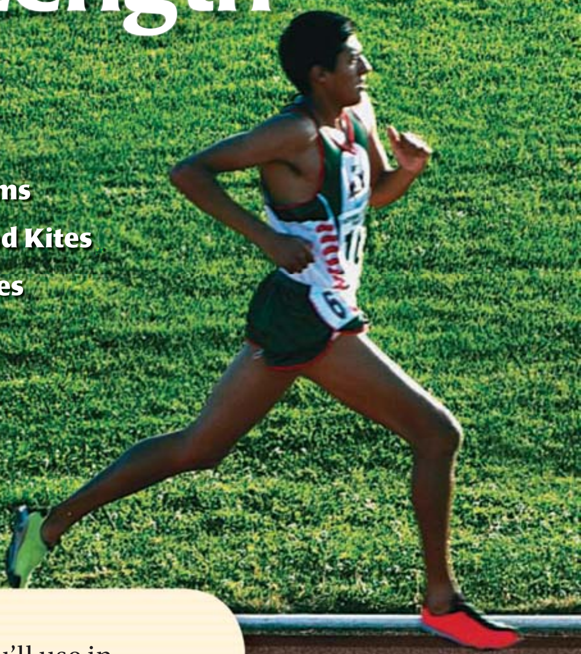
11 Measuring Length and Area

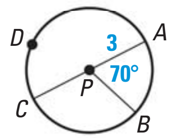


Before

In previous chapters, you learned the following skills, which you'll use in Chapter 11: applying properties of circles and polygons, using formulas, solving for lengths in right triangles, and using ratios and proportions.

Prerequisite Skills

VOCABULARY CHECK



Give the indicated measure for $\odot P$.

1. The radius 2. The diameter 3. $m\overset{\frown}{ADB}$

SKILLS AND ALGEBRA CHECK

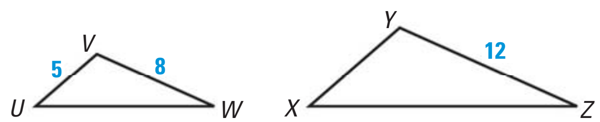
4. Use a formula to find the width w of the rectangle that has a perimeter of 24 centimeters and a length of 9 centimeters. *(Review p. 49 for 11.1.)*

In $\triangle ABC$, angle C is a right angle. Use the given information to find AC. *(Review pp. 433, 457, 473 for 11.1, 11.6.)*

5. $AB = 14$, $BC = 6$ 6. $m\angle A = 35°$, $AB = 25$ 7. $m\angle B = 60°$, $BC = 5$

8. Which special quadrilaterals have diagonals that bisect each other? *(Review pp. 533, 542 for 11.2.)*

9. Use a proportion to find XY if $\triangle UVW \sim \triangle XYZ$. *(Review p. 372 for 11.3.)*



@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 11, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 779. You will also use the key vocabulary listed below.

Big Ideas

1. **Using area formulas for polygons**
2. **Relating length, perimeter, and area ratios in similar polygons**
3. **Comparing measures for parts of circles and the whole circle**

KEY VOCABULARY

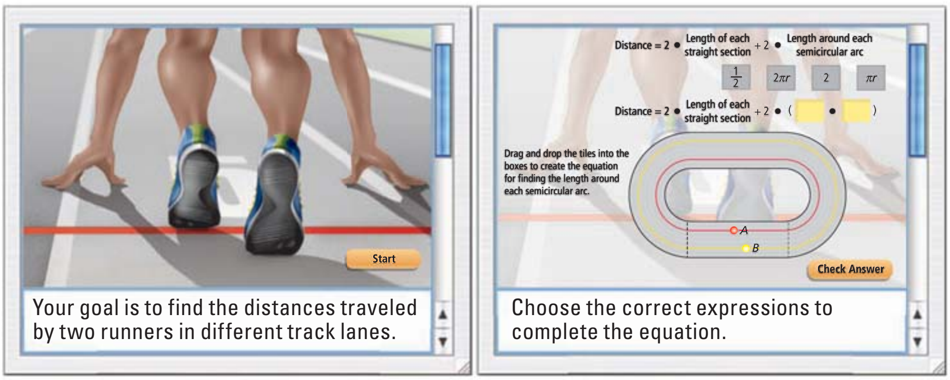
- bases of a parallelogram, *p. 720*
- height of a parallelogram, *p. 720*
- height of a trapezoid, *p. 730*
- circumference, *p. 746*
- arc length, *p. 747*
- sector of a circle, *p. 756*
- center of a polygon, *p. 762*
- radius of a polygon, *p. 762*
- apothem of a polygon, *p. 762*
- central angle of a regular polygon, *p. 762*
- probability, *p. 771*
- geometric probability, *p. 771*

Why?

You can apply formulas for perimeter, circumference, and area to find and compare measures. To find lengths along a running track, you can break the track into straight sides and semicircles.

Animated Geometry

The animation illustrated below for Example 5 on page 749 helps you answer this question: How far does a runner travel to go around a track?



Animated Geometry at classzone.com

Other animations for Chapter 11: pages 720, 739, 759, 765, and 771

11.1 Areas of Triangles and Parallelograms

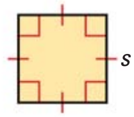
Before You learned properties of triangles and parallelograms.
Now You will find areas of triangles and parallelograms.
Why? So you can plan a jewelry making project, as in Ex. 44.

Key Vocabulary
- **bases of a parallelogram**
- **height of a parallelogram**
- **area,** *p. 49*
- **perimeter,** *p. 49*

POSTULATES — *For Your Notebook*

POSTULATE 24 Area of a Square Postulate

The area of a square is the square of the length of its side.



$A = s^2$

POSTULATE 25 Area Congruence Postulate

If two polygons are congruent, then they have the same area.

POSTULATE 26 Area Addition Postulate

The area of a region is the sum of the areas of its nonoverlapping parts.

RECTANGLES A rectangle that is b units by h units can be split into $b \cdot h$ unit squares, so the area formula for a rectangle follows from Postulates 24 and 26.

THEOREM — *For Your Notebook*

THEOREM 11.1 Area of a Rectangle

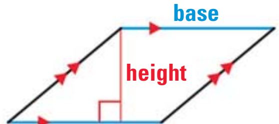
The area of a rectangle is the product of its base and height.

Justification: Ex. 46, p. 726

$A = bh$

READ DIAGRAMS
The word *base* can refer to a segment or to its length. The segment used for the height must be perpendicular to the bases used.

PARALLELOGRAMS Either pair of parallel sides can be used as the **bases** of a parallelogram. The **height** is the perpendicular distance between these bases.



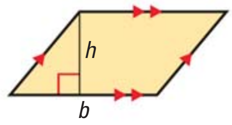
If you transform a rectangle to form other parallelograms with the same base and height, the area stays the same.

Animated Geometry at classzone.com

THEOREMS *For Your Notebook*

THEOREM 11.2 Area of a Parallelogram

The area of a parallelogram is the product of a base and its corresponding height.

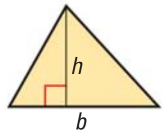


$A = bh$

Justification: Ex. 42, p. 725

THEOREM 11.3 Area of a Triangle

The area of a triangle is one half the product of a base and its corresponding height.

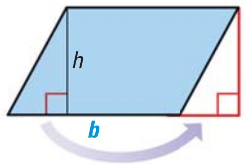


$A = \frac{1}{2}bh$

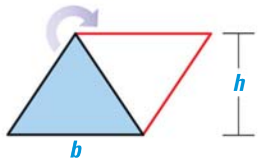
Justification: Ex. 43, p. 726

READ VOCABULARY
The *height* of a triangle is the length of the altitude drawn to the given *base*.

RELATING AREA FORMULAS As illustrated below, the area formula for a parallelogram is related to the formula for a rectangle, and the area formula for a triangle is related to the formula for a parallelogram. You will write a justification of these relationships in Exercises 42 and 43 on pages 725–726.



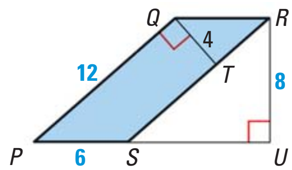
Area of ▱ = Area of Rectangle



Area of △ = $\frac{1}{2}$ • Area of ▱

EXAMPLE 1 Use a formula to find area

Find the area of ▱*PQRS*.



Solution

Method 1 Use $\overline{PS}$ as the base.
The base is extended to measure the height RU. So, $b = 6$ and $h = 8$.

Area $= bh = 6(8) = 48$ square units

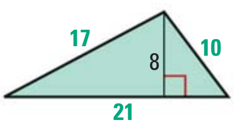
Method 2 Use $\overline{PQ}$ as the base.
Then the height is QT. So, $b = 12$ and $h = 4$.

Area $= bh = 12(4) = 48$ square units

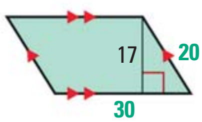
GUIDED PRACTICE for Example 1

Find the perimeter and area of the polygon.

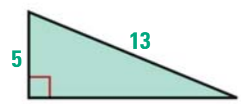
1.



2.

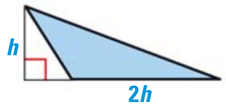


3.



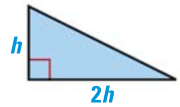
EXAMPLE 2 Solve for unknown measures

DRAW DIAGRAMS
Note that there are other ways you can draw the triangle described in Example 2.



ALGEBRA **The base of a triangle is twice its height. The area of the triangle is 36 square inches. Find the base and height.**

Let h represent the height of the triangle. Then the base is $2h$.



$A = \frac{1}{2}bh$ **Write formula.**

$36 = \frac{1}{2}(2h)(h)$ **Substitute 36 for A and $2h$ for b.**

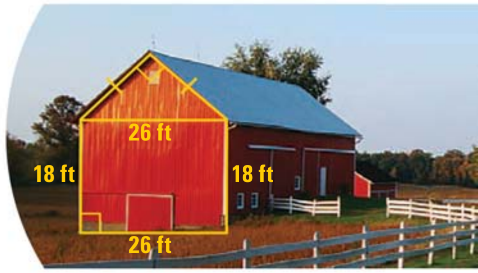
$36 = h^2$ **Simplify.**

$6 = h$ **Find positive square root of each side.**

▶ The height of the triangle is 6 inches, and the base is $6 \cdot 2 = 12$ inches.

EXAMPLE 3 Solve a multi-step problem

PAINTING You need to buy paint so that you can paint the side of a barn. A gallon of paint covers 350 square feet. How many gallons should you buy?



Solution

You can use a right triangle and a rectangle to approximate the area of the side of the barn.

ANOTHER WAY
In Example 3, you have a 45°-45°-90° triangle, so you can also find x by using trigonometry or special right angles.

STEP 1 **Find** the length x of each leg of the triangle.

$26^2 = x^2 + x^2$ **Use Pythagorean Theorem.**

$676 = 2x^2$ **Simplify.**

$\sqrt{338} = x$ **Solve for the positive value of x.**

STEP 2 **Find** the approximate area of the side of the barn.

Area = **Area of rectangle** + **Area of triangle**

$= 26(18) + \frac{1}{2} \cdot \left[(\sqrt{338})(\sqrt{338})\right] = 637 \text{ ft}^2$

STEP 3 **Determine** how many gallons of paint you need.

$637 \cancel{\text{ft}^2} \cdot \frac{1 \text{ gal}}{350 \cancel{\text{ft}^2}} \approx 1.82 \text{ gal}$ **Use unit analysis.**

▶ Round up so you will have enough paint. You need to buy 2 gallons of paint.

✓ GUIDED PRACTICE for Examples 2 and 3

4. A parallelogram has an area of 153 square inches and a height of 17 inches. What is the length of the base?

5. **WHAT IF?** In Example 3, suppose there is a 5 foot by 10 foot rectangular window on the side of the barn. What is the approximate area you need to paint?

11.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 23, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 21, 30, 39, and 45

SKILL PRACTICE

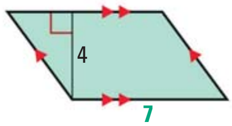
1. VOCABULARY Copy and complete: Either pair of parallel sides of a parallelogram can be called its __?__, and the perpendicular distance between these sides is called the __?__.

2. ★ WRITING What are the two formulas you have learned for the area of a rectangle? *Explain* why these formulas give the same results.

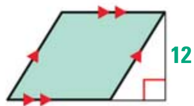
EXAMPLE 1 on p. 721 for Exs. 3–15

FINDING AREA Find the area of the polygon.

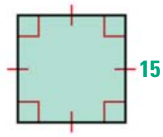
3.



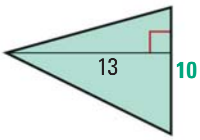
4.



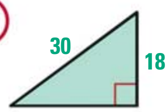
5.



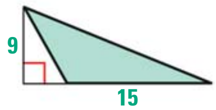
6.



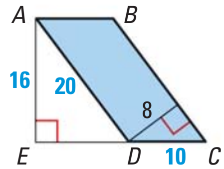
7.



8.



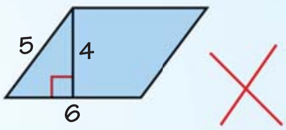
9. COMPARING METHODS Show two different ways to calculate the area of parallelogram *ABCD*. *Compare* your results.



ERROR ANALYSIS *Describe* and correct the error in finding the area of the parallelogram.

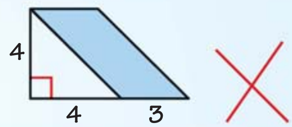
10.

A = bh
= (6)(5)
= 30



11.

A = bh
= (7)(4)
= 28



PYTHAGOREAN THEOREM The lengths of the hypotenuse and one leg of a right triangle are given. Find the perimeter and area of the triangle.

12. Hypotenuse: 15 in.; leg: 12 in.

13. Hypotenuse: 34 ft; leg: 16 ft

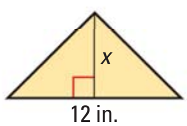
14. Hypotenuse: 85 m; leg: 84 m

15. Hypotenuse: 29 cm; leg: 20 cm

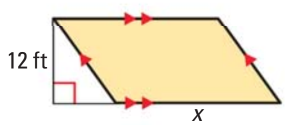
EXAMPLE 2 on p. 722 for Exs. 16–21

xy ALGEBRA Find the value of *x*.

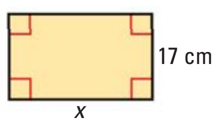
16. $A = 36 \text{ in.}^2$



17. $A = 276 \text{ ft}^2$



18. $A = 476 \text{ cm}^2$



19. *xy* **ALGEBRA** The area of a triangle is 4 square feet. The height of the triangle is half its base. Find the base and the height.

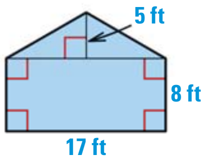
20. *xy* **ALGEBRA** The area of a parallelogram is 507 square centimeters, and its height is three times its base. Find the base and the height.

21. ★ **OPEN-ENDED MATH** A polygon has an area of 80 square meters and a height of 10 meters. Make scale drawings of three different triangles and three different parallelograms that match this description. Label the base and the height.

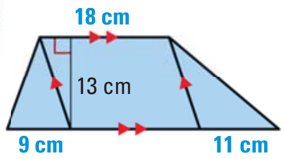
EXAMPLE 3
on p. 722
for Exs. 22–27

FINDING AREA **Find the area of the shaded polygon.**

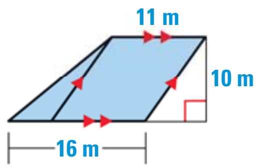
22.



23.



24.



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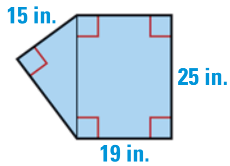
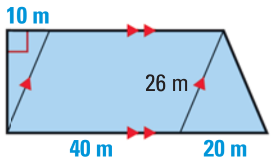
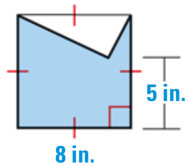


26.



27.



COORDINATE GRAPHING **Graph the points and connect them to form a polygon. Find the area of the polygon.**

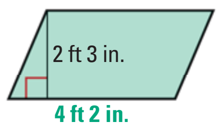
28. $A(3, 3)$, $B(10, 3)$, $C(8, -3)$, $D(1, -3)$

29. $E(-2, -2)$, $F(5, 1)$, $G(3, -2)$

30. ★ **MULTIPLE CHOICE** What is the area of the parallelogram shown at the right?

Ⓐ 8 ft^2 6 in.^{2} Ⓑ 1350 in.

Ⓒ 675 in.^{2} Ⓓ 9.375 ft^2



31. **TECHNOLOGY** Use geometry drawing software to draw a line ℓ and a line m parallel to ℓ. Then draw $\triangle ABC$ so that C is on line ℓ and $\overline{AB}$ is on line m. Find the base AB, the height CD, and the area of $\triangle ABC$. Move point C to change the shape of $\triangle ABC$. What do you notice about the base, height, and area of $\triangle ABC$?

32. **USING TRIGONOMETRY** In $\square ABCD$, base AD is 15 and AB is 8. What are the height and area of $\square ABCD$ if $m\angle DAB$ is 20°? if $m\angle DAB$ is 50°?

33. *xy* **ALGEBRA** Find the area of a right triangle with side lengths 12 centimeters, 35 centimeters, and 37 centimeters. Then find the length of the altitude drawn to the hypotenuse.

34. *xy* **ALGEBRA** Find the area of a triangle with side lengths 5 feet, 5 feet, and 8 feet. Then find the lengths of all three altitudes of the triangle.

35. **CHALLENGE** The vertices of quadrilateral $ABCD$ are $A(2, -2)$, $B(6, 4)$, $C(-1, 5)$, and $D(-5, 2)$. Without using the Distance Formula, find the area of $ABCD$. Show your steps.

PROBLEM SOLVING

36. **SAILING** Sails A and B are right triangles. The lengths of the legs of Sail A are 65 feet and 35 feet. The lengths of the legs of Sail B are 29.5 feet and 10.5 feet. Find the area of each sail to the nearest square foot. About how many times as great is the area of Sail A as the area of Sail B?

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EXAMPLE 3 on p. 722 for Ex. 37

37. **MOWING** You can mow 10 square yards of grass in one minute. How long does it take you to mow a triangular plot with height 25 yards and base 24 yards? How long does it take you to mow a rectangular plot with base 24 yards and height 36 yards?

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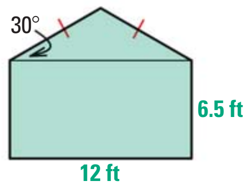
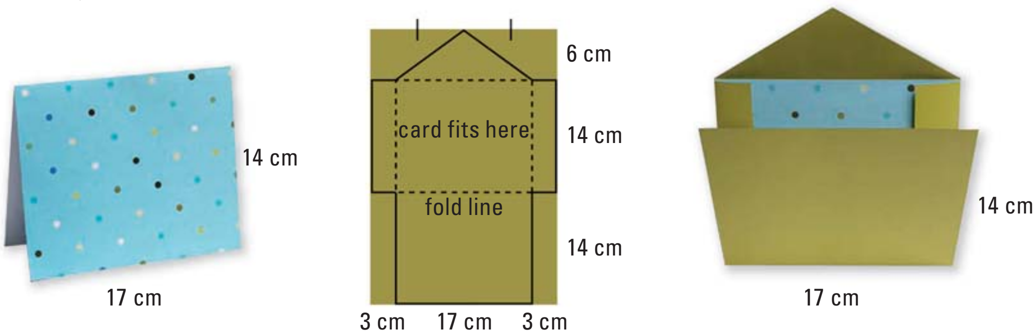
38. **CARPENTRY** You are making a table in the shape of a parallelogram to replace an old 24 inch by 15 inch rectangular table. You want the areas of two tables to be equal. The base of the parallelogram is 20 inches. What should the height be?

39. ★ **SHORT RESPONSE** A *4 inch square* is a square that has a side length of 4 inches. Does a 4 inch square have an area of 4 square inches? If not, what size square does have an area of 4 square inches? *Explain*.

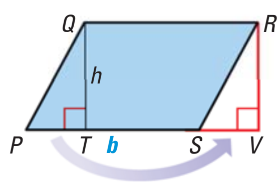
40. **PAINTING** You are earning money by painting a shed. You plan to paint two sides of the shed today. Each of the two sides has the dimensions shown at the right. You can paint 200 square feet per hour, and you charge $20 per hour. How much will you get paid for painting those two sides of the shed?



41. **ENVELOPES** The pattern below shows how to make an envelope to fit a card that is 17 centimeters by 14 centimeters. What are the dimensions of the rectangle you need to start with? What is the area of the paper that is actually used in the envelope? of the paper that is cut off?



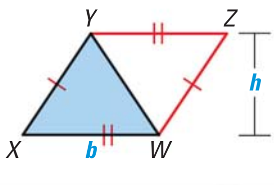
42. **JUSTIFYING THEOREM 11.2** You can use the area formula for a rectangle to justify the area formula for a parallelogram. First draw $\square PQRS$ with base b and height h, as shown. Then draw a segment perpendicular to $\overleftrightarrow{PS}$ through point R. Label point V.



   a. In the diagram, *explain* how you know that $\triangle PQT \cong \triangle SRV$.

   b. *Explain* how you know that the area of $PQRS$ is equal to the area of $QRVT$. How do you know that Area of $PQRS = bh$?

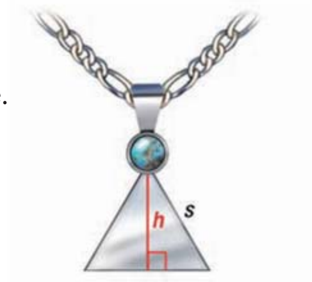
43. JUSTIFYING THEOREM 11.3 You can use the area formula for a parallelogram to justify the area formula for a triangle. Start with two congruent triangles with base b and height h. Place and label them as shown. *Explain* how you know that $XYZW$ is a parallelogram and that Area of $\triangle XYW = \frac{1}{2}bh$.



44. MULTI-STEP PROBLEM You have enough silver to make a pendant with an area of 4 square centimeters. The pendant will be an equilateral triangle. Let s be the side length of the triangle.

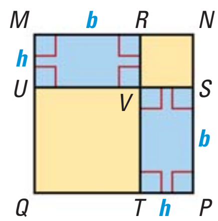
a. Find the height h of the triangle in terms of s. Then write a formula for the area of the triangle in terms of s.

b. Find the side length of the triangle. Round to the nearest centimeter.



45. ★ EXTENDED RESPONSE The base of a parallelogram is 7 feet and the height is 3 feet. *Explain* why the perimeter cannot be determined from the given information. Is there a least possible perimeter for the parallelogram? Is there a greatest possible perimeter? *Explain*.

46. JUSTIFYING THEOREM 11.1 You can use the diagram to show that the area of a rectangle is the product of its base b and height h.

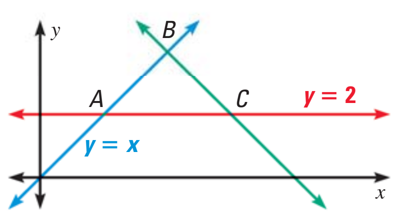


a. Figures $MRVU$ and $VSPT$ are congruent rectangles with base b and height h. *Explain* why $RNSV$, $UVTQ$, and $MNPQ$ are squares. Write expressions in terms of b and h for the areas of the squares.

b. Let A be the area of $MRVU$. Substitute A and the expressions from part (a) into the equation below. Solve to find an expression for A.

Area of $MNPQ$ = Area of $MRVU$ + Area of $UVTQ$ + Area of $RNSV$ + Area of $VSPT$

47. CHALLENGE An equation of $\overleftrightarrow{AB}$ is $y = x$. An equation of $\overleftrightarrow{AC}$ is $y = 2$. Suppose $\overleftrightarrow{BC}$ is placed so that $\triangle ABC$ is isosceles with an area of 4 square units. Find two different lines that fit these conditions. Give an equation for each line. Is there another line that could fit this requirement for $\overleftrightarrow{BC}$? *Explain*.

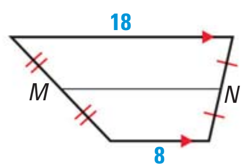


MIXED REVIEW

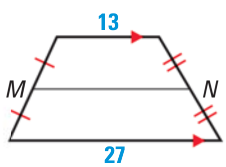
PREVIEW
Prepare for Lesson 11.2 in Exs. 48–50.

Find the length of the midsegment $\overline{MN}$ of the trapezoid. *(p. 542)*

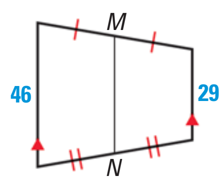
48.



49.



50.



