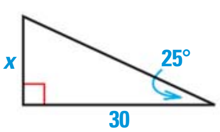
Prerequisite Skills

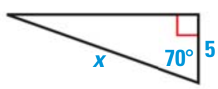
VOCABULARY CHECK

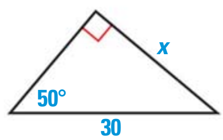
1. Copy and complete: The area of a regular polygon is given by the formula $A = \underline{\ ?\ }$.
2. *Explain* what it means for two polygons to be similar.

SKILLS AND ALGEBRA CHECK

Use trigonometry to find the value of *x*. *(Review pp. 466, 473 for 12.2–12.5.)*

3. 

4. 

5. 

Find the circumference and area of the circle with the given dimension. *(Review pp. 746, 755 for 12.2–12.5.)*

6. $r = 2$ m
7. $d = 3$ in.
8. $r = 2\sqrt{5}$ cm

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 12, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 856. You will also use the key vocabulary listed below.

Big Ideas

1. **Exploring solids and their properties**
2. **Solving problems using surface area and volume**
3. **Connecting similarity to solids**

KEY VOCABULARY

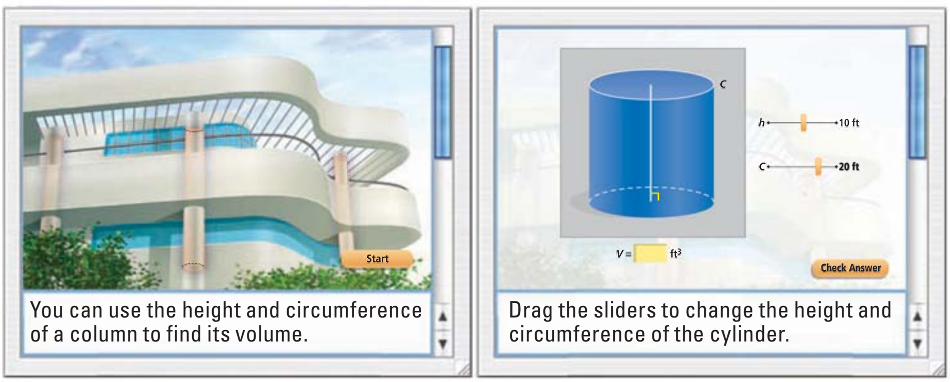
- polyhedron, *p. 794*
  face, edge, vertex
- Platonic solids, *p. 796*
- cross section, *p. 797*
- prism, *p. 803*
- surface area, *p. 803*
- lateral area, *p. 803*
- net, *p. 803*
- right prism, *p. 804*
- oblique prism, *p. 804*
- cylinder, *p. 805*
- right cylinder, *p. 805*
- pyramid, *p. 810*
- regular pyramid, *p. 810*
- cone, *p. 812*
- right cone, *p. 812*
- volume, *p. 819*
- sphere, *p. 838*
- great circle, *p. 839*
- hemisphere, *p. 839*
- similar solids, *p. 847*

Why?

Knowing how to use surface area and volume formulas can help you solve problems in three dimensions. For example, you can use a formula to find the volume of a column in a building.

Animated Geometry

The animation illustrated below for Exercise 31 on page 825 helps you answer this question: What is the volume of the column?



You can use the height and circumference of a column to find its volume.

Drag the sliders to change the height and circumference of the cylinder.

Animated **Geometry** at classzone.com

Other animations for Chapter 12: pages 795, 805, 821, 833, 841, and 852

12.1 Investigate Solids

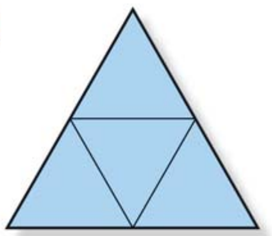
MATERIALS • poster board • scissors • tape • straightedge

QUESTION What solids can be made using congruent regular polygons?

Platonic solids, named after the Greek philosopher Plato (427 B.C.–347 B.C.), are solids that have the same congruent regular polygon as each *face*, or side of the solid.

EXPLORE 1 Make a solid using four equilateral triangles

STEP 1



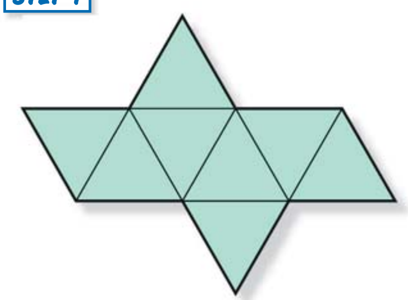
Make a net Copy the full-sized triangle from page 793 on poster board to make a template. Trace the triangle four times to make a *net* like the one shown.

STEP 2

Make a solid Cut out your net. Fold along the lines. Tape the edges together to form a solid. How many faces meet at each *vertex*?

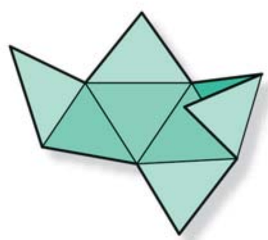
EXPLORE 2 Make a solid using eight equilateral triangles

STEP 1



Make a net Trace your triangle template from Explore 1 eight times to make a net like the one shown.

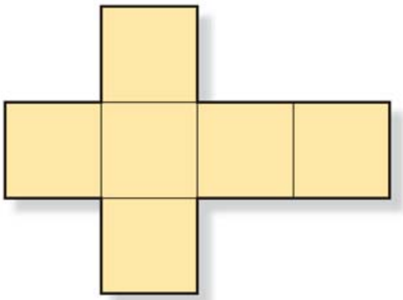
STEP 2



Make a solid Cut out your net. Fold along the lines. Tape the edges together to form a solid. How many faces meet at each vertex?

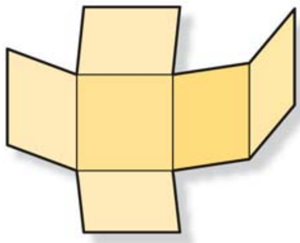
EXPLORE 3 Make a solid using six squares

STEP 1



Make a net Copy the full-sized square from the bottom of the page on poster board to make a template. Trace the square six times to make a net like the one shown.

STEP 2

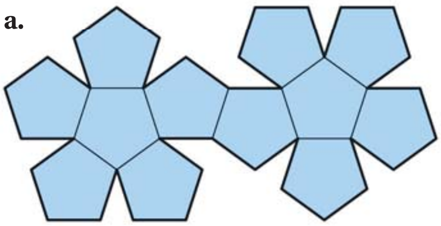


Make a solid Cut out your net. Fold along the lines. Tape the edges together to form a solid. How many faces meet at each vertex?

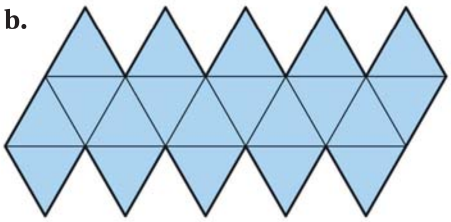
DRAW CONCLUSIONS Use your observations to complete these exercises

1. The two other convex solids that you can make using congruent, regular faces are shown below. For each of these solids, how many faces meet at each vertex?

a.

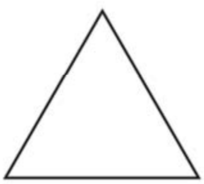


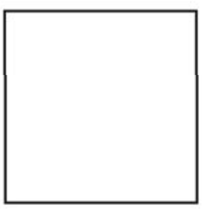
b.



2. *Explain* why it is not possible to make a solid that has six congruent equilateral triangles meeting at each vertex.
3. *Explain* why it is not possible to make a solid that has three congruent regular hexagons meeting at each vertex.
4. Count the number of vertices V, edges E, and faces F for each solid you made. Make a conjecture about the relationship between the sum $F + V$ and the value of E.

Templates:



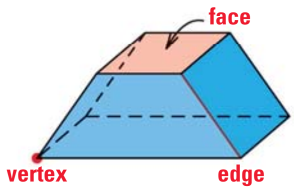


12.1 Explore Solids

Before	You identified polygons.
Now	You will identify solids.
Why	So you can analyze the frame of a house, as in Example 2.

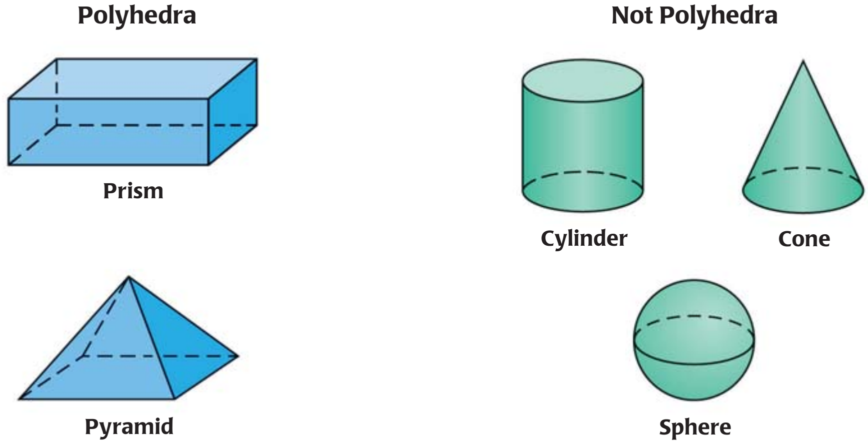
Key Vocabulary
- **polyhedron**
  face, edge, vertex
- **base**
- **regular polyhedron**
- **convex polyhedron**
- **Platonic solids**
- **cross section**

A **polyhedron** is a solid that is bounded by polygons, called **faces**, that enclose a single region of space. An **edge** of a polyhedron is a line segment formed by the intersection of two faces. A **vertex** of a polyhedron is a point where three or more edges meet. The plural of polyhedron is *polyhedra* or *polyhedrons*.

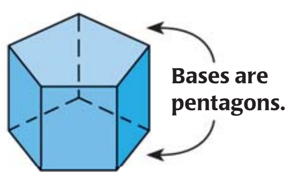


KEY CONCEPT — *For Your Notebook*

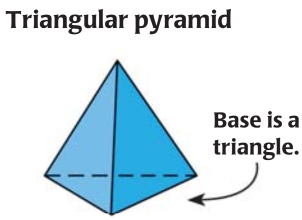
Types of Solids



CLASSIFYING SOLIDS Of the five solids above, the prism and the pyramid are polyhedra. To name a prism or a pyramid, use the shape of the *base*.



The two **bases** of a prism are congruent polygons in parallel planes.

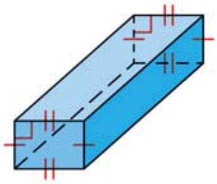


The **base** of a pyramid is a polygon.

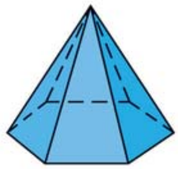
EXAMPLE 1 Identify and name polyhedra

Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges.

a.



b.



c.

Solution

a. The solid is formed by polygons, so it is a polyhedron. The two bases are congruent rectangles, so it is a rectangular prism. It has 6 faces, 8 vertices, and 12 edges.

b. The solid is formed by polygons, so it is a polyhedron. The base is a hexagon, so it is a hexagonal pyramid. It has 7 faces, consisting of 1 base, 3 visible triangular faces, and 3 non-visible triangular faces. The polyhedron has 7 faces, 7 vertices, and 12 edges.

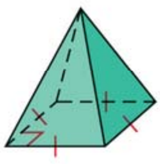
c. The cone has a curved surface, so it is not a polyhedron.

AnimatedGeometry at classzone.com

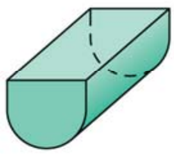
✓ GUIDED PRACTICE for Example 1

Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges.

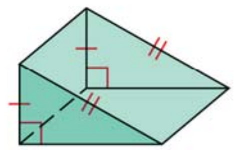
1.



2.



3.

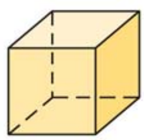


EULER'S THEOREM Notice in Example 1 that the sum of the number of faces and vertices of the polyhedra is two more than the number of edges. This suggests the following theorem, proved by the Swiss mathematician Leonhard Euler (pronounced "oi′-ler"), who lived from 1707 to 1783.

THEOREM *For Your Notebook*

THEOREM 12.1 Euler's Theorem

The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula $F + V = E + 2$.



$F = 6$, $V = 8$, $E = 12$
$6 + 8 = 12 + 2$

EXAMPLE 2 Use Euler's Theorem in a real-world situation

HOUSE CONSTRUCTION **Find the number of edges on the frame of the house.**

Solution

The frame has one face as its foundation, four that make up its walls, and two that make up its roof, for a total of 7 faces.

To find the number of vertices, notice that there are 5 vertices around each pentagonal wall, and there are no other vertices. So, the frame of the house has 10 vertices.

Use Euler's Theorem to find the number of edges.

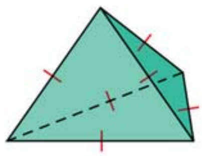
$F + V = E + 2$ **Euler's Theorem**

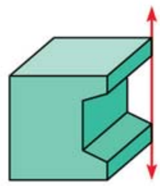
$7 + 10 = E + 2$ **Substitute known values.**

$15 = E$ **Solve for *E*.**

▶ The frame of the house has 15 edges.

REGULAR POLYHEDRA A polyhedron is **regular** if all of its faces are congruent regular polygons. A polyhedron is **convex** if any two points on its surface can be connected by a segment that lies entirely inside or on the polyhedron. If this segment goes outside the polyhedron, then the polyhedron is nonconvex, or *concave*.

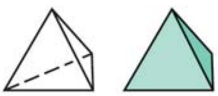

regular, convex

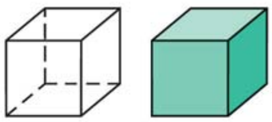

nonregular, concave

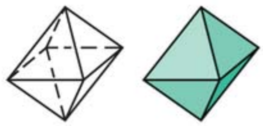
There are five regular polyhedra, called **Platonic solids** after the Greek philosopher Plato (c. 427 B.C.–347 B.C.). The five Platonic solids are shown.

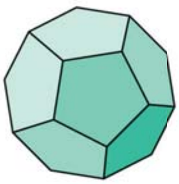
READ VOCABULARY

Notice that the names of four of the Platonic solids end in "hedron." *Hedron* is Greek for "side" or "face." Sometimes a cube is called a regular *hexahedron*.


Regular tetrahedron
4 faces


Cube
6 faces


Regular octahedron
8 faces

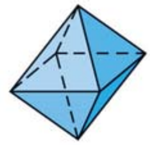

Regular dodecahedron
12 faces


Regular icosahedron
20 faces

There are only five regular polyhedra because the sum of the measures of the angles that meet at a vertex of a convex polyhedron must be less than 360°. This means that the only possible combinations of regular polygons at a vertex that will form a polyhedron are 3, 4, or 5 triangles, 3 squares, and 3 pentagons.

EXAMPLE 3 Use Euler's Theorem with Platonic solids

Find the number of faces, vertices, and edges of the regular octahedron. Check your answer using Euler's Theorem.



ANOTHER WAY

An octahedron has 8 faces, each of which has 3 vertices and 3 edges. Each vertex is shared by 4 faces; each edge is shared by 2 faces. They should only be counted once.

$V = \frac{8 \cdot 3}{4} = 6$

$E = \frac{8 \cdot 3}{2} = 12$

Solution

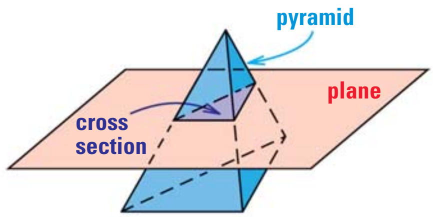
By counting on the diagram, the octahedron has 8 faces, 6 vertices, and 12 edges. Use Euler's Theorem to check.

$F + V = E + 2$ — **Euler's Theorem**

$8 + 6 = 12 + 2$ — **Substitute.**

$14 = 14$ ✓ — **This is a true statement. So, the solution checks.**

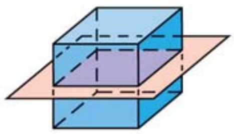
CROSS SECTIONS Imagine a plane slicing through a solid. The intersection of the plane and the solid is called a **cross section**. For example, the diagram shows that an intersection of a plane and a triangular pyramid is a triangle.



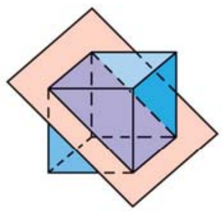
EXAMPLE 4 Describe cross sections

Describe the shape formed by the intersection of the plane and the cube.

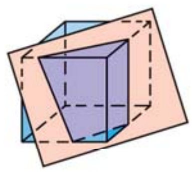
a.



b.



c.



Solution

a. The cross section is a square.

b. The cross section is a rectangle.

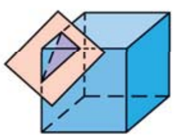
c. The cross section is a trapezoid.

✓ GUIDED PRACTICE for Examples 2, 3, and 4

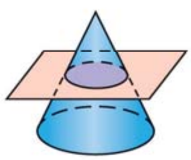
4. Find the number of faces, vertices, and edges of the regular dodecahedron on page 796. Check your answer using Euler's Theorem.

Describe **the shape formed by the intersection of the plane and the solid.**

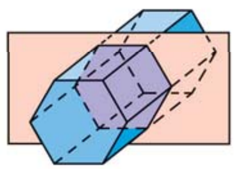
5.



6.



7.



12.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 25, and 35

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 21, 28, 30, 31, 39, and 41

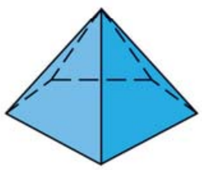
SKILL PRACTICE

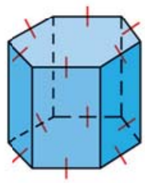
1. **VOCABULARY** Name the five Platonic solids and give the number of faces for each.

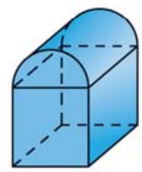
2. ★ **WRITING** State Euler's Theorem in words.

EXAMPLE 1 on p. 795 for Exs. 3–10

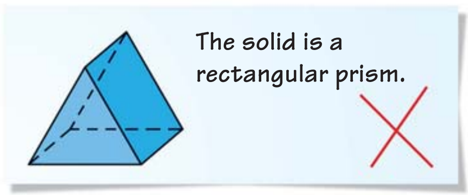
IDENTIFYING POLYHEDRA **Determine whether the solid is a polyhedron. If it is, name the polyhedron. *Explain* your reasoning.**

3. 

4. 

5. 

6. **ERROR ANALYSIS** *Describe* and correct the error in identifying the solid.



SKETCHING POLYHEDRA **Sketch the polyhedron.**

7. Rectangular prism
8. Triangular prism
9. Square pyramid
10. Pentagonal pyramid

EXAMPLES 2 and 3 on pp. 796–797 for Exs. 11–24

APPLYING EULER'S THEOREM **Use Euler's Theorem to find the value of n.**

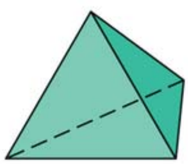
11. Faces: n
Vertices: 12
Edges: 18

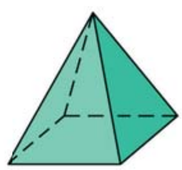
12. Faces: 5
Vertices: n
Edges: 8

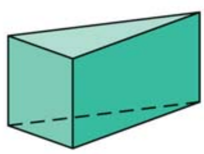
13. Faces: 10
Vertices: 16
Edges: n

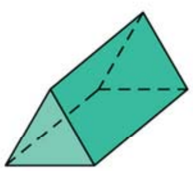
14. Faces: n
Vertices: 12
Edges: 30

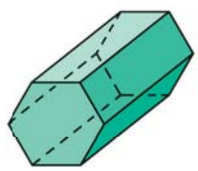
APPLYING EULER'S THEOREM **Find the number of faces, vertices, and edges of the polyhedron. Check your answer using Euler's Theorem.**

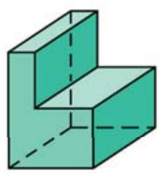
15. 

16. 

17. 

18. 

19. 

20. 

21. ★ **WRITING** *Explain* why a cube is also called a regular hexahedron.

PUZZLES **Determine whether the solid puzzle is *convex* or *concave*.**

22.

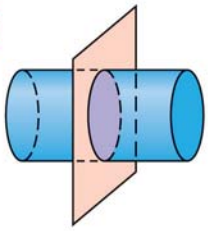
23.

24.

EXAMPLE 4
on p. 797
for Exs. 25–28

CROSS SECTIONS **Draw and *describe* the cross section formed by the intersection of the plane and the solid.**

25.



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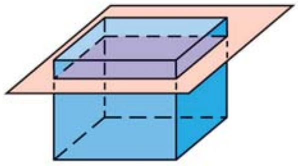

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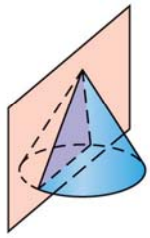
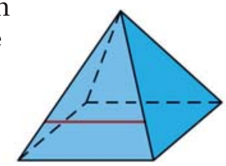


28. ★ **MULTIPLE CHOICE** What is the shape of the cross section formed by the plane parallel to the base that intersects the red line drawn on the square pyramid?



Ⓐ Square　　Ⓑ Triangle

Ⓒ Kite　　Ⓓ Trapezoid

29. **ERROR ANALYSIS** *Describe* and correct the error in determining that a tetrahedron has 4 faces, 4 edges, and 6 vertices.

30. ★ **MULTIPLE CHOICE** Which two solids have the same number of faces?

Ⓐ A triangular prism and a rectangular prism

Ⓑ A triangular pyramid and a rectangular prism

Ⓒ A triangular prism and a square pyramid

Ⓓ A triangular pyramid and a square pyramid

31. ★ **MULTIPLE CHOICE** How many faces, vertices, and edges does an octagonal prism have?

Ⓐ 8 faces, 6 vertices, and 12 edges

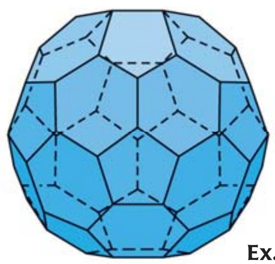
Ⓑ 8 faces, 12 vertices, and 18 edges

Ⓒ 10 faces, 12 vertices, and 20 edges

Ⓓ 10 faces, 16 vertices, and 24 edges

32. **EULER'S THEOREM** The solid shown has 32 faces and 90 edges. How many vertices does the solid have? *Explain* your reasoning.

33. **CHALLENGE** *Describe* how a plane can intersect a cube to form a hexagonal cross section.



Ex. 32

PROBLEM SOLVING

EXAMPLE 2 on p. 796 for Exs. 34–35

34. MUSIC The speaker shown at the right has 7 faces. Two faces are pentagons and 5 faces are rectangles.

a. Find the number of vertices.

b. Use Euler's Theorem to determine how many edges the speaker has.

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35. CRAFT BOXES The box shown at the right is a hexagonal prism. It has 8 faces. Two faces are hexagons and 6 faces are squares. Count the edges and vertices. Use Euler's Theorem to check your answer.

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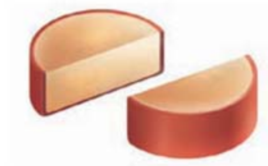
FOOD ***Describe*** **the shape that is formed by the cut made in the food shown.**

36. Watermelon

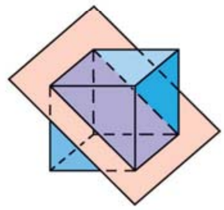
37. Bread

38. Cheese



39. ★ **SHORT RESPONSE** Name a polyhedron that has 4 vertices and 6 edges. Can you draw a polyhedron that has 4 vertices, 6 edges, and a different number of faces? *Explain* your reasoning.

40. MULTI-STEP PROBLEM The figure at the right shows a plane intersecting a cube through four of its vertices. An edge length of the cube is 6 inches.



a. *Describe* the shape formed by the cross section.

b. What is the perimeter of the cross section?

c. What is the area of the cross section?

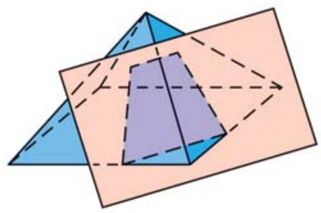
41. ★ **EXTENDED RESPONSE** Use the diagram of the square pyramid intersected by a plane.

a. *Describe* the shape of the cross section shown.

b. Can a plane intersect the pyramid at a point? If so, sketch the intersection.

c. *Describe* the shape of the cross section when the pyramid is sliced by a plane parallel to its base.

d. Is it possible to have a pentagon as a cross section of this pyramid? If so, draw the cross section.



42. PLATONIC SOLIDS Make a table of the number of faces, vertices, and edges for the five Platonic solids. Use Euler's Theorem to check each answer.

○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE

REASONING **Is it possible for a cross section of a cube to have the given shape? If yes, *describe* or sketch how the plane intersects the cube.**

43. Circle

44. Pentagon

45. Rhombus

46. Isosceles triangle

47. Regular hexagon

48. Scalene triangle

49. **CUBE** *Explain* how the numbers of faces, vertices, and edges of a cube change when you cut off each feature.

a. A corner **b.** An edge **c.** A face **d.** 3 corners

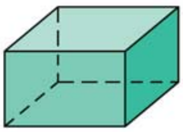
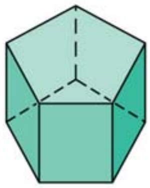
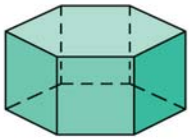
50. **TETRAHEDRON** *Explain* how the numbers of faces, vertices, and edges of a regular tetrahedron change when you cut off each feature.

a. A corner **b.** An edge **c.** A face **d.** 2 edges

51. **CHALLENGE** The *angle defect D* at a vertex of a polyhedron is defined as follows:

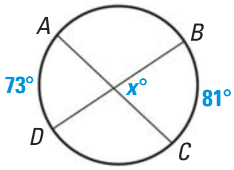
$D = 360° - $ (sum of all angle measures at the vertex)

Verify that for the figures with regular bases below, $DV = 720$ where V is the number of vertices.

MIXED REVIEW

Find the value of *x*. *(p. 680)*

52.


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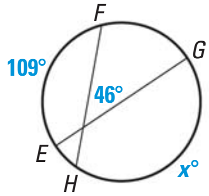

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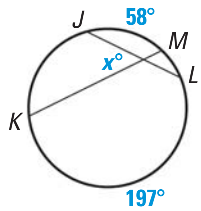


PREVIEW
Prepare for Lesson 12.2 in Exs. 55–60.

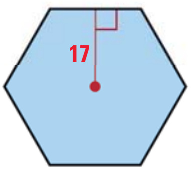
Use the given radius *r* or diameter *d* to find the circumference and area of the circle. Round your answers to two decimal places. *(p. 755)*

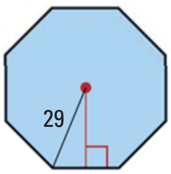
55. $r = 11$ cm

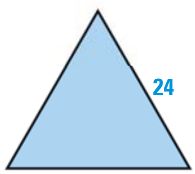
56. $d = 28$ in.

57. $d = 15$ ft

Find the perimeter and area of the regular polygon. Round your answers to two decimal places. *(p. 762)*

58.


59.


60.


12.2 Investigate Surface Area

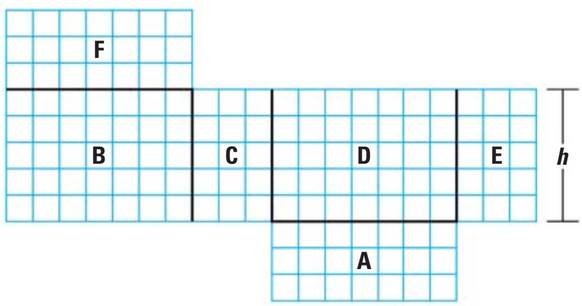
MATERIALS • graph paper • scissors • tape

QUESTION How can you find the surface area of a polyhedron?

A *net* is a pattern that can be folded to form a polyhedron. To find the *surface area* of a polyhedron, you can find the area of its net.

EXPLORE Create a polyhedron using a net

STEP 1 ***Draw a net*** Copy the net below on graph paper. Be sure to label the sections of the net.



STEP 2 ***Create a polyhedron*** Cut out the net and fold it along the black lines to form a polyhedron. Tape the edges together. Describe the polyhedron. Is it regular? Is it convex?

STEP 3 ***Find surface area*** The *surface area* of a polyhedron is the sum of the areas of its faces. Find the surface area of the polyhedron you just made. (Each square on the graph paper measures 1 unit by 1 unit.)

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Lay the net flat again and find the following measures.

   A: the area of Rectangle A

   P: the perimeter of Rectangle A

   h: the height of Rectangles B, C, D, and E

2. Use the values from Exercise 1 to find $2A + Ph$. *Compare* this value to the surface area you found in Step 3 above. What do you notice?

3. Make a conjecture about the surface area of a rectangular prism.

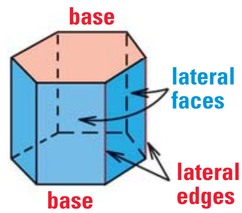
4. Use graph paper to draw the net of another rectangular prism. Fold the net to make sure that it forms a rectangular prism. Use your conjecture from Exercise 3 to calculate the surface area of the prism.

12.2 Surface Area of Prisms and Cylinders

Before	You found areas of polygons.
Now	You will find the surface areas of prisms and cylinders.
Why?	So you can find the surface area of a drum, as in Ex. 22.

Key Vocabulary
- **prism** lateral faces, lateral edges
- **surface area**
- **lateral area**
- **net**
- **right prism**
- **oblique prism**
- **cylinder**
- **right cylinder**

A **prism** is a polyhedron with two congruent faces, called *bases*, that lie in parallel planes. The other faces, called **lateral faces**, are parallelograms formed by connecting the corresponding vertices of the bases. The segments connecting these vertices are **lateral edges**. Prisms are classified by the shapes of their bases.



The **surface area** of a polyhedron is the sum of the areas of its faces. The **lateral area** of a polyhedron is the sum of the areas of its lateral faces.

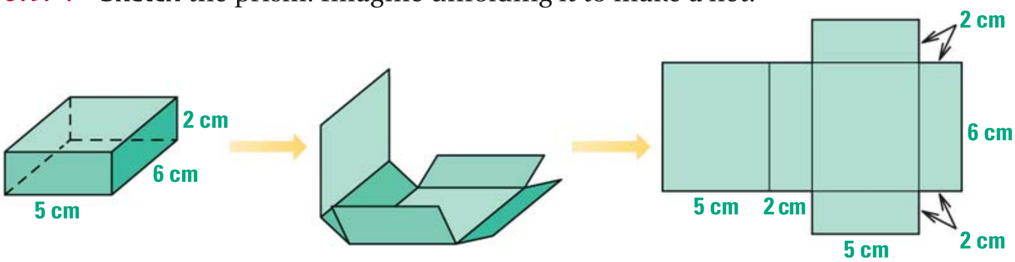
Imagine that you cut some edges of a polyhedron and unfold it. The two-dimensional representation of the faces is called a **net**. As you saw in the Activity on page 802, the surface area of a prism is equal to the area of its net.

EXAMPLE 1 Use the net of a prism

Find the surface area of a rectangular prism with height 2 centimeters, length 5 centimeters, and width 6 centimeters.

Solution

STEP 1 **Sketch** the prism. Imagine unfolding it to make a net.



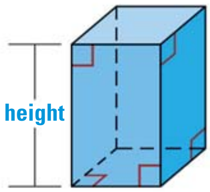
STEP 2 **Find** the areas of the rectangles that form the faces of the prism.

Congruent faces	Dimensions	Area of each face
Left and right faces	6 cm by 2 cm	$6 \cdot 2 = 12 \text{ cm}^2$
Front and back faces	5 cm by 2 cm	$5 \cdot 2 = 10 \text{ cm}^2$
Top and bottom faces	6 cm by 5 cm	$6 \cdot 5 = 30 \text{ cm}^2$

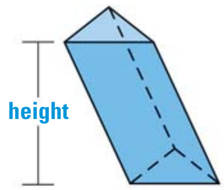
STEP 3 **Add** the areas of all the faces to find the surface area.

▶ The surface area of the prism is $S = 2(12) + 2(10) + 2(30) = 104 \text{ cm}^2$.

RIGHT PRISMS The height of a prism is the perpendicular distance between its bases. In a **right prism**, each lateral edge is perpendicular to both bases. A prism with lateral edges that are not perpendicular to the bases is an **oblique prism**.



Right rectangular prism



Oblique triangular prism

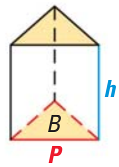
THEOREM *For Your Notebook*

THEOREM 12.2 Surface Area of a Right Prism

The surface area S of a right prism is

$S = 2B + Ph = aP + Ph,$

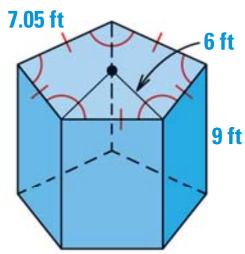
where a is the apothem of the base, B is the area of a base, P is the perimeter of a base, and h is the height.



$S = 2B + Ph = aP + Ph$

EXAMPLE 2 Find the surface area of a right prism

Find the surface area of the right pentagonal prism.



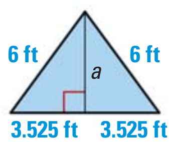
Solution

STEP 1 **Find** the perimeter and area of a base of the prism.

Each base is a regular pentagon.

Perimeter $P = 5(7.05) = 35.25$

Apothem $a = \sqrt{6^2 - 3.525^2} \approx 4.86$



REVIEW APOTHEM
For help with finding the apothem, see p. 762.

STEP 2 **Use** the formula for the surface area that uses the apothem.

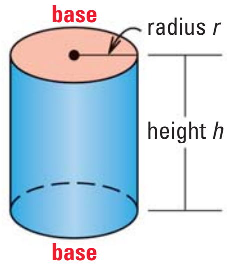
$S = aP + Ph$	**Surface area of a right prism**
$\approx (4.86)(35.25) + (35.25)(9)$	**Substitute known values.**
≈ 488.57	**Simplify.**

▶ The surface area of the right pentagonal prism is about 488.57 square feet.

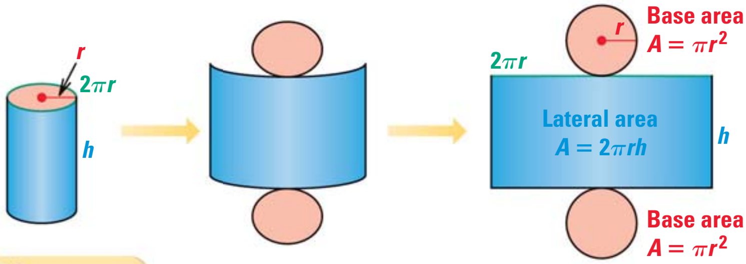
✓ GUIDED PRACTICE for Examples 1 and 2

1. Draw a net of a triangular prism.
2. Find the surface area of a right rectangular prism with height 7 inches, length 3 inches, and width 4 inches using (a) a net and (b) the formula for the surface area of a right prism.

CYLINDERS A **cylinder** is a solid with congruent circular bases that lie in parallel planes. The height of a cylinder is the perpendicular distance between its bases. The radius of a base is the *radius* of the cylinder. In a **right cylinder**, the segment joining the centers of the bases is perpendicular to the bases.



The lateral area of a cylinder is the area of its curved surface. It is equal to the product of the circumference and the height, or $2\pi rh$. The surface area of a cylinder is equal to the sum of the lateral area and the areas of the two bases.



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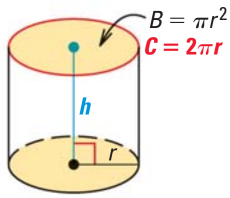
THEOREM — For Your Notebook

THEOREM 12.3 Surface Area of a Right Cylinder

The surface area S of a right cylinder is

$$S = 2B + Ch = 2\pi r^2 + 2\pi rh,$$

where B is the area of a base, C is the circumference of a base, r is the radius of a base, and h is the height.



$S = 2B + Ch = 2\pi r^2 + 2\pi rh$

EXAMPLE 3 Find the surface area of a cylinder

COMPACT DISCS You are wrapping a stack of 20 compact discs using a shrink wrap. Each disc is cylindrical with height 1.2 millimeters and radius 60 millimeters. What is the minimum amount of shrink wrap needed to cover the stack of 20 discs?

Solution

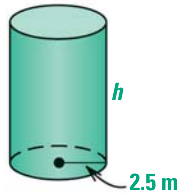
The 20 discs are stacked, so the height of the stack will be $20(1.2) = 24$ mm. The radius is 60 millimeters. The minimum amount of shrink wrap needed will be equal to the surface area of the stack of discs.

$S = 2\pi r^2 + 2\pi rh$	Surface area of a cylinder
$= 2\pi(60)^2 + 2\pi(60)(24)$	Substitute known values.
$\approx 31{,}667$	Use a calculator.

▶ You will need at least 31,667 square millimeters, or about 317 square centimeters of shrink wrap.

EXAMPLE 4 Find the height of a cylinder

Find the height of the right cylinder shown, which has a surface area of 157.08 square meters.



Solution

Substitute known values in the formula for the surface area of a right cylinder and solve for the height h.

$S = 2\pi r^2 + 2\pi rh$	Surface area of a cylinder
$157.08 = 2\pi(2.5)^2 + 2\pi(2.5)h$	Substitute known values.
$157.08 = 12.5\pi + 5\pi h$	Simplify.
$157.08 - 12.5\pi = 5\pi h$	Subtract 12.5π from each side.
$117.81 \approx 5\pi h$	Simplify. Use a calculator.
$7.5 \approx h$	Divide each side by 5π.

▶ The height of the cylinder is about 7.5 meters.

✓ GUIDED PRACTICE for Examples 3 and 4

3. Find the surface area of a right cylinder with height 18 centimeters and radius 10 centimeters. Round your answer to two decimal places.

4. Find the radius of a right cylinder with height 5 feet and surface area 208π square feet.

12.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 9, and 23

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 24, 25, and 26

SKILL PRACTICE

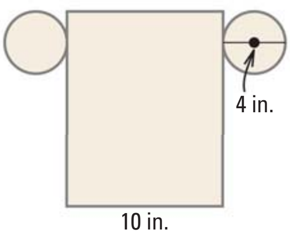
1. VOCABULARY Sketch a triangular prism. Identify its *bases*, *lateral faces*, and *lateral edges*.

2. ★ **WRITING** *Explain* how the formula $S = 2B + Ph$ applies to finding the surface area of both a right prism and a right cylinder.

EXAMPLE 1 on p. 803 for Exs. 3–5

USING NETS Find the surface area of the solid formed by the net. Round your answer to two decimal places.

3.



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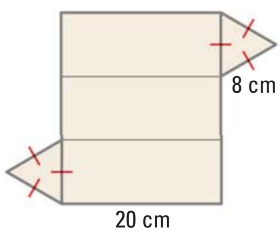

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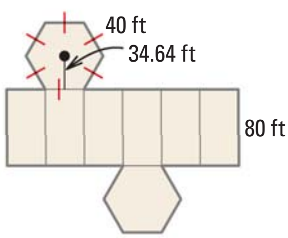
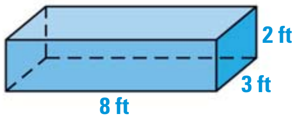


EXAMPLE 2
on p. 804
for Exs. 6–8

SURFACE AREA OF A PRISM **Find the surface area of the right prism. Round your answer to two decimal places.**

6.



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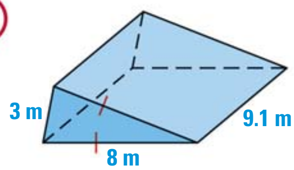

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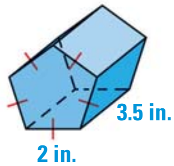


EXAMPLE 3
on p. 805
for Exs. 9–12

SURFACE AREA OF A CYLINDER **Find the surface area of the right cylinder using the given radius *r* and height *h*. Round your answer to two decimal places.**

9.

$r = 0.8$ in.
$h = 2$ in.

10.

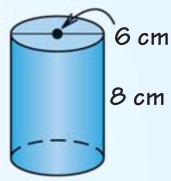
$r = 12$ mm
$h = 40$ mm

11.

$r = 8$ in.
$h = 8$ in.

12. **ERROR ANALYSIS** *Describe* and correct the error in finding the surface area of the right cylinder.

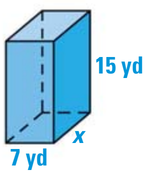
$$S = 2\pi(6^2) + 2\pi(6)(8)$$
$$= 2\pi(36) + 2\pi(48)$$
$$= 168\pi$$
$$\approx 528 \text{ cm}^2$$



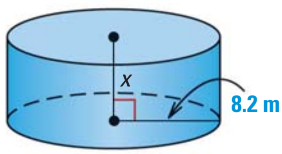
EXAMPLE 4
on p. 806
for Exs. 13–15

ALGEBRA **Solve for *x* given the surface area *S* of the right prism or right cylinder. Round your answer to two decimal places.**

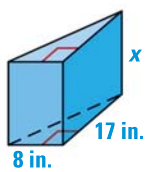
13. $S = 606 \text{ yd}^2$



14. $S = 1097 \text{ m}^2$



15. $S = 616 \text{ in.}^2$



16. **SURFACE AREA OF A PRISM** A triangular prism with a right triangular base has leg length 9 units and hypotenuse length 15 units. The height of the prism is 8 units. Sketch the prism and find its surface area.

17. ★ **MULTIPLE CHOICE** The length of each side of a cube is multiplied by 3. What is the change in the surface area of the cube?

(A) The surface area is 3 times the original surface area.

(B) The surface area is 6 times the original surface area.

(C) The surface area is 9 times the original surface area.

(D) The surface area is 27 times the original surface area.

18. **SURFACE AREA OF A CYLINDER** The radius and height of a right cylinder are each divided by $\sqrt{5}$. What is the change in surface area of the cylinder?

19. **SURFACE AREA OF A PRISM** Find the surface area of a right hexagonal prism with all edges measuring 10 inches.

20. **HEIGHT OF A CYLINDER** Find the height of a cylinder with a surface area of 108π square meters. The radius of the cylinder is twice the height.

21. **CHALLENGE** The *diagonal* of a cube is a segment whose endpoints are vertices that are not on the same face. Find the surface area of a cube with diagonal length 8 units. Round your answer to two decimal places.

PROBLEM SOLVING

EXAMPLE 3
on p. 805
for Ex. 22

22. **BASS DRUM** A bass drum has a diameter of 20 inches and a depth of 8 inches. Find the surface area of the drum.

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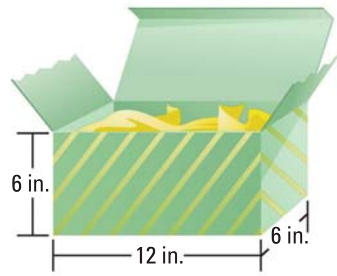
23. **GIFT BOX** An open gift box is shown at the right. When the gift box is closed, it has a length of 12 inches, a width of 6 inches, and a height of 6 inches.



a. What is the minimum amount of wrapping paper needed to cover the closed gift box?

b. Why is the area of the net of the box larger than the amount of paper found in part (a)?

c. When wrapping the box, why would you want more paper than the amount found in part (a)?

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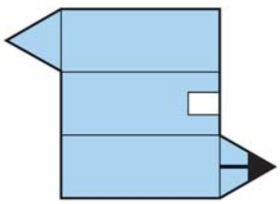
24. ★ **EXTENDED RESPONSE** A right cylinder has a radius of 4 feet and height of 10 feet.

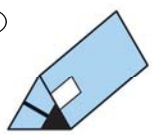
a. Find the surface area of the cylinder.

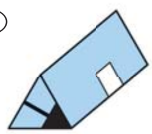
b. Suppose you can either *double the radius* or *double the height*. Which do you think will create a greater surface area?

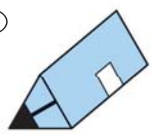
c. Check your answer in part (b) by calculating the new surface areas.

25. ★ **MULTIPLE CHOICE** Which three-dimensional figure does the net represent?



Ⓐ 

Ⓑ 

Ⓒ 

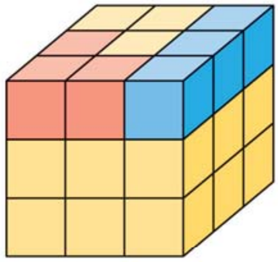
Ⓓ 

◯ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

26. ★ **SHORT RESPONSE** A company makes two types of recycling bins. One type is a right rectangular prism with length 14 inches, width 12 inches, and height 36 inches. The other type is a right cylinder with radius 6 inches and height 36 inches. Both types of bins are missing a base, so the bins have one open end. Which recycle bin requires more material to make? *Explain.*

27. **MULTI-STEP PROBLEM** Consider a cube that is built using 27 unit cubes as shown at the right.

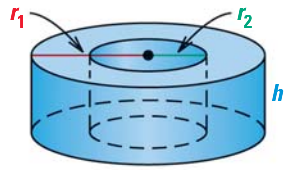


a. Find the surface area of the solid formed when the red unit cubes are removed from the solid shown.

b. Find the surface area of the solid formed when the blue unit cubes are removed from the solid shown.

c. Why are your answers different in parts (a) and (b)? *Explain.*

28. **SURFACE AREA OF A RING** The ring shown is a right cylinder of radius r_1 with a cylindrical hole of r_2. The ring has height h.

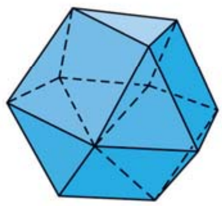


a. Find the surface area of the ring if r_1 is 12 meters, r_2 is 6 meters, and h is 8 meters. Round your answer to two decimal places.

b. Write a formula that can be used to find the surface area S of any cylindrical ring where $0 < r_2 < r_1$.

29. **DRAWING SOLIDS** A cube with edges 1 foot long has a cylindrical hole with diameter 4 inches drilled through one of its faces. The hole is drilled perpendicular to the face and goes completely through to the other side. Draw the figure and find its surface area.

30. **CHALLENGE** A cuboctahedron has 6 square faces and 8 equilateral triangle faces, as shown. A cuboctahedron can be made by slicing off the corners of a cube.



a. Sketch a net for the cuboctahedron.

b. Each edge of a cuboctahedron has a length of 5 millimeters. Find its surface area.

MIXED REVIEW

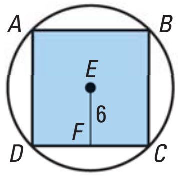
The sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides. *(p. 507)*

31. 1260° **32.** 1080° **33.** 720° **34.** 1800°

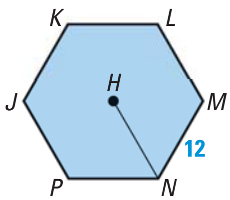
PREVIEW
Prepare for Lesson 12.3 in Exs. 35–37.

Find the area of the regular polygon. *(p. 762)*

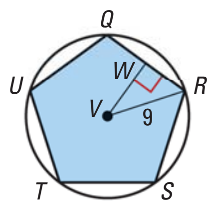
35.



36.



37.



12.3 Surface Area of Pyramids and Cones

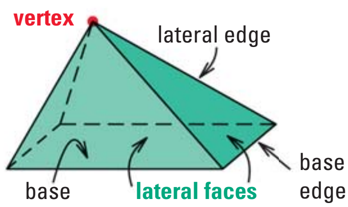
Before You found surface areas of prisms and cylinders.

Now You will find surface areas of pyramids and cones.

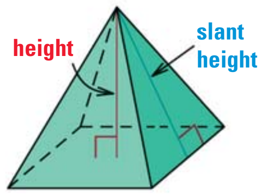
Why? So you can find the surface area of a volcano, as in Ex. 33.