The coordinates of $\triangle PQR$ are $P(-4, 1)$, $Q(2, 5)$, and $R(1, -4)$. Graph the image of the triangle after the translation. Use prime notation. *(p. 572)*

51. $(x, y) \rightarrow (x + 1, y + 4)$

52. $(x, y) \rightarrow (x + 3, y - 5)$

53. $(x, y) \rightarrow (x - 3, y - 2)$

54. $(x, y) \rightarrow (x - 2, y + 3)$

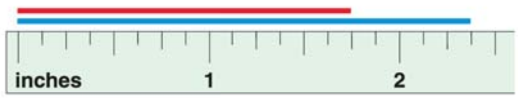
Extension
Use after Lesson 11.1

Determine Precision and Accuracy

GOAL Determine the precision and accuracy of measurements.

Key Vocabulary
- **unit of measure**
- **greatest possible error**
- **relative error**

All measurements are approximations. The length of each segment below, *to the nearest inch*, is 2 inches. The measurement is to the nearest inch, so the **unit of measure** is 1 inch.



If you are told that an object is 2 inches long, you know that its exact length is between $1\frac{1}{2}$ inches and $2\frac{1}{2}$ inches, or within $\frac{1}{2}$ inch of 2 inches. The **greatest possible error** of a measurement is equal to one half of the unit of measure.

When the unit of measure is smaller, the greatest possible error is smaller and the measurement is *more precise*. Using one-eighth inch as the unit of measure for the segments above gives lengths of $1\frac{6}{8}$ inches and $2\frac{3}{8}$ inches and a greatest possible error of $\frac{1}{16}$ inch.

EXAMPLE 1 Find greatest possible error

AMUSEMENT PARK The final drop of a log flume ride is listed in the park guide as 52.3 feet. Find the unit of measure and the greatest possible error.

Solution

The measurement 52.3 feet is given to the nearest tenth of a foot. So, the unit of measure is $\frac{1}{10}$ foot. The greatest possible error is half the unit of measure. Because $\frac{1}{2}\left(\frac{1}{10}\right) = \frac{1}{20} = 0.05$, the greatest possible error is 0.05 foot.

READ VOCABULARY
The *precision* of a measurement depends only on the unit of measure. The *accuracy* of a measurement depends on both the unit of measure and on the size of the object being measured.

RELATIVE ERROR The diameter of a bicycle tire is 26 inches. The diameter of a key ring is 1 inch. In each case, the greatest possible error is $\frac{1}{2}$ inch, but a half-inch error has a much greater effect on the diameter of a smaller object. The **relative error** of a measurement is the ratio $\frac{\text{greatest possible error}}{\text{measured length}}$.

Bicycle tire diameter	Key ring diameter
Rel. error $= \frac{0.5\text{ in.}}{26\text{ in.}} \approx 0.01923 \approx 1.9\%$	Rel. error $= \frac{0.5\text{ in.}}{1\text{ in.}} = 0.5 = 50\%$

The measurement with the smaller relative error is said to be *more accurate*.

EXAMPLE 2 Find relative error

PLAYING AREAS An air hockey table is 3.7 feet wide. An ice rink is 85 feet wide. Find the relative error of each measurement. Which measurement is more accurate?

	Air hockey table (3.7 feet)	Ice rink (85 feet)
Unit of measure	0.1 ft	1 ft
Greatest possible error $\frac{1}{2} \cdot$ (unit of measure)	$\frac{1}{2}(0.1 \text{ ft}) = 0.05 \text{ ft}$	$\frac{1}{2}(1 \text{ ft}) = 0.5 \text{ ft}$
Relative error $\frac{\text{greatest possible error}}{\text{measured length}}$	$\frac{0.05 \text{ ft}}{3.7 \text{ ft}} \approx 0.0135 \approx 1.4\%$	$\frac{0.5 \text{ ft}}{85 \text{ ft}} \approx 0.00588 \approx 0.6\%$

▶ The ice rink width has the smaller relative error, so it is more accurate.

PRACTICE

1. **VOCABULARY** *Describe* the difference between the *precision* of a measurement and the *accuracy* of a measurement. Give an example that illustrates the difference.

EXAMPLE 1 on p. 727 for Exs. 2–5

GREATEST POSSIBLE ERROR **Find the unit of measure. Then find the greatest possible error.**

2. 14.6 in.
3. 6 m
4. 8.217 km
5. $4\frac{5}{16}$ yd

EXAMPLE 2 on p. 728 for Exs. 6–9

RELATIVE ERROR **Find the relative error of the measurement.**

6. 4.0 cm
7. 28 in.
8. 4.6 m
9. 12.16 mm

10. **CHOOSING A UNIT** You are estimating the amount of paper needed to make book covers for your textbooks. Which unit of measure, 1 foot, 1 inch, or $\frac{1}{16}$ inch, should you use to measure your textbooks? *Explain*.

11. **REASONING** The greatest possible error of a measurement is $\frac{1}{16}$ inch. *Explain* how such a measurement could be more accurate in one situation than in another situation.

PRECISION AND ACCURACY **Tell which measurement is more precise. Then tell which of the two measurements is more accurate.**

12. 17 cm; 12 cm
13. 18.65 ft; 25.6 ft
14. 6.8 in.; 13.4 ft
15. 3.5 ft; 35 in.

16. **PERIMETER** A side of the eraser shown is a parallelogram. What is the greatest possible error for the length of each side of the parallelogram? for the perimeter of the parallelogram? Find the greatest and least possible perimeter of the parallelogram.

Investigating Geometry ACTIVITY *Use before Lesson 11.2*

11.2 Areas of Trapezoids and Kites

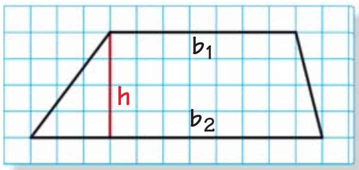
MATERIALS • graph paper • straightedge • scissors • tape

QUESTION How can you use a parallelogram to find other areas?

A trapezoid or a kite can be cut out and rearranged to form a parallelogram.

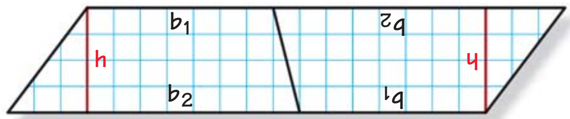
EXPLORE 1 Use two congruent trapezoids to form a parallelogram

STEP 1



Draw a trapezoid Fold graph paper in half and draw a trapezoid. Cut out two congruent trapezoids. Label as shown.

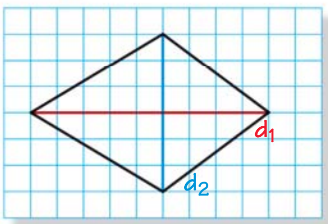
STEP 2



Create a parallelogram Arrange the two trapezoids from Step 1 to form a parallelogram. Then tape them together.

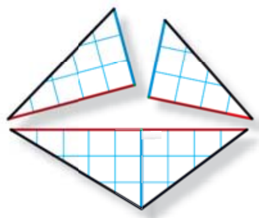
EXPLORE 2 Use one kite to form a rectangle

STEP 1



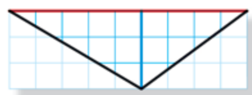
Draw a kite Draw a kite and its perpendicular diagonals. Label the diagonal that is a line of symmetry d_1. Label the other diagonal d_2.

STEP 2



Cut triangles Cut out the kite. Cut along d_1 to form two congruent triangles. Then cut one triangle along part of d_2 to form two right triangles.

STEP 3



Create a rectangle Turn over the right triangles. Place each with its hypotenuse along a side of the larger triangle to form a rectangle. Then tape the pieces together.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. In Explore 1, how does the area of one trapezoid compare to the area of the parallelogram formed from two trapezoids? Write expressions in terms of b_1, b_2, and h for the base, height, and area of the parallelogram. Then write a formula for the area of a trapezoid.

2. In Explore 2, how do the base and height of the rectangle compare to d_1 and d_2? Write an expression for the area of the rectangle in terms of d_1 and d_2. Then use that expression to write a formula for the area of a kite.

11.2 Areas of Trapezoids, Rhombuses, and Kites

Before You found areas of triangles and parallelograms.

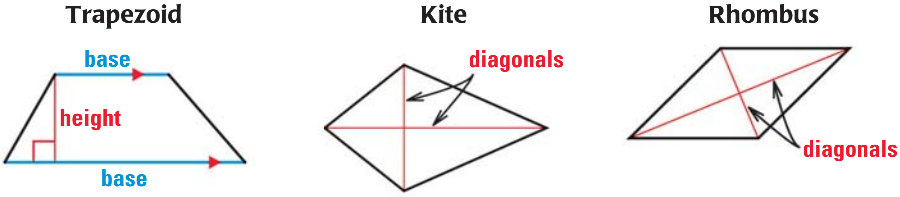
Now You will find areas of other types of quadrilaterals.

Why? So you can find the area of a free-throw lane, as in Example 1.

Key Vocabulary
- **height of a trapezoid**
- **diagonal,** *p. 507*
- **bases of a trapezoid,** *p. 542*

As you saw in the Activity on page 729, you can use the area formula for a parallelogram to develop area formulas for other special quadrilaterals. The areas of the figures below are related to the lengths of the marked segments.

The **height of a trapezoid** is the perpendicular distance between its bases.

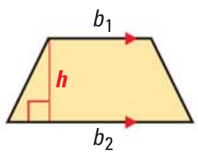


THEOREM *For Your Notebook*

THEOREM 11.4 Area of a Trapezoid

The area of a trapezoid is one half the product of the height and the sum of the lengths of the bases.

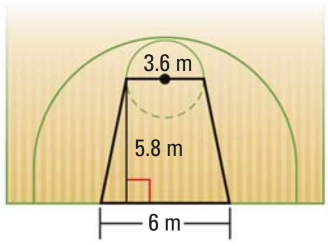
Proof: Ex. 40, p. 736



$A = \frac{1}{2}h(b_1 + b_2)$

EXAMPLE 1 Find the area of a trapezoid

BASKETBALL The free-throw lane on an international basketball court is shaped like a trapezoid. Find the area of the free-throw lane.



Solution

The height of the trapezoid is 5.8 meters. The lengths of the bases are 3.6 meters and 6 meters.

$A = \frac{1}{2}h(b_1 + b_2)$ **Formula for area of a trapezoid**

$= \frac{1}{2}(5.8)(3.6 + 6)$ **Substitute 5.8 for *h*, 3.6 for b_1, and 6 for b_2.**

$= 27.84$ **Simplify.**

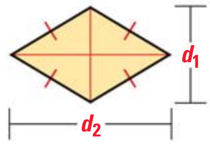
▶ The area of the free-throw lane is about 27.8 square meters.

ANOTHER WAY
In a trapezoid, the average of the lengths of the bases is also the length of the midsegment. So, you can also find the area by multiplying the midsegment by the height.

THEOREMS *For Your Notebook*

THEOREM 11.5 Area of a Rhombus

The area of a rhombus is one half the product of the lengths of its diagonals.

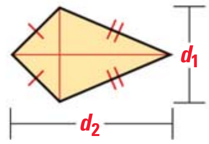


$A = \frac{1}{2}d_1d_2$

Justification: Ex. 39, p. 735

THEOREM 11.6 Area of a Kite

The area of a kite is one half the product of the lengths of its diagonals.



$A = \frac{1}{2}d_1d_2$

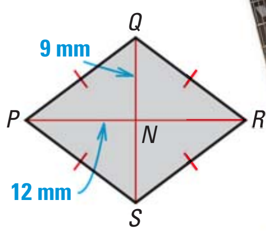
Proof: Ex. 41, p. 736

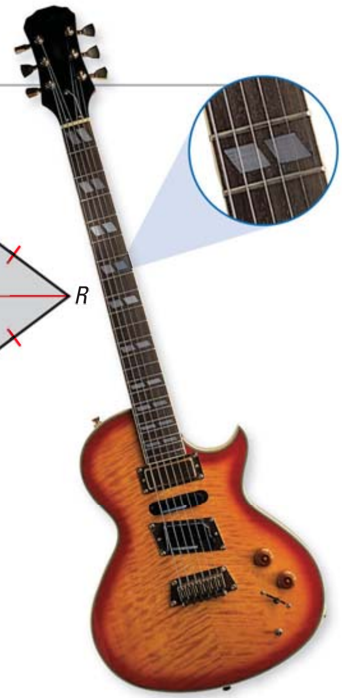
ANOTHER WAY

Remember that a rhombus is also a parallelogram, so you can also use the formula $A = bh$.

EXAMPLE 2 Find the area of a rhombus

MUSIC Rhombus $PQRS$ represents one of the inlays on the guitar in the photo. Find the area of the inlay.





Solution

STEP 1 **Find** the length of each diagonal. The diagonals of a rhombus bisect each other, so $QN = NS$ and $PN = NR$.

$QS = QN + NS = 9 + 9 = 18$ mm

$PR = PN + NR = 12 + 12 = 24$ mm

STEP 2 **Find** the area of the rhombus. Let d_1 represent QS and d_2 represent PR.

$A = \frac{1}{2}d_1d_2$ **Formula for area of a rhombus**

$= \frac{1}{2}(18)(24)$ **Substitute.**

$= 216$ **Simplify.**

▶ The area of the inlay is 216 square millimeters.

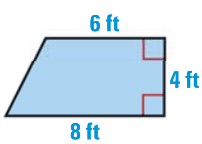
READ DIAGRAMS

When you read a diagram, look for information you need to find. The diagram gives the lengths of $\overline{QN}$ and $\overline{PN}$, but not the lengths of $\overline{QS}$ and $\overline{PR}$.

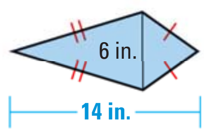
✓ GUIDED PRACTICE for Examples 1 and 2

Find the area of the figure.

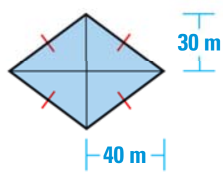
1.



2.



3.



★ EXAMPLE 3 Standardized Test Practice

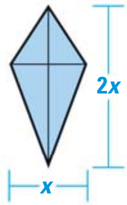
One diagonal of a kite is twice as long as the other diagonal. The area of the kite is 72.25 square inches. What are the lengths of the diagonals?

Ⓐ 6 in., 6 in. Ⓑ 8.5 in., 8.5 in. Ⓒ 8.5 in., 17 in. Ⓓ 6 in., 12 in.

ELIMINATE CHOICES
In Example 3, you can eliminate choices A and B because in each case, one diagonal is not twice as long as the other diagonal.

Solution

Draw and label a diagram. Let x be the length of one diagonal. The other diagonal is twice as long, so label it $2x$. Use the formula for the area of a kite to find the value of x.



$A = \frac{1}{2}d_1d_2$ **Formula for area of a kite**

$72.25 = \frac{1}{2}(x)(2x)$ **Substitute 72.25 for A, x for d_1, and $2x$ for d_2.**

$72.25 = x^2$ **Simplify.**

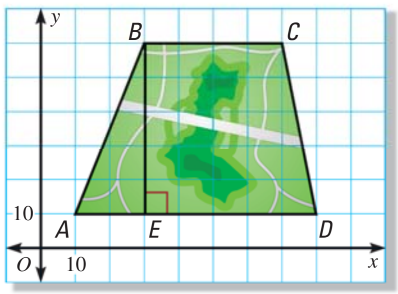
$8.5 = x$ **Find the positive square root of each side.**

The lengths of the diagonals are 8.5 inches and $2(8.5) = 17$ inches.

▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

EXAMPLE 4 Find an area in the coordinate plane

CITY PLANNING You have a map of a city park. Each grid square represents a 10 meter by 10 meter square. Find the area of the park.



Solution

STEP 1 **Find** the lengths of the bases and the height of trapezoid $ABCD$.

$b_1 = BC = |70 - 30| = 40$ m

$b_2 = AD = |80 - 10| = 70$ m

$h = BE = |60 - 10| = 50$ m

STEP 2 **Find** the area of $ABCD$.

$$A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(50)(40 + 70) = 2750$$

▶ The area of the park is 2750 square meters.

✓ GUIDED PRACTICE for Examples 3 and 4

4. The area of a kite is 80 square feet. One diagonal is 4 times as long as the other. Find the diagonal lengths.

5. Find the area of a rhombus with vertices $M(1, 3)$, $N(5, 5)$, $P(9, 3)$, and $Q(5, 1)$.

11.2 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 17, and 35

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 30, 39, and 42

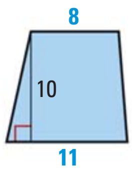
SKILL PRACTICE

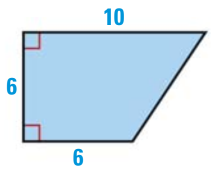
1. **VOCABULARY** Copy and complete: The perpendicular distance between the bases of a trapezoid is called the _?_ of the trapezoid.

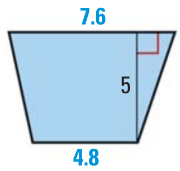
2. ★ **WRITING** Sketch a kite and its diagonals. *Describe* what you know about the segments and angles formed by the intersecting diagonals.

EXAMPLE 1 on p. 730 for Exs. 3–6

FINDING AREA **Find the area of the trapezoid.**

3. 

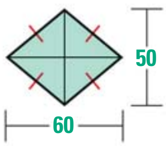
4. 

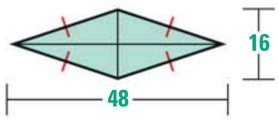
5. 

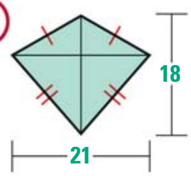
6. **DRAWING DIAGRAMS** The lengths of the bases of a trapezoid are 5.4 centimeters and 10.2 centimeters. The height is 8 centimeters. Draw and label a trapezoid that matches this description. Then find its area.

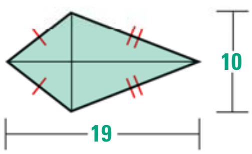
EXAMPLE 2 on p. 731 for Exs. 7–14

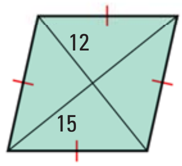
FINDING AREA **Find the area of the rhombus or kite.**

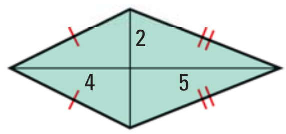
7. 

8. 

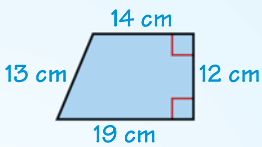
9. 

10. 

11. 

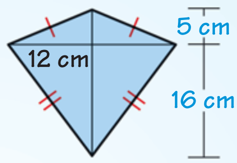
12. 

ERROR ANALYSIS ***Describe*** **and correct the error in finding the area.**

13. 

$A = \frac{1}{2}(13)(14 + 19)$

$= 214.5 \text{ cm}^2$

14. 

$A = \frac{1}{2}(12)(21)$

$= 126 \text{ cm}^2$

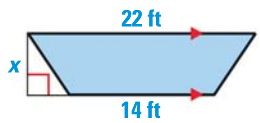
EXAMPLE 3 on p. 732 for Exs. 15–18

15. ★ **MULTIPLE CHOICE** One diagonal of a rhombus is three times as long as the other diagonal. The area of the rhombus is 24 square feet. What are the lengths of the diagonals?

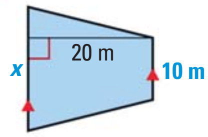
Ⓐ 8 ft, 11 ft Ⓑ 4 ft, 12 ft Ⓒ 2 ft, 6 ft Ⓓ 6 ft, 24 ft

ALGEBRA **Use the given information to find the value of x.**

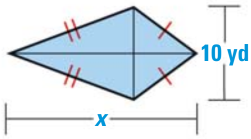
16. Area $= 108\text{ ft}^2$



17. Area $= 300\text{ m}^2$



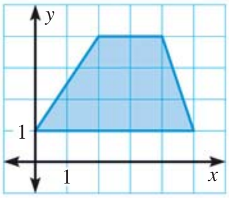
18. Area $= 100\text{ yd}^2$



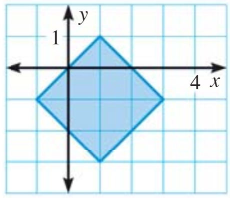
EXAMPLE 4
on p. 732
for Exs. 19–21

COORDINATE GEOMETRY **Find the area of the figure.**

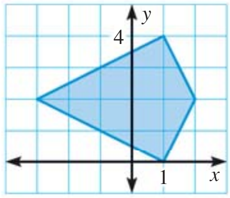
19.



20.



21.



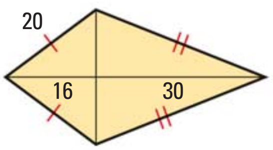
ALGEBRA **Find the lengths of the bases of the trapezoid described.**

22. The height is 3 feet. One base is twice as long as the other base. The area is 13.5 square feet.

23. One base is 8 centimeters longer than the other base. The height is 6 centimeters and the area is 54 square centimeters.

FINDING AREA **Find the area of the shaded region.**

24.



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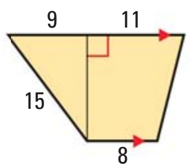

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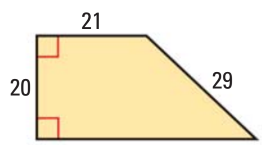

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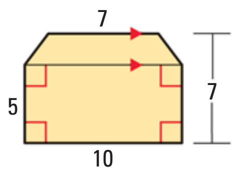

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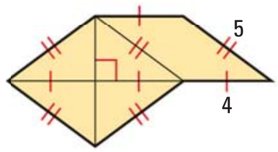
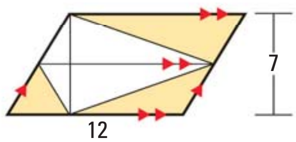


29.



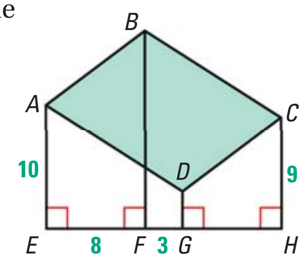
30. ★ **OPEN-ENDED MATH** Draw three examples of trapezoids that match this description: The height of the trapezoid is 3 units and its area is the same as the area of a parallelogram with height 3 units and base 8 units.

VISUALIZING **Sketch the figure. Then determine its perimeter and area.**

31. The figure is a trapezoid. It has two right angles. The lengths of its bases are 7 and 15. Its height is 6.

32. The figure is a rhombus. Its side length is 13. The length of one of its diagonals is 24.

33. **CHALLENGE** In the diagram shown at the right, $ABCD$ is a parallelogram and $BF = 16$. Find the area of $\square ABCD$. *Explain* your reasoning. (*Hint:* Draw auxiliary lines through point A and through point D that are parallel to $\overline{EH}$.)



◯ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

EXAMPLE 1 on p. 730 for Ex. 34

34. TRUCKS The windshield in a truck is in the shape of a trapezoid. The lengths of the bases of the trapezoid are 70 inches and 79 inches. The height is 35 inches. Find the area of the glass in the windshield.

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EXAMPLE 2 on p. 731 for Ex. 35

35. INTERNET You are creating a kite-shaped logo for your school's website. The diagonals of the logo are 8 millimeters and 5 millimeters long. Find the area of the logo. Draw two different possible shapes for the logo.

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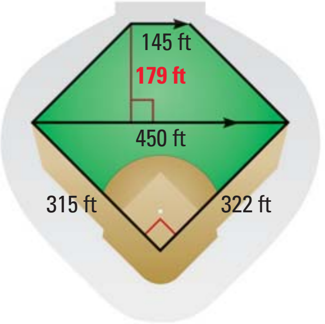
36. DESIGN You are designing a wall hanging that is in the shape of a rhombus. The area of the wall hanging is 432 square inches and the length of one diagonal is 36 inches. Find the length of the other diagonal.

37. MULTI-STEP PROBLEM As shown, a baseball stadium's playing field is shaped like a pentagon. To find the area of the playing field shown at the right, you can divide the field into two smaller polygons.

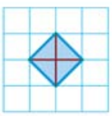
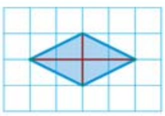
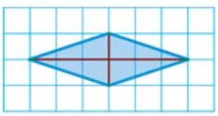
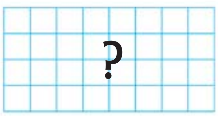


a. Classify the two polygons.

b. Find the area of the playing field in square feet. Then express your answer in square yards. Round to the nearest square foot.

38. VISUAL REASONING Follow the steps in parts (a)–(c).

a. **Analyze** Copy the table and extend it to include a column for $n = 5$. Complete the table for $n = 4$ and $n = 5$.

Rhombus number, *n*	1	2	3	4
Diagram				 ?
Area, *A*	2	4	6	?