Key Vocabulary
- pyramid
- vertex of a pyramid
- regular pyramid
- slant height
- cone
- vertex of a cone
- right cone
- lateral surface

A **pyramid** is a polyhedron in which the base is a polygon and the lateral faces are triangles with a common vertex, called the **vertex of the pyramid**. The intersection of two lateral faces is a *lateral edge*. The intersection of the base and a lateral face is a *base edge*. The height of the pyramid is the perpendicular distance between the base and the vertex.



Pyramid



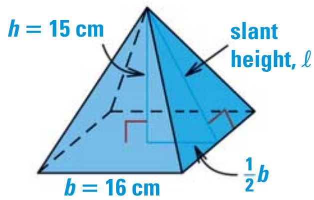
Regular pyramid

NAME PYRAMIDS
Pyramids are classified by the shapes of their bases.

A **regular pyramid** has a regular polygon for a base and the segment joining the vertex and the center of the base is perpendicular to the base. The lateral faces of a regular pyramid are congruent isosceles triangles. The **slant height** of a regular pyramid is the height of a lateral face of the regular pyramid. A nonregular pyramid does not have a slant height.

EXAMPLE 1 Find the area of a lateral face of a pyramid

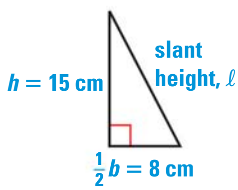
A regular square pyramid has a height of 15 centimeters and a base edge length of 16 centimeters. Find the area of each lateral face of the pyramid.



Solution

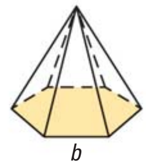
Use the Pythagorean Theorem to find the slant height ℓ.

$\ell^2 = h^2 + \left(\frac{1}{2}b\right)^2$	Write formula.
$\ell^2 = 15^2 + 8^2$	Substitute for h and $\frac{1}{2}b$.
$\ell^2 = 289$	Simplify.
$\ell = 17$	Find the positive square root.

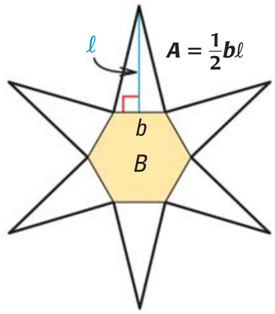


▸ The area of each triangular face is $A = \frac{1}{2}b\ell = \frac{1}{2}(16)(17) = 136$ square centimeters.

SURFACE AREA A regular hexagonal pyramid and its net are shown at the right. Let b represent the length of a base edge, and let ℓ represent the slant height of the pyramid.



The area of each lateral face is $\frac{1}{2}b\ell$ and the perimeter of the base is $P = 6b$. So, the surface area S is as follows.



$S =$ (Area of base) $+ 6$(Area of lateral face)

$S = B + 6\left(\frac{1}{2}b\ell\right)$ **Substitute.**

$S = B + \frac{1}{2}(6b)\ell$ **Rewrite $6\left(\frac{1}{2}b\ell\right)$ as $\frac{1}{2}(6b)\ell$.**

$S = B + \frac{1}{2}P\ell$ **Substitute P for $6b$.**

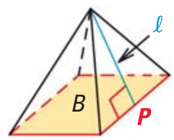
THEOREM — *For Your Notebook*

THEOREM 12.4 Surface Area of a Regular Pyramid

The surface area S of a regular pyramid is

$$S = B + \frac{1}{2}P\ell,$$

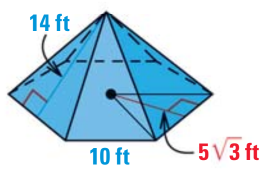
where B is the area of the base, P is the perimeter of the base, and ℓ is the slant height.



$S = B + \frac{1}{2}P\ell$

EXAMPLE 2 Find the surface area of a pyramid

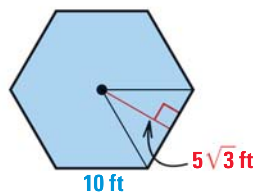
Find the surface area of the regular hexagonal pyramid.



Solution

REVIEW AREA
For help with finding the area of regular polygons, see p. 762.

First, find the area of the base using the formula for the area of a regular polygon, $\frac{1}{2}aP$. The apothem a of the hexagon is $5\sqrt{3}$ feet and the perimeter P is $6 \cdot 10 = 60$ feet. So, the area of the base B is $\frac{1}{2}(5\sqrt{3})(60) = 150\sqrt{3}$ square feet. Then, find the surface area.



$S = B + \frac{1}{2}P\ell$ **Formula for surface area of regular pyramid**

$= 150\sqrt{3} + \frac{1}{2}(60)(14)$ **Substitute known values.**

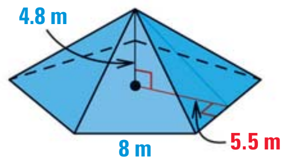
$= 150\sqrt{3} + 420$ **Simplify.**

≈ 679.81 **Use a calculator.**

▶ The surface area of the regular hexagonal pyramid is about 679.81 ft^2.

✓ GUIDED PRACTICE for Examples 1 and 2

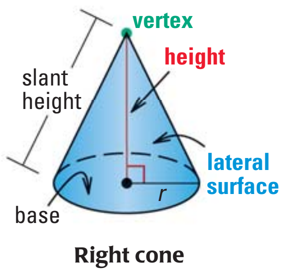
1. Find the area of each lateral face of the regular pentagonal pyramid shown.
2. Find the surface area of the regular pentagonal pyramid shown.



CONES A **cone** has a circular base and a **vertex** that is not in the same plane as the base. The radius of the base is the *radius* of the cone. The height is the perpendicular distance between the vertex and the base.

In a **right cone**, the segment joining the vertex and the center of the base is perpendicular to the base and the slant height is the distance between the vertex and a point on the base edge.

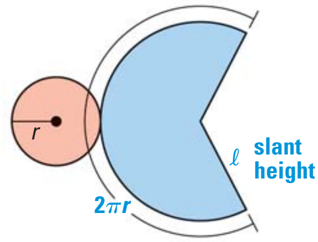
The **lateral surface** of a cone consists of all segments that connect the vertex with points on the base edge.



Right cone

SURFACE AREA When you cut along the slant height and base edge and lay a right cone flat, you get the net shown at the right.

The circular base has an area of πr^2 and the lateral surface is the sector of a circle. You can use a proportion to find the area of the sector, as shown below.



$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Arc length}}{\text{Circumference of circle}} \quad \textbf{Set up proportion.}$$

$$\frac{\text{Area of sector}}{\pi \ell^2} = \frac{2\pi r}{2\pi \ell} \quad \textbf{Substitute.}$$

$$\text{Area of sector} = \pi \ell^2 \cdot \frac{2\pi r}{2\pi \ell} \quad \textbf{Multiply each side by } \pi\ell^2.$$

$$\text{Area of sector} = \pi r \ell \quad \textbf{Simplify.}$$

The surface area of a cone is the sum of the base area, πr^2, and the lateral area, $\pi r \ell$. Notice that the quantity $\pi r \ell$ can be written as $\frac{1}{2}(2\pi r)\ell$, or $\frac{1}{2}C\ell$.

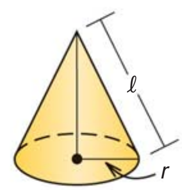
THEOREM — *For Your Notebook*

THEOREM 12.5 Surface Area of a Right Cone

The surface area S of a right cone is

$$S = B + \frac{1}{2}C\ell = \pi r^2 + \pi r \ell,$$

where B is the area of the base, C is the circumference of the base, r is the radius of the base, and ℓ is the slant height.



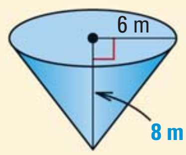
$$S = B + \frac{1}{2}C\ell = \pi r^2 + \pi r \ell$$

★ EXAMPLE 3 Standardized Test Practice

What is the surface area of the right cone?

Ⓐ $72\pi\ m^2$ Ⓑ $96\pi\ m^2$

Ⓒ $132\pi\ m^2$ Ⓓ $136\pi\ m^2$



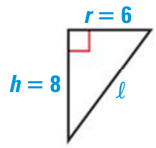
Solution

To find the slant height ℓ of the right cone, use the Pythagorean Theorem.

$\ell^2 = h^2 + r^2$ **Write formula.**

$\ell^2 = 8^2 + 6^2$ **Substitute.**

$\ell = 10$ **Find positive square root.**



ANOTHER WAY

You can use a Pythagorean triple to find ℓ.

$6 = 2 \cdot 3$ and $8 = 2 \cdot 4$, so $\ell = 2 \cdot 5 = 10$.

Use the formula for the surface area of a right cone.

$S = \pi r^2 + \pi r\ell$ **Formula for surface area of a right cone**

$= \pi(6^2) + \pi(6)(10)$ **Substitute.**

$= 96\pi$ **Simplify.**

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

EXAMPLE 4 Find the lateral area of a cone

TRAFFIC CONE The traffic cone can be approximated by a right cone with radius 5.7 inches and height 18 inches. Find the approximate lateral area of the traffic cone.

Solution

To find the slant height ℓ, use the Pythagorean Theorem.

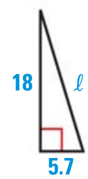
$\ell^2 = 18^2 + (5.7)^2$, so $\ell \approx 18.9$ inches.

Find the lateral area.

Lateral area $= \pi r\ell$ **Write formula.**

$= \pi(5.7)(18.9)$ **Substitute known values.**

≈ 338.4 **Simplify and use a calculator.**

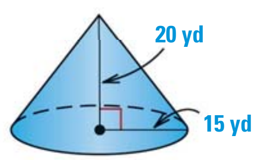


▶ The lateral area of the traffic cone is about 338.4 square inches.

✓ GUIDED PRACTICE for Examples 3 and 4

3. Find the lateral area of the right cone shown.

4. Find the surface area of the right cone shown.



12.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 11, and 29

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, and 31

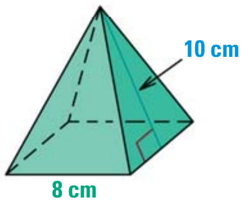
SKILL PRACTICE

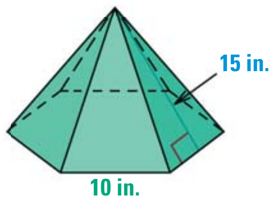
1. **VOCABULARY** Draw a regular square pyramid. Label its *height*, *slant height*, and *base*.

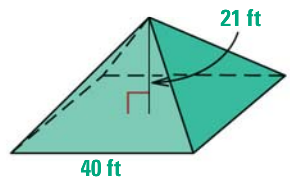
2. ★ **WRITING** *Compare* the height and slant height of a right cone.

EXAMPLE 1 on p. 810 for Exs. 3–5

AREA OF A LATERAL FACE **Find the area of each lateral face of the regular pyramid.**

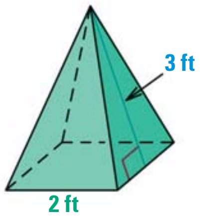
3. 

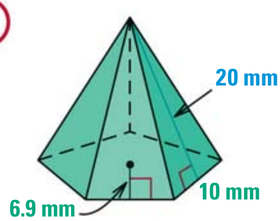
4. 

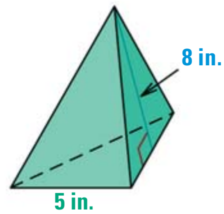
5. 

EXAMPLE 2 on p. 811 for Exs. 6–9

SURFACE AREA OF A PYRAMID **Find the surface area of the regular pyramid. Round your answer to two decimal places.**

6. 

7. 

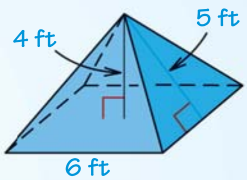
8. 

9. **ERROR ANALYSIS** *Describe* and correct the error in finding the surface area of the regular pyramid.

$$S = B + \frac{1}{2}P\ell$$

$$= 6^2 + \frac{1}{2}(24)(4)$$

$$= 84 \text{ ft}^2$$



EXAMPLES 3 and 4 on p. 813 for Exs. 10–17

LATERAL AREA OF A CONE **Find the lateral area of the right cone. Round your answer to two decimal places.**

10. 

$r = 7.5$ cm
$h = 25$ cm

11. 

$r = 1$ in.
$h = 4$ in.

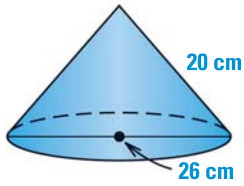
12. 

$d = 7$ in.
$h = 1$ ft

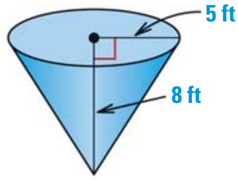
SURFACE AREA OF A CONE **Find the surface area of the right cone. Round your answer to two decimal places.**

13.

14.

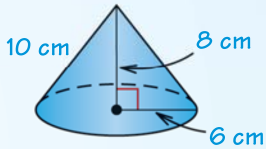


15.

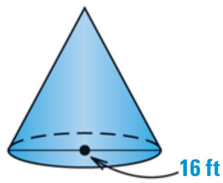


16. **ERROR ANALYSIS** *Describe* and correct the error in finding the surface area of the right cone.

$$S = \pi(r^2) + \pi r^2 \ell$$
$$= \pi(36) + \pi(36)(10)$$
$$= 396\pi \text{ cm}^2$$



17. ★ **MULTIPLE CHOICE** The surface area of the right cone is 200π square feet. What is the slant height of the cone?



Ⓐ 10.5 ft Ⓑ 17 ft

Ⓒ 23 ft Ⓓ 24 ft

VISUAL REASONING **In Exercises 18–21, sketch the described solid and find its surface area. Round your answer to two decimal places.**

18. A right cone has a radius of 15 feet and a slant height of 20 feet.

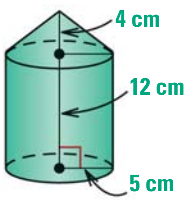
19. A right cone has a diameter of 16 meters and a height of 30 meters.

20. A regular pyramid has a slant height of 24 inches. Its base is an equilateral triangle with a base edge length of 10 inches.

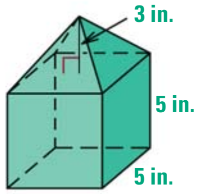
21. A regular pyramid has a hexagonal base with a base edge length of 6 centimeters and a slant height of 9 centimeters.

COMPOSITE SOLIDS **Find the surface area of the solid. The pyramids are regular and the cones are right. Round your answers to two decimal places, if necessary.**

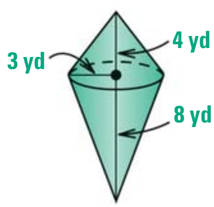
22.



23.



24.



25. **TETRAHEDRON** Find the surface area of a regular tetrahedron with edge length 4 centimeters.

26. **CHALLENGE** A right cone with a base of radius 4 inches and a regular pyramid with a square base both have a slant height of 5 inches. Both solids have the same surface area. Find the length of a base edge of the pyramid. Round your answer to the nearest hundredth of an inch.

PROBLEM SOLVING

EXAMPLE 2 on p. 811 for Ex. 27

27. CANDLES A candle is in the shape of a regular square pyramid with base edge length 6 inches. Its height is 4 inches. Find its surface area.

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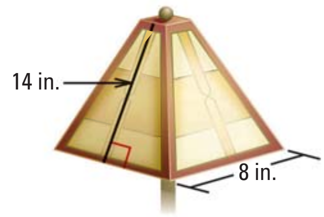
28. LAMPSHADE A glass lampshade is shaped like a regular square pyramid.



a. Approximate the lateral area of the lampshade shown.

b. *Explain* why your answer to part (a) is not the exact lateral area.

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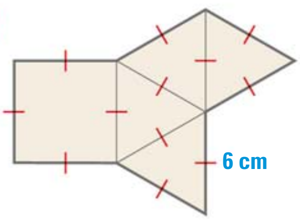
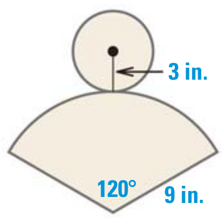
USING NETS Name the figure that is represented by the net. Then find its surface area. Round your answer to two decimal places.

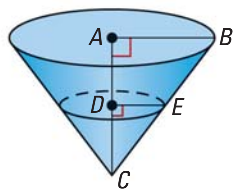
29.



30.



31. ★ SHORT RESPONSE In the figure, $AC = 4$, $AB = 3$, and $DC = 2$.

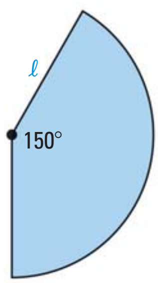


a. Prove $\triangle ABC \sim \triangle DEC$.

b. Find BC, DE, and EC.

c. Find the surface areas of the larger cone and the smaller cone in terms of π. *Compare* the surface areas using a percent.

32. MULTI-STEP PROBLEM The sector shown can be rolled to form the lateral surface of a right cone. The lateral surface area of the cone is 20 square meters.



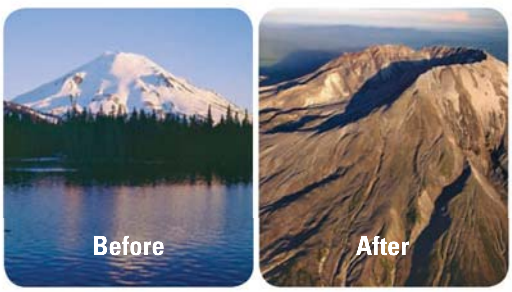
a. Write the formula for the area of a sector.

b. Use the formula in part (a) to find the slant height of the cone. *Explain* your reasoning.

c. Find the radius and height of the cone.

33. VOLCANOES Before 1980, Mount St. Helens was a conic volcano with a height from its base of about 1.08 miles and a base radius of about 3 miles. In 1980, the volcano erupted, reducing its height to about 0.83 mile.

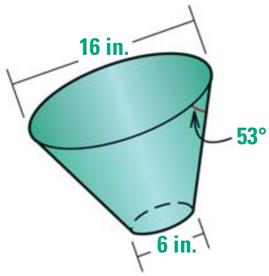
Approximate the lateral area of the volcano after 1980. (*Hint:* The ratio of the radius of the destroyed cone-shaped top to its height is the same as the ratio of the radius of the original volcano to its height.)



○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

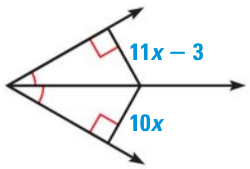
34. CHALLENGE An *Elizabethan collar* is used to prevent an animal from irritating a wound. The angle between the opening with a 16 inch diameter and the side of the collar is 53°. Find the surface area of the collar shown.



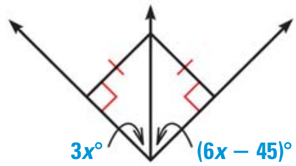
MIXED REVIEW

Find the value of *x*. *(p. 310)*

35.



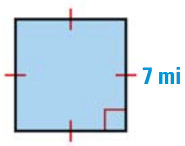
36.



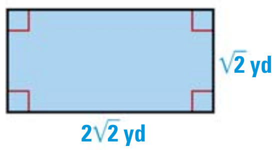
PREVIEW
Prepare for Lesson 12.4 in Exs. 37–39.

In Exercises 37–39, find the area of the polygon. *(pp. 720, 730)*

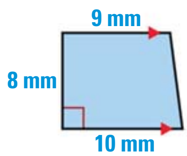
37.



38.



39.

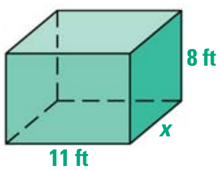


QUIZ *for Lessons 12.1–12.3*

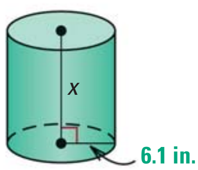
1. A polyhedron has 8 vertices and 12 edges. How many faces does the polyhedron have? *(p. 794)*

Solve for *x* given the surface area *S* of the right prism or right cylinder. Round your answer to two decimal places. *(p. 803)*

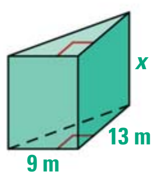
2. $S = 366 \text{ ft}^2$



3. $S = 717 \text{ in.}^2$

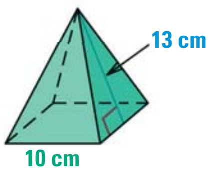


4. $S = 567 \text{ m}^2$

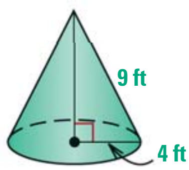


Find the surface area of the regular pyramid or right cone. Round your answer to two decimal places. *(p. 810)*

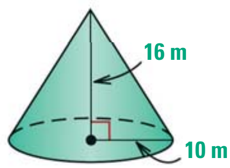
5.



6.



7.



MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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Lessons 12.1–12.3

1. **SHORT RESPONSE** Using Euler's Theorem, *explain* why it is not possible for a polyhedron to have 6 vertices and 7 edges.

2. **SHORT RESPONSE** *Describe* two methods of finding the surface area of a rectangular solid.

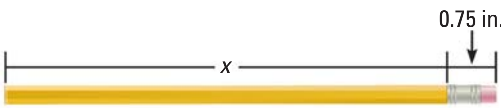
3. **EXTENDED RESPONSE** Some pencils are made from slats of wood that are machined into right regular hexagonal prisms.

   a. The formula for the surface area of a new unsharpened pencil without an eraser is

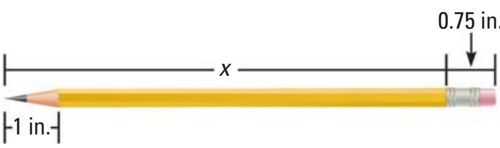
$$S = 3\sqrt{3}r^2 + 6rh.$$

   Tell what each variable in this formula represents.

   b. After a pencil is painted, a metal band that holds an eraser is wrapped around one end. Write a formula for the surface area of the visible portion of the pencil, shown below.

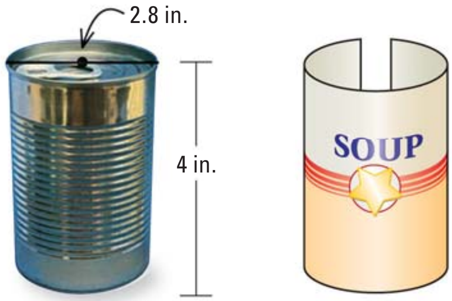


   c. After a pencil is sharpened, the end is shaped like a cone. Write a formula to find the surface area of the visible portion of the pencil, shown below.



   d. Use your formulas from parts (b) and (c) to write a formula for the difference of the surface areas of the two pencils. Define any variables in your formula.

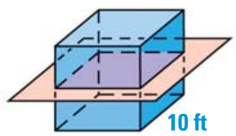
4. **GRIDDED ANSWER** The amount of paper needed for a soup can label is approximately equal to the lateral area of the can. Find the lateral area of the soup can in square inches. Round your answer to two decimal places.



5. **SHORT RESPONSE** If you know the diameter d and slant height ℓ of a right cone, how can you find the surface area of the cone?

6. **OPEN-ENDED** Identify an object in your school or home that is a rectangular prism. Measure its length, width, and height to the nearest quarter inch. Then approximate the surface area of the object.

7. **MULTI-STEP PROBLEM** The figure shows a plane intersecting a cube parallel to its base. The cube has a side length of 10 feet.



   a. Describe the shape formed by the cross section.

   b. Find the perimeter and area of the cross section.

   c. When the cross section is cut along its diagonal, what kind of triangles are formed?

   d. Find the area of one of the triangles formed in part (c).

8. **SHORT RESPONSE** A cone has a base radius of $3x$ units and a height of $4x$ units. The surface area of the cone is 1944π square units. Find the value of x. *Explain* your steps.

12.4 Volume of Prisms and Cylinders

Before You found surface areas of prisms and cylinders.

Now You will find volumes of prisms and cylinders.

Why So you can determine volume of water in an aquarium, as in Ex. 33.

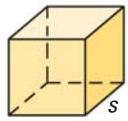
Key Vocabulary
- volume

The **volume** of a solid is the number of cubic units contained in its interior. Volume is measured in cubic units, such as cubic centimeters (cm^3).

POSTULATES — For Your Notebook

POSTULATE 27 Volume of a Cube Postulate

The volume of a cube is the cube of the length of its side.



$V = s^3$

POSTULATE 28 Volume Congruence Postulate

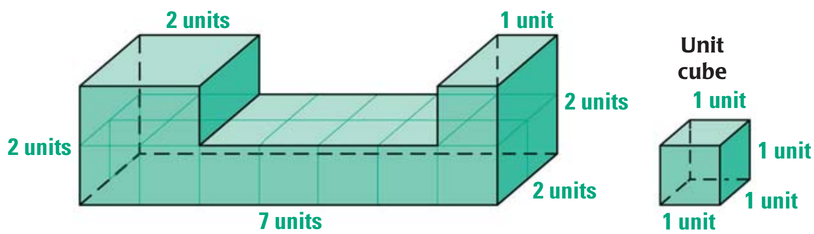
If two polyhedra are congruent, then they have the same volume.

POSTULATE 29 Volume Addition Postulate

The volume of a solid is the sum of the volumes of all its nonoverlapping parts.

EXAMPLE 1 Find the number of unit cubes

3-D PUZZLE **Find the volume of the puzzle piece in cubic units.**



Solution

To find the volume, find the number of unit cubes it contains. Separate the piece into three rectangular boxes as follows:

The *base* is 7 units by 2 units. So, it contains 7 • 2, or 14 unit cubes.

The *upper left box* is 2 units by 2 units. So, it contains 2 • 2, or 4 unit cubes.

The *upper right box* is 1 unit by 2 units. So, it contains 1 • 2, or 2 unit cubes.

▶ By the Volume Addition Postulate, the total volume of the puzzle piece is $14 + 4 + 2 = 20$ cubic units.

VOLUME FORMULAS The volume of any right prism or right cylinder can be found by multiplying the area of its base by its height.

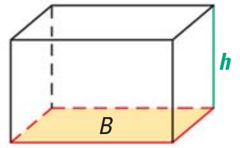
THEOREMS *For Your Notebook*

THEOREM 12.6 Volume of a Prism

The volume V of a prism is

$V = Bh$,

where B is the area of a base and h is the height.



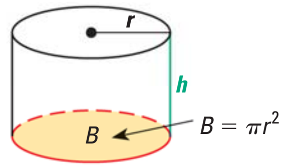
$V = Bh$

THEOREM 12.7 Volume of a Cylinder

The volume V of a cylinder is

$V = Bh = \pi r^2 h$,

where B is the area of a base, h is the height, and r is the radius of a base.

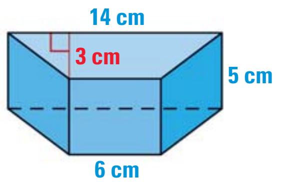


$V = Bh = \pi r^2 h$

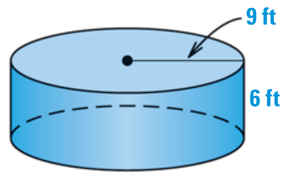
EXAMPLE 2 Find volumes of prisms and cylinders

Find the volume of the solid.

a. Right trapezoidal prism



b. Right cylinder



Solution

REVIEW AREA
For help with finding the area of a trapezoid, see p. 730.

a. The area of a base is $\frac{1}{2}(3)(6 + 14) = 30 \text{ cm}^2$ and $h = 5$ cm.

$V = Bh = 30(5) = 150 \text{ cm}^3$

b. The area of the base is $\pi \cdot 9^2$, or $81\pi \text{ ft}^2$. Use $h = 6$ ft to find the volume.

$V = Bh = 81\pi(6) = 486\pi \approx 1526.81 \text{ ft}^3$

EXAMPLE 3 Use volume of a prism

xy ALGEBRA The volume of the cube is 90 cubic inches. Find the value of x.

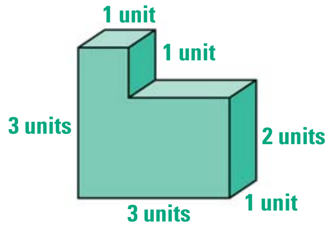
Solution

A side length of the cube is x inches.

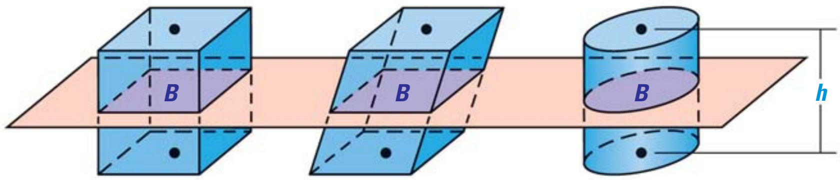
$V = x^3$	Formula for volume of a cube
$90 \text{ in.}^3 = x^3$	Substitute for V.
$4.48 \text{ in.} \approx x$	Find the cube root.

✓ **GUIDED PRACTICE** for Examples 1, 2, and 3

1. Find the volume of the puzzle piece shown in cubic units.
2. Find the volume of a square prism that has a base edge length of 5 feet and a height of 12 feet.
3. The volume of a right cylinder is 684π cubic inches and the height is 18 inches. Find the radius.



USING CAVALIERI'S PRINCIPLE Consider the solids below. All three have equal heights h and equal cross-sectional areas B. Mathematician Bonaventura Cavalieri (1598–1647) claimed that all three of the solids have the same volume. This principle is stated below.



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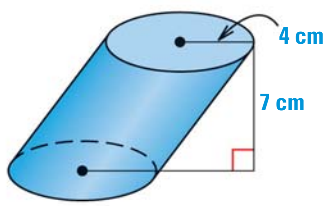
THEOREM — For Your Notebook

THEOREM 12.8 Cavalieri's Principle

If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

EXAMPLE 4 Find the volume of an oblique cylinder

Find the volume of the oblique cylinder.



Solution

Cavalieri's Principle allows you to use Theorem 12.7 to find the volume of the oblique cylinder.

$V = \pi r^2 h$	Formula for volume of a cylinder
$= \pi(4^2)(7)$	Substitute known values.
$= 112\pi$	Simplify.
≈ 351.86	Use a calculator.

▶ The volume of the oblique cylinder is about 351.86 cm^3.

APPLY THEOREMS
Cavalieri's Principle tells you that the volume formulas on page 820 work for oblique prisms and cylinders.

EXAMPLE 5 Solve a real-world problem

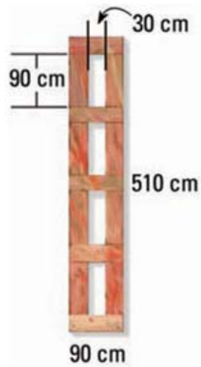
SCULPTURE The sculpture is made up of 13 beams. In centimeters, suppose the dimensions of each beam are 30 by 30 by 90. Find its volume.

Romartyr, Hamburg, 1989 © Carl Andre/ licensed by VAGA, NY

ANOTHER WAY
For alternative methods for solving the problem in Example 5, turn to page 826 for the **Problem Solving Workshop**.

Solution

The area of the base B can be found by subtracting the area of the small rectangles from the area of the large rectangle.



B = Area of large rectangle − 4 • Area of small rectangle

$= 90 \cdot 510 - 4(30 \cdot 90)$

$= 35{,}100 \text{ cm}^2$

Use the formula for the volume of a prism.

$V = Bh$ — **Formula for volume of a prism**

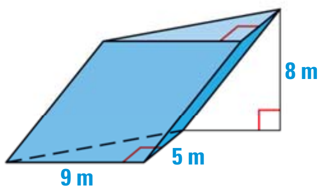
$= 35{,}100(30)$ — **Substitute.**

$= 1{,}053{,}000 \text{ cm}^3$ — **Simplify.**

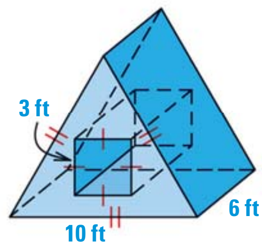
▶ The volume of the sculpture is $1{,}053{,}000 \text{ cm}^3$, or 1.053 m^3.

✓ GUIDED PRACTICE for Examples 4 and 5

4. Find the volume of the oblique prism shown below.



5. Find the volume of the solid shown below.



12.4 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 11, and 29
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 3, 21, and 33

SKILL PRACTICE

1. **VOCABULARY** In what type of units is the volume of a solid measured?

2. ★ **WRITING** Two solids have the same surface area. Do they have the same volume? *Explain* your reasoning.

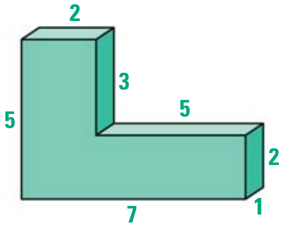
EXAMPLE 1
on p. 819 for Exs. 3–6

3. ★ **MULTIPLE CHOICE** How many 3 inch cubes can fit completely in a box that is 15 inches long, 9 inches wide, and 3 inches tall?

Ⓐ 15 Ⓑ 45 Ⓒ 135 Ⓓ 405

USING UNIT CUBES **Find the volume of the solid by determining how many unit cubes are contained in the solid.**

4.



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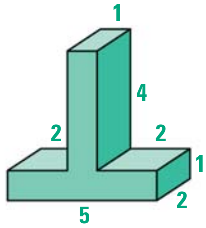

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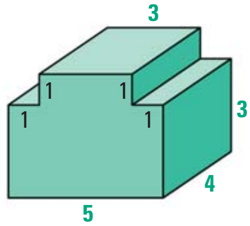
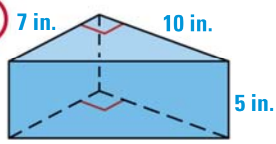


EXAMPLE 2
on p. 820
for Exs. 7–13

FINDING VOLUME **Find the volume of the right prism or right cylinder. Round your answer to two decimal places.**

7.



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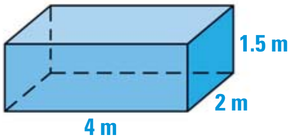

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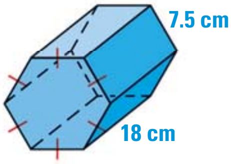

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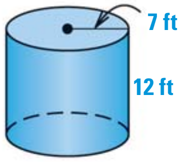

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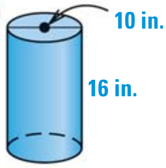

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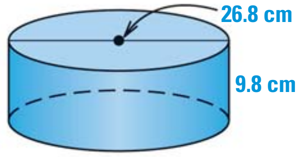


13. **ERROR ANALYSIS** *Describe* and correct the error in finding the volume of a right cylinder with radius 4 feet and height 3 feet.

$$V = 2\pi rh$$
$$= 2\pi(4)(3)$$
$$= 24\pi \text{ ft}^3$$

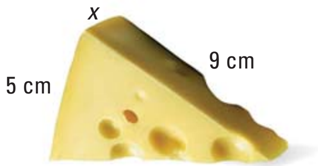
14. **FINDING VOLUME** Sketch a rectangular prism with height 3 feet, width 11 inches, and length 7 feet. Find its volume.

EXAMPLE 3
on p. 820
for Exs. 15–17

xy ALGEBRA **Find the length *x* using the given volume *V*.**

15. $V = 1000 \text{ in.}^3$

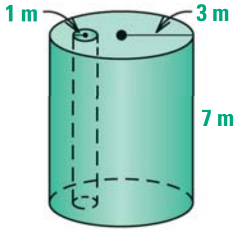
16. $V = 45 \text{ cm}^3$



17. $V = 128\pi \text{ in.}^3$

COMPOSITE SOLIDS **Find the volume of the solid. The prisms and cylinders are right. Round your answer to two decimal places, if necessary.**

18.



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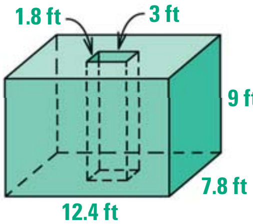

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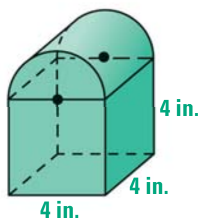


21. ★ **MULTIPLE CHOICE** What is the height of a cylinder with radius 4 feet and volume 64π cubic feet?

(A) 4 feet (B) 8 feet (C) 16 feet (D) 256 feet

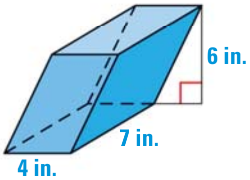
22. **FINDING HEIGHT** The bases of a right prism are right triangles with side lengths of 3 inches, 4 inches, and 5 inches. The volume of the prism is 96 cubic inches. What is the height of the prism?

23. **FINDING DIAMETER** A cylinder has height 8 centimeters and volume 1005.5 cubic centimeters. What is the diameter of the cylinder?

EXAMPLE 4
on p. 821
for Exs. 24–26

VOLUME OF AN OBLIQUE SOLID **Use Cavalieri's Principle to find the volume of the oblique prism or cylinder. Round your answer to two decimal places.**

24.



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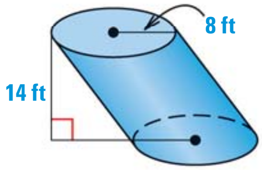

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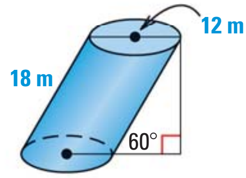

27. **CHALLENGE** The bases of a right prism are rhombuses with diagonals 12 meters and 16 meters long. The height of the prism is 8 meters. Find the lateral area, surface area, and volume of the prism.

PROBLEM SOLVING

EXAMPLE 5
on p. 822
for Exs. 28–30

28. **JEWELRY** The bead at the right is a rectangular prism of length 17 millimeters, width 9 millimeters, and height 5 millimeters. A 3 millimeter wide hole is drilled through the smallest face. Find the volume of the bead.

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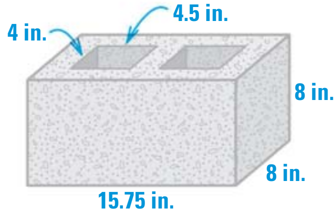
29. **MULTI-STEP PROBLEM** In the concrete block shown, the holes are 8 inches deep.



a. Find the volume of the block using the Volume Addition Postulate.

b. Find the volume of the block using the formula in Theorem 12.6.

c. *Compare* your answers in parts (a) and (b).

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30. **OCEANOGRAPHY** The Blue Hole is a cylindrical trench located on Lighthouse Reef Atoll, an island off the coast of Central America. It is approximately 1000 feet wide and 400 feet deep.

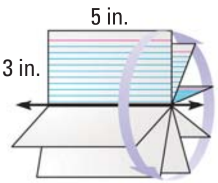
a. Find the volume of the Blue Hole.

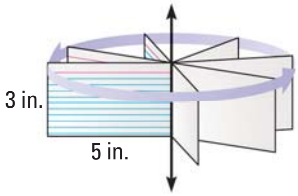
b. About how many gallons of water does the Blue Hole contain? ($1\text{ ft}^3 = 7.48$ gallons)

31. **ARCHITECTURE** A cylindrical column in the building shown has circumference 10 feet and height 20 feet. Find its volume. Round your answer to two decimal places.

AnimatedGeometry at classzone.com

32. **ROTATIONS** A 3 inch by 5 inch index card is rotated around a horizontal line and a vertical line to produce two different solids, as shown. Which solid has a greater volume? *Explain* your reasoning.





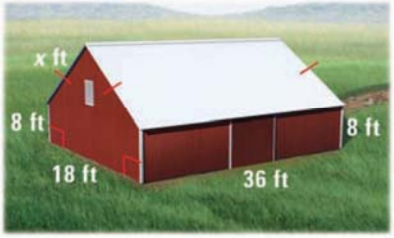
33. ★ **EXTENDED RESPONSE** An aquarium shaped like a rectangular prism has length 30 inches, width 10 inches, and height 20 inches.

   a. **Calculate** You fill the aquarium $\frac{3}{4}$ full with water. What is the volume of the water?

   b. **Interpret** When you submerge a rock in the aquarium, the water level rises 0.25 inch. Find the volume of the rock.

   c. **Interpret** How many rocks of the same size as the rock in part (b) can you place in the aquarium before water spills out?

34. **CHALLENGE** A barn is in the shape of a pentagonal prism with the dimensions shown. The volume of the barn is 9072 cubic feet. Find the dimensions of each half of the roof.

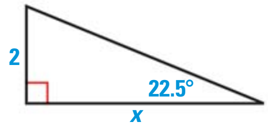


MIXED REVIEW

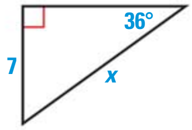
PREVIEW
Prepare for Lesson 12.5 in Exs. 35–40.

Find the value of *x*. Round your answer to two decimal places. *(pp. 466, 473)*

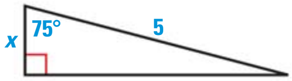
35.



36.



37.



Find the area of the figure described. Round your answer to two decimal places. *(pp. 755, 762)*

38. A circle with radius 9.5 inches

39. An equilateral triangle with perimeter 78 meters and apothem 7.5 meters

40. A regular pentagon with radius 10.6 inches

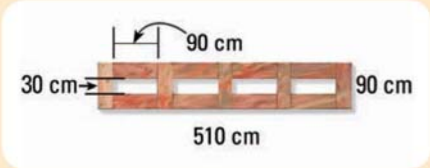
Using ALTERNATIVE METHODS

Another Way to Solve Example 5, page 822

MULTIPLE REPRESENTATIONS In Lesson 12.4, you used volume postulates and theorems to find volumes of prisms and cylinders. Now, you will learn two different ways to solve Example 5 on page 822.

PROBLEM

SCULPTURE The sculpture is made up of 13 beams. In centimeters, suppose the dimensions of each beam are 30 by 30 by 90. Find its volume.



METHOD 1

Finding Volume by Subtracting Empty Spaces One alternative approach is to compute the volume of the prism formed if the holes in the sculpture were filled. Then, to get the correct volume, you must subtract the volume of the four holes.

STEP 1 **Read** the problem. In centimeters, each beam measures 30 by 30 by 90.

The dimensions of the entire sculpture are 30 by 90 by $(4 \cdot 90 + 5 \cdot 30)$, or 30 by 90 by 510.

The dimensions of each hole are equal to the dimensions of one beam.

STEP 2 **Apply** the Volume Addition Postulate. The volume of the sculpture is equal to the volume of the larger prism minus 4 times the volume of a hole.

$$\begin{aligned}\text{Volume } V \text{ of sculpture} &= \text{Volume of larger prism} - \text{Volume of 4 holes}\\ &= 30 \cdot 90 \cdot 510 - 4(30 \cdot 30 \cdot 90)\\ &= 1{,}377{,}000 - 4 \cdot 81{,}000\\ &= 1{,}377{,}000 - 324{,}000\\ &= 1{,}053{,}000\end{aligned}$$

▶ The volume of the sculpture is 1,053,000 cubic centimeters, or 1.053 cubic meters.

STEP 3 **Check** page 822 to verify your new answer, and confirm that it is the same.

METHOD 2 **Finding Volume of Pieces** Another alternative approach is to use the dimensions of each beam.

STEP 1 **Look** at the sculpture. Notice that the sculpture consists of 13 beams, each with the same dimensions. Therefore, the volume of the sculpture will be 13 times the volume of one beam.

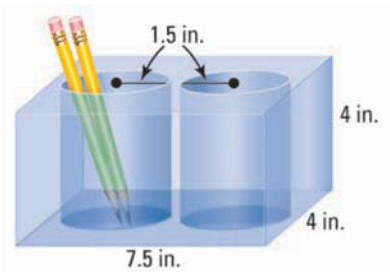
STEP 2 **Write** an expression for the volume of the sculpture and find the volume.

$$\begin{aligned}\text{Volume of sculpture} &= 13(\text{Volume of one beam})\\ &= 13(30 \cdot 30 \cdot 90)\\ &= 13 \cdot 81{,}000\\ &= 1{,}053{,}000\end{aligned}$$

▶ The volume of the sculpture is 1,053,000 cm^3, or 1.053 m^3.

PRACTICE

1. **PENCIL HOLDER** The pencil holder has the dimensions shown.



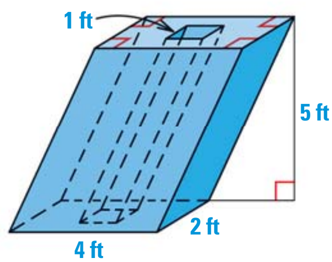
   a. Find its volume using the Volume Addition Postulate.

   b. Use its base area to find its volume.

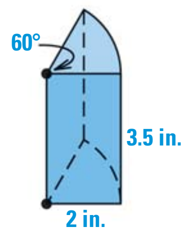
2. **ERROR ANALYSIS** A student solving Exercise 1 claims that the surface area is found by subtracting four times the base area of the cylinders from the surface area of the rectangular prism. *Describe* and correct the student's error.

3. **REASONING** You drill a circular hole of radius r through the base of a cylinder of radius R. Assume the hole is drilled completely through to the other base. You want the volume of the hole to be half the volume of the cylinder. Express r as a function of R.

4. **FINDING VOLUME** Find the volume of the solid shown below. Assume the hole has square cross sections.



5. **FINDING VOLUME** Find the volume of the solid shown to the right.



6. **SURFACE AREA** Refer to the diagram of the sculpture on page 826.

   a. *Describe* a method to find the surface area of the sculpture.

   b. *Explain* why adding the individual surface areas of the beams will give an incorrect result for the total surface area.

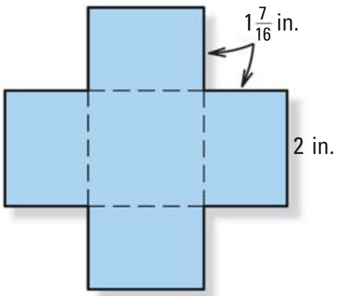
12.5 Investigate the Volume of a Pyramid

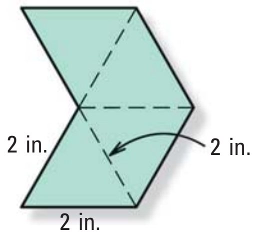
MATERIALS • ruler • poster board • scissors • tape • uncooked rice

QUESTION **How is the volume of a pyramid related to the volume of a prism with the same base and height?**

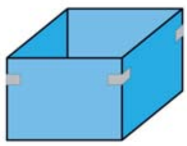
EXPLORE **Compare the volume of a prism and a pyramid using nets**

STEP 1 ***Draw nets*** Use a ruler to draw the two nets shown below on poster board. (Use $1\frac{7}{16}$ inches to approximate $\sqrt{2}$ inches.)





STEP 2 ***Create an open prism and an open pyramid*** Cut out the nets. Fold along the dotted lines to form an open prism and an open pyramid, as shown below. Tape each solid to hold it in place, making sure that the edges do not overlap.



STEP 3 ***Compare volumes*** Fill the pyramid with uncooked rice and pour it into the prism. Repeat this as many times as needed to fill the prism. How many times did you fill the pyramid? What does this tell you about the volume of the solids?

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. *Compare* the area of the base of the pyramid to the area of the base of the prism. Placing the pyramid inside the prism will help. What do you notice?
2. *Compare* the heights of the solids. What do you notice?
3. Make a conjecture about the ratio of the volumes of the solids.
4. Use your conjecture to write a formula for the volume of a pyramid that uses the formula for the volume of a prism.

12.5 Volume of Pyramids and Cones

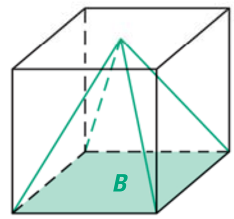
Before You found surface areas of pyramids and cones.

Now You will find volumes of pyramids and cones.

Why? So you can find the edge length of a pyramid, as in Example 2.