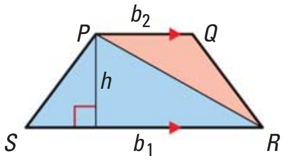
b. **Use Algebra** *Describe* the relationship between the rhombus number n and the area of the rhombus. Then write an algebraic rule for finding the area of the nth rhombus.

c. **Compare** In each rhombus, the length of one diagonal (d_1) is 2. What is the length of the other diagonal (d_2) for the nth rhombus? Use the formula for the area of a rhombus to write a rule for finding the area of the nth rhombus. *Compare* this rule with the one you wrote in part (b).

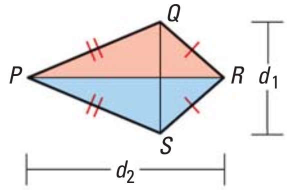
39. ★ SHORT RESPONSE Look back at the Activity on page 729. *Explain* how the results for kites in Explore 2 can be used to justify Theorem 11.5, the formula for the area of a rhombus.

PROVING THEOREMS 11.4 AND 11.6 **Use the triangle area formula and the triangles in the diagram to write a plan for the proof.**

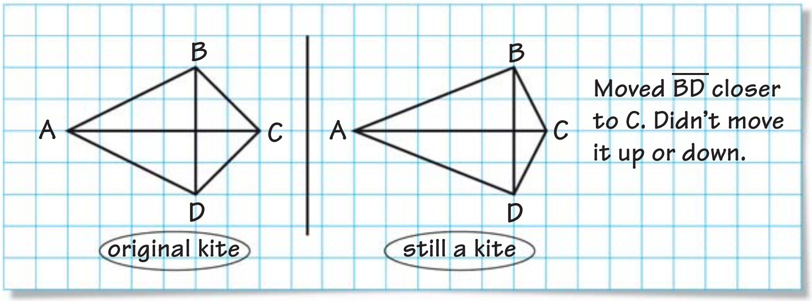
40. Show that the area A of the trapezoid shown is $\frac{1}{2}h(b_1 + b_2)$.



41. Show that the area A of the kite shown is $\frac{1}{2}d_1d_2$.



42. ★ **EXTENDED RESPONSE** You will explore the effect of moving a diagonal.

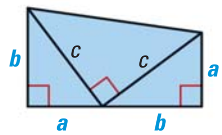


a. Investigate Draw a kite in which the longer diagonal is horizontal. Suppose this diagonal is fixed and you can slide the vertical diagonal left or right and up or down. You can keep sliding as long as the diagonals continue to intersect. Draw and identify each type of figure you can form.

b. Justify Is it possible to form any shapes that are not quadrilaterals? *Explain.*

c. Compare Compare the areas of the different shapes you found in part (b). What do you notice about the areas? *Explain.*

43. **CHALLENGE** James A. Garfield, the twentieth president of the United States, discovered a proof of the Pythagorean Theorem in 1876. His proof involved the fact that a trapezoid can be formed from two congruent right triangles and an isosceles right triangle. Use the diagram to show that $a^2 + b^2 = c^2$.



MIXED REVIEW

Solve for the indicated variable. Write a reason for each step. *(p. 105)*

44. $d = rt$; solve for t

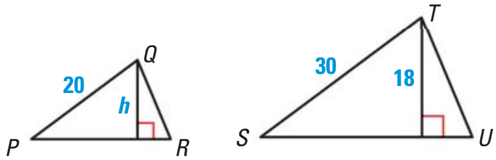
45. $A = \frac{1}{2}bh$; solve for h

46. $P = 2\ell + 2w$; solve for w

47. Find the angle measures of an isosceles triangle if the measure of a base angle is 4 times the measure of the vertex angle. *(p. 264)*

PREVIEW
Prepare for Lesson 11.3 in Ex. 48.

48. In the diagram at the right, $\triangle PQR \sim \triangle STU$. The perimeter of $\triangle STU$ is 81 inches. Find the height h and the perimeter of $\triangle PQR$. *(p. 372)*



11.3 Perimeter and Area of Similar Figures

Before You used ratios to find perimeters of similar figures.

Now You will use ratios to find areas of similar figures.

Why So you can apply similarity in cooking, as in Example 3.

Key Vocabulary
- **regular polygon,** *p. 43*
- **corresponding sides,** *p. 225*
- **similar polygons,** *p. 372*

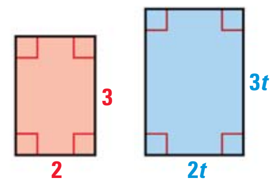
In Chapter 6 you learned that if two polygons are similar, then the ratio of their perimeters, or of any two corresponding lengths, is equal to the ratio of their corresponding side lengths. As shown below, the areas have a different ratio.

Ratio of perimeters

$$\frac{\text{Blue}}{\text{Red}} = \frac{10t}{10} = t$$

Ratio of areas

$$\frac{\text{Blue}}{\text{Red}} = \frac{6t^2}{6} = t^2$$



THEOREM *For Your Notebook*

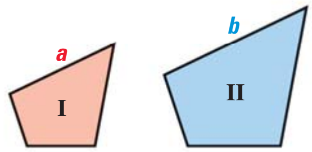
THEOREM 11.7 Areas of Similar Polygons

If two polygons are similar with the lengths of corresponding sides in the ratio of $a : b$, then the ratio of their areas is $a^2 : b^2$.

$$\frac{\text{Side length of Polygon I}}{\text{Side length of Polygon II}} = \frac{a}{b}$$

$$\frac{\text{Area of Polygon I}}{\text{Area of Polygon II}} = \frac{a^2}{b^2}$$

Justification: Ex. 30, p. 742



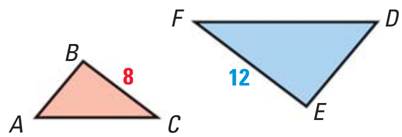
Polygon I ~ Polygon II

EXAMPLE 1 Find ratios of similar polygons

In the diagram, $\triangle ABC \sim \triangle DEF$. Find the indicated ratio.

a. Ratio (red to blue) of the perimeters

b. Ratio (red to blue) of the areas



INTERPRET RATIOS
You can also compare the measures with fractions. The perimeter of $\triangle ABC$ is two thirds of the perimeter of $\triangle DEF$. The area of $\triangle ABC$ is four ninths of the area of $\triangle DEF$.

Solution

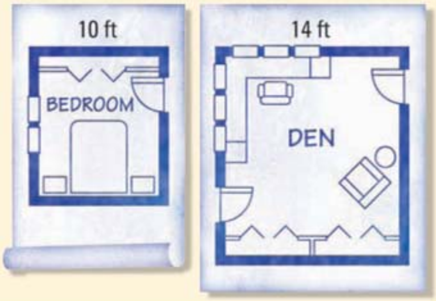
The ratio of the lengths of corresponding sides is $\frac{8}{12} = \frac{2}{3}$, or $2:3$.

a. By Theorem 6.1 on page 374, the ratio of the perimeters is $2:3$.

b. By Theorem 11.7 above, the ratio of the areas is $2^2 : 3^2$, or $4:9$.

EXAMPLE 2 Standardized Test Practice

You are installing the same carpet in a bedroom and den. The floors of the rooms are similar. The carpet for the bedroom costs \$225. Carpet is sold by the square foot. How much does it cost to carpet the den?



Ⓐ \$115　　Ⓑ \$161

Ⓒ \$315　　Ⓓ \$441

USE ESTIMATION

The cost for the den is $\frac{49}{25}$ times the cost for the bedroom. Because $\frac{49}{25}$ is a little less than 2, the cost for the den is a little less than twice \$225. The only possible choice is D.

Solution

The ratio of a side length of the den to the corresponding side length of the bedroom is 14 : 10, or 7 : 5. So, the ratio of the areas is $7^2 : 5^2$, or 49 : 25. This ratio is also the ratio of the carpeting costs. Let x be the cost for the den.

$\frac{49}{25} = \frac{x}{225}$ ← **cost of carpet for den** / ← **cost of carpet for bedroom**

$x = 441$　　**Solve for x.**

▶ It costs \$441 to carpet the den. The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 1 and 2

1. The perimeter of $\triangle ABC$ is 16 feet, and its area is 64 feet. The perimeter of $\triangle DEF$ is 12 feet. Given $\triangle ABC \sim \triangle DEF$, find the ratio of the area of $\triangle ABC$ to the area of $\triangle DEF$. Then find the area of $\triangle DEF$.

EXAMPLE 3 Use a ratio of areas

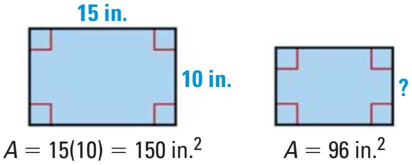
COOKING A large rectangular baking pan is 15 inches long and 10 inches wide. A smaller pan is similar to the large pan. The area of the smaller pan is 96 square inches. Find the width of the smaller pan.

ANOTHER WAY

For an alternative method for solving the problem in Example 3, turn to page 744 for the **Problem Solving Workshop**.

Solution

First draw a diagram to represent the problem. Label dimensions and areas.



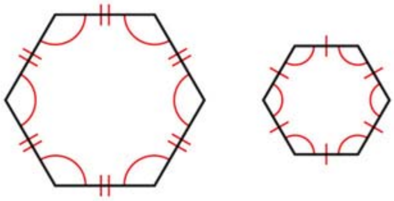
Then use Theorem 11.7. If the area ratio is $a^2 : b^2$, then the length ratio is $a : b$.

$\frac{\text{Area of smaller pan}}{\text{Area of large pan}} = \frac{96}{150} = \frac{16}{25}$　　**Write ratio of known areas. Then simplify.**

$\frac{\text{Length in smaller pan}}{\text{Length in large pan}} = \frac{4}{5}$　　**Find square root of area ratio.**

▶ Any length in the smaller pan is $\frac{4}{5}$, or 0.8, of the corresponding length in the large pan. So, the width of the smaller pan is 0.8(10 inches) = 8 inches.

REGULAR POLYGONS Consider two regular polygons with the same number of sides. All of the angles are congruent. The lengths of all pairs of corresponding sides are in the same ratio. So, any two such polygons are similar. Also, any two circles are similar.



EXAMPLE 4 Solve a multi-step problem

GAZEBO The floor of the gazebo shown is a regular octagon. Each side of the floor is 8 feet, and the area is about 309 square feet. You build a small model gazebo in the shape of a regular octagon. The perimeter of the floor of the model gazebo is 24 inches. Find the area of the floor of the model gazebo to the nearest tenth of a square inch.

Solution

All regular octagons are similar, so the floor of the model is similar to the floor of the full-sized gazebo.

ANOTHER WAY
In Step 1, instead of finding the perimeter of the full-sized and comparing perimeters, you can find the side length of the model and compare side lengths. $24 \div 8 = 3$, so the ratio of side lengths is $\frac{8\text{ ft.}}{3\text{ in.}} = \frac{96\text{ in.}}{3\text{ in.}} = \frac{32}{1}$.

STEP 1 **Find** the ratio of the lengths of the two floors by finding the ratio of the perimeters. Use the same units for both lengths in the ratio.

$$\frac{\text{Perimeter of full-sized}}{\text{Perimeter of model}} = \frac{8(8\text{ ft})}{24\text{ in.}} = \frac{64\text{ ft}}{24\text{ in.}} = \frac{64\text{ ft}}{2\text{ ft}} = \frac{32}{1}$$

So, the ratio of corresponding lengths (full-sized to model) is 32 : 1.

STEP 2 **Calculate** the area of the model gazebo's floor. Let x be this area.

$$\frac{(\text{Length in full-sized})^2}{(\text{Length in model})^2} = \frac{\text{Area of full-sized}}{\text{Area of model}} \qquad \textbf{Theorem 11.7}$$

$$\frac{32^2}{1^2} = \frac{309\text{ ft}^2}{x\text{ ft}^2} \qquad \textbf{Substitute.}$$

$$1024x = 309 \qquad \textbf{Cross Products Property}$$

$$x \approx 0.302\text{ ft}^2 \qquad \textbf{Solve for } \boldsymbol{x}\textbf{.}$$

STEP 3 **Convert** the area to square inches.

$$0.302\ \cancel{\text{ft}^2} \cdot \frac{144\text{ in.}^2}{1\ \cancel{\text{ft}^2}} \approx 43.5\text{ in.}^2$$

▶ The area of the floor of the model gazebo is about 43.5 square inches.

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✓ GUIDED PRACTICE for Examples 3 and 4

2. The ratio of the areas of two regular decagons is 20 : 36. What is the ratio of their corresponding side lengths in simplest radical form?

3. Rectangles I and II are similar. The perimeter of Rectangle I is 66 inches. Rectangle II is 35 feet long and 20 feet wide. Show the steps you would use to find the ratio of the areas and then find the area of Rectangle I.

11.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 17, and 27

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 12, 18, 28, 32, and 33

SKILL PRACTICE

1. **VOCABULARY** Sketch two similar triangles. Use your sketch to explain what is meant by *corresponding side lengths*.

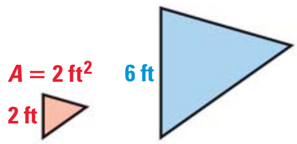
2. ★ **WRITING** Two regular n-gons are similar. The ratio of their side lengths is is 3 : 4. Do you need to know the value of n to find the ratio of the perimeters or the ratio of the areas of the polygons? *Explain*.

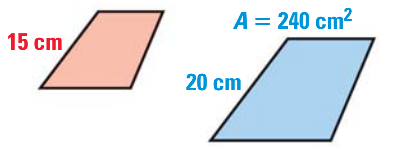
EXAMPLES 1 and 2 on pp. 737–738 for Exs. 3–8

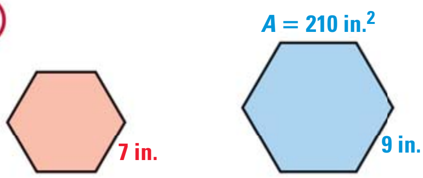
FINDING RATIOS **Copy and complete the table of ratios for similar polygons.**

	Ratio of corresponding side lengths	Ratio of perimeters	Ratio of areas
3.	6 : 11	?	?
4.	?	20 : 36 = ?	?

RATIOS AND AREAS **Corresponding lengths in similar figures are given. Find the ratios (red to blue) of the perimeters and areas. Find the unknown area.**

5.


6.


7.


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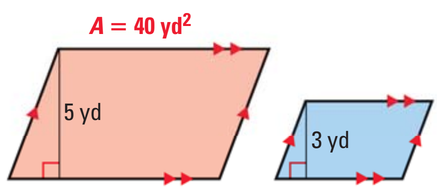


EXAMPLE 3 on p. 738 for Exs. 9–15

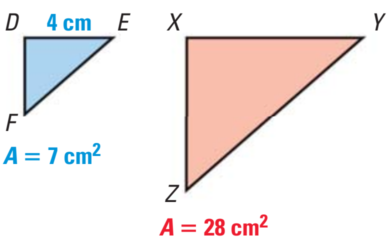
FINDING LENGTH RATIOS **The ratio of the areas of two similar figures is given. Write the ratio of the lengths of corresponding sides.**

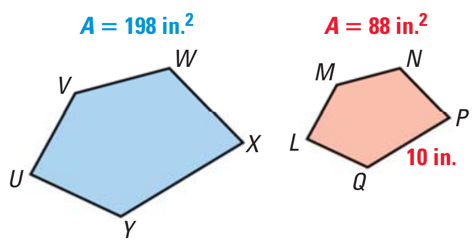
9. Ratio of areas = 49 : 16
10. Ratio of areas = 16 : 121
11. Ratio of areas = 121 : 144

12. ★ **MULTIPLE CHOICE** The area of $\triangle LMN$ is 18 ft^2 and the area of $\triangle FGH$ is 24 ft^2. If $\triangle LMN \sim \triangle FGH$, what is the ratio of LM to FG?

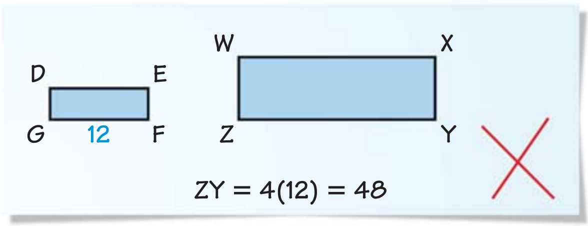
   (A) 3 : 4 (B) 9 : 16 (C) $\sqrt{3}$: 2 (D) 4 : 3

FINDING SIDE LENGTHS **Use the given area to find *XY*.**

13. $\triangle DEF \sim \triangle XYZ$


14. $UVWXY \sim LMNPQ$


15. **ERROR ANALYSIS** In the diagram, Rectangles *DEFG* and *WXYZ* are similar. The ratio of the area of *DEFG* to the area of *WXYZ* is 1 : 4. *Describe* and correct the error in finding *ZY*.



EXAMPLE 4 on p. 739 for Exs. 16–17

16. **REGULAR PENTAGONS** Regular pentagon *QRSTU* has a side length of 12 centimeters and an area of about 248 square centimeters. Regular pentagon *VWXYZ* has a perimeter of 140 centimeters. Find its area.

17. **RHOMBUSES** Rhombuses *MNPQ* and *RSTU* are similar. The area of *RSTU* is 28 square feet. The diagonals of *MNPQ* are 25 feet long and 14 feet long. Find the area of *MNPQ*. Then use the ratio of the areas to find the lengths of the diagonals of *RSTU*.

18. ★ **SHORT RESPONSE** You enlarge the same figure three different ways. In each case, the enlarged figure is similar to the original. List the enlargements in order from smallest to largest. *Explain.*

Case 1 The side lengths of the original figure are multiplied by 3.
Case 2 The perimeter of the original figure is multiplied by 4.
Case 3 The area of the original figure is multiplied by 5.

REASONING In Exercises 19 and 20, copy and complete the statement using *always*, *sometimes*, or *never*. *Explain* your reasoning.

19. Doubling the side length of a square __?__ doubles the area.

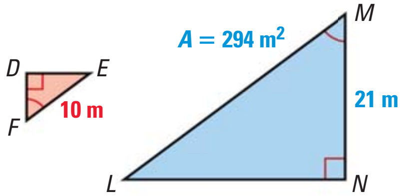
20. Two similar octagons __?__ have the same perimeter.

21. **FINDING AREA** The sides of $\triangle ABC$ are 4.5 feet, 7.5 feet, and 9 feet long. The area is about 17 square feet. *Explain* how to use the area of $\triangle ABC$ to find the area of a $\triangle DEF$ with side lengths 6 feet, 10 feet, and 12 feet.

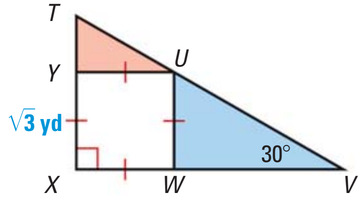
22. **RECTANGLES** Rectangles *ABCD* and *DEFG* are similar. The length of *ABCD* is 24 feet and the perimeter is 84 square feet. The width of *DEFG* is 3 yards. Find the ratio of the area of *ABCD* to the area of *DEFG*.

SIMILAR TRIANGLES *Explain* why the red and blue triangles are similar. Find the ratio (red to blue) of the areas of the triangles. Show your steps.

23.



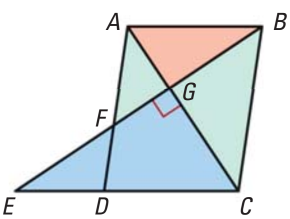
24.



25. **CHALLENGE** In the diagram shown, *ABCD* is a parallelogram. The ratio of the area of $\triangle AGB$ to the area of $\triangle CGE$ is 9 : 25, $CG = 10$, and $GE = 15$.

a. Find *AG*, *GB*, *GF*, and *FE*. Show your methods.

b. Give two area ratios other than 9 : 25 or 25 : 9 for pairs of similar triangles in the figure. *Explain.*



PROBLEM SOLVING

26. **BANNER** Two rectangular banners from this year's music festival are shown. Organizers of next year's festival want to design a new banner that will be similar to the banner whose dimensions are given in the photograph. The length of the longest side of the new banner will be 5 feet. Find the area of the new banner.

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EXAMPLE 3 on p. 738 for Ex. 27

27. **PATIO** A new patio will be an irregular hexagon. The patio will have two long parallel sides and an area of 360 square feet. The area of a similar shaped patio is 250 square feet, and its long parallel sides are 12.5 feet apart. What will be the corresponding distance on the new patio?

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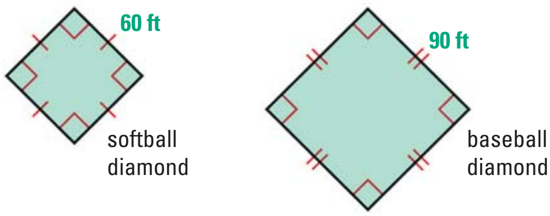
28. ★ **MULTIPLE CHOICE** You need 20 pounds of grass seed to plant grass inside the baseball diamond shown. About how many pounds do you need to plant grass inside the softball diamond?

Ⓐ 6 Ⓑ 9

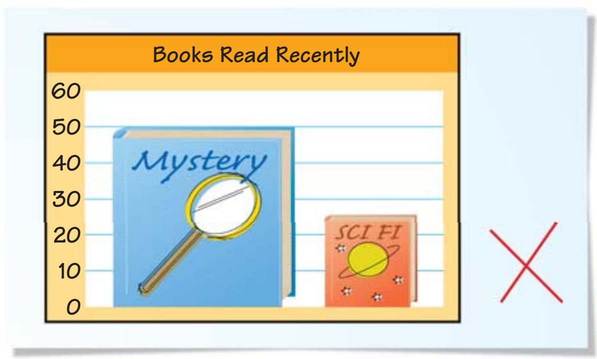
Ⓒ 13 Ⓓ 20



29. **MULTI-STEP PROBLEM** Use graph paper for parts (a) and (b).
    a. Draw a triangle and label its vertices. Find the area of the triangle.
    b. Mark and label the midpoints of each side of the triangle. Connect the midpoints to form a smaller triangle. Show that the larger and smaller triangles are similar. Then use the fact that the triangles are similar to find the area of the smaller triangle.

30. **JUSTIFYING THEOREM 11.7** Choose a type of polygon for which you know the area formula. Use algebra and the area formula to prove Theorem 11.7 for that polygon. (*Hint:* Use the ratio for the corresponding side lengths in two similar polygons to express each dimension in one polygon as $\frac{a}{b}$ times the corresponding dimension in the other polygon.)

31. **MISLEADING GRAPHS** A student wants to show that the students in a science class prefer mysteries to science fiction books. Over a two month period, the students in the class read 50 mysteries, but only 25 science fiction books. The student makes a bar graph of these data. *Explain* why the graph is visually misleading. Show how the student could redraw the bar graph.

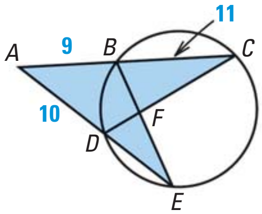


○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

32. ★ **OPEN-ENDED MATH** The ratio of the areas of two similar polygons is 9 : 6. Draw two polygons that fit this description. Find the ratio of their perimeters. Then write the ratio in simplest radical form.

33. ★ **EXTENDED RESPONSE** Use the diagram shown at the right.

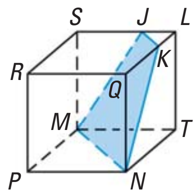


a. Name as many pairs of similar triangles as you can. *Explain* your reasoning.

b. Find the ratio of the areas for one pair of similar triangles.

c. Show two ways to find the length of $\overline{DE}$.

34. **CHALLENGE** In the diagram, the solid figure is a cube. Quadrilateral $JKNM$ is on a plane that cuts through the cube, with $JL = KL$.



a. *Explain* how you know that $\triangle JKL \sim \triangle MNP$.

b. Suppose $\frac{JK}{MN} = \frac{1}{3}$. Find the ratio of the area of $\triangle JKL$ to the area of one face of the cube.

c. Find the ratio of the area of $\triangle JKL$ to the area of pentagon $JKQRS$.

MIXED REVIEW

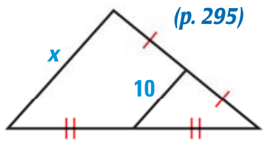
PREVIEW
Prepare for Lesson 11.4 in Exs. 35–38.

Find the circumference of the circle with the given radius r or diameter d. Use $\pi \approx 3.14$. Round your answers to the nearest hundredth. *(p. 49)*

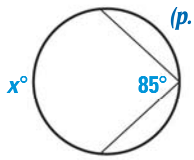
35. $d = 4$ cm **36.** $d = 10$ ft **37.** $r = 2.5$ yd **38.** $r = 3.1$ m

Find the value of x.