Key Vocabulary
- **pyramid,** *p. 810*
- **cone,** *p. 812*
- **volume,** *p. 819*

Recall that the volume of a prism is Bh, where B is the area of a base and h is the height. In the figure at the right, you can see that the volume of a pyramid must be less than the volume of a prism with the same base area and height. As suggested by the Activity on page 828, the volume of a pyramid is one third the volume of a prism.



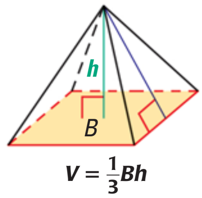
THEOREMS *For Your Notebook*

THEOREM 12.9 Volume of a Pyramid

The volume V of a pyramid is

$$V = \frac{1}{3}Bh,$$

where B is the area of the base and h is the height.

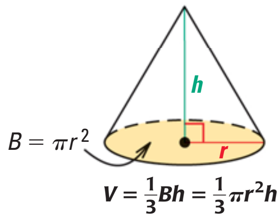


THEOREM 12.10 Volume of a Cone

The volume V of a cone is

$$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h,$$

where B is the area of the base, h is the height, and r is the radius of the base.

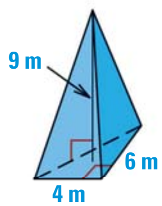


EXAMPLE 1 Find the volume of a solid

APPLY FORMULAS
The formulas given in Theorems 12.9 and 12.10 apply to right and oblique pyramids and cones. This follows from Cavalieri's Principle, stated on page 821.

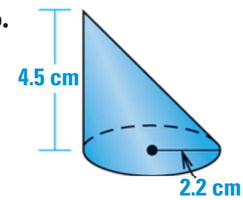
Find the volume of the solid.

a.



$$V = \frac{1}{3}Bh$$
$$= \frac{1}{3}\left(\frac{1}{2} \cdot 4 \cdot 6\right)(9)$$
$$= 36 \text{ m}^3$$

b.



$$V = \frac{1}{3}Bh$$
$$= \frac{1}{3}(\pi r^2)h$$
$$= \frac{1}{3}(\pi \cdot 2.2^2)(4.5)$$
$$= 7.26\pi$$
$$\approx 22.81 \text{ cm}^3$$

EXAMPLE 2 Use volume of a pyramid

xy **ALGEBRA** **Originally, the pyramid had height 144 meters and volume 2,226,450 cubic meters. Find the side length of the square base.**

Khafre's Pyramid, Egypt

Solution

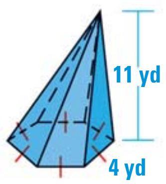
$V = \frac{1}{3}Bh$	**Write formula.**
$2{,}226{,}450 = \frac{1}{3}(x^2)(144)$	**Substitute.**
$6{,}679{,}350 = 144x^2$	**Multiply each side by 3.**
$46{,}384 \approx x^2$	**Divide each side by 144.**
$215 \approx x$	**Find the positive square root.**

▶ Originally, the side length of the base was about 215 meters.

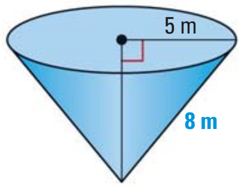
✓ GUIDED PRACTICE for Examples 1 and 2

Find the volume of the solid. Round your answer to two decimal places, if necessary.

1. Hexagonal pyramid



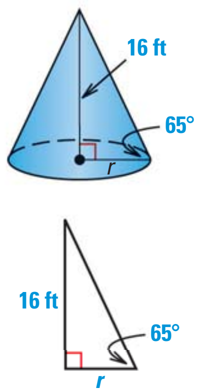
2. Right cone



3. The volume of a right cone is 1350π cubic meters and the radius is 18 meters. Find the height of the cone.

EXAMPLE 3 Use trigonometry to find the volume of a cone

Find the volume of the right cone.



Solution

To find the radius r of the base, use trigonometry.

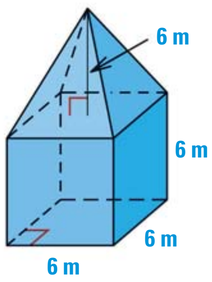
$\tan 65° = \frac{\text{opp.}}{\text{adj.}}$	**Write ratio.**
$\tan 65° = \frac{16}{r}$	**Substitute.**
$r = \frac{16}{\tan 65°} \approx 7.46$	**Solve for *r*.**

Use the formula for the volume of a cone.

$$V = \frac{1}{3}(\pi r^2)h \approx \frac{1}{3}\pi(7.46^2)(16) \approx 932.45 \text{ ft}^3$$

EXAMPLE 4 Find volume of a composite solid

Find the volume of the solid shown.



Solution

Volume of solid	=	Volume of cube	+	Volume of pyramid

$= s^3 + \frac{1}{3}Bh$ **Write formulas.**

$= 6^3 + \frac{1}{3}(6)^2 \cdot 6$ **Substitute.**

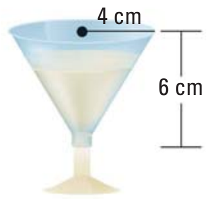
$= 216 + 72$ **Simplify.**

$= 288$ **Add.**

▶ The volume of the solid is 288 cubic meters.

EXAMPLE 5 Solve a multi-step problem

SCIENCE You are using the funnel shown to measure the coarseness of a particular type of sand. It takes 2.8 seconds for the sand to empty out of the funnel. Find the flow rate of the sand in milliliters per second. (1 mL = 1 cm^3)



Solution

STEP 1 **Find** the volume of the funnel using the formula for the volume of a cone.

$$V = \frac{1}{3}(\pi r^2)h = \frac{1}{3}\pi(4^2)(6) \approx 101 \text{ cm}^3 = 101 \text{ mL}$$

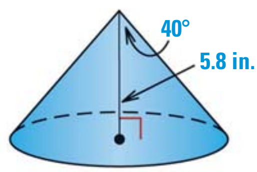
STEP 2 **Divide** the volume of the funnel by the time it takes the sand to empty out of the funnel.

$$\frac{101 \text{ mL}}{2.8 \text{ s}} \approx 36.07 \text{ mL/s}$$

▶ The flow rate of the sand is about 36.07 milliliters per second.

✓ GUIDED PRACTICE for Examples 3, 4, and 5

4. Find the volume of the cone at the right. Round your answer to two decimal places.



5. A right cylinder with radius 3 centimeters and height 10 centimeters has a right cone on top of it with the same base and height 5 centimeters. Find the volume of the solid. Round your answer to two decimal places.

6. **WHAT IF?** In Example 5, suppose a different type of sand is used that takes 3.2 seconds to empty out of the funnel. Find its flow rate.

12.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 17, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 11, 18, and 35

◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

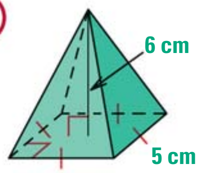
SKILL PRACTICE

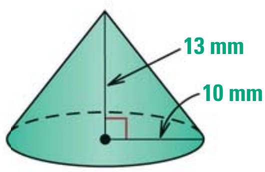
1. **VOCABULARY** *Explain* the difference between a *triangular prism* and a *triangular pyramid*. Draw an example of each.

2. ★ **WRITING** *Compare* the volume of a square pyramid to the volume of a square prism with the same base and height as the pyramid.

EXAMPLE 1 on p. 829 for Exs. 3–11

VOLUME OF A SOLID **Find the volume of the solid. Round your answer to two decimal places.**

3.


4.


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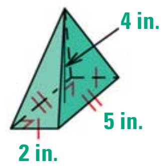
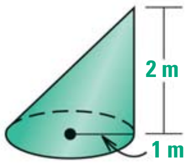
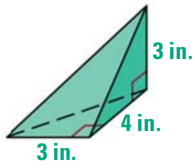
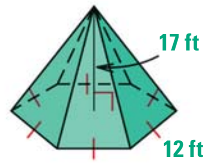


6.


7.


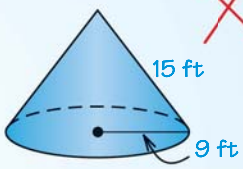
8.


ERROR ANALYSIS *Describe* **and correct the error in finding the volume of the right cone or pyramid.**

9.
$V = \frac{1}{3}\pi(9^2)(15)$

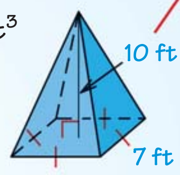
$= 405\pi$

$\approx 1272 \text{ ft}^3$



10.
$V = \frac{1}{2}(49)(10)$

$= 245 \text{ ft}^3$

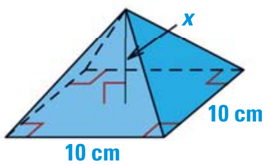


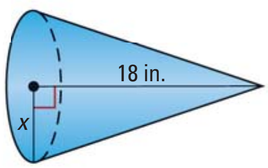
11. ★ **MULTIPLE CHOICE** The volume of a pyramid is 45 cubic feet and the height is 9 feet. What is the area of the base?

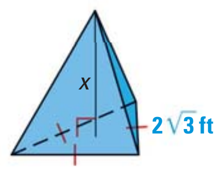
Ⓐ 3.87 ft^2 Ⓑ 5 ft^2 Ⓒ 10 ft^2 Ⓓ 15 ft^2

EXAMPLE 2 on p. 830 for Exs. 12–14

ALGEBRA **Find the value of *x*.**

12. Volume = 200 cm^3


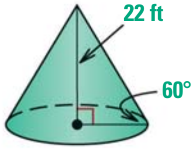
13. Volume = $216\pi \text{ in.}^3$


14. Volume = $7\sqrt{3} \text{ ft}^3$


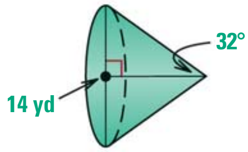
EXAMPLE 3
on p. 830
for Exs. 15–19

VOLUME OF A CONE **Find the volume of the right cone. Round your answer to two decimal places.**

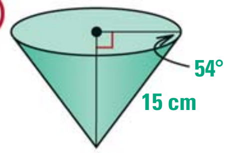
15.



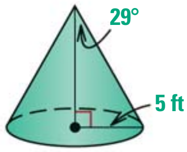
16.



17.



18. ★ **MULTIPLE CHOICE** What is the approximate volume of the cone?



Ⓐ 47.23 ft^3 Ⓑ 236.14 ft^3

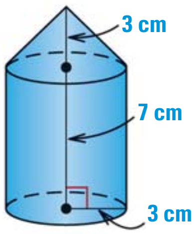
Ⓒ 269.92 ft^3 Ⓓ 354.21 ft^3

19. **HEIGHT OF A CONE** A cone with a diameter of 8 centimeters has volume 143.6 cubic centimeters. Find the height of the cone. Round your answer to two decimal places.

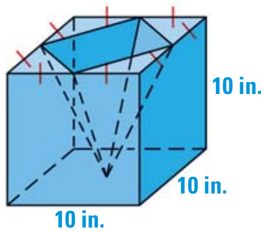
EXAMPLE 4
on p. 831
for Exs. 20–25

COMPOSITE SOLIDS **Find the volume of the solid. The prisms, pyramids, and cones are right. Round your answer to two decimal places.**

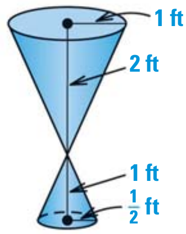
20.



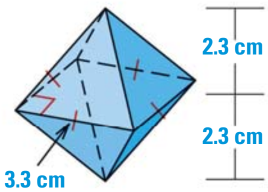
21.



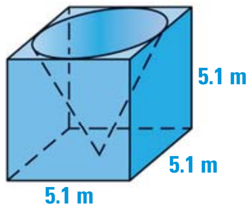
22.



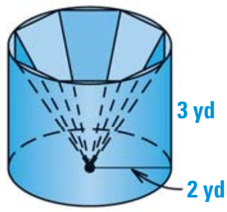
23.



24.



25.



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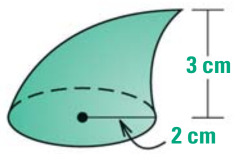
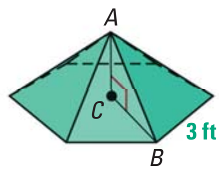
26. **FINDING VOLUME** The figure at the right is a cone that has been warped but whose cross sections still have the same area as a right cone with equal base area and height. Find the volume of this solid.



27. **FINDING VOLUME** Sketch a regular square pyramid with base edge length 5 meters inscribed in a cone with height 7 meters. Find the volume of the cone. *Explain* your reasoning.

28. **CHALLENGE** Find the volume of the regular hexagonal pyramid. Round your answer to the nearest hundredth of a cubic foot. In the diagram, $m\angle ABC = 35°$.



EXAMPLE 5 on p. 831 for Ex. 30

29. CAKE DECORATION A pastry bag filled with frosting has height 12 inches and radius 4 inches. A cake decorator can make 15 flowers using one bag of frosting.

a. How much frosting is in the pastry bag? Round your answer to the nearest cubic inch.

b. How many cubic inches of frosting are used to make each flower?

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POPCORN A snack stand serves a small order of popcorn in a cone-shaped cup and a large order of popcorn in a cylindrical cup.

30. Find the volume of the small cup.

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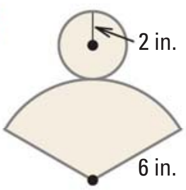
31. How many small cups of popcorn do you have to buy to equal the amount of popcorn in a large container? Do not perform any calculations. *Explain.*

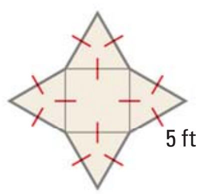
32. Which container gives you more popcorn for your money? *Explain.*

USING NETS In Exercises 33 and 34, use the net to sketch the solid. Then find the volume of the solid. Round your answer to two decimal places.

33.



34.



35. ★ **EXTENDED RESPONSE** A pyramid has height 10 feet and a square base with side length 7 feet.

a. How does the volume of the pyramid change if the base stays the same and the height is doubled?

b. How does the volume of the pyramid change if the height stays the same and the side length of the base is doubled?

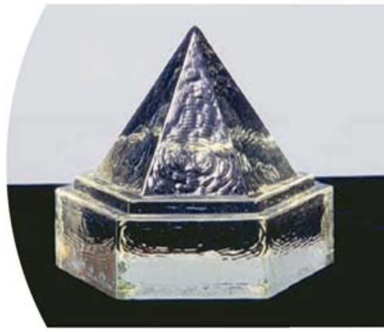
c. *Explain* why your answers to parts (a) and (b) are true for any height and side length.

36. AUTOMATIC FEEDER Assume the automatic pet feeder is a right cylinder on top of a right cone of the same radius. (1 cup = 14.4 in.^{3})

a. Calculate the amount of food in cups that can be placed in the feeder.

b. A cat eats one third of a cup of food, twice per day. How many days will the feeder have food without refilling it?

37. **NAUTICAL PRISMS** The nautical deck prism shown is composed of the following three solids: a regular hexagonal prism with edge length 3.5 inches and height 1.5 inches, a regular hexagonal prism with edge length 3.25 inches and height 0.25 inch, and a regular hexagonal pyramid with edge length 3 inches and height 3 inches. Find the volume of the deck prism.



38. **MULTI-STEP PROBLEM** Calculus can be used to show that the average value of r^2 of a circular cross section of a cone is $\frac{r_b^2}{3}$, where r_b is the radius of the base.

a. Find the average area of a circular cross section of a cone whose base has radius R.

b. Show that the volume of the cone can be expressed as follows:

V_{cone} = (Average area of a circular cross section) • (Height of cone)

39. **MULTIPLE REPRESENTATIONS** Water flows into a reservoir shaped like a right cone at the rate of 1.8 cubic meters per minute. The height and diameter of the reservoir are equal.

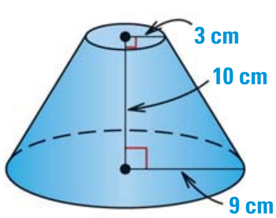
a. **Using Algebra** As the water flows into the reservoir, the relationship $h = 2r$ is always true. Using this fact, show that $V = \frac{\pi h^3}{12}$.

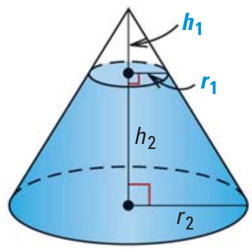
b. **Making a Table** Make a table that gives the height h of the water after 1, 2, 3, 4, and 5 minutes.

c. **Drawing a Graph** Make a graph of height versus time. Is there a linear relationship between the height of the water and time? *Explain.*

FRUSTUM **A frustum of a cone is the part of the cone that lies between the base and a plane parallel to the base, as shown. Use the information to complete Exercises 40 and 41.**

One method for calculating the volume of a frustum is to add the areas of the two bases to their geometric mean, then multiply the result by $\frac{1}{3}$ the height.





40. Use the measurements in the diagram at the left above to calculate the volume of the frustum.

41. Complete parts (a) and (b) below to write a formula for the volume of a frustum that has bases with radii r_1 and r_2 and a height h_2.

a. Use similar triangles to find the value of h_1 in terms of h_2, r_1, and r_2.

b. Write a formula in terms of h_2, r_1, and r_2 for V_{frustum} = (Original volume) − (Removed volume).

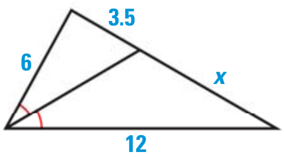
c. Show that your formula in part (b) is equivalent to the formula involving geometric mean described above.

42. **CHALLENGE** A square pyramid is inscribed in a right cylinder so that the base of the pyramid is on a base of the cylinder, and the vertex of the pyramid is on the other base of the cylinder. The cylinder has radius 6 feet and height 12 feet. Find the volume of the pyramid. Round your answer to two decimal places.

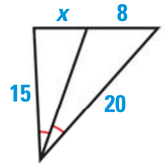
MIXED REVIEW

In Exercises 43–45, find the value of *x*. *(p. 397)*

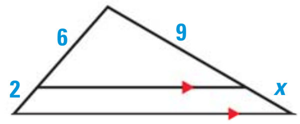
43.



44.



45.

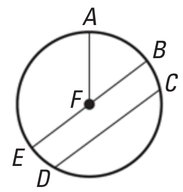


PREVIEW
Prepare for Lesson 12.6 in Exs. 46–52.

46. Copy the diagram at the right. Name a radius, diameter, and chord. *(p. 651)*

47. Name a minor arc of ⊙*F*. *(p. 659)*

48. Name a major arc of ⊙*F*. *(p. 659)*



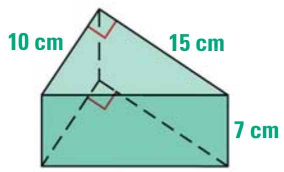
Find the area of the circle with the given radius *r*, diameter *d*, or circumference *C*. *(p. 755)*

49. $r = 3$ m **50.** $d = 7$ mi **51.** $r = 0.4$ cm **52.** $C = 8\pi$ in.

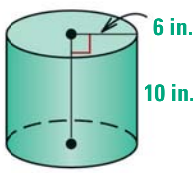
QUIZ *for Lessons 12.4–12.5*

Find the volume of the figure. Round your answer to two decimal places, if necessary. *(pp. 819, 829)*

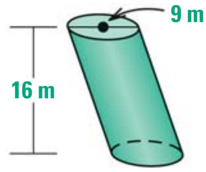
1.



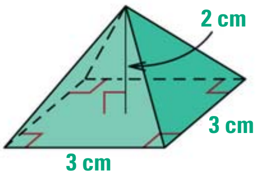
2.



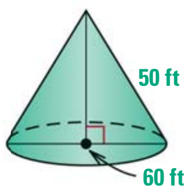
3.



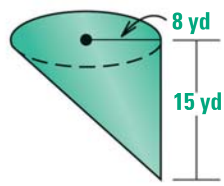
4.



5.



6.



7. Suppose you fill up a cone-shaped cup with water. You then pour the water into a cylindrical cup with the same radius. Both cups have a height of 6 inches. Without doing any calculation, determine how high the water level will be in the cylindrical cup once all of the water is poured into it. *Explain* your reasoning. *(p. 829)*

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12.5 Minimize Surface Area

MATERIALS • computer

QUESTION How can you find the minimum surface area of a solid with a given volume?

A manufacturer needs a cylindrical container with a volume of 72 cubic centimeters. You have been asked to find the dimensions of such a container so that it has a minimum surface area.

EXAMPLE Use a spreadsheet

STEP 1 ***Make a table*** Make a table with the four column headings shown in Step 4. The first column is for the given volume V. In cell A2, enter 72. In cell A3, enter the formula "=A2".

STEP 2 ***Enter radius*** The second column is for the radius r. Cell B2 stores the starting value for r. So, enter 2 into cell B2. In cell B3, use the formula "=B2 + 0.05" to increase r in increments of 0.05 centimeter.

STEP 3 ***Enter formula for height*** The third column is for the height. In cell C2, enter the formula "=A2/(PI()*B2^2)". *Note:* Your spreadsheet might use a different expression for π.

STEP 4 ***Enter formula for surface area*** The fourth column is for the surface area. In cell D2, enter the formula "=2*PI()*B2^2+2*PI()*B2*C2".

	A	B	C	D
1	Volume V	Radius r	Height$=V/(\pi r^2)$	Surface area $S=2\pi r^2+2\pi r$
2	72.00	2.00	=A2/(PI()*B2^2)	=2*PI()*B2^2+2*PI()*B2*C2
3	=A2	=B2+0.05		

STEP 5 ***Create more rows*** Use the *Fill Down* feature to create more rows. Rows 3 and 4 of your spreadsheet should resemble the one below.

	A	B	C	D
...				
3	72.00	2.05	5.45	96.65
4	72.00	2.10	5.20	96.28

PRACTICE

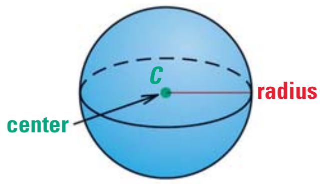
1. From the data in your spreadsheet, which dimensions yield a minimum surface area for the given volume? *Explain* how you know.
2. **WHAT IF?** Find the dimensions that give the minimum surface area if the volume of a cylinder is instead 200π cubic centimeters.

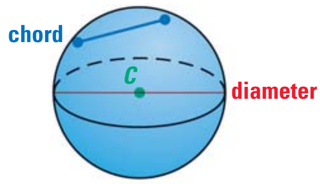
12.6 Surface Area and Volume of Spheres

Before You found surface areas and volumes of polyhedra.
Now You will find surface areas and volumes of spheres.
Why? So you can find the volume of a tennis ball, as in Ex. 33.

Key Vocabulary
- **sphere**
  center, radius, chord, diameter
- **great circle**
- **hemispheres**

A **sphere** is the set of all points in space equidistant from a given point. This point is called the **center** of the sphere. A **radius** of a sphere is a segment from the center to a point on the sphere. A **chord** of a sphere is a segment whose endpoints are on the sphere. A **diameter** of a sphere is a chord that contains the center.





As with circles, the terms radius and diameter also represent distances, and the diameter is twice the radius.

THEOREM *For Your Notebook*

THEOREM 12.11 Surface Area of a Sphere

The surface area S of a sphere is

$S = 4\pi r^2$,

where r is the radius of the sphere.

$S = 4\pi r^2$

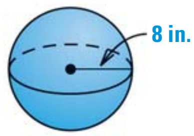
USE FORMULAS
If you understand how a formula is derived, then it will be easier for you to remember the formula.

SURFACE AREA FORMULA To understand how the formula for the surface area of a sphere is derived, think of a baseball. The surface area of a baseball is sewn from two congruent shapes, each of which resembles two joined circles, as shown.

So, the entire covering of the baseball consists of four circles, each with radius r. The area A of a circle with radius r is $A = \pi r^2$. So, the area of the covering can be approximated by $4\pi r^2$. This is the formula for the surface area of a sphere.

EXAMPLE 1 Find the surface area of a sphere

Find the surface area of the sphere.



Solution

$S = 4\pi r^2$ **Formula for surface area of a sphere**

$= 4\pi(8^2)$ **Substitute 8 for *r*.**

$= 256\pi$ **Simplify.**

≈ 804.25 **Use a calculator.**

▶ The surface area of the sphere is about 804.25 square inches.

★ EXAMPLE 2 Standardized Test Practice

The surface area of the sphere is 20.25π square centimeters. What is the diameter of the sphere?

(A) 2.25 cm (B) 4.5 cm

(C) 5.5 cm (D) 20.25 cm

$S = 20.25\pi$ cm²

Solution

$S = 4\pi r^2$ **Formula for surface area of a sphere**

$20.25\pi = 4\pi r^2$ **Substitute 20.25π for *S*.**

$5.0625 = r^2$ **Divide each side by 4π.**

$2.25 = r$ **Find the positive square root.**

The diameter of the sphere is $2r = 2 \cdot 2.25 = 4.5$ centimeters.

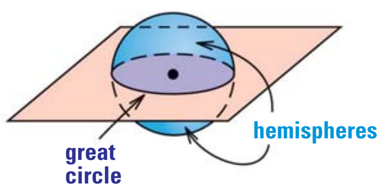
▶ The correct answer is B. (A) (B) (C) (D)

AVOID ERRORS
Be sure to multiply the value of *r* by 2 to find the diameter.

✓ GUIDED PRACTICE for Examples 1 and 2

1. The diameter of a sphere is 40 feet. Find the surface area of the sphere.
2. The surface area of a sphere is 30π square meters. Find the radius of the sphere.

GREAT CIRCLES If a plane intersects a sphere, the intersection is either a single point or a circle. If the plane contains the center of the sphere, then the intersection is a **great circle** of the sphere. The circumference of a great circle is the circumference of the sphere. Every great circle of a sphere separates the sphere into two congruent halves called **hemispheres**.



EXAMPLE 3 Use the circumference of a sphere

EXTREME SPORTS In a sport called *sphereing*, a person rolls down a hill inside an inflatable ball surrounded by another ball. The diameter of the outer ball is 12 feet. Find the surface area of the outer ball.

Solution

The diameter of the outer sphere is 12 feet, so the radius is $\frac{12}{2} = 6$ feet.

Use the formula for the surface area of a sphere.

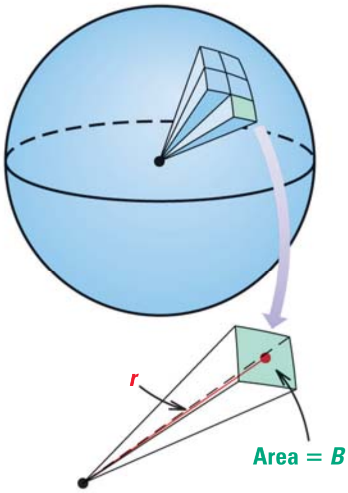
$$S = 4\pi r^2 = 4\pi(6^2) = 144\pi$$

▶ The surface area of the outer ball is 144π, or about 452.39 square feet.

✓ GUIDED PRACTICE for Example 3

3. In Example 3, the circumference of the inner ball is 6π feet. Find the surface area of the inner ball. Round your answer to two decimal places.

VOLUME FORMULA Imagine that the interior of a sphere with radius r is approximated by n pyramids, each with a base area of B and a height of r. The volume of each pyramid is $\frac{1}{3}Br$ and the sum of the base areas is nB. The surface area of the sphere is approximately equal to nB, or $4\pi r^2$. So, you can approximate the volume V of the sphere as follows.



$V \approx n\left(\frac{1}{3}Br\right)$	Each pyramid has a volume of $\frac{1}{3}Br$.
$\approx \frac{1}{3}(nB)r$	Regroup factors.
$= \frac{1}{3}(4\pi r^2)r$	Substitute $4\pi r^2$ for nB.
$= \frac{4}{3}\pi r^3$	Simplify.

THEOREM *For Your Notebook*

THEOREM 12.12 Volume of a Sphere

The volume V of a sphere is

$$V = \frac{4}{3}\pi r^3,$$

where r is the radius of the sphere.

EXAMPLE 4 Find the volume of a sphere

The soccer ball has a diameter of 9 inches. Find its volume.

Solution

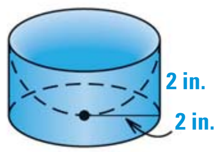
The diameter of the ball is 9 inches, so the radius is $\frac{9}{2} = 4.5$ inches.

$V = \frac{4}{3}\pi r^3$	Formula for volume of a sphere
$= \frac{4}{3}\pi(4.5)^3$	Substitute.
$= 121.5\pi$	Simplify.
≈ 381.70	Use a calculator.

▶ The volume of the soccer ball is 121.5π, or about 381.70 cubic inches.

EXAMPLE 5 Find the volume of a composite solid

Find the volume of the composite solid.



Solution

Volume of solid = Volume of cylinder − Volume of hemisphere

$= \pi r^2 h - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$	Formulas for volume
$= \pi(2)^2(2) - \frac{2}{3}\pi(2)^3$	Substitute.
$= 8\pi - \frac{2}{3}(8\pi)$	Multiply.
$= \frac{24}{3}\pi - \frac{16}{3}\pi$	Rewrite fractions using least common denominator.
$= \frac{8}{3}\pi$	Simplify.

▶ The volume of the solid is $\frac{8}{3}\pi$, or about 8.38 cubic inches.

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✓ GUIDED PRACTICE for Examples 4 and 5

4. The radius of a sphere is 5 yards. Find the volume of the sphere. Round your answer to two decimal places.

5. A solid consists of a hemisphere of radius 1 meter on top of a cone with the same radius and height 5 meters. Find the volume of the solid. Round your answer to two decimal places.

12.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 13, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 20, 28, 33, and 34

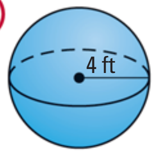
SKILL PRACTICE

1. **VOCABULARY** What are the formulas for finding the surface area of a sphere and the volume of a sphere?

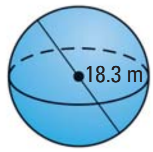
2. ★ **WRITING** When a plane intersects a sphere, what point in the sphere must the plane contain for the intersection to be a great circle? *Explain.*

EXAMPLE 1 on p. 839 for Exs. 3–5

FINDING SURFACE AREA **Find the surface area of the sphere. Round your answer to two decimal places.**

3. 

4. 

5. 

EXAMPLE 2 on p. 839 for Ex. 6

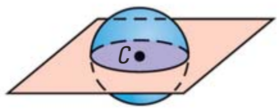
6. ★ **MULTIPLE CHOICE** What is the approximate radius of a sphere with surface area 32π square meters?

Ⓐ 2 meters Ⓑ 2.83 meters Ⓒ 4.90 meters Ⓓ 8 meters

EXAMPLE 3 on p. 840 for Exs. 7–11

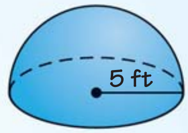
USING A GREAT CIRCLE **In Exercises 7–9, use the sphere below. The center of the sphere is C and its circumference is 9.6π inches.**

7. Find the radius of the sphere.
8. Find the diameter of the sphere.
9. Find the surface area of one hemisphere.



10. **ERROR ANALYSIS** *Describe* and correct the error in finding the surface area of a hemisphere with radius 5 feet.

$$S = 4\pi r^2$$
$$= 4\pi(5)^2$$
$$= 100\pi$$
$$\approx 314.16 \text{ ft}^2$$



11. **GREAT CIRCLE** The circumference of a great circle of a sphere is 48.4π centimeters. What is the surface area of the sphere?

EXAMPLE 4 on p. 841 for Exs. 12–15

FINDING VOLUME **Find the volume of the sphere using the given radius r or diameter d. Round your answer to two decimal places.**

12. $r = 6$ in.

13. $r = 40$ mm

14. $d = 5$ cm

15. **ERROR ANALYSIS** *Describe* and correct the error in finding the volume of a sphere with diameter 16 feet.

$$V = \frac{4}{3}\pi r^2$$
$$= \frac{4}{3}\pi(8)^2$$
$$= 85.33\pi \approx 268.08 \text{ ft}^2$$

USING VOLUME In Exercises 16–18, find the radius of a sphere with the given volume *V*. Round your answers to two decimal places.

16. $V = 1436.76 \text{ m}^3$

17. $V = 91.95 \text{ cm}^3$

18. $V = 20{,}814.37 \text{ in.}^3$

19. **FINDING A DIAMETER** The volume of a sphere is 36π cubic feet. What is the diameter of the sphere?

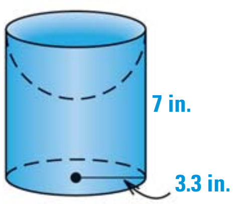
20. ★ **MULTIPLE CHOICE** Let V be the volume of a sphere, S be the surface area of the sphere, and r be the radius of the sphere. Which equation represents the relationship between these three measures?

(A) $V = \frac{rS}{3}$ (B) $V = \frac{r^2S}{3}$ (C) $V = \frac{3}{2}rS$ (D) $V = \frac{3}{2}r^2S$

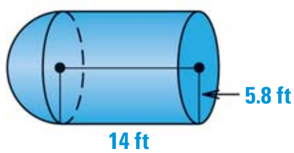
EXAMPLE 5
on p. 841
for Exs. 21–23

COMPOSITE SOLIDS Find the surface area and the volume of the solid. The cylinders and cones are right. Round your answer to two decimal places.

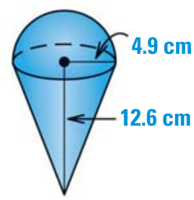
21.



22.



23.



USING A TABLE Copy and complete the table below. Leave your answers in terms of π.

	Radius of sphere	Circumference of great circle	Surface area of sphere	Volume of sphere
24.	10 ft	?	?	?
25.	?	26π in.	?	?
26.	?	?	$2500\pi \text{ cm}^2$	?
27.	?	?	?	$12{,}348\pi \text{ m}^3$

28. ★ **MULTIPLE CHOICE** A sphere is inscribed in a cube of volume 64 cubic centimeters. What is the surface area of the sphere?

(A) $4\pi \text{ cm}^2$ (B) $\frac{32}{3}\pi \text{ cm}^2$ (C) $16\pi \text{ cm}^2$ (D) $64\pi \text{ cm}^2$

29. **CHALLENGE** The volume of a right cylinder is the same as the volume of a sphere. The radius of the sphere is 1 inch.

a. Give three possibilities for the dimensions of the cylinder.

b. Show that the surface area of the cylinder is sometimes greater than the surface area of the sphere.

PROBLEM SOLVING

EXAMPLE 5 on p. 841 for Ex. 30

30. GRAIN SILO A grain silo has the dimensions shown. The top of the silo is a hemispherical shape. Find the volume of the grain silo.

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31. GEOGRAPHY The circumference of Earth is about 24,855 miles. Find the surface area of the Western Hemisphere of Earth.

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32. MULTI-STEP PROBLEM A ball has volume 1427.54 cubic centimeters.

a. Find the radius of the ball. Round your answer to two decimal places.

b. Find the surface area of the ball. Round your answer to two decimal places.

33. ★ SHORT RESPONSE Tennis balls are stored in a cylindrical container with height 8.625 inches and radius 1.43 inches.

a. The circumference of a tennis ball is 8 inches. Find the volume of a tennis ball.

b. There are 3 tennis balls in the container. Find the amount of space within the cylinder not taken up by the tennis balls.

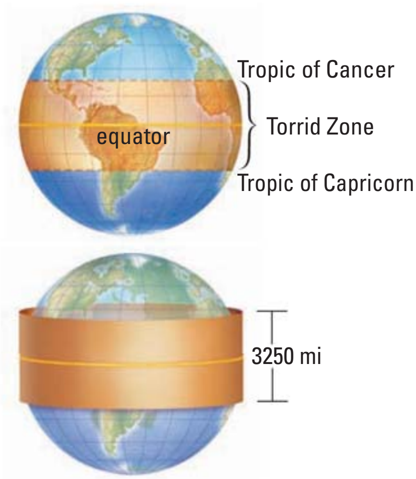
34. ★ EXTENDED RESPONSE A partially filled balloon has circumference 27π centimeters. Assume the balloon is a sphere.

a. Calculate Find the volume of the balloon.

b. Predict Suppose you double the radius by increasing the air in the balloon. *Explain* what you expect to happen to the volume.

c. Justify Find the volume of the balloon with the radius doubled. Was your prediction from part (b) correct? What is the ratio of this volume to the original volume?

35. GEOGRAPHY The Torrid Zone on Earth is the area between the Tropic of Cancer and the Tropic of Capricorn, as shown. The distance between these two tropics is about 3250 miles. You can think of this distance as the height of a cylindrical belt around Earth at the equator, as shown.



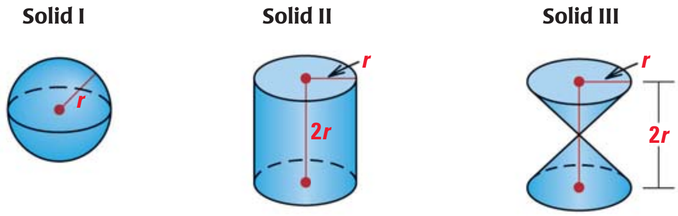
a. Estimate the surface area of the Torrid Zone and the surface area of Earth. (Earth's radius is about 3963 miles at the equator.)

b. A meteorite is equally likely to hit anywhere on Earth. Estimate the probability that a meteorite will land in the Torrid Zone.

○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

36. REASONING List the following three solids in order of (a) surface area, and (b) volume, from least to greatest.



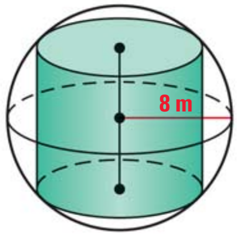
37. ROTATION A circle with diameter 18 inches is rotated about its diameter. Find the surface area and the volume of the solid formed.

38. TECHNOLOGY A cylinder with height $2x$ is inscribed in a sphere with radius 8 meters. The center of the sphere is the midpoint of the altitude that joins the centers of the bases of the cylinder.

a. Show that the volume V of the cylinder is $2\pi x(64 - x^2)$.

b. Use a graphing calculator to graph $V = 2\pi x(64 - x^2)$ for values of x between 0 and 8. Find the value of x that gives the maximum value of V.

c. Use the value for x from part (b) to find the maximum volume of the cylinder.



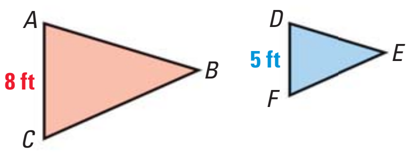
39. CHALLENGE A sphere with radius 2 centimeters is inscribed in a right cone with height 6 centimeters. Find the surface area and the volume of the cone.

MIXED REVIEW

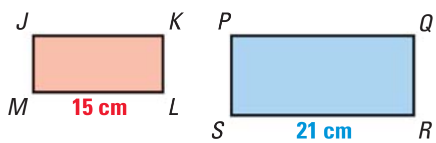
PREVIEW
Prepare for Lesson 12.7 in Exs. 40–41.

In Exercises 40 and 41, the polygons are similar. Find the ratio (red to blue) of their areas. Find the unknown area. Round your answer to two decimal places. *(p. 737)*

40. Area of $\triangle ABC = 42\text{ ft}^2$
Area of $\triangle DEF$ = _?_

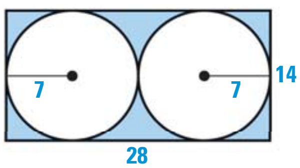


41. Area of $PQRS = 195\text{ cm}^2$
Area of $JKLM$ = _?_

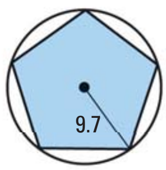


Find the probability that a randomly chosen point in the figure lies in the shaded region. *(p. 771)*

42.



43.



44. A cone is inscribed in a right cylinder with volume 330 cubic units. Find the volume of the cone. *(pp. 819, 829)*

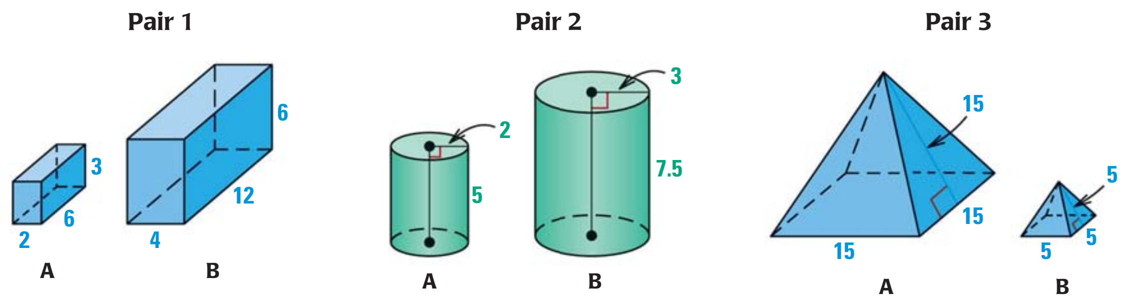
12.7 Investigate Similar Solids

MATERIALS • paper • pencil

QUESTION How are the surface areas and volumes of similar solids related?

EXPLORE Compare the surface areas and volumes of similar solids

The solids shown below are *similar*.



STEP 1 ***Make a table*** Copy and complete the table below.

	Scale factor of Solid A to Solid B	Surface area of Solid A, S_A	Surface area of Solid B, S_B	$\frac{S_A}{S_B}$
Pair 1	$\frac{1}{2}$	?	?	?
Pair 2	?	?	63π	?
Pair 3	?	?	?	$\frac{9}{1}$

STEP 2 ***Insert columns*** Insert columns for V_A, V_B, and $\frac{V_A}{V_B}$. Use the dimensions of the solids to find V_A, the volume of Solid A, and V_B, the volume of Solid B. Then find the ratio of these volumes.

STEP 3 ***Compare ratios*** Compare the ratios $\frac{S_A}{S_B}$ and $\frac{V_A}{V_B}$ to the scale factor.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Make a conjecture about how the surface areas and volumes of similar solids are related to the scale factor.
2. Use your conjecture to write a ratio of surface areas and volumes if the dimensions of two similar rectangular prisms are ℓ, w, h, and $k\ell$, kw, kh.

12.7 Explore Similar Solids

Before	You used properties of similar polygons.
Now	You will use properties of similar solids.
Why	So you can determine a ratio of volumes, as in Ex. 26.

Key Vocabulary
- **similar solids**

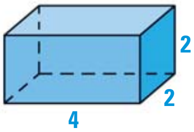
Two solids of the same type with equal ratios of corresponding linear measures, such as heights or radii, are called **similar solids**. The common ratio is called the *scale factor* of one solid to the other solid. Any two cubes are similar, as well as any two spheres.

Similar cylinders   Nonsimilar cylinders

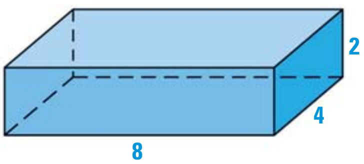
The green cylinders shown above are not similar. Their heights are equal, so they have a 1 : 1 ratio. The radii are different, however, so there is no common ratio.

EXAMPLE 1 Identify similar solids

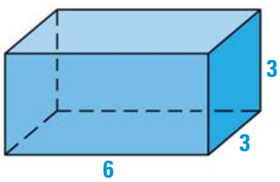
Tell whether the given right rectangular prism is similar to the right rectangular prism shown at the right.



a.



b.



Solution

a. Lengths $\frac{4}{8} = \frac{1}{2}$ Widths $\frac{2}{4} = \frac{1}{2}$ Heights $\frac{2}{2} = \frac{1}{1}$

▶ The prisms are not similar because the ratios of corresponding linear measures are not all equal.

b. Lengths $\frac{4}{6} = \frac{2}{3}$ Widths $\frac{2}{3}$ Heights $\frac{2}{3}$

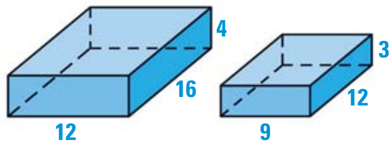
▶ The prisms are similar because the ratios of corresponding linear measures are all equal. The scale factor is 2 : 3.

COMPARE RATIOS
To compare the ratios of corresponding side lengths, write the ratios as fractions in simplest form.

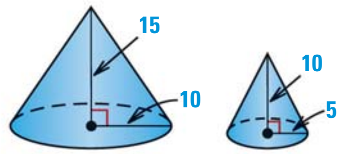
GUIDED PRACTICE for Example 1

Tell whether the pair of right solids is similar. *Explain* your reasoning.

1.



2.



SIMILAR SOLIDS THEOREM The surface areas S and volumes V of the similar solids in Example 1, part (b), are as follows.

Prism	Dimensions	Surface area, $S = 2B + Ph$	Volume, $V = Bh$
Smaller	4 by 2 by 2	$S = 2(8) + 12(2) = 40$	$V = 8(2) = 16$
Larger	6 by 3 by 3	$S = 2(18) + 18(3) = 90$	$V = 18(3) = 54$

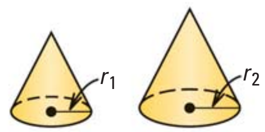
The ratio of side lengths is $2:3$. Notice that the ratio of surface areas is $40:90$, or $4:9$, which can be written as $2^2:3^2$, and the ratio of volumes is $16:54$, or $8:27$, which can be written as $2^3:3^3$. This leads to the following theorem.

READ VOCABULARY
In Theorem 12.13, areas can refer to any pair of corresponding areas in the similar solids, such as lateral areas, base areas, and surface areas.

THEOREM *For Your Notebook*

THEOREM 12.13 Similar Solids Theorem

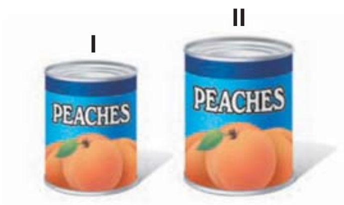
If two similar solids have a scale factor of $a:b$, then corresponding areas have a ratio of $a^2:b^2$, and corresponding volumes have a ratio of $a^3:b^3$.



$\frac{r_1}{r_2} = \frac{a}{b}$, $\frac{S_1}{S_2} = \frac{a^2}{b^2}$, $\frac{V_1}{V_2} = \frac{a^3}{b^3}$

EXAMPLE 2 Use the scale factor of similar solids

PACKAGING The cans shown are similar with a scale factor of $87:100$. Find the surface area and volume of the larger can.



$S = 51.84$ in.^{2}
$V = 28.27$ in.^{3}

Solution

Use Theorem 12.13 to write and solve two proportions.

$$\frac{\text{Surface area of I}}{\text{Surface area of II}} = \frac{a^2}{b^2} \qquad \frac{\text{Volume of I}}{\text{Volume of II}} = \frac{a^3}{b^3}$$

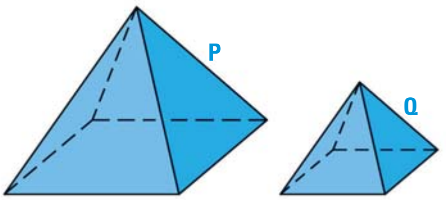
$$\frac{51.84}{\text{Surface area of II}} = \frac{87^2}{100^2} \qquad \frac{28.27}{\text{Volume of II}} = \frac{87^3}{100^3}$$

$$\text{Surface area of II} \approx 68.49 \qquad \text{Volume of II} \approx 42.93$$

▶ The surface area of the larger can is about 68.49 square inches, and the volume of the larger can is about 42.93 cubic inches.

EXAMPLE 3 Find the scale factor

The pyramids are similar. Pyramid P has a volume of 1000 cubic inches and Pyramid Q has a volume of 216 cubic inches. Find the scale factor of Pyramid P to Pyramid Q.



Solution

Use Theorem 12.13 to find the ratio of the two volumes.