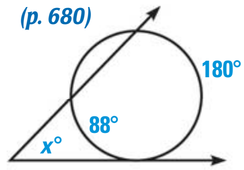
39. *(p. 295)*



40. *(p. 672)*



41. *(p. 680)*

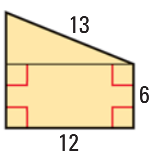


QUIZ for Lessons 11.1–11.3

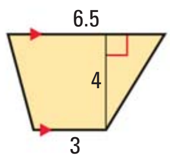
1. The height of ▱$ABCD$ is 3 times its base. Its area is 108 square feet. Find the base and the height. *(p. 720)*

Find the area of the figure.

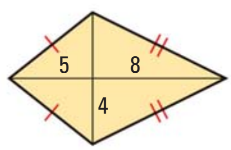
2. *(p. 720)*



3. *(p. 730)*



4. *(p. 730)*



5. The ratio of the lengths of corresponding sides of two similar heptagons is 7 : 20. Find the ratio of their perimeters and their areas. *(p. 737)*

6. Triangles PQR and XYZ are similar. The area of $\triangle PQR$ is 1200 ft^2 and the area of $\triangle XYZ$ is 48 ft^2. Given $PQ = 50$ ft, find XY. *(p. 737)*

PROBLEM SOLVING WORKSHOP
LESSON 11.3

Using ALTERNATIVE METHODS

Another Way to Solve Example 3, page 738

MULTIPLE REPRESENTATIONS In Example 3 on page 738, you used proportional reasoning to solve a problem about cooking. You can also solve the problem by using an area formula.

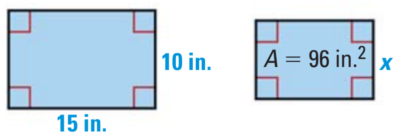
PROBLEM

COOKING A large rectangular baking pan is 15 inches long and 10 inches wide. A smaller pan is similar to the large pan. The area of the smaller pan is 96 square inches. Find the width of the smaller pan.

METHOD

Using a Formula You can use what you know about side lengths of similar figures to find the width of the pan.

STEP 1 **Use** the given dimensions of the large pan to write expressions for the dimensions of the smaller pan. Let x represent the width of the smaller pan.



The length of the larger pan is 1.5 times its width. So, the length of the smaller pan is also 1.5 times its width, or $1.5x$.

STEP 2 **Use** the formula for the area of a rectangle to write an equation.

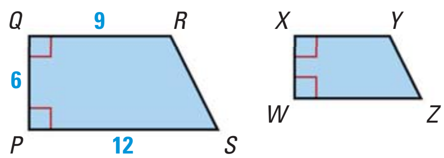
$A = \ell w$	Formula for area of a rectangle
$96 = 1.5x \cdot x$	Substitute $1.5x$ for ℓ and x for w.
$8 = x$	Solve for a positive value of x.

▶ The width of the smaller pan is 8 inches.

PRACTICE

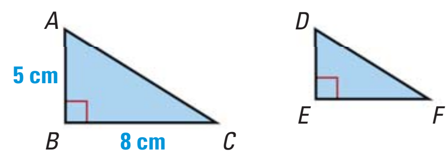
1. **COOKING** A third pan is similar to the large pan shown above and has 1.44 times its area. Find the length of the third pan.

2. **TRAPEZOIDS** Trapezoid $PQRS$ is similar to trapezoid $WXYZ$. The area of $WXYZ$ is 28 square units. Find WZ.



3. **SQUARES** One square has sides of length s. If another square has twice the area of the first square, what is its side length?

4. **REASONING** $\triangle ABC \sim \triangle DEF$ and the area of $\triangle DEF$ is 11.25 square centimeters. Find DE and DF. *Explain* your reasoning.



MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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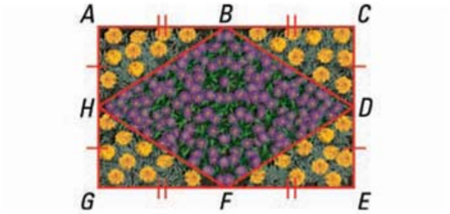
Lessons 11.1–11.3

1. **MULTI-STEP PROBLEM** The diagram below represents a rectangular flower bed. In the diagram, $AG = 9.5$ feet and $GE = 15$ feet.



   a. *Explain* how you know that $BDFH$ is a rhombus.

   b. Find the area of rectangle $ACEG$ and the area of rhombus $BDFH$.

   c. You want to plant asters inside rhombus $BDFH$ and marigolds in the other parts of the flower bed. It costs about $.30 per square foot to plant marigolds and about $.40 per square foot to plant asters. How much will you spend on flowers?

2. **OPEN-ENDED** A polygon has an area of 48 square meters and a height of 8 meters. Draw three different triangles that fit this description and three different parallelograms. *Explain* your thinking.

3. **EXTENDED RESPONSE** You are tiling a 12 foot by 21 foot rectangular floor. Prices are shown below for two sizes of square tiles.

   a. How many small tiles would you need for the floor? How many large tiles?

   b. Find the cost of buying large tiles for the floor and the cost of buying small tiles for the floor. Which tile should you use if you want to spend as little as possible?

   c. *Compare* the side lengths, the areas, and the costs of the two tiles. Is the cost per tile based on side length or on area? *Explain*.

4. **SHORT RESPONSE** What happens to the area of a rhombus if you double the length of each diagonal? if you triple the length of each diagonal? *Explain* what happens to the area of a rhombus if each diagonal is multiplied by the same number n.

5. **MULTI-STEP PROBLEM** The pool shown is a right triangle with legs of length 40 feet and 41 feet. The path around the pool is 40 inches wide.

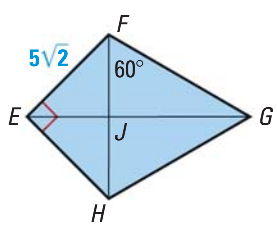
   a. Find the area of $\triangle STU$.

   b. In the diagram, $\triangle PQR \sim \triangle STU$, and the scale factor of the two triangles is 1.3 : 1. Find the perimeter of $\triangle PQR$.

   c. Find the area of $\triangle PQR$. Then find the area of the path around the pool.

6. **GRIDDED ANSWER** In trapezoid $ABCD$, $\overline{AB} \parallel \overline{CD}$, $m\angle D = 90°$, $AD = 5$ inches, and $CD = 3 \cdot AB$. The area of trapezoid $ABCD$ is 1250 square inches. Find the length (in inches) of $\overline{CD}$.

7. **EXTENDED RESPONSE** In the diagram below, $\triangle EFH$ is an isosceles right triangle, and $\triangle FGH$ is an equilateral triangle.



   a. Find FH. *Explain* your reasoning.

   b. Find EG. *Explain* your reasoning.

   c. Find the area of $EFGH$.

11.4 Circumference and Arc Length

Before You found the circumference of a circle.

Now You will find arc lengths and other measures.

Why? So you can find a running distance, as in Example 5.

Key Vocabulary
- **circumference**
- **arc length**
- **radius,** *p. 651*
- **diameter,** *p. 651*
- **measure of an arc,** *p. 659*

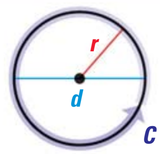
The **circumference** of a circle is the distance around the circle. For all circles, the ratio of the circumference to the diameter is the same. This ratio is known as π, or *pi*. In Chapter 1, you used 3.14 to approximate the value of π. Throughout this chapter, you should use the π key on a calculator, then round to the hundredths place unless instructed otherwise.

THEOREM *For Your Notebook*

THEOREM 11.8 Circumference of a Circle

The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$, where d is the diameter of the circle and r is the radius of the circle.

Justification: Ex. 2, p. 769



$C = \pi d = 2\pi r$

EXAMPLE 1 Use the formula for circumference

Find the indicated measure.

a. Circumference of a circle with radius 9 centimeters

b. Radius of a circle with circumference 26 meters

Solution

a. $C = 2\pi r$ — Write circumference formula.

$= 2 \cdot \pi \cdot 9$ — Substitute 9 for r.

$= 18\pi$ — Simplify.

≈ 56.55 — Use a calculator.

▶ The circumference is about 56.55 centimeters.

b. $C = 2\pi r$ — Write circumference formula.

$26 = 2\pi r$ — Substitute 26 for C.

$\frac{26}{2\pi} = r$ — Divide each side by 2π.

$4.14 \approx r$ — Use a calculator.

▶ The radius is about 4.14 meters.

ANOTHER WAY
You can give an exact measure in terms of π. In Example 1, part (a), the exact circumference is 18π. The exact radius in Example 1, part (b) is $\frac{26}{2\pi}$, or $\frac{13}{\pi}$.

EXAMPLE 2 Use circumference to find distance traveled

TIRE REVOLUTIONS The dimensions of a car tire are shown at the right. To the nearest foot, how far does the tire travel when it makes 15 revolutions?

Solution

STEP 1 **Find** the diameter of the tire.

$d = 15 + 2(5.5) = 26$ in.

STEP 2 **Find** the circumference of the tire.

$C = \pi d = \pi(26) \approx 81.68$ in.

STEP 3 **Find** the distance the tire travels in 15 revolutions. In one revolution, the tire travels a distance equal to its circumference. In 15 revolutions, the tire travels a distance equal to 15 times its circumference.

Distance traveled = Number of revolutions • Circumference

$\approx 15 \cdot 81.68$ in.

$= 1225.2$ in.

AVOID ERRORS
Always pay attention to units. In Example 2, you need to convert units to get a correct answer.

STEP 4 **Use** unit analysis. Change 1225.2 inches to feet.

$$1225.2 \cancel{\text{in.}} \cdot \frac{1 \text{ ft}}{12 \cancel{\text{in.}}} = 102.1 \text{ ft}$$

▶ The tire travels approximately 102 feet.

✓ GUIDED PRACTICE for Examples 1 and 2

1. Find the circumference of a circle with diameter 5 inches. Find the diameter of a circle with circumference 17 feet.

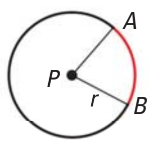
2. A car tire has a diameter of 28 inches. How many revolutions does the tire make while traveling 500 feet?

ARC LENGTH An **arc length** is a portion of the circumference of a circle. You can use the measure of the arc (in degrees) to find its length (in linear units).

COROLLARY *For Your Notebook*

ARC LENGTH COROLLARY

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.



$$\frac{\text{Arc length of } \overparen{AB}}{2\pi r} = \frac{m\overparen{AB}}{360^\circ}, \text{ or Arc length of } \overparen{AB} = \frac{m\overparen{AB}}{360^\circ} \cdot 2\pi r$$

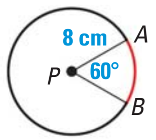
EXAMPLE 3 Find arc lengths

INTERPRET DIAGRAMS

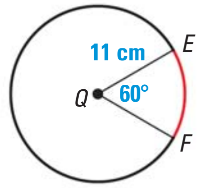
In Example 3, $\widehat{AB}$ and $\widehat{EF}$ have the same measure. However, they have different lengths because they are in circles with different circumferences.

Find the length of each red arc.

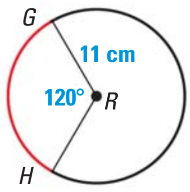
a.



b.



c.



Solution

a. Arc length of $\widehat{AB} = \frac{60°}{360°} \cdot 2\pi(8) \approx 8.38$ centimeters

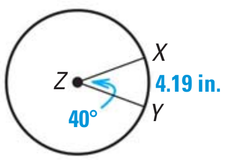
b. Arc length of $\widehat{EF} = \frac{60°}{360°} \cdot 2\pi(11) \approx 11.52$ centimeters

c. Arc length of $\widehat{GH} = \frac{120°}{360°} \cdot 2\pi(11) \approx 23.04$ centimeters

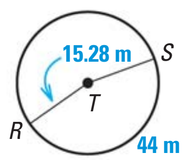
EXAMPLE 4 Use arc lengths to find measures

Find the indicated measure.

a. Circumference C of $\odot Z$



b. $m\widehat{RS}$



Solution

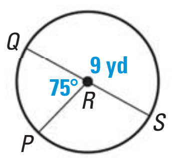
a.
$$\frac{\text{Arc length of } \widehat{XY}}{C} = \frac{m\widehat{XY}}{360°}$$
$$\frac{4.19}{C} = \frac{40°}{360°}$$
$$\frac{4.19}{C} = \frac{1}{9}$$
▶ $37.71 = C$

b.
$$\frac{\text{Arc length of } \widehat{RS}}{2\pi r} = \frac{m\widehat{RS}}{360°}$$
$$\frac{44}{2\pi(15.28)} = \frac{m\widehat{RS}}{360°}$$
$$360° \cdot \frac{44}{2\pi(15.28)} = m\widehat{RS}$$
▶ $165° \approx m\widehat{RS}$

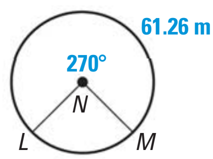
✓ GUIDED PRACTICE for Examples 3 and 4

Find the indicated measure.

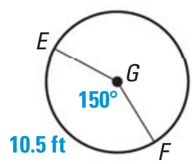
3. Length of $\widehat{PQ}$



4. Circumference of $\odot N$

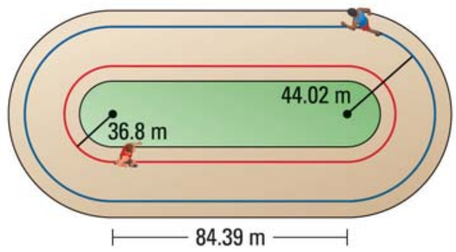


5. Radius of $\odot G$



EXAMPLE 5 Use arc length to find distances

TRACK The curves at the ends of the track shown are 180° arcs of circles. The radius of the arc for a runner on the red path shown is 36.8 meters. About how far does this runner travel to go once around the track? Round to the nearest tenth of a meter.



Solution

USE FORMULAS
The arc length of a semicircle is half the circumference of the circle with the same radius. So, the arc length of a semicircle is $\frac{1}{2} \cdot 2\pi r$, or πr.

The path of a runner is made of two straight sections and two semicircles. To find the total distance, find the sum of the lengths of each part.

Distance = 2 • Length of each straight section + 2 • Length of each semicircle

$$= 2(84.39) + 2 \cdot \left(\frac{1}{2} \cdot 2\pi \cdot 36.8\right)$$

$$\approx 400.0 \text{ meters}$$

▶ The runner on the red path travels about 400 meters.

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✓ GUIDED PRACTICE for Example 5

6. In Example 5, the radius of the arc for a runner on the blue path is 44.02 meters, as shown in the diagram. About how far does this runner travel to go once around the track? Round to the nearest tenth of a meter.

11.4 EXERCISES

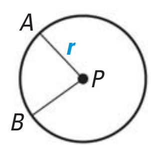
HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 23, 25, and 35

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 31, 32, and 38

SKILL PRACTICE

In Exercises 1 and 2, refer to the diagram of $\odot P$ shown.



1. VOCABULARY Copy and complete the equation: $\frac{?}{2\pi r} = \frac{m\overset{\frown}{AB}}{?}$.

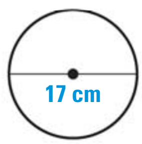
2. ★ WRITING *Describe* the difference between the *arc measure* and the *arc length* of $\overset{\frown}{AB}$.

EXAMPLE 1
on p. 746 for Exs. 3–7

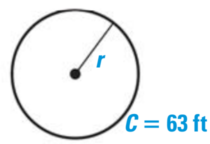
USING CIRCUMFERENCE **Use the diagram to find the indicated measure.**

3. Find the circumference.

4. Find the circumference.



5. Find the radius.



FINDING EXACT MEASURES **Find the indicated measure.**

6. The exact circumference of a circle with diameter 5 inches

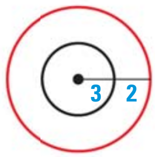
7. The exact radius of a circle with circumference 28π meters

EXAMPLE 2
on p. 747
for Exs. 8–10

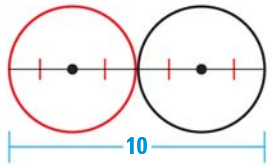
FINDING CIRCUMFERENCE **Find the circumference of the red circle.**

8.

9.



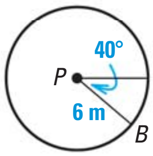
10.



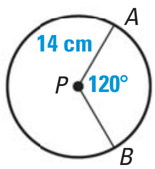
EXAMPLE 3
on p. 748
for Exs. 11–20

FINDING ARC LENGTHS **Find the length of $\widehat{AB}$.**

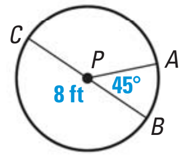
11.



12.

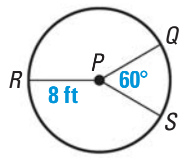


13.



14. **ERROR ANALYSIS** A student says that two arcs from different circles have the same arc length if their central angles have the same measure. *Explain* the error in the student's reasoning.

FINDING MEASURES **In $\odot P$ shown at the right, $\angle QPR \cong \angle RPS$. Find the indicated measure.**



15. $m\widehat{QRS}$

16. Length of $\widehat{QRS}$

17. $m\widehat{QR}$

18. $m\widehat{RSQ}$

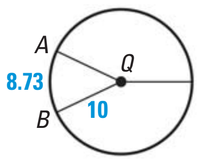
19. Length of $\widehat{QR}$

20. Length of $\widehat{RSQ}$

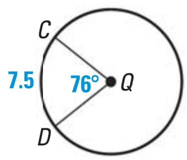
EXAMPLE 4
on p. 748
for Exs. 21–23

USING ARC LENGTH **Find the indicated measure.**

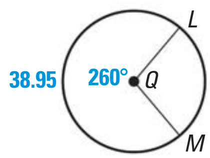
21. $m\widehat{AB}$



22. Circumference of $\odot Q$



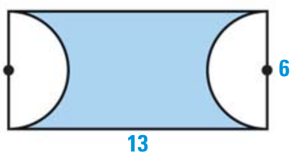
23. Radius of $\odot Q$



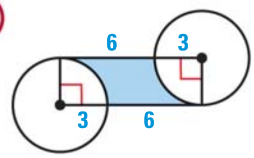
EXAMPLE 5
on p. 749
for Exs. 24–25

FINDING PERIMETERS **Find the perimeter of the shaded region.**

24.



25.



COORDINATE GEOMETRY **The equation of a circle is given. Find the circumference of the circle. Write the circumference in terms of π.**

26. $x^2 + y^2 = 16$

27. $(x + 2)^2 + (y - 3)^2 = 9$

28. $x^2 + y^2 = 18$

29. **xy ALGEBRA** Solve the formula $C = 2\pi r$ for r. Solve the formula $C = \pi d$ for d. Use the rewritten formulas to find r and d when $C = 26\pi$.

○ = WORKED-OUT SOLUTIONS on p. WS1

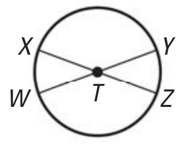
★ = STANDARDIZED TEST PRACTICE

30. **FINDING VALUES** In the table below, $\widehat{AB}$ refers to the arc of a circle. Copy and complete the table.

Radius	?	2	0.8	4.2	?	$4\sqrt{2}$
$m\widehat{AB}$	45°	60°	?	183°	90°	?
Length of $\widehat{AB}$	4	?	0.3	?	3.22	2.86

31. ★ **SHORT RESPONSE** Suppose $\widehat{EF}$ is an arc on a circle with radius r. Let $x°$ be the measure of $\widehat{EF}$. *Describe* the effect on the length of $\widehat{EF}$ if you (a) double the radius of the circle, and (b) double the measure of $\widehat{EF}$.

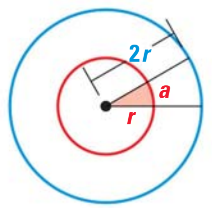
32. ★ **MULTIPLE CHOICE** In the diagram, $\overline{WY}$ and $\overline{XZ}$ are diameters of $\odot T$, and $WY = XZ = 6$. If $m\widehat{XY} = 140°$, what is the length of $\widehat{YZ}$?



Ⓐ $\frac{2}{3}\pi$ Ⓑ $\frac{4}{3}\pi$ Ⓒ 6π Ⓓ 4π

33. **CHALLENGE** Find the circumference of a circle inscribed in a rhombus with diagonals that are 12 centimeters and 16 centimeters long. *Explain*.

34. **FINDING CIRCUMFERENCE** In the diagram, the measure of the shaded red angle is 30°. The arc length a is 2. *Explain* how to find the circumference of the blue circle without finding the radius of either the red or the blue circles.



PROBLEM SOLVING

35. **TREES** A group of students wants to find the diameter of the trunk of a young sequoia tree. The students wrap a rope around the tree trunk, then measure the length of rope needed to wrap one time around the trunk. This length is 21 feet 8 inches. *Explain* how they can use this length to estimate the diameter of the tree trunk to the nearest half foot.

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36. **INSCRIBED SQUARE** A square with side length 6 units is inscribed in a circle so that all four vertices are on the circle. Draw a sketch to represent this problem. Find the circumference of the circle.

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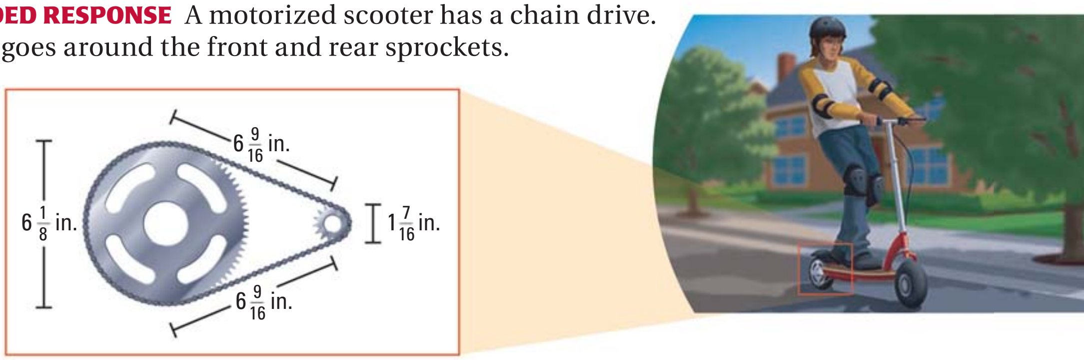
EXAMPLE 2 on p. 747 for Ex. 37

37. **MEASURING WHEEL** As shown, a measuring wheel is used to calculate the length of a path. The diameter of the wheel is 8 inches. The wheel rotates 87 times along the length of the path. About how long is the path?

38. ★ **EXTENDED RESPONSE** A motorized scooter has a chain drive. The chain goes around the front and rear sprockets.

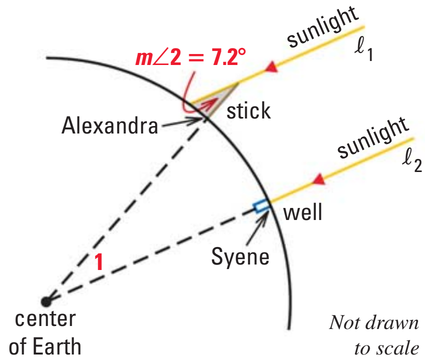


a. About how long is the chain? *Explain*.

b. Each sprocket has teeth that grip the chain. There are 76 teeth on the larger sprocket, and 15 teeth on the smaller sprocket. About how many teeth are gripping the chain at any given time? *Explain*.

39. **SCIENCE** Over 2000 years ago, the Greek scholar Eratosthenes estimated Earth's circumference by assuming that the Sun's rays are parallel. He chose a day when the Sun shone straight down into a well in the city of Syene. At noon, he measured the angle the Sun's rays made with a vertical stick in the city of Alexandria. Eratosthenes assumed that the distance from Syene to Alexandria was equal to about 575 miles.

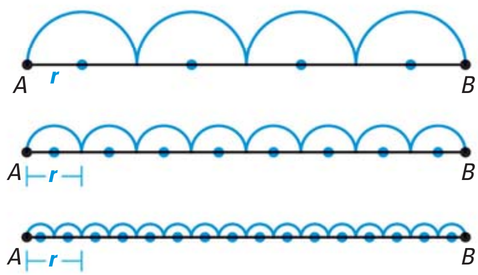
Find $m\angle 1$. Then estimate Earth's circumference.



CHALLENGE **Suppose $\overline{AB}$ is divided into four congruent segments, and semicircles with radius r are drawn.**

40. What is the sum of the four arc lengths if the radius of each arc is r?

41. Suppose that $\overline{AB}$ is divided into n congruent segments and that semicircles are drawn, as shown. What will the sum of the arc lengths be for 8 segments? for 16 segments? for n segments? *Explain* your thinking.



MIXED REVIEW

PREVIEW

Prepare for Lesson 11.5 in Exs. 42–45.