$\frac{a^3}{b^3} = \frac{1000}{216}$ **Write ratio of volumes.**

$\frac{a}{b} = \frac{10}{6}$ **Find cube roots.**

$\frac{a}{b} = \frac{5}{3}$ **Simplify.**

▶ The scale factor of Pyramid P to Pyramid Q is 5 : 3.

EXAMPLE 4 Compare similar solids

CONSUMER ECONOMICS A store sells balls of yarn in two different sizes. The diameter of the larger ball is twice the diameter of the smaller ball. If the balls of yarn cost \$7.50 and \$1.50, respectively, which ball of yarn is the better buy?

Solution

STEP 1 **Compute** the ratio of volumes using the diameters.

$$\frac{\text{Volume of large ball}}{\text{Volume of small ball}} = \frac{2^3}{1^3} = \frac{8}{1}, \text{ or } 8:1$$

STEP 2 **Find** the ratio of costs.

$$\frac{\text{Price of large ball}}{\text{Volume of small ball}} = \frac{\$7.50}{\$1.50} = \frac{5}{1}, \text{ or } 5:1$$

STEP 3 **Compare** the ratios in Steps 1 and 2.

If the ratios were the same, neither ball would be a better buy. Comparing the smaller ball to the larger one, the price increase is less than the volume increase. So, you get more yarn for your dollar if you buy the larger ball of yarn.

▶ The larger ball of yarn is the better buy.

✓ GUIDED PRACTICE for Examples 2, 3, and 4

3. Cube C has a surface area of 54 square units and Cube D has a surface area of 150 square units. Find the scale factor of C to D. Find the edge length of C, and use the scale factor to find the volume of D.

4. **WHAT IF?** In Example 4, calculate a new price for the larger ball of yarn so that neither ball would be a better buy than the other.

12.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 9, and 27

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 16, 28, 31, and 33

◆ = **MULTIPLE REPRESENTATIONS** Ex. 34

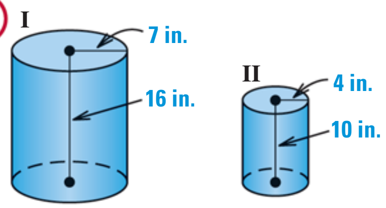
SKILL PRACTICE

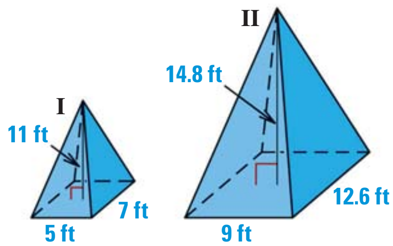
1. **VOCABULARY** What does it mean for two solids to be similar?

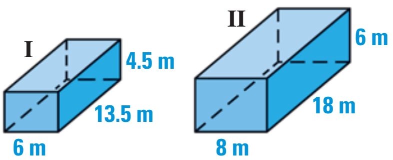
2. ★ **WRITING** How are the volumes of similar solids related?

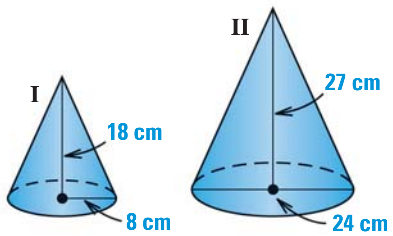
EXAMPLE 1 on p. 847 for Exs. 3–7

IDENTIFYING SIMILAR SOLIDS **Tell whether the pair of right solids is similar. *Explain* your reasoning.**

3. 

4. 

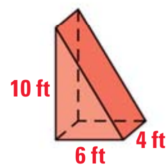
5. 

6. 

7. ★ **MULTIPLE CHOICE** Which set of dimensions corresponds to a triangular prism that is similar to the prism shown?

   Ⓐ 2 feet by 1 foot by 5 feet Ⓑ 4 feet by 2 feet by 8 feet

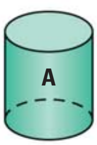
   Ⓒ 9 feet by 6 feet by 20 feet Ⓓ 15 feet by 10 feet by 25 feet



EXAMPLE 2 on p. 848 for Exs. 8–11

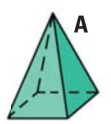
USING SCALE FACTOR **Solid A (shown) is similar to Solid B (not shown) with the given scale factor of A to B. Find the surface area and volume of Solid B.**

8. Scale factor of 1 : 2



$S = 150\pi$ in.^{2}
$V = 250\pi$ in.^{3}

9. Scale factor of 3 : 1



$S = 1500$ m^2
$V = 3434.6$ m^3

10. Scale factor of 5 : 2

$S = 2356.2$ cm^2
$V = 7450.9$ cm^3

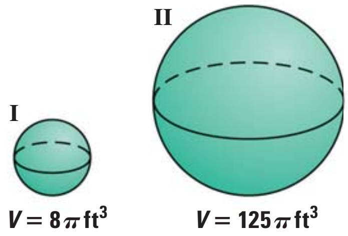
11. **ERROR ANALYSIS** The scale factor of two similar solids is 1 : 4. The volume of the smaller Solid A is 500π. *Describe* and correct the error in writing an equation to find the volume of the larger Solid B.

$$\frac{500\pi}{\text{Volume of B}} = \frac{1^2}{4^2}$$

EXAMPLE 3
on p. 849
for Exs. 12–18

FINDING SCALE FACTOR **In Exercises 12–15, Solid I is similar to Solid II. Find the scale factor of Solid I to Solid II.**

12.



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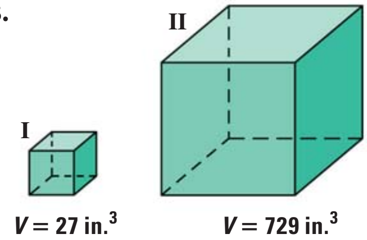

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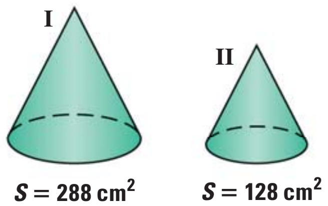

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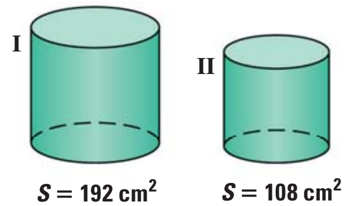


16. ★ **MULTIPLE CHOICE** The volumes of two similar cones are 8π and 27π. What is the ratio of the lateral areas of the cones?

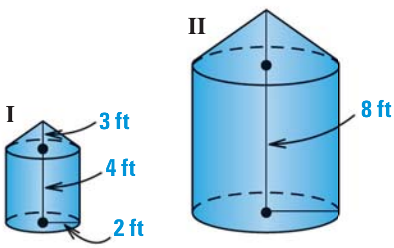
Ⓐ $\frac{8}{27}$ Ⓑ $\frac{1}{3}$ Ⓒ $\frac{4}{9}$ Ⓓ $\frac{2}{3}$

17. **FINDING A RATIO** Two spheres have volumes 2π cubic feet and 16π cubic feet. What is the ratio of the surface area of the smaller sphere to the surface area of the larger sphere?

18. **FINDING SURFACE AREA** Two cylinders have a scale factor of 2 : 3. The smaller cylinder has a surface area of 78π square meters. Find the surface area of the larger cylinder.

COMPOSITE SOLIDS **In Exercises 19–22, Solid I is similar to Solid II. Find the surface area and volume of Solid II.**

19.



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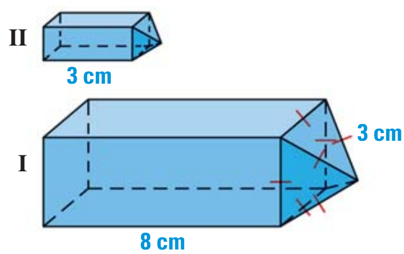

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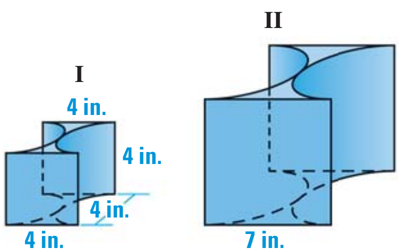

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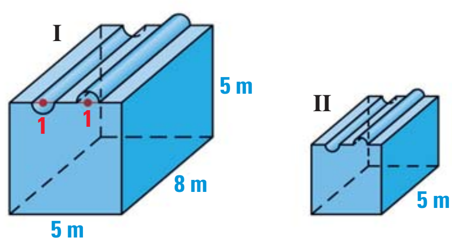


23. **ALGEBRA** Two similar cylinders have surface areas of 54π square feet and 384π square feet. The height of each cylinder is equal to its diameter. Find the radius and height of both cylinders.

24. **CHALLENGE** A plane parallel to the base of a cone divides the cone into two pieces with the dimensions shown. Find each ratio described.

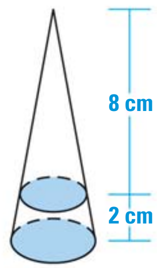
a. The area of the top shaded circle to the area of the bottom shaded circle

b. The slant height of the top part of the cone to the slant height of the whole cone

c. The lateral area of the top part of the cone to the lateral area of the whole cone

d. The volume of the top part of the cone to the volume of the whole cone

e. The volume of the top part of the cone to the volume of the bottom part



PROBLEM SOLVING

EXAMPLE 4 on p. 849 for Exs. 25–27

25. **COFFEE MUGS** The heights of two similar coffee mugs are 3.5 inches and 4 inches. The larger mug holds 12 fluid ounces. What is the capacity of the smaller mug?

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26. **ARCHITECTURE** You have a pair of binoculars that is similar in shape to the structure on page 847. Your binoculars are 6 inches high, and the height of the structure is 45 feet. Find the ratio of the volume of your binoculars to the volume of the structure.

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27. **PARTY PLANNING** Two similar punch bowls have a scale factor of 3 : 4. The amount of lemonade to be added is proportional to the volume. How much lemonade does the smaller bowl require if the larger bowl requires 64 fluid ounces?

28. ★ **OPEN-ENDED MATH** Using the scale factor 2 : 5, sketch a pair of solids in the correct proportions. Label the dimensions of the solids.

29. **MULTI-STEP PROBLEM** Two oranges are both spheres with diameters 3.2 inches and 4 inches. The skin on both oranges has an average thickness of $\frac{1}{8}$ inch.

a. Find the volume of each unpeeled orange.

b. *Compare* the ratio of the diameters to the ratio of the volumes.

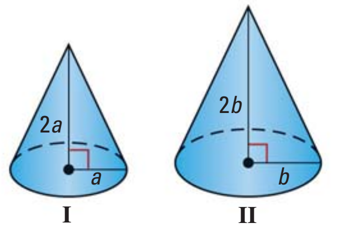
c. Find the diameter of each orange after being peeled.

d. *Compare* the ratio of surface areas of the peeled oranges to the ratio of the volumes of the peeled oranges.

Animated Geometry at classzone.com

○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE ◆ = MULTIPLE REPRESENTATIONS

30. **ALGEBRA** Use the two similar cones shown.



a. What is the scale factor of Cone I to Cone II? What should the ratio of the volume of Cone I to the volume of Cone II be?

b. Write an expression for the volume of each solid.

c. Write and simplify an expression for the ratio of the volume of Cone I to the volume of Cone II. Does your answer agree with your answer to part (a)? *Explain*.

31. ★ **EXTENDED RESPONSE** The scale factor of the model car at the right to the actual car is 1 : 18.

a. The model has length 8 inches. What is the length of the actual car?

b. Each tire of the model has a surface area of 12.1 square inches. What is the surface area of each tire of the actual car?

c. The actual car's engine has volume 8748 cubic inches. Find the volume of the model car's engine.

32. **USING VOLUMES** Two similar cylinders have volumes 16π and 432π. The larger cylinder has lateral area 72π. Find the lateral area of the smaller cylinder.

33. ★ **SHORT RESPONSE** A snow figure is made using three balls of snow with diameters 25 centimeters, 35 centimeters, and 45 centimeters. The smallest weighs about 1.2 kilograms. Find the total weight of the snow used to make the snow figure. *Explain* your reasoning.

34. **MULTIPLE REPRESENTATIONS** A gas is enclosed in a cubical container with side length s in centimeters. Its temperature remains constant while the side length varies. By the *Ideal Gas Law*, the pressure P in atmospheres (atm) of the gas varies inversely with its volume.

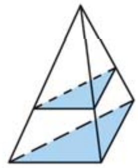
a. **Writing an Equation** Write an equation relating P and s. You will need to introduce a constant of variation k.

b. **Making a Table** Copy and complete the table below for various side lengths. Express the pressure P in terms of the constant k.

Side length s (cm)	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
Pressure P (atm)	?	$8k$	k	?	?

c. **Drawing a Graph** For this particular gas, $k = 1$. Use your table to sketch a graph of P versus s. Place P on the vertical axis and s on the horizontal axis. Does the graph show a linear relationship? *Explain*.

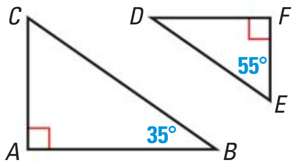
35. **CHALLENGE** A plane parallel to the base of a pyramid separates the pyramid into two pieces with equal volumes. The height of the pyramid is 12 feet. Find the height of the top piece.



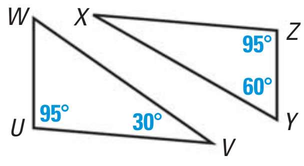
MIXED REVIEW

Determine whether the triangles are similar. If they are, write a similarity statement. *(p. 381)*

36.



37.



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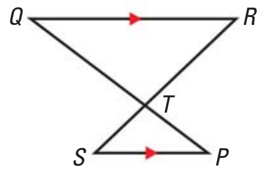


The sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides. *(p. 507)*

39. 900° **40.** 180° **41.** 540° **42.** 1080°

Write a standard equation of the circle with the given center and radius. *(p. 699)*

43. Center (2, 5), radius 4 **44.** Center (−3, 2), radius 6

Sketch the described solid and find its surface area. Round your answer to two decimal places, if necessary. *(p. 803)*

45. Right rectangular prism with length 8 feet, width 6 feet, and height 3 feet

46. Right regular pentagonal prism with all edges measuring 12 millimeters

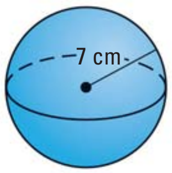
47. Right cylinder with radius 4 inches and height 4 inches

48. Right cylinder with diameter 9 centimeters and height 7 centimeters

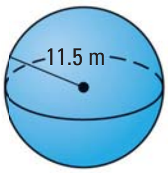
QUIZ *for Lessons 12.6–12.7*

Find the surface area and volume of the sphere. Round your answers to two decimal places. *(p. 838)*

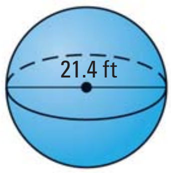
1.



2.

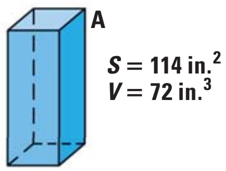


3.

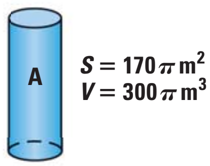


Solid A (shown) is similar to Solid B (not shown) with the given scale factor of A to B. Find the surface area *S* and volume *V* of Solid B. *(p. 847)*

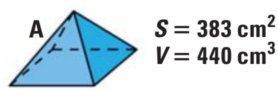
4. Scale factor of 1 : 3



5. Scale factor of 2 : 3



6. Scale factor of 5 : 4



7. Two similar cones have volumes 729π cubic feet and 343π cubic feet. What is the scale factor of the larger cone to the smaller cone? *(p. 847)*

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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Lessons 12.4–12.7

1. **MULTI-STEP PROBLEM** You have a container in the shape of a right rectangular prism with inside dimensions of length 24 inches, width 16 inches, and height 20 inches.

a. Find the volume of the inside of the container.

b. You are going to fill the container with boxes of cookies that are congruent right rectangular prisms. Each box has length 8 inches, width 2 inches, and height 3 inches. Find the volume of one box of cookies.

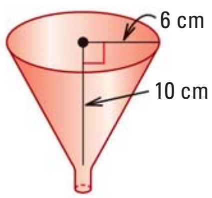
c. How many boxes of cookies will fit inside the cardboard container?

2. **SHORT RESPONSE** You have a cup in the shape of a cylinder with inside dimensions of diameter 2.5 inches and height 7 inches.

a. Find the volume of the inside of the cup.

b. You have an 18 ounce bottle of orange juice that you want to pour into the cup. Will all of the juice fit? *Explain* your reasoning. ($1 \text{ in.}^3 \approx 0.554$ fluid ounces)

3. **EXTENDED RESPONSE** You have a funnel with the dimensions shown.



a. Find the approximate volume of the funnel.

b. You are going to use the funnel to put oil in a car. Oil flows out of the funnel at a rate of 45 milliliters per second. How long will it take to empty the funnel when it is full of oil? $(1 \text{ mL} = 1 \text{ cm}^3)$

c. How long would it take to empty a funnel with radius 10 cm and height 6 cm?

d. *Explain* why you can claim that the time calculated in part (c) is greater than the time calculated in part (b) without doing any calculations.

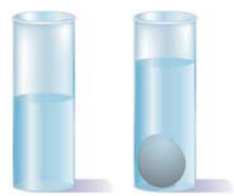
4. **EXTENDED RESPONSE** An official men's basketball has circumference 29.5 inches. An official women's basketball has circumference 28.5 inches.

a. Find the surface area and volume of the men's basketball.

b. Find the surface area and volume of the women's basketball using the formulas for surface area and volume of a sphere.

c. Use your answers in part (a) and the Similar Solids Theorem to find the surface area and volume of the women's basketball. Do your results match your answers in part (b)?

5. **GRIDDED ANSWER** To accurately measure the radius of a spherical rock, you place the rock into a cylindrical glass containing water. When you do so, the water level rises $\frac{9}{64}$ inch. The radius of the glass is 2 inches. What is the radius of the rock?



6. **SHORT RESPONSE** Sketch a rectangular prism and label its dimensions. Change the dimensions of the prism so that its surface area increases and its volume decreases.

7. **SHORT RESPONSE** A hemisphere and a right cone have the same radius and the height of the cone is equal to the radius. *Compare* the volumes of the solids.

8. **SHORT RESPONSE** *Explain* why the height of a right cone is always less than its slant height. Include a diagram in your answer.

12 CHAPTER SUMMARY

BIG IDEAS

For Your Notebook

Big Idea 1

Exploring Solids and Their Properties

Euler's Theorem is useful when finding the number of faces, edges, or vertices on a polyhedron, especially when one of those quantities is difficult to count by hand.

For example, suppose you want to find the number of edges on a regular icosahedron, which has 20 faces. You count 12 vertices on the solid. To calculate the number of edges, use Euler's Theorem:

$F + V = E + 2$ **Write Euler's Theorem.**

$20 + 12 = E + 2$ **Substitute known values.**

$30 = E$ **Solve for *E*.**

Big Idea 2

Solving Problems Using Surface Area and Volume

Figure	Surface Area	Volume
Right prism	$S = 2B + Ph$	$V = Bh$
Right cylinder	$S = 2B + Ch$	$V = Bh$
Regular pyramid	$S = B + \frac{1}{2}P\ell$	$V = \frac{1}{3}Bh$
Right cone	$S = B + \frac{1}{2}C\ell$	$V = \frac{1}{3}Bh$
Sphere	$S = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

The volume formulas for prisms, cylinders, pyramids, and cones can be used for oblique solids.

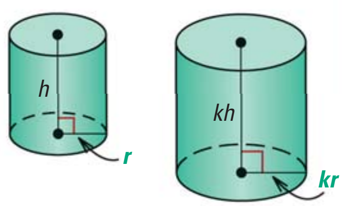
While many of the above formulas can be written in terms of more detailed variables, it is more important to remember the more general formulas for a greater understanding of why they are true.

Big Idea 3

Connecting Similarity to Solids

The similarity concepts learned in Chapter 6 can be extended to 3-dimensional figures as well.

Suppose you have a right cylindrical can whose surface area and volume are known. You are then given a new can whose linear dimensions are k times the dimensions of the original can. If the surface area of the original can is S and the volume of the original can is V, then the surface area and volume of the new can can be expressed as k^2S and k^3V, respectively.



12 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- polyhedron, *p. 794*
  face, edge, vertex, base
- regular polyhedron, *p. 796*
- convex polyhedron, *p. 796*
- Platonic solids, *p. 796*
- tetrahedron, *p. 796*
- cube, *p. 796*
- octahedron, *p. 796*
- dodecahedron, *p. 796*
- icosahedron, *p. 796*
- cross section, *p. 797*
- prism, *p. 803*
  lateral faces, lateral edges
- surface area, *p. 803*
- lateral area, *p. 803*
- net, *p. 803*
- right prism, *p. 804*
- oblique prism, *p. 804*
- cylinder, *p. 805*
- right cylinder, *p. 805*
- pyramid, *p. 810*
- vertex of a pyramid, *p. 810*
- regular pyramid, *p. 810*
- slant height, *p. 810*
- cone, *p. 812*
- vertex of a cone, *p. 812*
- right cone, *p. 812*
- lateral surface, *p. 812*
- volume, *p. 819*
- sphere, *p. 838*
  center, radius, chord, diameter
- great circle, *p. 839*
- hemisphere, *p. 839*
- similar solids, *p. 847*

VOCABULARY EXERCISES

1. Copy and complete: A __?__ is the set of all points in space equidistant from a given point.

2. **WRITING** Sketch a right rectangular prism and an oblique rectangular prism. *Compare* the prisms.

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 12.

12.1 Explore Solids   *pp. 794–801*

EXAMPLE

A polyhedron has 16 vertices and 24 edges. How many faces does the polyhedron have?

$F + V = E + 2$ **Euler's Theorem**

$F + 16 = 24 + 2$ **Substitute known values.**

$F = 10$ **Solve for *F*.**

▶ The polyhedron has 10 faces.

EXERCISES

EXAMPLES 2 and 3 on pp. 796–797 for Exs. 3–5

Use Euler's Theorem to find the value of n.

3. Faces: 20
Vertices: n
Edges: 30

4. Faces: n
Vertices: 6
Edges: 12

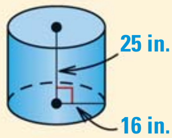
5. Faces: 14
Vertices: 24
Edges: n

12 CHAPTER REVIEW

12.2 Surface Area of Prisms and Cylinders *pp. 803–809*

EXAMPLE

Find the surface area of the right cylinder.



$S = 2\pi r^2 + 2\pi rh$ **Write formula.**

$= 2\pi(16)^2 + 2\pi(16)(25)$ **Substitute for r and h.**

$= 1312\pi$ **Simplify.**

≈ 4121.77 **Use a calculator.**

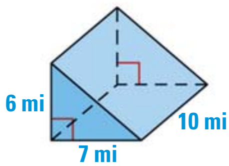
▶ The surface area of the cylinder is about 4121.77 square inches.

EXERCISES

EXAMPLES 2, 3, and 4 on pp. 804–806 for Exs. 6–9

Find the surface area of the right prism or right cylinder. Round your answer to two decimal places, if necessary.

6.



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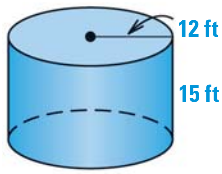

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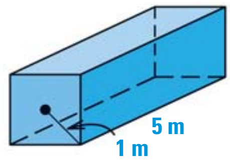

9. A cylinder has a surface area of 44π square meters and a radius of 2 meters. Find the height of the cylinder.

12.3 Surface Area of Pyramids and Cones *pp. 810–817*

EXAMPLE

Find the lateral area of the right cone.

Lateral area $= \pi r\ell$ **Write formula.**

$= \pi(6)(16)$ **Substitute for r and ℓ.**

$= 96\pi$ **Simplify.**

≈ 301.59 **Use a calculator.**

▶ The lateral area of the cone is about 301.59 square centimeters.

EXERCISES

EXAMPLES 1, 2, and 4 on pp. 810–813 for Exs. 10–12

10. Find the surface area of a right square pyramid with base edge length 2 feet and height 5 feet.

11. The surface area of a cone with height 15 centimeters is 500π square centimeters. Find the radius of the base of the cone. Round your answer to two decimal places.

12. Find the surface area of a right octagonal pyramid with height 2.5 yards, and its base has apothem length 1.5 yards.

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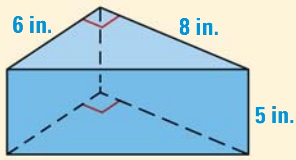
Chapter Review Practice

12.4 Volume of Prisms and Cylinders

pp. 819–825

EXAMPLE

Find the volume of the right triangular prism.



The area of the base is $B = \frac{1}{2}(6)(8) = 24$ square inches. Use $h = 5$ to find the volume.

$V = Bh$ **Write formula.**

$= 24(5)$ **Substitute for B and h.**

$= 120$ **Simplify.**

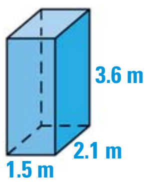
▶ The volume of the prism is 120 cubic inches.

EXERCISES

EXAMPLES 2 and 4 on pp. 820–821 for Exs. 13–15

Find the volume of the right prism or oblique cylinder. Round your answer to two decimal places.

13.



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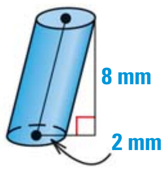

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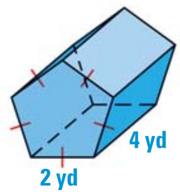
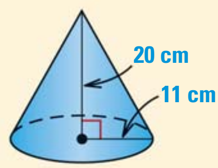


12.5 Volume of Pyramids and Cones

pp. 829–836

EXAMPLE

Find the volume of the right cone.



The area of the base is $B = \pi r^2 = \pi(11)^2 \approx 380.13 \text{ cm}^2$.
Use $h = 20$ to find the volume.

$V = \frac{1}{3}Bh$ **Write formula.**

$\approx \frac{1}{3}(380.13)(20)$ **Substitute for B and h.**

≈ 2534.20 **Simplify.**

▶ The volume of the cone is about 2534.20 cubic centimeters.

EXERCISES

EXAMPLES 1 and 2 on pp. 829–830 for Exs. 16–17

16. A cone with diameter 16 centimeters has height 15 centimeters. Find the volume of the cone. Round your answer to two decimal places.

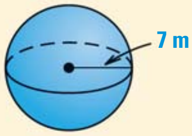
17. The volume of a pyramid is 60 cubic inches and the height is 15 inches. Find the area of the base.

12 CHAPTER REVIEW

12.6 Surface Area and Volume of Spheres pp. 838–845

EXAMPLE

Find the surface area of the sphere.



$S = 4\pi r^2$ **Write formula.**

$= 4\pi(7)^2$ **Substitute 7 for *r*.**

$= 196\pi$ **Simplify.**

▸ The surface area of the sphere is 196π, or about 615.75 square meters.

EXERCISES

EXAMPLES 1, 4, and 5 on pp. 839, 841 for Exs. 18–19

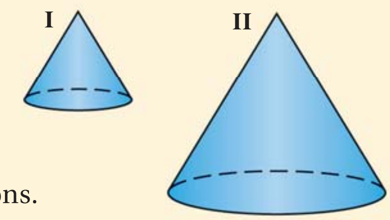
18. ASTRONOMY The shape of Pluto can be approximated as a sphere of diameter 2390 kilometers. Find the surface area and volume of Pluto. Round your answer to two decimal places.

19. A solid is composed of a cube with side length 6 meters and a hemisphere with diameter 6 meters. Find the volume of the solid. Round your answer to two decimal places.

12.7 Explore Similar Solids pp. 847–854

EXAMPLE

The cones are similar with a scale factor of 1 : 2. Find the surface area and volume of Cone II given that the surface area of Cone I is 384π square inches and the volume of Cone I is 768π cubic inches.



Use Theorem 12.13 to write and solve two proportions.

$$\frac{\text{Surface area of I}}{\text{Surface area of II}} = \frac{a^2}{b^2} \qquad \frac{\text{Volume of I}}{\text{Volume of II}} = \frac{a^3}{b^3}$$

$$\frac{384\pi}{\text{Surface area of II}} = \frac{1^2}{2^2} \qquad \frac{768\pi}{\text{Volume of II}} = \frac{1^3}{2^3}$$

$$\text{Surface area of II} = 1536\pi \text{ in.}^2 \qquad \text{Volume of II} = 6144\pi \text{ in.}^3$$

▸ The surface area of Cone II is 1536π, or about 4825.48 square inches, and the volume of Cone II is 6144π, or about 19,301.93 cubic inches.

EXERCISES

EXAMPLE 2 on p. 848 for Exs. 20–22

Solid A is similar to Solid B with the given scale factor of A to B. The surface area and volume of Solid A are given. Find the surface area and volume of Solid B.

20. Scale factor of 1 : 4
$S = 62 \text{ cm}^2$
$V = 30 \text{ cm}^3$

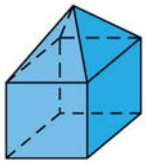
21. Scale factor of 1 : 3
$S = 112\pi \text{ m}^2$
$V = 160\pi \text{ m}^3$

22. Scale factor of 2 : 5
$S = 144\pi \text{ yd}^2$
$V = 288\pi \text{ yd}^3$

12 CHAPTER TEST

Find the number of faces, vertices, and edges of the polyhedron. Check your answer using Euler's Theorem.

1.



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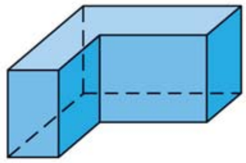

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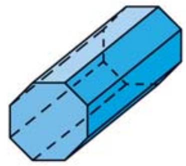
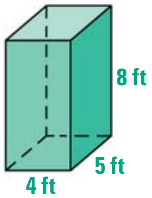


Find the surface area of the solid. The prisms, pyramids, cylinders, and cones are right. Round your answer to two decimal places, if necessary.

4.



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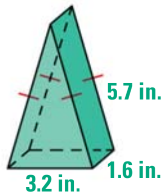

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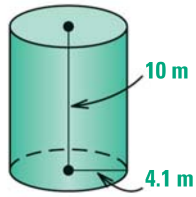

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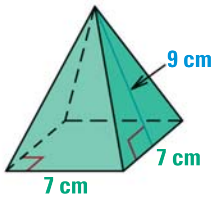

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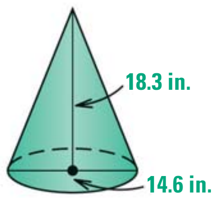

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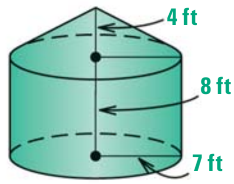
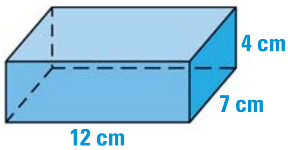


Find the volume of the right prism or right cylinder. Round your answer to two decimal places, if necessary.

10.



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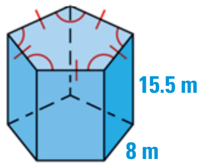

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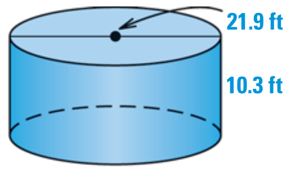
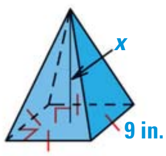
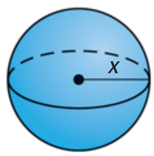


In Exercises 13–15, solve for x.

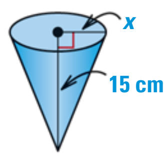
13. Volume $= 324 \text{ in.}^3$



14. Volume $= \frac{32\pi}{3} \text{ ft}^3$



15. Volume $= 180\pi \text{ cm}^3$



16. MARBLES The diameter of the marble shown is 35 millimeters. Find the surface area and volume of the marble.

17. PACKAGING Two similar cylindrical cans have a scale factor of 2 : 3. The smaller can has surface area 308π square inches and volume 735π cubic inches. Find the surface area and volume of the larger can.

CONTEXT-BASED MULTIPLE CHOICE QUESTIONS

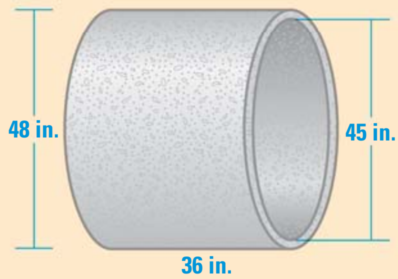
Some of the information you need to solve a context-based multiple choice question may appear in a table, a diagram, or a graph.

PROBLEM 1

One cubic foot of concrete weighs about 150 pounds. What is the approximate weight of the cylindrical section of concrete pipe shown?

Ⓐ 145 lb Ⓑ 686 lb

Ⓒ 2738 lb Ⓓ 5653 lb



Plan

INTERPRET THE DIAGRAM The pipe is a cylinder with length 36 inches and diameter 48 inches. The hollow center is also a cylinder with length 36 inches and diameter 45 inches. Find the volume of concrete used (in cubic feet). Then multiply by 150 pounds per cubic foot to find the weight of the concrete.

Solution

STEP 1 Find the volume of concrete used in the pipe.

Find the volume of a cylinder with diameter 48 inches and height 36 inches.

$V = \pi r^2 h = \pi(24^2)(36) \approx 65{,}144 \text{ in.}^3$

Find the volume of a cylinder with diameter 45 inches and height 36 inches.

$V = \pi r^2 h = \pi(22.5^2)(36) \approx 57{,}256 \text{ in.}^3$

To find the volume of concrete used in the pipe, subtract the smaller volume from the larger volume.

Volume of concrete used in pipe $\approx 65{,}144 - 57{,}256 = 7889 \text{ in.}^3$

STEP 2 Convert the volume to cubic feet.

Use unit analysis to convert 7889 cubic inches to cubic feet. There are 12 inches in 1 foot, so there are $12^3 = 1728$ cubic inches in 1 cubic foot.

$$7889 \cancel{\text{in.}^3} \cdot \frac{1 \text{ ft}^3}{1728 \cancel{\text{in.}^3}} \approx 4.57 \text{ ft}^3$$

STEP 3 Find the weight of the pipe.

To find the weight of the pipe, multiply the volume of the concrete used in the pipe by the weight of one cubic foot of concrete.

$$\text{Weight of pipe} \approx 4.57 \cancel{\text{ft}^3} \cdot \frac{150 \text{ lb}}{1 \cancel{\text{ft}^3}} = 685.5 \text{ lb}$$

The weight of the pipe is about 686 pounds.

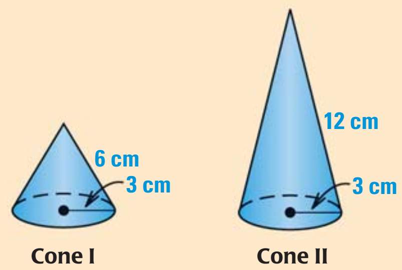
The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

PROBLEM 2

What is the ratio of the surface area of Cone I to the surface area of Cone II?

Ⓐ 1 : 2 Ⓑ 1 : 4

Ⓒ 3 : 5 Ⓓ 3 : 8



Plan

INTERPRET THE DIAGRAM The diagram shows that the cones have the same radius, but different slant heights. Find and compare the surface areas.

Solution

STEP 1 Find the surface area of each cone.

Use the formula for the surface area of a cone.

$$\text{Surface area of Cone I} = \pi r^2 + \pi r \ell = \pi(3^2) + \pi(3)(6) = 9\pi + 18\pi = 27\pi$$

$$\text{Surface area of Cone II} = \pi r^2 + \pi r \ell = \pi(3^2) + \pi(3)(12) = 9\pi + 36\pi = 45\pi$$

STEP 2 Compare the surface areas.

Write a ratio.

$$\frac{\text{Surface area of Cone I}}{\text{Surface area of Cone II}} = \frac{27\pi}{45\pi} = \frac{3}{5}, \text{ or } 3:5$$

The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

PRACTICE

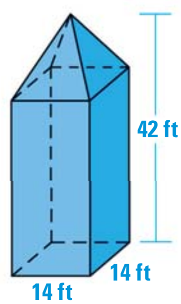
1. The amount a cannister can hold is proportional to its volume. The large cylindrical cannister in the table holds 2 kilograms of flour. About how many kilograms does the similar small cannister hold?

Size	Diameter
Small	24 cm
Medium	30 cm
Large	37.5 cm

Ⓐ 0.5 kg Ⓑ 1 kg

Ⓒ 1.3 kg Ⓓ 1.6 kg

2. The solid shown is made of a rectangular prism and a square pyramid. The height of the pyramid is one third the height of the prism. What is the volume of the solid?



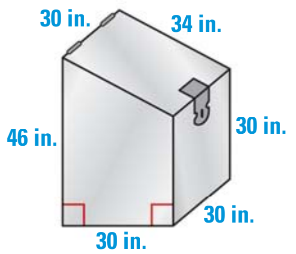
Ⓐ $457\frac{1}{3}\text{ ft}^3$ Ⓑ $6402\frac{2}{3}\text{ ft}^3$

Ⓒ 6860 ft^3 Ⓓ $10{,}976\text{ ft}^3$

12 ★ Standardized TEST PRACTICE

MULTIPLE CHOICE

In Exercises 1 and 2, use the diagram, which shows a bin for storing wood.



1. The bin is a prism. What is the shape of the base of the prism?

   Ⓐ Triangle　Ⓑ Rectangle

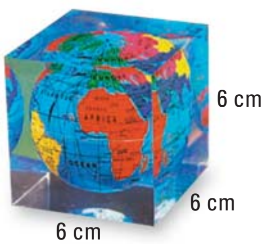
   Ⓒ Square　Ⓓ Trapezoid

2. What is the surface area of the bin?

   Ⓐ 3060 in.^{2}　Ⓑ 6480 in.^{2}

   Ⓒ 6960 in.^{2}　Ⓓ 8760 in.^{2}

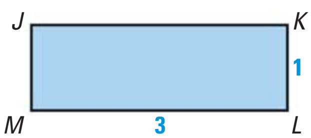
3. In the paperweight shown, a sphere with diameter 5 centimeters is embedded in a glass cube. What percent of the volume of the paperweight is taken up by the sphere?



   Ⓐ About 30%　Ⓑ About 40%

   Ⓒ About 50%　Ⓓ About 60%

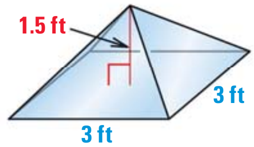
4. What is the volume of the solid formed when rectangle $JKLM$ is rotated 360° about $\overline{KL}$?



   Ⓐ π　Ⓑ 3π

   Ⓒ 6π　Ⓓ 9π

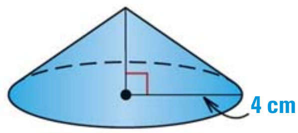
5. The skylight shown is made of four glass panes that are congruent isosceles triangles. One square foot of the glass used in the skylight weighs 3.25 pounds. What is the approximate total weight of the glass used in the four panes?



   Ⓐ 10 lb　Ⓑ 15 lb

   Ⓒ 29 lb　Ⓓ 41 lb

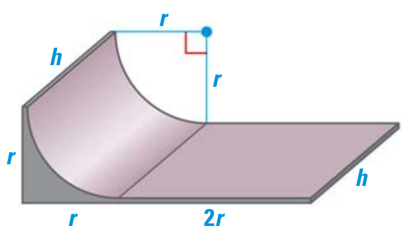
6. The volume of the right cone shown below is 16π cubic centimeters. What is the surface area of the cone?



   Ⓐ 12π cm^2　Ⓑ 18π cm^2

   Ⓒ 36π cm^2　Ⓓ 72π cm^2

7. The shaded surface of the skateboard ramp shown is divided into a flat rectangular portion and a curved portion. The curved portion is one fourth of a cylinder with radius r feet and height h feet. Which equation can be used to find the area of the top surface of the ramp?



   Ⓐ $2rh + 2\pi r^2$　Ⓑ $2rh + 2\pi rh$

   Ⓒ $2rh + \frac{1}{4}\pi r^2$　Ⓓ $2rh + \frac{1}{2}\pi rh$

GRIDDED ANSWER

8. The scale factor of two similar triangular prisms is 3 : 5. The volume of the larger prism is 175 cubic inches. What is the volume (in cubic inches) of the smaller prism?

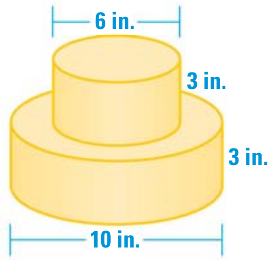
9. Two identical octagonal pyramids are joined together at their bases. The resulting polyhedron has 16 congruent triangular faces and 10 vertices. How many edges does it have?

10. The surface area of Sphere A is 27 square meters. The surface area of Sphere B is 48 square meters. What is the ratio of the diameter of Sphere A to the diameter of Sphere B, expressed as a decimal?

11. The volume of a square pyramid is 54 cubic meters. The height of the pyramid is 2 times the length of a side of its base. What is the height (in meters) of the pyramid?

SHORT RESPONSE

12. Two cake layers are right cylinders, as shown. The top and sides of each layer will be frosted, including the portion of the top of the larger layer that is under the smaller layer. One can of frosting covers 100 square inches. How many cans do you need to frost the cake?



13. The height of Cylinder B is twice the height of Cylinder A. The diameter of Cylinder B is half the diameter of Cylinder A. Let r be the radius and let h be the height of Cylinder A. Write expressions for the radius and height of Cylinder B. Which cylinder has a greater volume? *Explain.*

EXTENDED RESPONSE

14. A cylindrical oil tank for home use has the dimensions shown.

a. Find the volume of the tank to the nearest tenth of a cubic foot.

b. Use the fact that 1 cubic foot = 7.48 gallons to find how many gallons of oil are needed to fill the tank.

c. A homeowner uses about 1000 gallons of oil in a year. Assuming the tank is empty each time it was filled, how many times does the tank need to be filled during the year?

15. A manufacturer is deciding whether to package a product in a container shaped like a prism or one shaped like a cylinder. The manufacturer wants to use the least amount of material possible. The prism is 4 inches tall and has a square base with side length 3 inches. The height of the cylinder is 5 inches, and its radius is 1.6 inches.

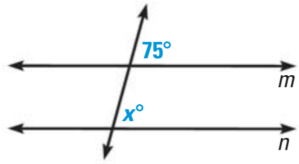
a. Find the surface area and volume of each container. If necessary, round to the nearest tenth.

b. For each container, find the ratio of the volume to the surface area. *Explain* why the manufacturer should compare the ratios before making a decision.

CUMULATIVE REVIEW Chapters 1–12

Find the value of x that makes $m \parallel n$. *(p. 161)*

1.



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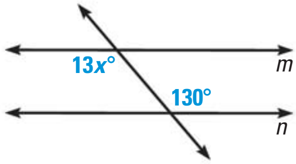

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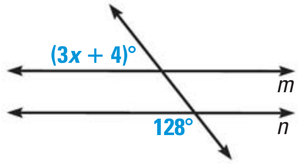
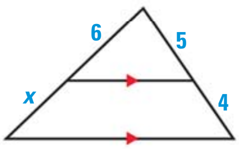


Find the value of the variable. *(p. 397)*

4.



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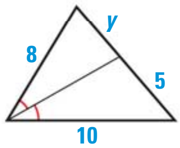

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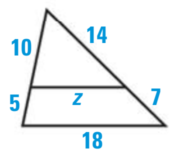
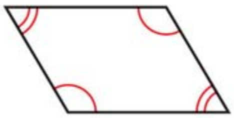


Explain **how you know that the quadrilateral is a parallelogram.** *(p. 522)*

7.



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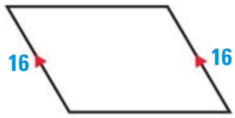
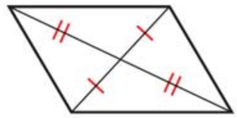
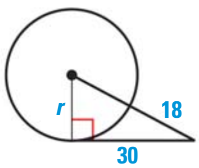


9.



Find the value of the variable. *(pp. 651, 672, 690)*

10.



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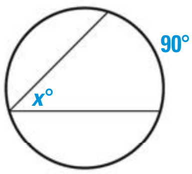

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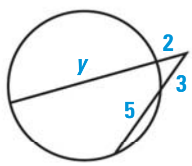
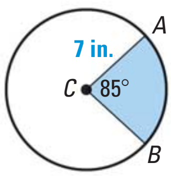
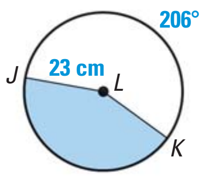


Find the area of the shaded region. *(p. 755)*

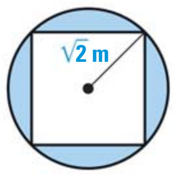
13.



14.

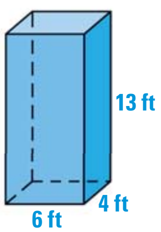


15.

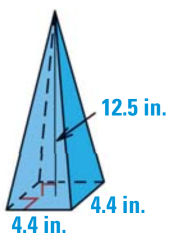


Find the surface area and volume of the right solid. Round your answer to two decimal places. *(pp. 803, 810, 819, 829)*

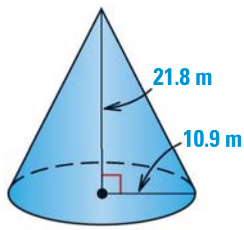
16.



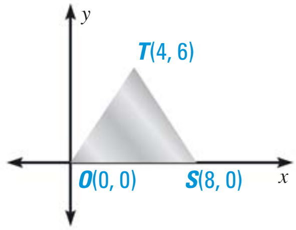
17.



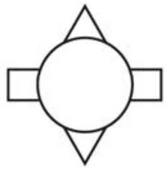
18.



19. **PHYSICS** Find the coordinates of point P that will allow the triangular plate of uniform thickness to be balanced on a point. *(p. 319)*



20. **SYMMETRY** Copy the figure on the right. Determine whether the figure has *line symmetry* and whether it has *rotational symmetry*. Identify all lines of symmetry and angles of rotation that map the figure onto itself. *(p. 619)*

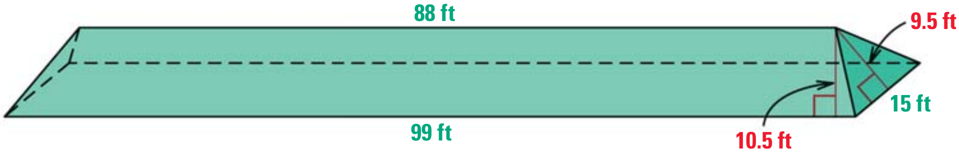


21. **TWO-WAY RADIOS** You and your friend want to test a pair of two-way radios. The radios are expected to transmit voices up to 6 miles. Your location is identified by the point $(-2, 4)$ on a coordinate plane where units are measured in miles. *(p. 699)*

   a. Write an inequality that represents the area expected to be covered by the radios.

   b. Determine whether your friend should be able to hear your voice when your friend is located at $(2, 0)$, $(3, 9)$, $(-6, -1)$, $(-6, 8)$, and $(-7, 5)$. *Explain* your reasoning.

22. **COVERED BRIDGE** A covered bridge has a roof with the dimensions shown. The top ridge of the roof is parallel to the base of the roof. The hidden back and left sides are the same as the front and right sides. Find the total area of the roof. *(pp. 720, 730)*



23. **CANDLES** The candle shown has diameter 2 inches and height 5.5 inches. *(pp. 803, 819)*

   a. Find the surface area and volume of the candle. Round your answer to two decimal places.

   b. The candle has a burning time of about 30 hours. Find the approximate volume of the candle after it has burned for 18 hours.

24. **GEOGRAPHY** The diameter of Earth is about 7920 miles. If approximately 70 percent of Earth's surface is covered by water, how many square miles of water are on Earth's surface? Round your answer to two decimal places. *(p. 838)*