Find the area of a circle with radius r. Round to the nearest hundredth. *(p. 49)*

42. $r = 6$ cm

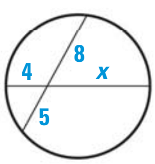
43. $r = 4.2$ in.

44. $r = 8\frac{3}{4}$ mi

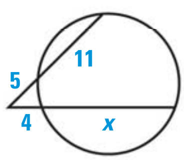
45. $r = 1\frac{3}{8}$ in.

Find the value of x. *(p. 689)*

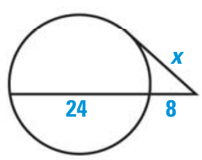
46.



47.



48.



Extension

Use after Lesson 11.4

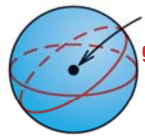
Geometry on a Sphere

GOAL Compare Euclidean and spherical geometries.

Key Vocabulary
- great circle

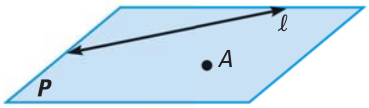
In Euclidean geometry, a plane is a flat surface that extends without end in all directions, and a line in the plane is a set of points that extends without end in two directions. Geometry on a sphere is different.

In *spherical geometry*, a plane is the surface of a sphere. A line is defined as a **great circle**, which is a circle on the sphere whose center is the center of the sphere.



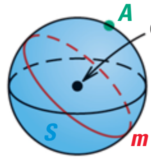
KEY CONCEPT *For Your Notebook*

Euclidean Geometry



Plane P contains line ℓ and point A not on the line ℓ.

Spherical Geometry



Sphere S contains great circle m and point A not on m. Great circle m is a line.

HISTORY NOTE
Spherical geometry is sometimes called *Riemann geometry* after Bernhard Riemann, who wrote the first description of it in 1854.

Some properties and postulates in Euclidean geometry are true in spherical geometry. Others are not, or are true only under certain circumstances. For example, in Euclidean geometry, Postulate 5 states that through any two points there exists exactly one line. On a sphere, this postulate is true only for points that are not the endpoints of a diameter of the sphere.

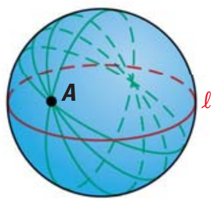
EXAMPLE 1 Compare Euclidean and spherical geometry

Tell whether the following postulate in Euclidean geometry is also true in spherical geometry. Draw a diagram to support your answer.

Parallel Postulate: If there is a line ℓ and a point A not on the line, then there is exactly one line through the point A parallel to the given line ℓ.

Solution

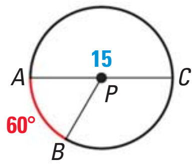
Parallel lines do not intersect. The sphere shows a line ℓ (a great circle) and a point A not on ℓ. Several lines are drawn through A. Each great circle containing A intersects ℓ. So, there can be no line parallel to ℓ. The parallel postulate is not true in spherical geometry.



DISTANCES In Euclidean geometry, there is exactly one distance that can be measured between any two points. On a sphere, there are two distances that can be measured between two points. These distances are the lengths of the major and minor arcs of the great circle drawn through the points.

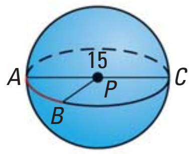
READ DIAGRAMS

The diagram below is a cross section of the sphere in Example 2. It shows $\overset{\frown}{AB}$ and $\overset{\frown}{ACB}$ on a great circle.



EXAMPLE 2 Find distances on a sphere

The diameter of the sphere shown is 15, and $m\overset{\frown}{AB} = 60°$. Find the distances between A and B.



Solution

Find the lengths of the minor arc $\overset{\frown}{AB}$ and the major arc $\overset{\frown}{ACB}$ of the great circle shown. In each case, let x be the arc length.

$$\frac{\text{Arc length of } \overset{\frown}{AB}}{2\pi r} = \frac{m\overset{\frown}{AB}}{360°} \qquad \frac{\text{Arc length of } \overset{\frown}{ACB}}{2\pi r} = \frac{m\overset{\frown}{ACB}}{360°}$$

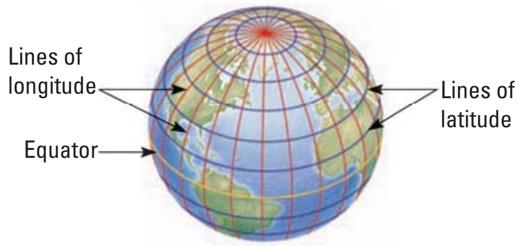
$$\frac{x}{15\pi} = \frac{60°}{360°} \qquad \frac{x}{15\pi} = \frac{360° - 60°}{360°}$$

$$x = 2.5\pi \qquad x = 12.5\pi$$

▶ The distances are 2.5π and 12.5π.

PRACTICE

1. **WRITING** Lines of latitude and longitude are used to identify positions on Earth. Which of the lines shown in the figure are great circles. Which are not? *Explain* your reasoning.



EXAMPLE 1 on p. 753 for Exs. 2–3

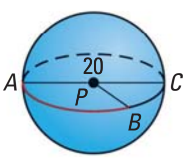
2. **COMPARING GEOMETRIES** Draw sketches to show that there is more than one line through the endpoints of a diameter of a sphere, but only one line through two points that are *not* endpoints of a diameter.

3. **COMPARING GEOMETRIES** The following statement is true in Euclidean geometry: If two lines intersect, then their intersection is exactly one point. Rewrite this statement to be true for lines on a sphere. *Explain*.

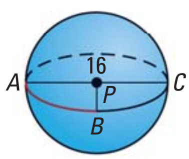
EXAMPLE 2 on p. 754 for Exs. 4–6

FINDING DISTANCES Use the diagram and the given arc measure to find the distances between points A and B. Leave your answers in terms of π.

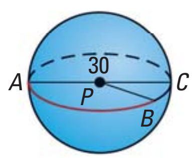
4. $m\overset{\frown}{AB} = 120°$



5. $m\overset{\frown}{AB} = 90°$



6. $m\overset{\frown}{AB} = 140°$



11.5 Areas of Circles and Sectors

Before You found circumferences of circles.

Now You will find the areas of circles and sectors.

Why So you can estimate walking distances, as in Ex. 38.

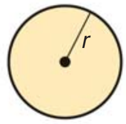
Key Vocabulary
- **sector of a circle**

In Chapter 1, you used the formula for the area of a circle. This formula is presented below as Theorem 11.9.

THEOREM *For Your Notebook*

THEOREM 11.9 Area of a Circle

The area of a circle is π times the square of the radius.



$A = \pi r^2$

Justification: Ex. 43, p. 761; Ex. 3, p. 769

EXAMPLE 1 Use the formula for area of a circle

Find the indicated measure.

a. Area

$r = 2.5$ cm

b. Diameter

$A = 113.1 \text{ cm}^2$

Solution

a. $A = \pi r^2$ — **Write formula for the area of a circle.**

$= \pi \cdot (2.5)^2$ — **Substitute 2.5 for r.**

$= 6.25\pi$ — **Simplify.**

≈ 19.63 — **Use a calculator.**

▶ The area of $\odot A$ is about 19.63 square centimeters.

b. $A = \pi r^2$ — **Write formula for the area of a circle.**

$113.1 = \pi r^2$ — **Substitute 113.1 for A.**

$\frac{113.1}{\pi} = r^2$ — **Divide each side by π.**

$6 \approx r$ — **Find the positive square root of each side.**

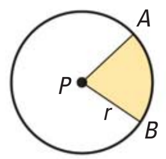
▶ The radius is about 6 inches, so the diameter is about 12 centimeters.

SECTORS A **sector of a circle** is the region bounded by two radii of the circle and their intercepted arc. In the diagram below, sector APB is bounded by $\overline{AP}$, $\overline{BP}$, and $\overset{\frown}{AB}$. Theorem 11.10 gives a method for finding the area of a sector.

THEOREM *For Your Notebook*

THEOREM 11.10 Area of a Sector

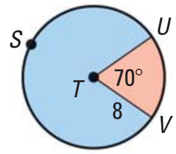
The ratio of the area of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc to 360°.



$$\frac{\text{Area of sector } APB}{\pi r^2} = \frac{m\overset{\frown}{AB}}{360°}, \text{ or Area of sector } APB = \frac{m\overset{\frown}{AB}}{360°} \cdot \pi r^2$$

EXAMPLE 2 Find areas of sectors

Find the areas of the sectors formed by $\angle UTV$.



Solution

STEP 1 **Find** the measures of the minor and major arcs.

Because $m\angle UTV = 70°$, $m\overset{\frown}{UV} = 70°$ and $m\overset{\frown}{USV} = 360° - 70° = 290°$.

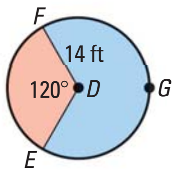
STEP 2 **Find** the areas of the small and large sectors.

Area of small sector	$= \frac{m\overset{\frown}{UV}}{360°} \cdot \pi r^2$	Write formula for area of a sector.
	$= \frac{70°}{360°} \cdot \pi \cdot 8^2$	Substitute.
	≈ 39.10	Use a calculator.
Area of large sector	$= \frac{m\overset{\frown}{USV}}{360°} \cdot \pi r^2$	Write formula for area of a sector.
	$= \frac{290°}{360°} \cdot \pi \cdot 8^2$	Substitute.
	≈ 161.97	Use a calculator.

▶ The areas of the small and large sectors are about 39.10 square units and 161.97 square units, respectively.

✓ GUIDED PRACTICE for Examples 1 and 2

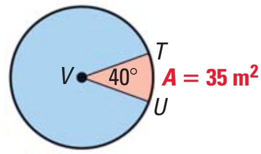
Use the diagram to find the indicated measure.



1. Area of $\odot D$
2. Area of red sector
3. Area of blue sector

EXAMPLE 3 Use the Area of a Sector Theorem

Use the diagram to find the area of $\odot V$.



Solution

$$\text{Area of sector } TVU = \frac{m\overset{\frown}{TU}}{360^\circ} \cdot \text{Area of } \odot V$$ **Write formula for area of a sector.**

$$35 = \frac{40^\circ}{360^\circ} \cdot \text{Area of } \odot V$$ **Substitute.**

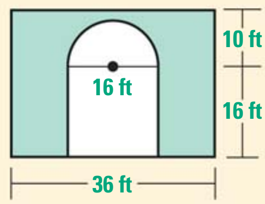
$$315 = \text{Area of } \odot V$$ **Solve for Area of $\odot V$.**

▶ The area of $\odot V$ is 315 square meters.

EXAMPLE 4 Standardized Test Practice

A rectangular wall has an entrance cut into it. You want to paint the wall. To the nearest square foot, what is the area of the region you need to paint?

Ⓐ 357 ft^2 Ⓑ 479 ft^2
Ⓒ 579 ft^2 Ⓓ 936 ft^2



Solution

AVOID ERRORS
Use the radius (8 ft), not the diameter (16 ft) when you calculate the area of the semicircle.

The area you need to paint is the area of the rectangle minus the area of the entrance. The entrance can be divided into a **semicircle** and a **square**.

$$\text{Area of wall} = \text{Area of rectangle} - (\text{Area of semicircle} + \text{Area of square})$$

$$= 36(26) - \left[\frac{180^\circ}{360^\circ} \cdot (\pi \cdot 8^2) + 16^2\right]$$

$$= 936 - [32\pi + 256]$$

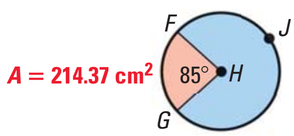
$$\approx 579.47$$

The area is about 579 square feet.

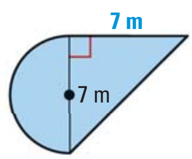
▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

GUIDED PRACTICE for Examples 3 and 4

4. Find the area of $\odot H$.



5. Find the area of the figure.



6. If you know the area and radius of a sector of a circle, can you find the measure of the intercepted arc? *Explain.*

11.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 17, and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 19, 40, and 42

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A __?__ of a circle is the region bounded by two radii of the circle and their intercepted arc.

2. ★ **WRITING** Suppose you double the arc measure of a sector in a given circle. Will the area of the sector also be doubled? *Explain.*

EXAMPLE 1 on p. 755 for Exs. 3–9

FINDING AREA **Find the exact area of a circle with the given radius *r* or diameter *d*. Then find the area to the nearest hundredth.**

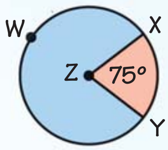
3. $r = 5$ in.
4. $d = 16$ ft
5. $d = 23$ cm
6. $r = 1.5$ km

USING AREA **In Exercises 7–9, find the indicated measure.**

7. The area of a circle is 154 square meters. Find the radius.
8. The area of a circle is 380 square inches. Find the radius.
9. The area of a circle is 676π square centimeters. Find the diameter.

EXAMPLE 2 on p. 756 for Exs. 10–13

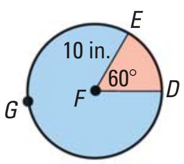
10. **ERROR ANALYSIS** In the diagram at the right, the area of $\odot Z$ is 48 square feet. A student writes a proportion to find the area of sector XZY. *Describe* and correct the error in writing the proportion. Then find the area of sector XZY.

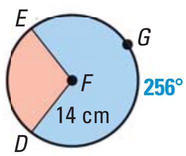


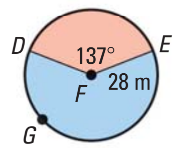
Let n be the area of sector XZY.

$\frac{n}{360°} = \frac{48}{285°}$

FINDING AREA OF SECTORS **Find the areas of the sectors formed by $\angle DFE$.**

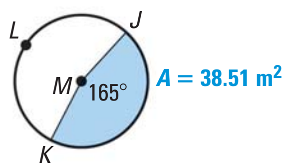
11.


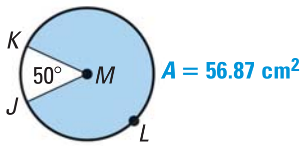
12.


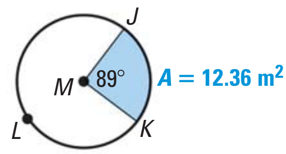
13.


EXAMPLE 3 on p. 757 for Exs. 14–16

USING AREA OF A SECTOR **Use the diagram to find the indicated measure.**

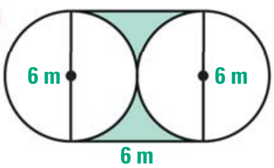
14. Find the area of $\odot M$.


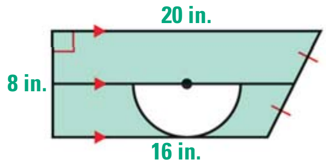
15. Find the area of $\odot M$.


16. Find the radius of $\odot M$.


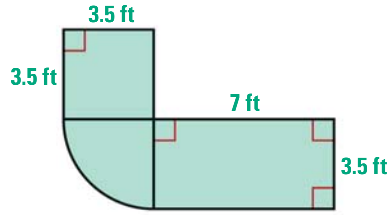
EXAMPLE 4 on p. 757 for Exs. 17–19

FINDING AREA **Find the area of the shaded region.**

17.


18.


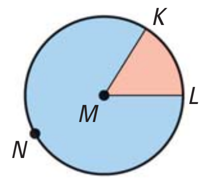
19. ★ **MULTIPLE CHOICE** The diagram shows the shape of a putting green at a miniature golf course. One part of the green is a sector of a circle. To the nearest square foot, what is the area of the putting green?



Ⓐ 46 ft^2 Ⓑ 49 ft^2

Ⓒ 56 ft^2 Ⓓ 75 ft^2

FINDING MEASURES **The area of $\odot M$ is 260.67 square inches. The area of sector KML is 42 square inches. Find the indicated measure.**



20. Radius of $\odot M$

21. Circumference of $\odot M$

22. $m\overset{\frown}{KL}$

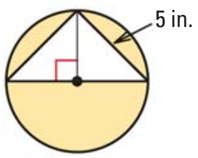
23. Perimeter of blue region

24. Length of $\overset{\frown}{KL}$

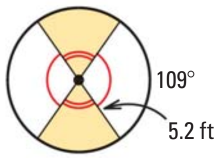
25. Perimeter of red region

FINDING AREA **Find the area of the shaded region.**

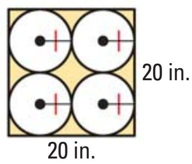
26.



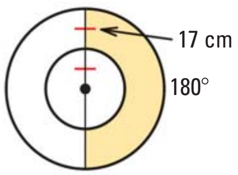
27.



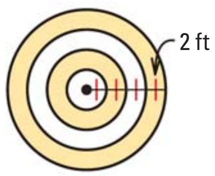
28.



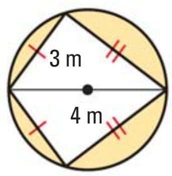
29.



30.

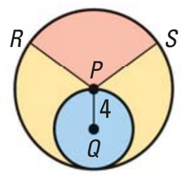


31.



Animated **Geometry** at classzone.com

32. **TANGENT CIRCLES** In the diagram at the right, $\odot Q$ and $\odot P$ are tangent, and P lies on $\odot Q$. The measure of $\overset{\frown}{RS}$ is 108°. Find the area of the red region, the area of the blue region, and the area of the yellow region. Leave your answers in terms of π.

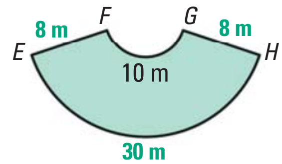


33. **SIMILARITY** Look back at the Perimeters of Similar Polygons Theorem on page 374 and the Areas of Similar Polygons Theorem on page 737. How would you rewrite these theorems to apply to circles? *Explain*.

34. **ERROR ANALYSIS** The ratio of the lengths of two arcs in a circle is 2 : 1. A student claims that the ratio of the areas of the sectors bounded by these arcs is 4 : 1, because $\left(\frac{2}{1}\right)^2 = \frac{4}{1}$. *Describe* and correct the error.

35. **DRAWING A DIAGRAM** A square is inscribed in a circle. The same square is also circumscribed about a smaller circle. Draw a diagram. Find the ratio of the area of the large circle to the area of the small circle.

36. **CHALLENGE** In the diagram at the right, $\overset{\frown}{FG}$ and $\overset{\frown}{EH}$ are arcs of concentric circles, and $\overline{EF}$ and $\overline{GH}$ lie on radii of the larger circle. Find the area of the shaded region.



PROBLEM SOLVING

EXAMPLE 1 on p. 755 for Ex. 37

37. METEOROLOGY The *eye of a hurricane* is a relatively calm circular region in the center of the storm. The diameter of the eye is typically about 20 miles. If the eye of a hurricane is 20 miles in diameter, what is the area of the land that is underneath the eye?

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38. WALKING The area of a circular pond is about 138,656 square feet. You are going to walk around the entire edge of the pond. About how far will you walk? Give your answer to the nearest foot.

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39. CIRCLE GRAPH The table shows how students get to school.

Method	% of Students
Bus	65%
Walk	25%
Other	10%

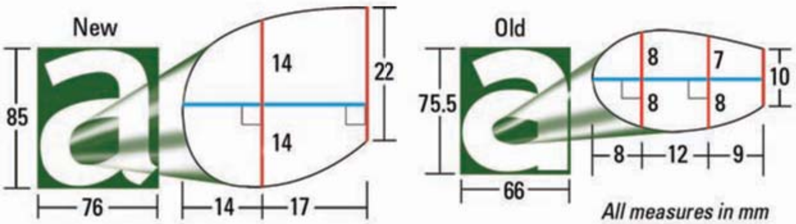
a. *Explain* why a circle graph is appropriate for the data.

b. You will represent each method by a sector of a circle graph. Find the central angle to use for each sector. Then use a protractor and a compass to construct the graph. Use a radius of 2 inches.

c. Find the area of each sector in your graph.

40. ★ **SHORT RESPONSE** It takes about $\frac{1}{4}$ cup of dough to make a tortilla with a 6 inch diameter. How much dough does it take to make a tortilla with a 12 inch diameter? *Explain* your reasoning.

41. HIGHWAY SIGNS A new typeface has been designed to make highway signs more readable. One change was to redesign the form of the letters to increase the space inside letters.



a. Estimate the interior area for the old and the new “a.” Then find the percent increase in interior area.

b. Do you think the change in interior area is just a result of a change in height and width of the letter *a*? *Explain*.

42. ★ **EXTENDED RESPONSE** A circular pizza with a 12 inch diameter is enough for you and 2 friends. You want to buy pizza for yourself and 7 friends. A 10 inch diameter pizza with one topping costs \$6.99 and a 14 inch diameter pizza with one topping costs \$12.99. How many 10 inch and 14 inch pizzas should you buy in each situation below? *Explain*.

a. You want to spend as little money as possible.

b. You want to have three pizzas, each with a different topping.

c. You want to have as much of the thick outer crust as possible.

43. **JUSTIFYING THEOREM 11.9** You can follow the steps below to justify the formula for the area of a circle with radius *r*.

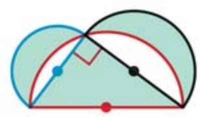
Divide a circle into 16 congruent sectors. Cut out the sectors.

Rearrange the 16 sectors to form a shape resembling a parallelogram.

a. Write expressions in terms of *r* for the approximate height and base of the parallelogram. Then write an expression for its area.

b. *Explain* how your answers to part (a) justify Theorem 11.9.

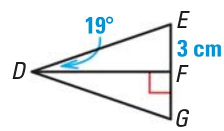
44. **CHALLENGE** Semicircles with diameters equal to the three sides of a right triangle are drawn, as shown. Prove that the sum of the area of the two shaded crescents equals the area of the triangle.



MIXED REVIEW

PREVIEW
Prepare for Lesson 11.6 In Exs. 45–47.

Triangle *DEG* is isosceles with altitude $\overline{DF}$. Find the given measurement. *Explain* your reasoning. *(p. 319)*



45. $m\angle DFG$ **46.** $m\angle FDG$ **47.** FG

Sketch the indicated figure. Draw all of its lines of symmetry. *(p. 619)*

48. Isosceles trapezoid **49.** Regular hexagon

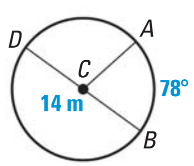
Graph $\triangle ABC$. Then find its area. *(p. 720)*

50. $A(2, 2), B(9, 2), C(4, 16)$ **51.** $A(-8, 3), B(-3, 3), C(-1, -10)$

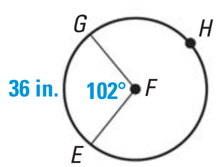
QUIZ *for Lessons 11.4–11.5*

Find the indicated measure. *(p. 746)*

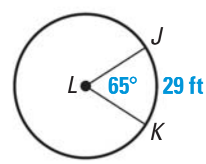
1. Length of $\widehat{AB}$



2. Circumference of $\odot F$

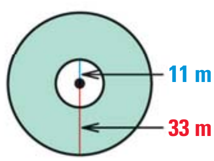


3. Radius of $\odot L$

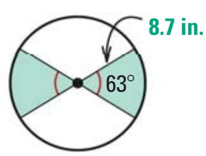


Find the area of the shaded region. *(p. 755)*

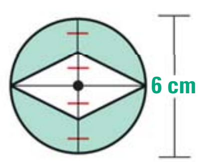
4.



5.



6.



11.6 Areas of Regular Polygons

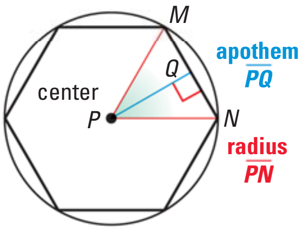
Before You found areas of circles.

Now You will find areas of regular polygons inscribed in circles.

Why? So you can understand the structure of a honeycomb, as in Ex. 44.

Key Vocabulary
- center of a polygon
- radius of a polygon
- apothem of a polygon
- central angle of a regular polygon

The diagram shows a regular polygon inscribed in a circle. The **center of the polygon** and the **radius of the polygon** are the center and the radius of its circumscribed circle.



$\angle MPN$ is a central angle.

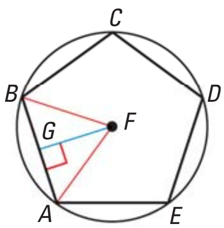
The distance from the center to any side of the polygon is called the **apothem of the polygon**. The apothem is the height to the base of an isosceles triangle that has two radii as legs.

A **central angle of a regular polygon** is an angle formed by two radii drawn to consecutive vertices of the polygon. To find the measure of each central angle, divide 360° by the number of sides.

EXAMPLE 1 Find angle measures in a regular polygon

In the diagram, *ABCDE* is a regular pentagon inscribed in $\odot F$. Find each angle measure.

a. $m\angle AFB$ **b.** $m\angle AFG$ **c.** $m\angle GAF$



Solution

a. $\angle AFB$ is a central angle, so $m\angle AFB = \frac{360°}{5}$, or 72°.

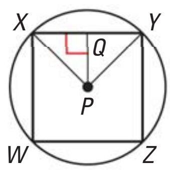
b. $\overline{FG}$ is an apothem, which makes it an altitude of isosceles $\triangle AFB$.
So, $\overline{FG}$ bisects $\angle AFB$ and $m\angle AFG = \frac{1}{2} m\angle AFB = 36°$.

c. The sum of the measures of right $\triangle GAF$ is 180°.
So, $90° + 36° + m\angle GAF = 180°$, and $m\angle GAF = 54°$.

READ DIAGRAMS
A segment whose length is *the apothem* is sometimes called *an apothem*. The segment is an altitude of an isosceles triangle, so it is also a median and angle bisector of the isosceles triangle.

✓ GUIDED PRACTICE for Example 1

In the diagram, *WXYZ* is a square inscribed in $\odot P$.

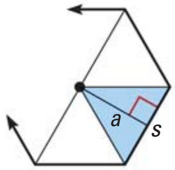


1. Identify the center, a radius, an apothem, and a central angle of the polygon.
2. Find $m\angle XPY$, $m\angle XPQ$, and $m\angle PXQ$.

AREA OF AN n-GON You can find the area of any regular n-gon by dividing it into congruent triangles.

A = Area of one triangle • Number of triangles

$= \left(\frac{1}{2} \cdot s \cdot a\right) \cdot n$	**Base of triangle is s and height of triangle is a. Number of triangles is n.**
$= \frac{1}{2} \cdot a \cdot (n \cdot s)$	**Commutative and Associative Properties of Equality**
$= \frac{1}{2} a \cdot P$	**There are n congruent sides of length s, so perimeter P is $n \cdot s$.**

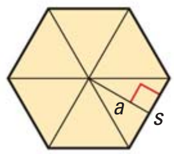


READ DIAGRAMS
In this book, a point shown inside a regular polygon marks the center of the circle that can be circumscribed about the polygon.

THEOREM — *For Your Notebook*

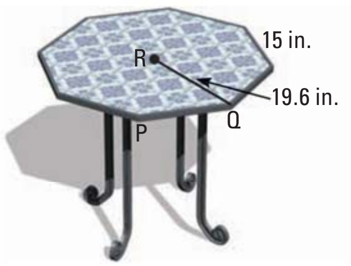
THEOREM 11.11 Area of a Regular Polygon

The area of a regular n-gon with side length s is half the product of the apothem a and the perimeter P, so $A = \frac{1}{2}aP$, or $A = \frac{1}{2}a \cdot ns$.



EXAMPLE 2 Find the area of a regular polygon

DECORATING You are decorating the top of a table by covering it with small ceramic tiles. The table top is a regular octagon with 15 inch sides and a radius of about 19.6 inches. What is the area you are covering?



Solution

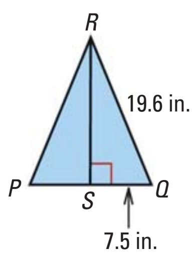
STEP 1 **Find** the perimeter P of the table top.
An octagon has 8 sides, so $P = 8(15) = 120$ inches.

STEP 2 **Find** the apothem a. The apothem is height RS of $\triangle PQR$.
Because $\triangle PQR$ is isosceles, altitude $\overline{RS}$ bisects $\overline{QP}$.

So, $QS = \frac{1}{2}(QP) = \frac{1}{2}(15) = 7.5$ inches.

To find RS, use the Pythagorean Theorem for $\triangle RQS$.

$a = RS \approx \sqrt{19.6^2 - 7.5^2} = \sqrt{327.91} \approx 18.108$



STEP 3 **Find** the area A of the table top.

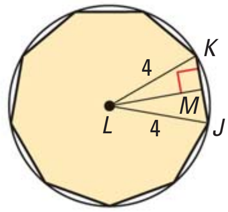
$A = \frac{1}{2}aP$	**Formula for area of regular polygon**
$\approx \frac{1}{2}(18.108)(120)$	**Substitute.**
≈ 1086.5	**Simplify.**

▶ So, the area you are covering with tiles is about 1086.5 square inches.

ROUNDING
In general, your answer will be more accurate if you avoid rounding until the last step. Round your final answers to the nearest tenth unless you are told otherwise.

EXAMPLE 3 Find the perimeter and area of a regular polygon

A regular nonagon is inscribed in a circle with radius 4 units. Find the perimeter and area of the nonagon.



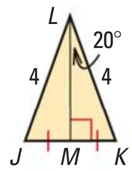
Solution

The measure of central $\angle JLK$ is $\frac{360°}{9}$, or 40°. Apothem $\overline{LM}$ bisects the central angle, so $m\angle KLM$ is 20°. To find the lengths of the legs, use trigonometric ratios for right $\triangle KLM$.

$$\sin 20° = \frac{MK}{LK} \qquad \cos 20° = \frac{LM}{LK}$$

$$\sin 20° = \frac{MK}{4} \qquad \cos 20° = \frac{LM}{4}$$

$$4 \cdot \sin 20° = MK \qquad 4 \cdot \cos 20° = LM$$



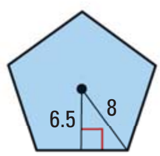
The regular nonagon has side length $s = 2MK = 2(4 \cdot \sin 20°) = 8 \cdot \sin 20°$ and apothem $a = LM = 4 \cdot \cos 20°$.

▶ So, the perimeter is $P = 9s = 9(8 \cdot \sin 20°) = 72 \cdot \sin 20° \approx 24.6$ units, and the area is $A = \frac{1}{2}aP = \frac{1}{2}(4 \cdot \cos 20°)(72 \cdot \sin 20°) \approx 46.3$ square units.

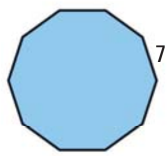
✓ GUIDED PRACTICE for Examples 2 and 3

Find the perimeter and the area of the regular polygon.

3.



4.



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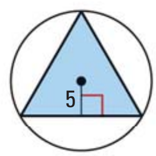


6. Which of Exercises 3–5 above can be solved using special right triangles?

CONCEPT SUMMARY *For Your Notebook*

Finding Lengths in a Regular *n*-gon

To find the area of a regular *n*-gon with radius *r*, you may need to first find the apothem ***a*** or the side length ***s***.

You can use . . .	. . . when you know *n* and . . .	. . . as in . . .
Pythagorean Theorem: $\left(\frac{1}{2}s\right)^2 + a^2 = r^2$	Two measures: *r* and ***a***, or *r* and ***s***	Example 2 and Guided Practice Ex. 3.
Special Right Triangles	Any one measure: *r* or ***a*** or ***s*** **And** the value of *n* is 3, 4, or 6	Guided Practice Ex. 5.
Trigonometry	Any one measure: *r* or ***a*** or ***s***	Example 3 and Guided Practice Exs. 4 and 5.

11.6 EXERCISES

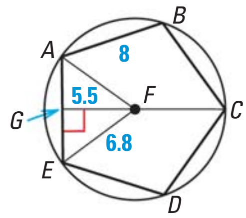
HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 21, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 5, 18, 22, and 44

SKILL PRACTICE

VOCABULARY **In Exercises 1–4, use the diagram shown.**



1. Identify the *center* of regular polygon *ABCDE*.
2. Identify a *central angle* of the polygon.
3. What is the *radius* of the polygon?
4. What is the *apothem*?

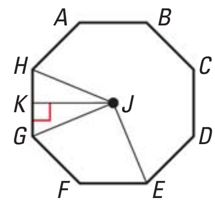
5. ★ **WRITING** *Explain* how to find the measure of a *central angle* of a regular polygon with n sides.

EXAMPLE 1 on p. 762 for Exs. 6–13

MEASURES OF CENTRAL ANGLES **Find the measure of a central angle of a regular polygon with the given number of sides. Round answers to the nearest tenth of a degree, if necessary.**

6. 10 sides
7. 18 sides
8. 24 sides
9. 7 sides

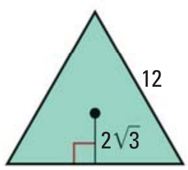
FINDING ANGLE MEASURES **Find the given angle measure for the regular octagon shown.**

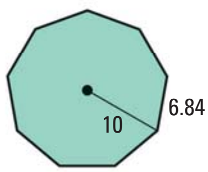


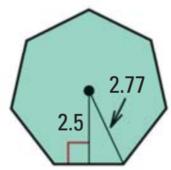
10. $m\angle GJH$
11. $m\angle GJK$
12. $m\angle KGJ$
13. $m\angle EJH$

EXAMPLE 2 on p. 763 for Exs. 14–17

FINDING AREA **Find the area of the regular polygon.**

14. 

15. 

16. 

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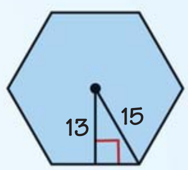
17. **ERROR ANALYSIS** *Describe* and correct the error in finding the area of the regular hexagon.

$\sqrt{15^2 - 13^2} \approx 7.5$

$A = \frac{1}{2}a \cdot ns$

$A = \frac{1}{2}(13)(6)(7.5) = 292.5$



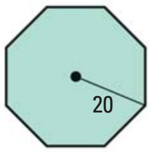
EXAMPLE 3 on p. 764 for Exs. 18–25

18. ★ **MULTIPLE CHOICE** Which expression gives the apothem for a regular dodecagon with side length 8?

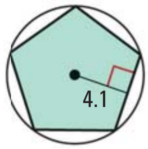
(A) $a = \dfrac{4}{\tan 30°}$ (B) $a = \dfrac{4}{\tan 15°}$ (C) $a = \dfrac{8}{\tan 15°}$ (D) $a = 8 \cdot \cos 15°$

PERIMETER AND AREA **Find the perimeter and area of the regular polygon.**

19.



20.

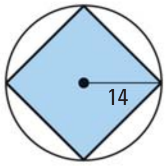


21.

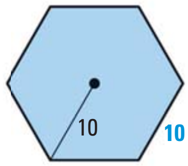
22. ★ **SHORT RESPONSE** The perimeter of a regular nonagon is 18 inches. Is that enough information to find the area? If so, find the area and *explain* your steps. If not, *explain* why not.

CHOOSE A METHOD **Identify any unknown length(s) you need to know to find the area of the regular polygon. Which methods in the table on page 764 can you use to find those lengths? Choose a method and find the area.**

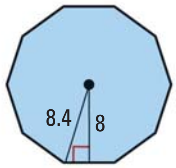
23.



24.



25.

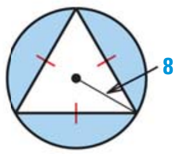


26. **INSCRIBED SQUARE** Find the area of the *unshaded* region in Exercise 23.

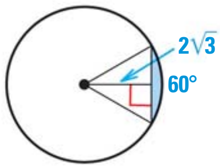
POLYGONS IN CIRCLES **Find the area of the shaded region.**

27.

28.



29.



30. **COORDINATE GEOMETRY** Find the area of a regular pentagon inscribed in a circle whose equation is given by $(x - 4)^2 + (y + 2)^2 = 25$.

REASONING **Decide whether the statement is *true* or *false*. *Explain*.**

31. The area of a regular n-gon of fixed radius r increases as n increases.

32. The apothem of a regular polygon is always less than the radius.

33. The radius of a regular polygon is always less than the side length.

34. **FORMULAS** In Exercise 44 on page 726, the formula $A = \frac{\sqrt{3}s^2}{4}$ for the area A of an equilateral triangle with side length s was developed. Show that the formulas for the area of a triangle and for the area of a regular polygon, $A = \frac{1}{2}bh$ and $A = \frac{1}{2}a \cdot ns$, also result in this formula when they are applied to an equilateral triangle with side length s.

35. **CHALLENGE** An equilateral triangle is shown inside a square inside a regular pentagon inside a regular hexagon. Write an expression for the exact area of the shaded regions in the figure. Then find the approximate area of the entire shaded region, rounded to the nearest whole unit.

○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

PROBLEM SOLVING

EXAMPLE 3
on p. 764
for Ex. 36

36. BASALTIC COLUMNS Basaltic columns are geological formations that result from rapidly cooling lava. The Giant's Causeway in Ireland, pictured here, contains many hexagonal columns. Suppose that one of the columns is in the shape of a regular hexagon with radius 8 inches.

a. What is the apothem of the column?

b. Find the perimeter and area of the column. Round the area to the nearest square inch.

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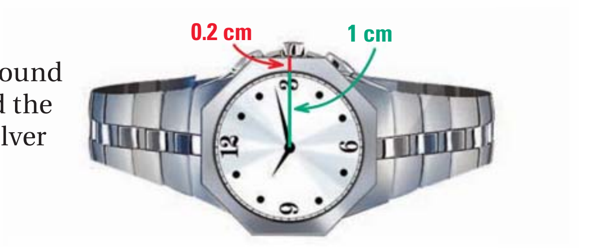
37. WATCH A watch has a circular face on a background that is a regular octagon. Find the apothem and the area of the octagon. Then find the area of the silver border around the circular face.



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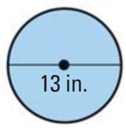
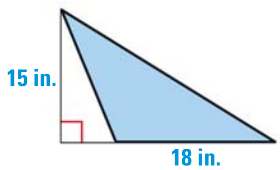
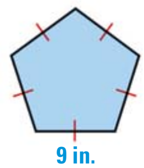
38. COMPARING AREAS *Predict* which figure has the greatest area and which has the smallest area. Check by finding the area of each figure.

a.


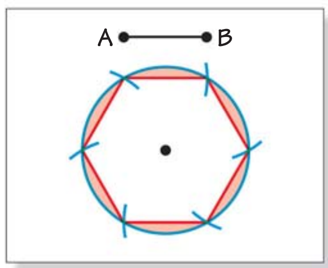
b.


c.


39. CRAFTS You want to make two wooden trivets, a large one and a small one. Both trivets will be shaped like regular pentagons. The perimeter of the small trivet is 15 inches, and the perimeter of the large trivet is 25 inches. Find the area of the small trivet. Then use the Areas of Similar Polygons Theorem to find the area of the large trivet. Round your answers to the nearest tenth.

40. CONSTRUCTION Use a ruler and compass.

a. Draw $\overline{AB}$ with a length of 1 inch. Open the compass to 1 inch and draw a circle with that radius. Using the same compass setting, mark off equal parts along the circle. Then connect the six points where the compass marks and circle intersect to draw a regular hexagon as shown.



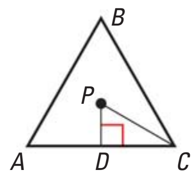
b. What is the area of the hexagon? of the shaded region?

c. *Explain* how to construct an equilateral triangle.

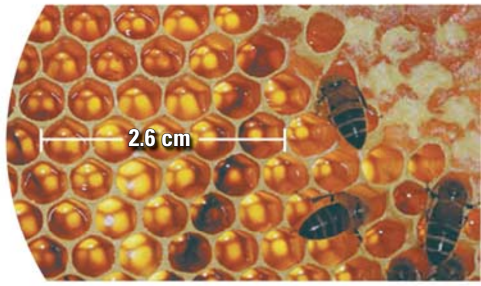
41. HEXAGONS AND TRIANGLES Show that a regular hexagon can be divided into six equilateral triangles with the same side length.

42. ALTERNATIVE METHODS Find the area of a regular hexagon with side length 2 and apothem $\sqrt{3}$ in at least four different ways.

43. **APPLYING TRIANGLE PROPERTIES** In Chapter 5, you learned properties of special segments in triangles. Use what you know about special segments in triangles to show that radius CP in equilateral $\triangle ABC$ is twice the apothem DP.



44. ★ **EXTENDED RESPONSE** Assume that each honeycomb cell is a regular hexagon. The distance is measured through the center of each cell.



a. Find the average distance across a cell in centimeters.

b. Find the area of a "typical" cell in square centimeters. Show your steps.

c. What is the area of 100 cells in square centimeters? in square decimeters? (1 decimeter = 10 centimeters.)

d. Scientists are often interested in the number of cells per square decimeter. *Explain* how to rewrite your results in this form.

45. **CONSTANT PERIMETER** Use a piece of string that is 60 centimeters long.

a. Arrange the string to form an equilateral triangle and find the area. Next form a square and find the area. Then do the same for a regular pentagon, a regular hexagon, and a regular decagon. What is happening to the area?

b. Predict and then find the areas of a regular 60-gon and a regular 120-gon.

c. Graph the area A as a function of the number of sides n. The graph approaches a limiting value. What shape do you think will have the greatest area? What will that area be?

46. **CHALLENGE** Two regular polygons both have n sides. One of the polygons is inscribed in, and the other is circumscribed about, a circle of radius r. Find the area between the two polygons in terms of n and r.

MIXED REVIEW

PREVIEW
Prepare for Lesson 11.7 in Exs. 47–51.

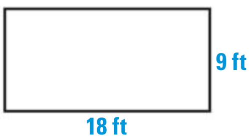
A jar contains 10 red marbles, 6 blue marbles, and 2 white marbles. Find the probability of the event described. *(p. 893)*

47. You randomly choose one red marble from the jar, put it back in the jar, and then randomly choose a red marble.

48. You randomly choose one blue marble from the jar, keep it, and then randomly choose one white marble.

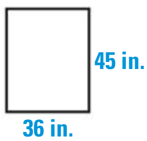
Find the ratio of the width to the length of the rectangle. Then simplify the ratio. *(p. 356)*

49.



50.

51.



52. The vertices of quadrilateral $ABCD$ are $A(-3, 3)$, $B(1, 1)$, $C(1, -3)$, and $D(-3, -1)$. Draw $ABCD$ and determine whether it is a parallelogram. *(p. 522)*

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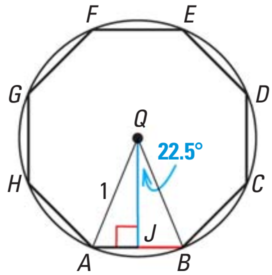
Keystrokes

11.6 Perimeter and Area of Polygons

MATERIALS • computer

QUESTION **How can you use a spreadsheet to find perimeters and areas of regular *n*-gons?**

First consider a regular octagon with radius 1.



Because there are 8 central angles, $m\angle JQB$ is $\frac{1}{2}\left(\frac{360°}{8}\right) = \frac{180°}{8}$, or 22.5°.

You can express the side length and apothem using trigonometric functions.

$\sin 22.5° = \frac{JB}{QB} = \frac{JB}{1} = JB$ $\qquad$ $\cos 22.5° = \frac{QJ}{QB} = \frac{QJ}{1} = QJ$

So, **side length** $\boldsymbol{s} = 2(JB) = 2 \cdot \sin 22.5°$ $\qquad$ So, **apothem** $\boldsymbol{a}$ is $QJ = \cos 22.5°$

Perimeter $\boldsymbol{P} = 8s = 8(2 \cdot \sin 22.5°) = 16 \cdot \sin 22.5°$

Area $\boldsymbol{A} = \frac{1}{2}aP = \frac{1}{2}(\cos 22.5°)(16 \cdot \sin 22.5°) = 8(\cos 22.5°)(\sin 22.5°)$

Using these steps for any regular n-gon inscribed in a circle of radius 1 gives

$$P = 2n \cdot \sin\left(\frac{180°}{n}\right) \quad \text{and} \quad A = n \cdot \sin\left(\frac{180°}{n}\right) \cdot \cos\left(\frac{180°}{n}\right).$$

EXAMPLE **Use a spreadsheet to find measures of regular *n*-gons**

STEP 1 ***Make a table*** Use a spreadsheet to make a table with three columns.

	A	B	C
1	Number of sides	Perimeter	Area
2	n	2*n*sin(180/n)	n*sin(180/n)*cos(180/n)
3	3	=2*A3*sin(180/A3)	=A3*sin(180/A3)*cos(180/A3)
4	=A3+1	=2*A4*sin(180/A4)	=A4*sin(180/A4)*cos(180/A4)

If your spreadsheet uses radian measure, use "pi()" instead of "180."

STEP 2 ***Enter formulas*** Enter the formulas shown in cells A4, B3, and C3. Then use the Fill Down feature to create more rows.

PRACTICE

1. What shape do the regular n-gons approach as the value of n gets very large? *Explain* your reasoning.
2. What value do the perimeters approach as the value of n gets very large? *Explain* how this result justifies the formula for the circumference of a circle.
3. What value do the areas approach as the value of n gets very large? *Explain* how this result justifies the formula for the area of a circle.

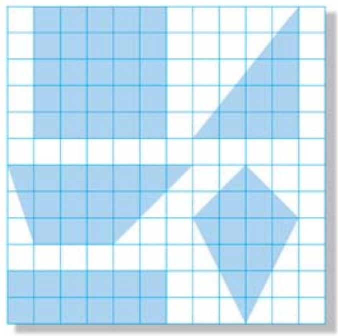
11.7 Investigate Geometric Probability

MATERIALS • graph paper • small dried bean

QUESTION How do theoretical and experimental probabilities compare?

EXPLORE Find geometric probabilities

STEP 1 ***Draw a target*** On a piece of graph paper, make a target by drawing some polygons. Choose polygons whose area you can calculate and make them as large as possible. Shade in the polygons. An example is shown.



Sample target

STEP 2 ***Calculate theoretical probability*** Calculate the *theoretical* probability that a randomly tossed bean that lands on the target will land in a shaded region.

$$\text{Theoretical probability} = \frac{\text{Sum of areas of polygons}}{\text{Area of paper}}$$

STEP 3 ***Perform an experiment*** Place the target on the floor against a wall. Toss a dried bean so that it hits the wall and then bounces onto the target. Determine whether the bean lands on a shaded or unshaded region of the target. If the bean lands so that it lies in both a shaded and unshaded region, use the region in which most of the bean lies. If the bean does not land completely on the target, repeat the toss.

STEP 4 ***Make a table*** Record the results of the toss in a table. Repeat until you have recorded the results of 50 tosses.

Toss	Shaded area	Unshaded area
1	X	
2		X
. . .	. . .	. . .
50	X	

STEP 5 ***Calculate experimental probability*** Use the results from your table to calculate the *experimental* probability that a randomly tossed bean that lands on the target will land in a shaded region.

$$\text{Experimental probability} = \frac{\text{Number of times a bean landed on a shaded region}}{\text{Total number of tosses}}$$

DRAW CONCLUSIONS Use your observations to complete these exercises

1. *Compare* the theoretical probability from Step 2 with the experimental probability from Step 5. What do you notice?
2. Repeat Steps 3–5, this time using only 10 tosses. Calculate the experimental probability for those 10 tosses. *Compare* the experimental probability and the theoretical probability.
3. **REASONING** How does the number of tosses affect the relationship between the experimental and theoretical probabilities? *Explain.*

11.7 Use Geometric Probability

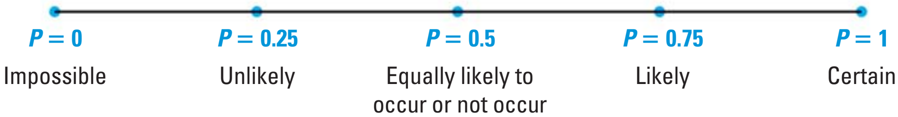
Before You found lengths and areas.

Now You will use lengths and areas to find geometric probabilities.

Why? So you can calculate real-world probabilities, as in Example 2.

Key Vocabulary
- **probability**
- **geometric probability**

The **probability** of an event is a measure of the likelihood that the event will occur. It is a number between 0 and 1, inclusive, and can be expressed as a fraction, decimal, or percent. The probability of event A is written as $P(A)$.



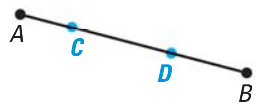
In a previous course, you may have found probability by calculating the ratio of the number of favorable outcomes to the total number of possible outcomes. In this lesson, you will find *geometric probabilities*.

A **geometric probability** is a ratio that involves a geometric measure such as length or area.

KEY CONCEPT *For Your Notebook*

Probability and Length

Let $\overline{AB}$ be a segment that contains the segment $\overline{CD}$. If a point K on $\overline{AB}$ is chosen at random, then the probability that it is on $\overline{CD}$ is the ratio of the length of $\overline{CD}$ to the length of $\overline{AB}$.



$$P(K \text{ is on } \overline{CD}) = \frac{\text{Length of } \overline{CD}}{\text{Length of } \overline{AB}}$$

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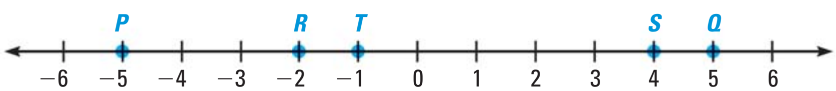
EXAMPLE 1 Use lengths to find a geometric probability

USE A FORMULA

To apply the geometric probability formulas on this page and on page 772, you need to know that every point on the segment or in the region is *equally likely* to be chosen.

Find the probability that a point chosen at random on $\overline{PQ}$ is on $\overline{RS}$.



Solution

$$P(\text{Point is on } \overline{RS}) = \frac{\text{Length of } \overline{RS}}{\text{Length of } \overline{PQ}} = \frac{|4-(-2)|}{|5-(-5)|} = \frac{6}{10} = \frac{3}{5}, \text{ 0.6, or } 60\%.$$

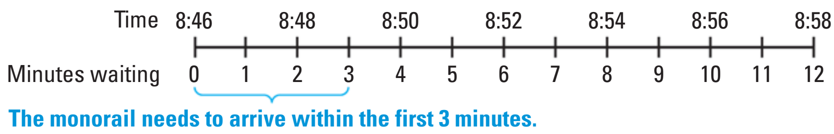
EXAMPLE 2 Use a segment to model a real-world probability

MONORAIL A monorail runs every 12 minutes. The ride from the station near your home to the station near your work takes 9 minutes. One morning, you arrive at the station near your home at 8:46. You want to get to the station near your work by 8:58. What is the probability you will get there by 8:58?

Solution

STEP 1 **Find** the longest you can wait for the monorail and still get to the station near your work by 8:58. The ride takes 9 minutes, so you need to catch the monorail no later than 9 minutes before 8:58, or by 8:49. The longest you can wait is 3 minutes (8:49 − 8:46 = 3 min).

STEP 2 **Model** the situation. The monorail runs every 12 minutes, so it will arrive in 12 minutes or less. You need it to arrive within 3 minutes.



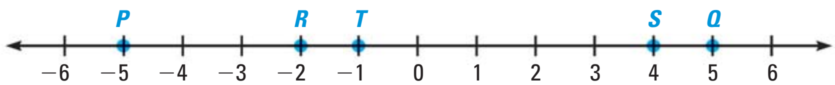
STEP 3 **Find** the probability.

$$P(\text{You get to the station by 8:58}) = \frac{\text{Favorable waiting time}}{\text{Maximum waiting time}} = \frac{3}{12} = \frac{1}{4}$$

▸ The probability that you will get to the station by 8:58. is $\frac{1}{4}$, or 25%.

✓ GUIDED PRACTICE for Examples 1 and 2

Find the probability that a point chosen at random on $\overline{PQ}$ is on the given segment. Express your answer as a fraction, a decimal, and a percent.



1. $\overline{RT}$ **2.** $\overline{TS}$ **3.** $\overline{PT}$ **4.** $\overline{RQ}$

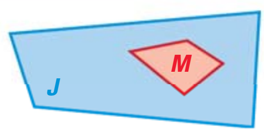
5. **WHAT IF?** In Example 2, suppose you arrive at the station near your home at 8:43. What is the probability that you will get to the station near your work by 8:58?

PROBABILITY AND AREA Another formula for geometric probability involves the ratio of the areas of two regions.

KEY CONCEPT *For Your Notebook*

Probability and Area

Let J be a region that contains region M. If a point K in J is chosen at random, then the probability that it is in region M is the ratio of the area of M to the area of J.



$$P(K \text{ is in region } M) = \frac{\text{Area of } M}{\text{Area of } J}$$

EXAMPLE 3 Use areas to find a geometric probability

ARCHERY The diameter of the target shown at the right is 80 centimeters. The diameter of the red circle on the target is 16 centimeters. An arrow is shot and hits the target. If the arrow is equally likely to land on any point on the target, what is the probability that it lands in the red circle?

Solution

ANOTHER WAY
All circles are similar and the Area of Similar Polygons Theorem also applies to circles. The ratio of radii is 8 : 40, or 1 : 5, so the ratio of areas is $1^2 : 5^2$, or 1 : 25.

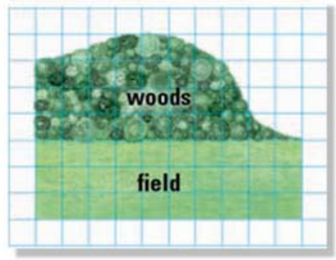
Find the ratio of the area of the red circle to the area of the target.

$$P(\text{arrow lands in red region}) = \frac{\text{Area of red circle}}{\text{Area of target}} = \frac{\pi(8^2)}{\pi(40^2)} = \frac{64\pi}{1600\pi} = \frac{1}{25}$$

▶ The probability that the arrow lands in the red region is $\frac{1}{25}$, or 4%.

EXAMPLE 4 Estimate area on a grid to find a probability

SCALE DRAWING Your dog dropped a ball in a park. A scale drawing of the park is shown. If the ball is equally likely to be anywhere in the park, estimate the probability that it is in the field.

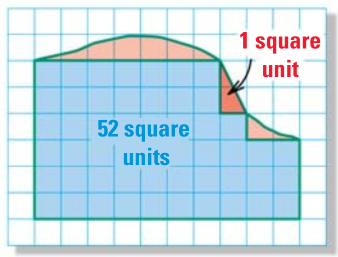


Solution

STEP 1 **Find** the area of the field. The shape is a rectangle, so the area is $bh = 10 \cdot 3 = 30$ square units.

STEP 2 **Find** the total area of the park.

Count the squares that are fully covered. There are 30 squares in the field and 22 in the woods. So, there are 52 full squares.



Make groups of partially covered squares so the combined area of each group is about 1 square unit. The total area of the partial squares is about 6 or 7 square units. So, use $52 + 6.5 = 58.5$ square units for the total area.

CHECK RESULTS
The ball must be either in the field or in the woods, so check that the probabilities in Example 4 and Guided Practice Exercise 7 add up to 100%.

STEP 3 **Write** a ratio of the areas to find the probability.

$$P(\text{ball in field}) = \frac{\text{Area of field}}{\text{Total area of park}} \approx \frac{30}{58.5} = \frac{300}{585} = \frac{20}{39}$$

▶ The probability that the ball is in the field is about $\frac{20}{39}$, or 51.3%.

✓ GUIDED PRACTICE for Examples 3 and 4

6. In the target in Example 3, each ring is 8 centimeters wide. Find the probability that an arrow lands in the black region.
7. In Example 4, estimate the probability that the ball is in the woods.

11.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 9, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 23, 34, and 35

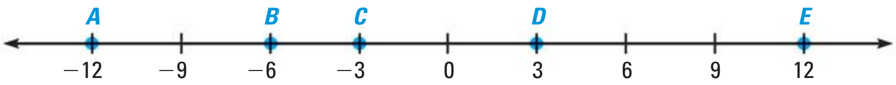
SKILL PRACTICE

1. **VOCABULARY** Copy and complete: If an event cannot occur, its probability is _?_. If an event is certain to occur, its probability is _?_.

2. ★ **WRITING** *Compare* a geometric probability and a probability found by dividing the number of favorable outcomes by the total number of possible outcomes.

EXAMPLE 1 on p. 771 for Exs. 3–7

PROBABILITY ON A SEGMENT In Exercises 3–6, find the probability that a point K, selected randomly on $\overline{AE}$, is on the given segment. Express your answer as a fraction, decimal, and percent.

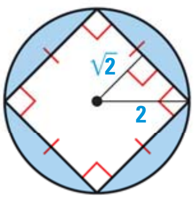


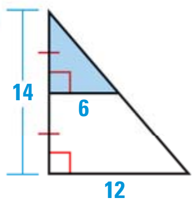
3. $\overline{AD}$ 4. $\overline{BC}$ 5. $\overline{DE}$ 6. $\overline{AE}$

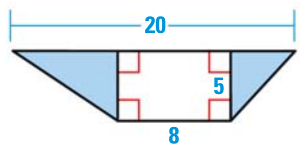
7. ★ **WRITING** Look at your answers to Exercises 3 and 5. *Describe* how the two probabilities are related.

EXAMPLE 3 on p. 773 for Exs. 8–11

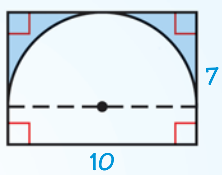
FIND A GEOMETRIC PROBABILITY Find the probability that a randomly chosen point in the figure lies in the shaded region.

8.


9.


10.


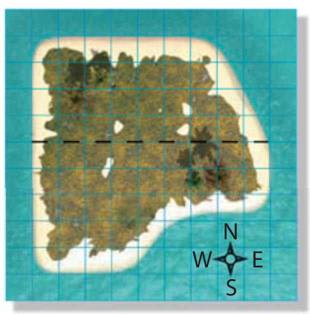
11. **ERROR ANALYSIS** Three sides of the rectangle are tangent to the semicircle. *Describe* and correct the error in finding the probability that a randomly chosen point in the figure lies in the shaded region.



$$\frac{10(7) - \frac{1}{2}\pi(5)^2}{10(7)} = \frac{70 - 12.5\pi}{70} \approx 43.9\%$$

EXAMPLE 4 on p. 773 for Exs. 12–14

ESTIMATING AREA Use the scale drawing.



12. What is the approximate area of the north side of the island? the south side of the island? the whole island?

13. Find the probability that a randomly chosen location on the island lies on the north side.

14. Find the probability that a randomly chosen location on the island lies on the south side.

15. **SIMILAR TRIANGLES** In Exercise 9, how do you know that the shaded triangle is similar to the whole triangle? *Explain* how you can use the Area of Similar Polygons Theorem to find the desired probability.

xy ALGEBRA In Exercises 16–19, find the probability that a point chosen at random on the segment satisfies the inequality.

16. $x - 6 \le 1$

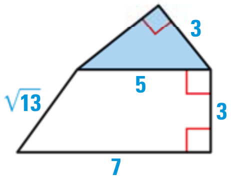
17. $1 \le 2x - 3 \le 5$

18. $\frac{x}{2} \ge 7$

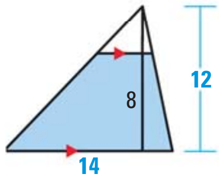
19. $3x \le 27$

FIND A GEOMETRIC PROBABILITY Find the probability that a randomly chosen point in the figure lies in the shaded region. *Explain* your steps.

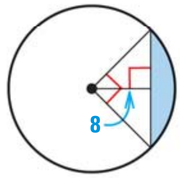
20.



21.



22.



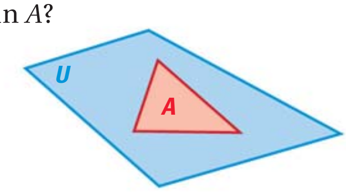
23. ★ **MULTIPLE CHOICE** A point X is chosen at random in region U, and U includes region A. What is the probability that X is not in A?

Ⓐ $\dfrac{\text{Area of } A}{\text{Area of } U}$

Ⓑ $\dfrac{\text{Area of } A}{\text{Area of } U - \text{Area of } A}$

Ⓒ $\dfrac{1}{\text{Area of } A}$

Ⓓ $\dfrac{\text{Area of } U - \text{Area of } A}{\text{Area of } U}$



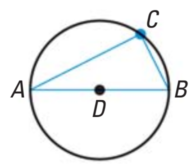
24. **ARCS AND SECTORS** A sector of a circle intercepts an arc of 80°. Find the probability that a randomly chosen point on the circle lies on the arc. Find the probability that a randomly chosen point in the circle lies in the sector. *Explain* why the probabilities do not depend on the radius.

INSCRIBED POLYGONS Find the probability that a randomly chosen point in the circle described lies in the inscribed polygon.

25. Regular hexagon inscribed in circle with circumference $C \approx 188.5$

26. Regular octagon inscribed in circle with radius r

27. **INSCRIBED ANGLES** Points A and B are the endpoints of a diameter of $\odot D$. Point C is chosen at random from the other points on the circle. What is the probability that $\triangle ABC$ is a right triangle? What is the probability that $m\angle CAB \le 45°$?

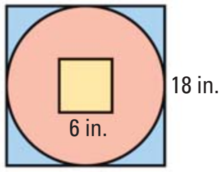


28. **COORDINATE GRAPHS** Graph the system of inequalities $0 \le x \le 2$, $0 \le y \le 3$, and $y \ge x$. If a point (x, y) is chosen at random in the solution region, what is the probability that $x^2 + y^2 \ge 4$?

29. **CHALLENGE** You carry out a series of steps to paint a walking stick. In the first step, you paint half the length of the stick. For each following step, you paint half of the remaining unpainted portion of the stick. After n steps, you choose a point at random on the stick. Find a value of n so that the probability of choosing a point on the painted portion of the stick after the nth step is greater than 99.95%.

PROBLEM SOLVING

30. DARTBOARD A dart is thrown and hits the target shown. If the dart is equally likely to hit any point on the target, what is the probability that it hits inside the inner square? that it hits outside the inner square but inside the circle?



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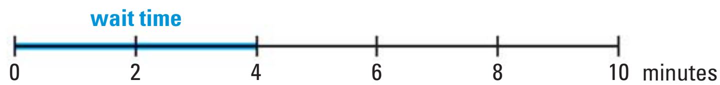
EXAMPLE 2 on p. 772 for Exs. 31–33

31. TRANSPORTATION A fair provides a shuttle bus from a parking lot to the fair entrance. Buses arrive at the parking lot every 10 minutes. They wait for 4 minutes while passengers get on and get off. Then the buses depart.



a. What is the probability that there is a bus waiting when a passenger arrives at a random time?

b. What is the probability that there is not a bus waiting when a passenger arrives at a random time?

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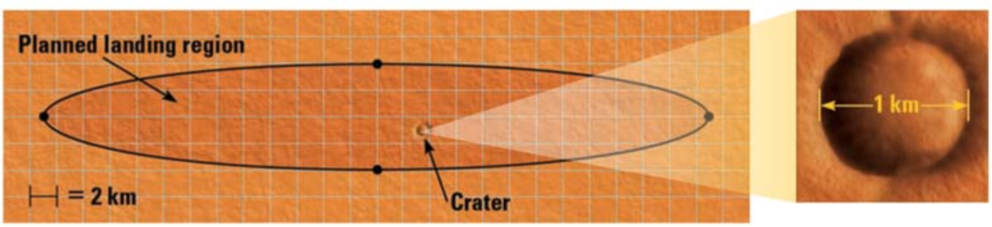
32. FIRE ALARM Suppose that your school day is from 8:00 A.M. until 3:00 P.M. You eat lunch at 12:00 P.M. If there is a fire drill at a random time during the day, what is the probability that it begins before lunch?

33. PHONE CALL You are expecting a call from a friend anytime between 7:00 P.M. and 8:00 P.M. You are practicing the drums and cannot hear the phone from 6:55 P.M. to 7:10 P.M. What is the probability that you missed your friend's call?

34. ★ EXTENDED RESPONSE Scientists lost contact with the space probe Beagle 2 when it was landing on Mars in 2003. They have been unable to locate it since. Early in the search, some scientists thought that it was possible, though unlikely, that Beagle had landed in a circular crater inside the planned landing region. The diameter of the crater is 1 km.



a. In the scale drawing, each square has side length 2 kilometers. Estimate the area of the planned landing region. *Explain* your steps.

b. Estimate the probability of Beagle 2 landing in the crater if it was equally likely to land anywhere in the planned landing region.

35. ★ SHORT RESPONSE If the central angle of a sector of a circle stays the same and the radius of the circle doubles, what can you conclude about the probability of a randomly selected point being in the sector? *Explain*. Include an example with your explanation.

36. **PROBABILITY AND LENGTH** A 6 inch long rope is cut into two pieces at a random point. Find the probability both pieces are at least 1 inch long.

37. **COMPOUND EVENTS** You throw two darts at the dartboard in Exercise 30 on page 776. Each dart hits the dartboard. The throws are independent of each other. Find the probability of the compound event described.

a. Both darts hit the yellow square.

b. The first dart hits the yellow square and the second hits outside the circle.

c. Both darts hit inside the circle but outside the yellow square.

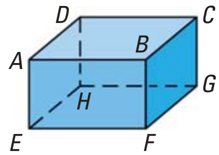
38. **CHALLENGE** A researcher used a 1 hour tape to record birdcalls. Eight minutes after the recorder was turned on, a 5 minute birdcall began. Later, the researcher accidentally erased 10 continuous minutes of the tape. What is the probability that part of the birdcall was erased? What is the probability that all of the birdcall was erased?

MIXED REVIEW

PREVIEW
Prepare for Lesson 12.1 in Exs. 39–41.

39. Draw a concave hexagon and a concave pentagon. *(p. 42)*

Think of each segment shown as part of a line.

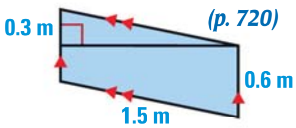


40. Name the intersection of plane *DCH* and plane *ADE*. *(p. 96)*

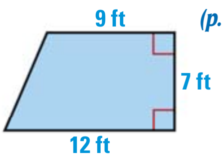
41. Name a plane that appears to be parallel to plane *ADH*. *(p. 147)*

Find the area of the polygon.

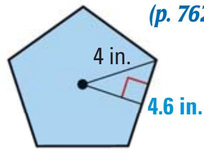
42. *(p. 720)*



43. *(p. 730)*



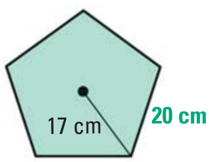
44. *(p. 762)*



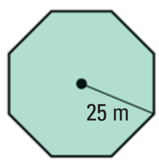
QUIZ for Lessons 11.6–11.7

Find the area of the regular polygon. *(p. 762)*

1.

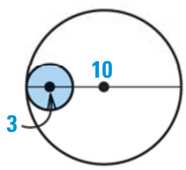


2.

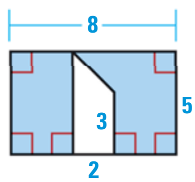


Find the probability that a randomly chosen point in the figure lies in the shaded region. *(p. 771)*

3.



4.



STATE TEST PRACTICE
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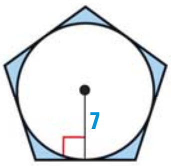
Lessons 11.4–11.7

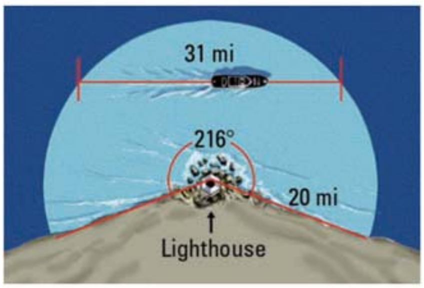
1. **MULTI-STEP PROBLEM** The Hobby-Eberly optical telescope is located in Fort Davis, Texas. The telescope's primary mirror is made of 91 small mirrors that form a hexagon. Each small mirror is a regular hexagon with side length 0.5 meter.

   a. Find the apothem of a small mirror.
   b. Find the area of one of the small mirrors.
   c. Find the area of the primary mirror.

2. **GRIDDED ANSWER** As shown, a circle is inscribed in a regular pentagon. The circle and the pentagon have the same center. Find the area of the shaded region. Round to the nearest tenth.



3. **EXTENDED RESPONSE** The diagram shows a projected beam of light from a lighthouse.

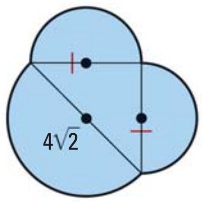


   a. Find the area of the water's surface that is illuminated by the lighthouse.
   b. A boat traveling along a straight line is illuminated by the lighthouse for about 31 miles. Find the closest distance between the lighthouse and the boat. *Explain* your steps.

4. **SHORT RESPONSE** At a school fundraiser, a glass jar with a circular base is filled with water. A circular red dish is placed at the bottom of the jar. A person donates a coin by dropping it into the jar. If the coin lands in the dish, the person wins a small prize.

   a. Suppose a coin tossed into the jar has an equally likely chance of landing anywhere on the bottom of the jar, including in the dish. What is the probability that it will land in the dish?
   b. Suppose 400 coins are dropped into the jar. About how many prizes would you expect people to win? *Explain*.

5. **SHORT RESPONSE** The figure is made of a right triangle and three semicircles. Write expressions for the perimeter and area of the figure in terms of π. *Explain* your reasoning.



6. **OPEN-ENDED** In general, a fan with a greater area does a better job of moving air and cooling you. The fan below is a sector of a cardboard circle. Give an example of a cardboard fan with a smaller radius that will do a better job of cooling you. The intercepted arc should be less than 180°.

11 CHAPTER SUMMARY

BIG IDEAS
For Your Notebook

Big Idea 1

Using Area Formulas for Polygons

Polygon	Formula	
Triangle	$A = \frac{1}{2}bh,$	with base b and height h
Parallelogram	$A = bh,$	with base b and height h
Trapezoid	$A = \frac{1}{2}h(b_1 + b_2),$	with bases b_1 and b_2 and height h
Rhombus	$A = \frac{1}{2}d_1d_2,$	with diagonals d_1 and d_2
Kite	$A = \frac{1}{2}d_1d_2,$	with diagonals d_1 and d_2
Regular polygon	$A = \frac{1}{2}a \cdot ns,$	with apothem a, n sides, and side length s

Sometimes you need to use the Pythagorean Theorem, special right triangles, or trigonometry to find a length in a polygon before you can find its area.

Big Idea 2

Relating Length, Perimeter, and Area Ratios in Similar Polygons

You can use ratios of corresponding measures to find other ratios of measures. You can solve proportions to find unknown lengths or areas.

If two figures are similar and . . .	then . . .
the ratio of side lengths is $a:b$	• the ratio of perimeters is also $a:b$. • the ratio of areas is $a^2:b^2$.
the ratio of perimeters is $c:d$	• the ratio of side lengths is also $c:d$. • the ratio of areas is $c^2:d^2$.
the ratio of areas is $e:f$	• the ratio of side lengths is $\sqrt{e}:\sqrt{f}$. • the ratio of perimeters is $\sqrt{e}:\sqrt{f}$.

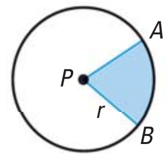
Big Idea 3

Comparing Measures for Parts of Circles and the Whole Circle

Given $\odot P$ with radius r, you can use proportional reasoning to find measures of parts of the circle.

Arc length $\quad \dfrac{\text{Arc length of } \overset{\frown}{AB}}{2\pi r} = \dfrac{m\overset{\frown}{AB}}{360^\circ}$ ← Part ← Whole

Area of sector $\quad \dfrac{\text{Area of sector } APB}{\pi r^2} = \dfrac{m\overset{\frown}{AB}}{360^\circ}$ ← Part ← Whole



11 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

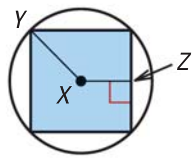
- bases of a parallelogram, *p. 720*
- height of a parallelogram, *p. 720*
- height of a trapezoid, *p. 730*
- circumference, *p. 746*
- arc length, *p. 747*
- sector of a circle, *p. 756*
- center of a polygon, *p. 762*
- radius of a polygon, *p. 762*
- apothem of a polygon, *p. 762*
- central angle of a regular polygon, *p. 762*
- probability, *p. 771*
- geometric probability, *p. 771*

VOCABULARY EXERCISES

1. Copy and complete: A *sector of a circle* is the region bounded by __?__.

2. **WRITING** *Explain* the relationship between the height of a parallelogram and the bases of a parallelogram.

The diagram shows a square inscribed in a circle. Name an example of the given segment.



3. An apothem of the square **4.** A radius of the square

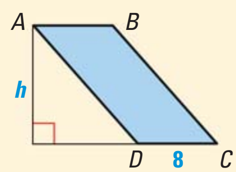
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 11.

11.1 Areas of Triangles and Parallelograms *pp. 720–726*

EXAMPLE

The area of ▱*ABCD* is 96 square units. Find its height *h*.



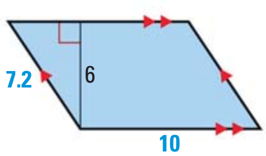
$A = bh$	Formula for area of a parallelogram
$96 = 8h$	Substitute 96 for *A* and 8 for *b*.
$h = 12$	Solve.

EXAMPLES 1, 2, and 3 on pp. 721–722 for Exs. 5–8

EXERCISES

Find the area of the polygon.

5.



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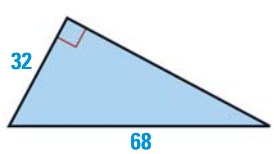
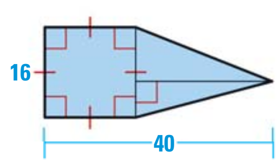


7.



8. The area of a triangle is 147 square inches and its height is 1.5 times its base. Find the base and the height of the triangle.

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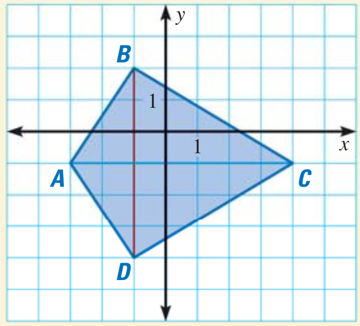
Chapter Review Practice

11.2 Areas of Trapezoids, Rhombuses, and Kites *pp. 730–736*

EXAMPLE

Find the area of the kite.



Find the lengths of the diagonals of the kite.

$d_1 = BD = |2 - (-4)| = 6$

$d_2 = AC = |4 - (-3)| = 7$

Find the area of $ABCD$.

$A = \frac{1}{2}d_1d_2$ — Formula for area of a kite

$= \frac{1}{2}(6)(7) = 21$ — Substitute and simplify.

▶ The area of the kite is 21 square units.

EXERCISES

EXAMPLE 4 on p. 732 for Exs. 9–11

Graph the polygon with the given vertices and find its area.

9. $L(2, 2)$, $M(6, 2)$, $N(8, 4)$, $P(4, 4)$

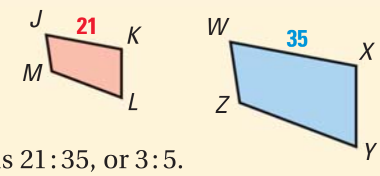
10. $Q(-3, 0)$, $R(-2, 3)$, $S(-1, 0)$, $T(-2, -2)$

11. $D(-1, 4)$, $E(5, 4)$, $F(3, -2)$, $G(1, -2)$

11.3 Perimeter and Area of Similar Figures *pp. 737–743*

EXAMPLE

Quadrilaterals *JKLM* and *WXYZ* are similar. Find the ratios (red to blue) of the perimeters and of the areas.



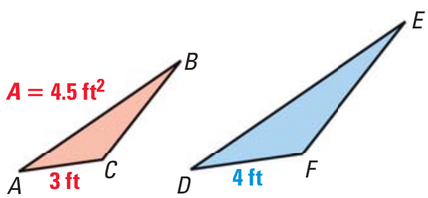
The ratio of the lengths of the corresponding sides is 21 : 35, or 3 : 5. Using Theorem 6.1, the ratio of the perimeters is 3 : 5. Using Theorem 11.7, the ratio of the areas is $3^2 : 5^2$, or 9 : 25.

EXERCISES

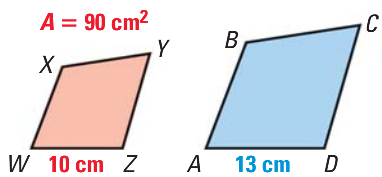
EXAMPLES 1, 2, and 3 on pp. 737–738 for Exs. 12–14

The polygons are similar. Find the ratio (red to blue) of the perimeters and of the areas. Then find the unknown area.

12. $\triangle ABC \sim \triangle DEF$



13. $WXYZ \sim ABCD$



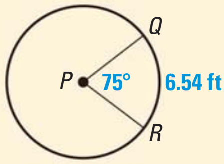
14. The ratio of the areas of two similar figures is 144 : 49. Write the ratio of the lengths of corresponding sides.

11 CHAPTER REVIEW

11.4 Circumference and Arc Length *pp. 746–752*

EXAMPLE

The arc length of $\overparen{QR}$ is 6.54 feet. Find the radius of $\odot P$.



$$\frac{\text{Arc length of } \overparen{QR}}{2\pi r} = \frac{m\overparen{QR}}{360^\circ}$$ **Arc Length Corollary**

$$\frac{6.54}{2\pi r} = \frac{75^\circ}{360^\circ}$$ **Substitute.**

$$6.54(360^\circ) = 75^\circ(2\pi r)$$ **Cross Products Property**

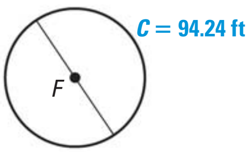
$$r \approx 5.00 \text{ ft}$$ **Solve.**

EXERCISES

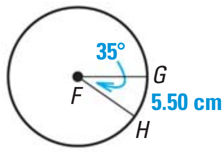
EXAMPLES 1, 3, and 4 on pp. 746, 748 for Exs. 15–17

Find the indicated measure.

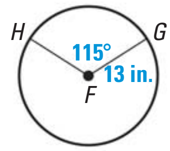
15. Diameter of $\odot F$



16. Circumference of $\odot F$



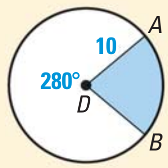
17. Length of $\overparen{GH}$



11.5 Areas of Circles and Sectors *pp. 755–761*

EXAMPLE

Find the area of sector *ADB*.



First find the measure of the minor arc.

$m\angle ADB = 360^\circ - 280^\circ = 80^\circ$, so $m\overparen{AB} = 80^\circ$.

$$\text{Area of sector } ADB = \frac{m\overparen{AB}}{360^\circ} \cdot \pi r^2$$ **Formula for area of a sector**

$$= \frac{80^\circ}{360^\circ} \cdot \pi \cdot 10^2$$ **Substitute.**

$$\approx 69.81 \text{ units}^2$$ **Use a calculator.**

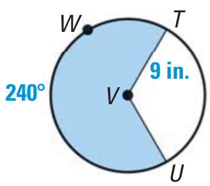
▶ The area of the small sector is about 69.81 square units.

EXERCISES

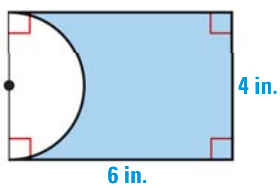
EXAMPLES 2, 3, and 4 on pp. 756–757 for Exs. 18–20

Find the area of the blue shaded region.

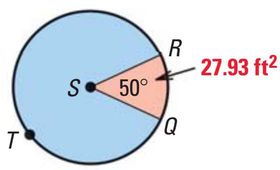
18.



19.



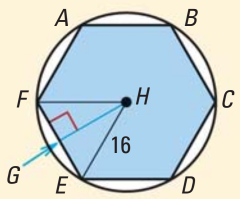
20.



11.6 Areas of Regular Polygons

pp. 762–768

EXAMPLE



A regular hexagon is inscribed in $\odot H$. Find (a) $m\angle EHG$, and (b) the area of the hexagon.

a. $\angle FHE$ is a central angle, so $m\angle FHE = \frac{360°}{6} = 60°$.
Apothem $\overline{GH}$ bisects $\angle FHE$. So, $m\angle EHG = 30°$.

b. Because $\triangle EHG$ is a 30°-60°-90° triangle, $GE = \frac{1}{2} \cdot HE = 8$ and $GH = \sqrt{3} \cdot GE = 8\sqrt{3}$. So, $s = 16$ and $a = 8\sqrt{3}$. Then use the area formula.

$$A = \frac{1}{2}a \cdot ns = \frac{1}{2}(8\sqrt{3})(6)(16) \approx 665.1 \text{ square units}$$

EXERCISES

EXAMPLES 2 and 3 on pp. 763–764 for Exs. 21–22

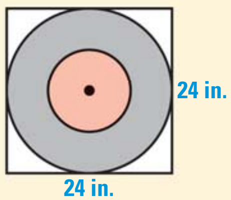
21. PLATTER A platter is in the shape of a regular octagon. Find the perimeter and area of the platter if its apothem is 6 inches.

22. PUZZLE A jigsaw puzzle is in the shape of a regular pentagon. Find its area if its radius is 17 centimeters and its side length is 20 centimeters.

11.7 Use Geometric Probability

pp. 771–777

EXAMPLE



A dart is thrown and hits the square dartboard shown. The dart is equally likely to land on any point on the board. Find the probability that the dart lands in the white region outside the concentric circles.

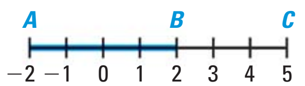
$$P(\text{dart lands in white region}) = \frac{\text{Area of white region}}{\text{Area of dart board}} = \frac{24^2 - \pi(12^2)}{24^2} \approx 0.215$$

▸ The probability that the dart lands in the white region is about 21.5%.

EXERCISES

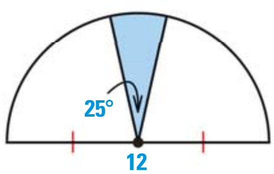
EXAMPLES 1 and 3 on pp. 771, 773 for Exs. 23–26

23. A point K is selected randomly on $\overline{AC}$ at the right. What is the probability that K is on $\overline{AB}$?

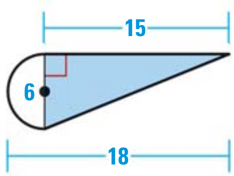


Find the probability that a randomly chosen point in the figure lies in the shaded region.

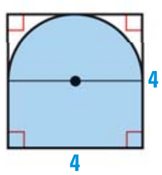
24.



25.



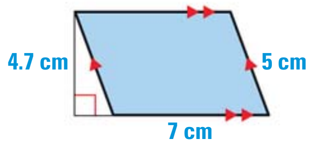
26.



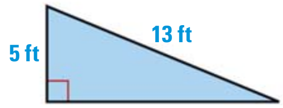
11 CHAPTER TEST

In Exercises 1–6, find the area of the shaded polygon.

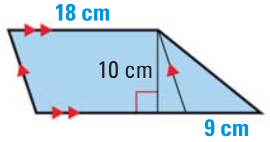
1.



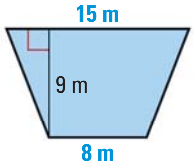
2.



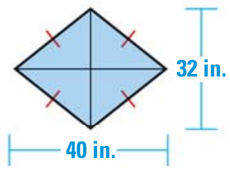
3.



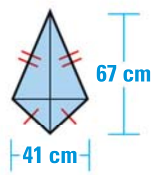
4.



5.



6.



7. The base of a parallelogram is 3 times its height. The area of the parallelogram is 108 square inches. Find the base and the height.

Quadrilaterals *ABCD* and *EFGH* are similar. The perimeter of *ABCD* is 40 inches and the perimeter of *EFGH* is 16 inches.

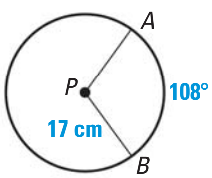
8. Find the ratio of the perimeters of *ABCD* to *EFGH*.

9. Find the ratio of the corresponding side lengths of *ABCD* to *EFGH*.

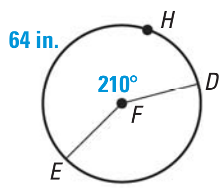
10. Find the ratio of the areas of *ABCD* to *EFGH*.

Find the indicated measure for the circle shown.

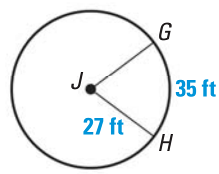
11. Length of $\widehat{AB}$



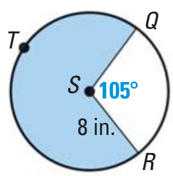
12. Circumference of $\odot F$



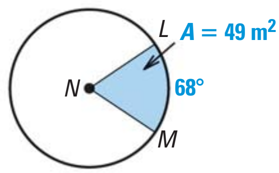
13. $m\widehat{GH}$



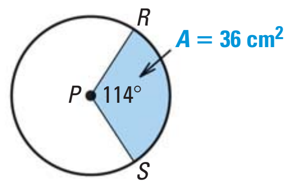
14. Area of shaded sector



15. Area of $\odot N$

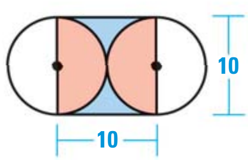


16. Radius of $\odot P$



17. **TILING** A floor tile is in the shape of a regular hexagon and has a perimeter of 18 inches. Find the side length, apothem, and area of the tile.

Find the probability that a randomly chosen point in the figure lies in the region described.



18. In the red region

19. In the blue region

11 ALGEBRA REVIEW

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USE ALGEBRAIC MODELS TO SOLVE PROBLEMS

EXAMPLE 1 *Write and solve an algebraic model for a problem*

FUNDRAISER You are baking cakes to sell at a fundraiser. It costs \$3 to make each cake, and you plan to sell the cakes for \$8 each. You spent \$20 on pans and utensils. How many cakes do you need to sell to make a profit of \$50?

Solution

Let x represent the number of cakes sold.

Income − Expenses = Profit	**Write verbal model.**
$8x - (3x + 20) = 50$	**Substitute $8x$ for income, $3x + 20$ for expenses, and 50 for profit.**
$8x - 3x - 20 = 50$	**Distributive Property**
$5x - 20 = 50$	**Combine like terms.**
$x = 14$	**Solve for x.**

▸ You need to sell 14 cakes to make a profit of \$50.

EXERCISES

EXAMPLE 1 for Exs. 1–7

Write an algebraic model to represent the situation. Then solve the problem.

1. **BICYCLES** You ride your bike 14.25 miles in 90 minutes. At this rate, how far can you bike in 2 hours?

2. **SHOPPING** Alma spent \$39 on a shirt and a jacket. The shirt cost \$12. Find the original cost of a jacket if Alma bought it on sale for 25% off.

3. **CELL PHONES** Your cell phone provider charges \$29.50 per month for 200 minutes. You pay \$.25 per minute for each minute over 200 minutes. In May, your bill was \$32.75. How many additional minutes did you use?

4. **EXERCISE** Jaime burns 12.1 calories per minute running and 7.6 calories per minute swimming. He wants to burn at least 400 calories and plans to swim for 20 minutes. How long does he need to run to meet his goal?

5. **CARS** You buy a car for \$18,000. The value of the car decreases 10% each year. What will the value of the car be after 5 years?

6. **TICKETS** Student tickets for a show cost \$5 and adult tickets cost \$8. At one show, \$2065 was collected in ticket sales. If 62 more student tickets were sold than adult tickets, how many of each type of ticket was sold?

7. **TENNIS** The height h in feet of a tennis ball is $h = -16t^2 + 47t + 6$, where t is the time in seconds after being hit. If the ball is not first hit by another player, how long does it take to reach the ground?

Scoring Rubric

Full Credit
- solution is complete and correct

Partial Credit
- solution is complete but has errors,

or
- solution is without error but incomplete

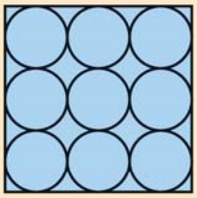
No Credit
- no solution is given,

or
- solution makes no sense

EXTENDED RESPONSE QUESTIONS

PROBLEM

You are making circular signs for a pep rally at your school. You can cut 4 circles with diameter 10 inches from a cardboard square that is 20 inches long on each side, or 9 circles with diameter 12 inches from a cardboard square that is 36 inches long on each side.



a. For each cardboard square, find the area of the cardboard that is used for the signs. Round to the nearest square inch. Show your work.

b. You want to waste as little of a cardboard square as possible. Does it matter which size of cardboard you use? If so, which size of cardboard should you choose if you want to use a greater percent of the cardboard's area for the signs? *Explain*.

Below are sample solutions to the problem. Read each solution and the comments in blue to see why the sample represents *full credit*, *partial credit*, or *no credit*.

SAMPLE 1: Full credit solution

In part (a), the student's work is shown and the calculations are correct.

a. For each cardboard square, multiply the number of circles by the area of one circle.

For the 20 inch square, the radius of each of the 4 circles is 5 inches.

$$\text{Area of 4 circles} = 4 \cdot \pi r^2 = 4 \cdot \pi(5)^2 \approx 314 \text{ in.}^2$$

For the 36 inch square, the radius of each of the 9 circles is 6 inches.

$$\text{Area of 9 circles} = 9 \cdot \pi r^2 = 9 \cdot \pi(6)^2 \approx 1018 \text{ in.}^2$$

The reasoning in part (b) is correct and the answer is correct.

b. For each cardboard square, find the percent of the cardboard square's area that is used for the circles.

$$\text{Percent for 20 inch square: } \frac{\text{Area of 4 circles}}{\text{Area of cardboard}} = \frac{314}{20^2} = 0.785 = 78.5\%$$

$$\text{Percent for 36 inch square: } \frac{\text{Area of 9 circles}}{\text{Area of cardboard}} = \frac{1018}{36^2} \approx 0.785 = 78.5\%$$

It doesn't matter which size of cardboard you use. In each case, you will use about 78.5% of the cardboard's area.

SAMPLE 2: Partial credit solution

In part (a), the answer is incomplete because the student does not find the area of all the circles.

The reasoning in part (b) is correct, but the answer is wrong because the student did not consider the area of all the circles.

a. Use the formula $A = \pi r^2$ to find the area of each circle. Divide each diameter in half to get the radius of the circle.

Area of 10 inch diameter circle $= \pi(5)^2 \approx 79 \text{ in.}^2$
Area of 12 inch diameter circle $= \pi(6)^2 \approx 113 \text{ in.}^2$

b. Find and compare the percents.

$$\frac{\text{Area of circles}}{\text{Area of 20 in. square}} = \frac{79}{20^2} = 0.1975 = 19.75\%$$

$$\frac{\text{Area of circles}}{\text{Area of 36 in. square}} = \frac{113}{36^2} \approx 0.0872 = 8.72\%$$

You use 19.75% of the 20 inch cardboard's area, but only 8.72% of the 36 inch cardboard's area. So, you should use the 20 inch cardboard.

SAMPLE 3: No credit solution

In part (a), the wrong formula is used.

In part (b), the reasoning and the answer are incorrect.

a. Area $= \pi d = \pi(10) \approx 31 \text{ in.}^2$ Multiply by 4 to get 124 in.^2
Area $= \pi d = \pi(12) \approx 38 \text{ in.}^2$ Multiply by 9 to get 342 in.^2

b. You use 342 in.^2 of cardboard for 9 signs, and only 124 in.^2 for 4 signs. You should use the 36 inch cardboard because you will use more of it.

PRACTICE Apply the Scoring Rubric

1. A student's solution to the problem on the previous page is given below. Score the solution as *full credit*, *partial credit*, or *no credit*. *Explain* your reasoning. If you choose *partial credit* or *no credit*, *explain* how you would change the solution so that it earns a score of full credit.

a. There are two sizes of circles you can make. Find the area of each.

Area of a circle made from the 20 inch square $= \pi(5)^2 \approx 78.5 \text{ in.}^2$
Area of a circle made from the 36 inch square $= \pi(6)^2 \approx 113.1 \text{ in.}^2$

Then multiply each area by the number of circles that have that area.

Area of circles in 20 inch square $\approx 4 \cdot 78.5 = 314 \text{ in.}^2$
Area of circles in 36 inch square $\approx 9 \cdot 113.1 \approx 1018 \text{ in.}^2$

b. Find the percent of each square's area that is used for the signs.

$$\frac{\text{Area of 4 circles}}{\text{Area of 20 in. square}} = \frac{314}{20} = 15.7\%$$

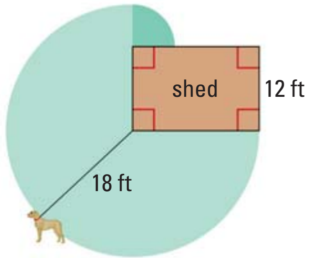
$$\frac{\text{Area of 9 circles}}{\text{Area of 36 in. square}} = \frac{1018}{36} \approx 28.3\%$$

Because 28.3% > 15.7%, you use a greater percent of the cardboard's area when you use the 36 inch square.

11 ★ Standardized TEST PRACTICE

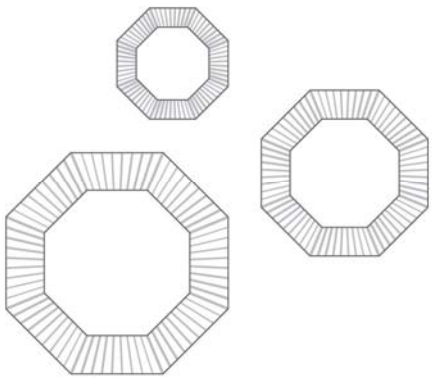
EXTENDED RESPONSE

1. A dog is tied to the corner of a shed with a leash. The leash prevents the dog from moving more than 18 feet from the corner. In the diagram, the shaded sectors show the region over which the dog can roam.
   a. Find the area of the sector with radius 18 feet.
   b. What is the radius of the smaller sector? Find its area. *Explain*.
   c. Find the area over which the dog can move. *Explain*.

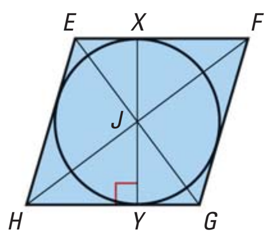


2. A circle passes through the points (3, 0), (9, 0), (6, 3), and (6, −3).
   a. Graph the circle in a coordinate plane. Give the coordinates of its center.
   b. Sketch the image of the circle after a dilation centered at the origin with a scale factor of 2. How are the coordinates of the center of the dilated circle related to the coordinates of the center of the original circle? *Explain*.
   c. How are the circumferences of the circle and its image after the dilation related? How are the areas related? *Explain*.

3. A caterer uses a set of three different-sized trays. Each tray is a regular octagon. The areas of the trays are in the ratio 2 : 3 : 4.
   a. The area of the smallest tray is about 483 square centimeters. Find the areas of the other trays to the nearest square centimeter. *Explain* your reasoning.
   b. The perimeter of the smallest tray is 80 centimeters. Find the approximate perimeters of the other trays. Round to the nearest tenth of a centimeter. *Explain* your reasoning.

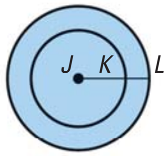


4. In the diagram, the diagonals of rhombus $EFGH$ intersect at point J, $EG = 6$, and $FH = 8$. A circle with center J is inscribed in $EFGH$, and $\overline{XY}$ is a diameter of $\odot J$.
   a. Find EF. *Explain* your reasoning.
   b. Use the formula for the area of a rhombus to find the area of $EFGH$.
   c. Use the formula for the area of a parallelogram to write an equation relating the area of $EFGH$ from part (b) to EF and XY.
   d. Find XY. Then find the area of the inscribed circle. *Explain* your reasoning.



MULTIPLE CHOICE

5. In the diagram, J is the center of two circles, and K lies on $\overline{JL}$. Given $JL = 6$ and $KL = 2$, what is the ratio of the area of the smaller circle to the area of the larger circle?



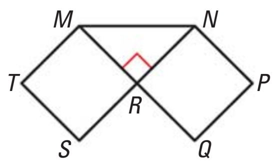
Ⓐ $\sqrt{2} : \sqrt{3}$

Ⓑ $1:3$

Ⓒ $2:3$

Ⓓ $4:9$

6. In the diagram, $TMRS$ and $RNPQ$ are congruent squares, and $\triangle MNR$ is a right triangle. What is the probability that a randomly chosen point on the diagram lies inside $\triangle MNR$?



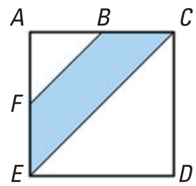
Ⓐ 0.2 Ⓑ 0.25

Ⓒ 0.5 Ⓓ 0.75

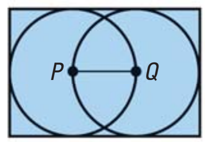
GRIDDED ANSWER

7. You are buying fertilizer for a lawn that is shaped like a parallelogram. Two sides of the parallelogram are each 300 feet long, and the perpendicular distance between these sides is 150 feet. One bag of fertilizer covers 5000 square feet and costs $14. How much (in dollars) will you spend?

8. In square $ACDE$, $ED = 2$, $AB = BC$, and $AF = FE$. What is the area (in square units) of the shaded region?

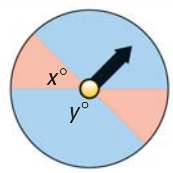


9. In the diagram, a rectangle's sides are tangent to two circles with centers at points P and Q. The circumference of each circle is 8π square units. What is the area (in square units) of the rectangle?



SHORT RESPONSE

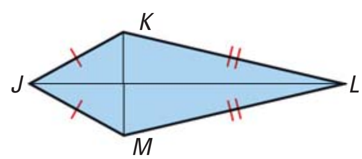
10. You are designing a spinner for a board game. An arrow is attached to the center of a circle with diameter 7 inches. The arrow is spun until it stops. The arrow has an equally likely chance of stopping anywhere.



a. If $x° = 45°$, what is the probability that the arrow points to a red sector? *Explain*.

b. You want to change the spinner so the probability that the arrow points to a blue sector is half the probability that it points to a red sector. What values should you use for x and y? *Explain*.

11. In quadrilateral $JKLM$, $JL = 3 \cdot KM$. The area of $JKLM$ is 54 square centimeters.



a. Find JL and KM.

b. Quadrilateral $NPQR$ is similar to $JKLM$, and its area is 486 square centimeters. Sketch $NPQR$ and its diagonals. Then find the length of $\overline{NQ}$. *Explain* your reasoning.