10 Properties of Circles

Before

In previous chapters, you learned the following skills, which you'll use in Chapter 10: classifying triangles, finding angle measures, and solving equations.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

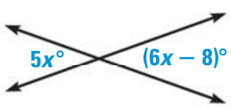
1. Two similar triangles have congruent corresponding angles and _?_ corresponding sides.
2. Two angles whose sides form two pairs of opposite rays are called _?_ .
3. The _?_ of an angle is all of the points between the sides of the angle.

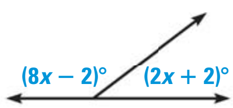
SKILLS AND ALGEBRA CHECK

Use the Converse of the Pythagorean Theorem to classify the triangle. *(Review p. 441 for 10.1.)*

4. 0.6, 0.8, 0.9
5. 11, 12, 17
6. 1.5, 2, 2.5

Find the value of the variable. *(Review pp. 24, 35 for 10.2, 10.4.)*

7.


8.


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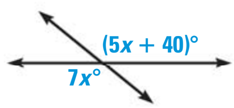


@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 10, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 707. You will also use the key vocabulary listed below.

Big Ideas

1. Using properties of segments that intersect circles
2. Applying angle relationships in circles
3. Using circles in the coordinate plane

KEY VOCABULARY

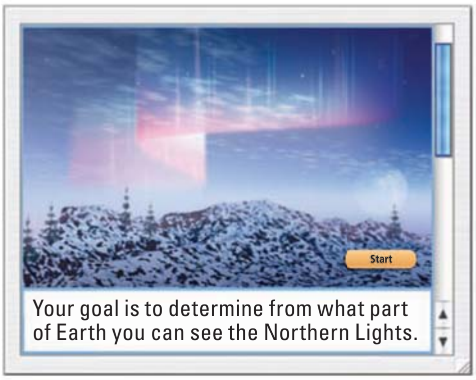
- circle, *p. 651*
  center, radius, diameter
- chord, *p. 651*
- secant, *p. 651*
- tangent, *p. 651*
- central angle, *p. 659*
- minor arc, *p. 659*
- major arc, *p. 659*
- semicircle, *p. 659*
- congruent circles, *p. 660*
- congruent arcs, *p. 660*
- inscribed angle, *p. 672*
- intercepted arc, *p. 672*
- standard equation of a circle, *p. 699*

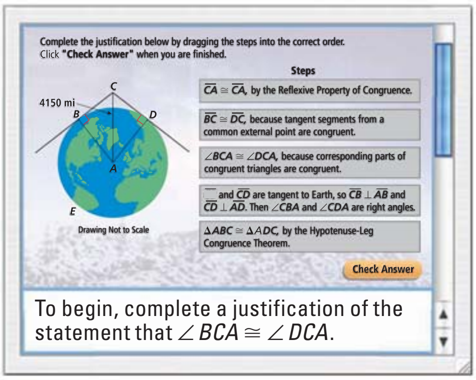
Why?

Circles can be used to model a wide variety of natural phenomena. You can use properties of circles to investigate the Northern Lights.

Animated Geometry

The animation illustrated below for Example 4 on page 682 helps you answer this question: From what part of Earth are the Northern Lights visible?





Animated Geometry at classzone.com

Other animations for Chapter 10: pages 655, 661, 671, 691, and 701

10.1 Explore Tangent Segments

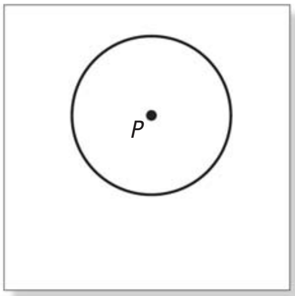
MATERIALS • compass • ruler

QUESTION How are the lengths of tangent segments related?

A line can intersect a circle at 0, 1, or 2 points. If a line is in the plane of a circle and intersects the circle at 1 point, the line is a *tangent*.

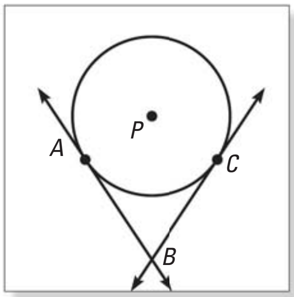
EXPLORE Draw tangents to a circle

STEP 1



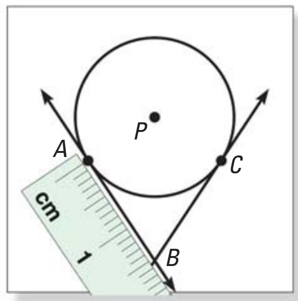
Draw a circle Use a compass to draw a circle. Label the center *P*.

STEP 2



Draw tangents Draw lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CB}$ so that they intersect $\odot P$ only at *A* and *C*, respectively. These lines are called *tangents*.

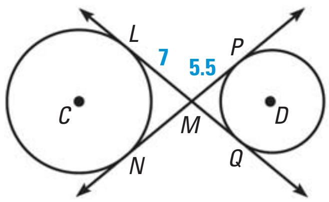
STEP 3



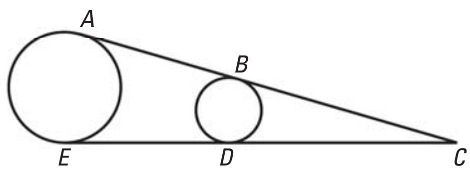
Measure segments $\overline{AB}$ and $\overline{CB}$ are called *tangent segments*. Measure and compare the lengths of the tangent segments.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Repeat Steps 1–3 with three different circles.
2. Use your results from Exercise 1 to make a conjecture about the lengths of tangent segments that have a common endpoint.
3. In the diagram, *L*, *Q*, *N*, and *P* are points of tangency. Use your conjecture from Exercise 2 to find *LQ* and *NP* if $LM = 7$ and $MP = 5.5$.



4. In the diagram below, *A*, *B*, *D*, and *E* are points of tangency. Use your conjecture from Exercise 2 to explain why $\overline{AB} \cong \overline{ED}$.



10.1 Use Properties of Tangents

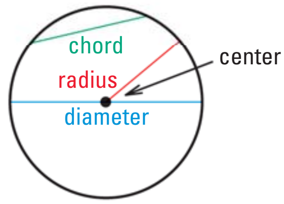
Before You found the circumference and area of circles.

Now You will use properties of a tangent to a circle.

Why? So you can find the range of a GPS satellite, as in Ex. 37.

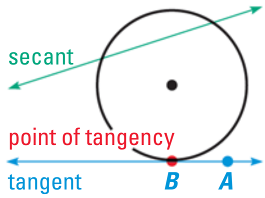
Key Vocabulary
- **circle**
  center, radius, diameter
- **chord**
- **secant**
- **tangent**

A **circle** is the set of all points in a plane that are equidistant from a given point called the **center** of the circle. A circle with center P is called "circle P" and can be written $\odot P$. A segment whose endpoints are the center and any point on the circle is a **radius**.



A **chord** is a segment whose endpoints are on a circle. A **diameter** is a chord that contains the center of the circle.

A **secant** is a line that intersects a circle in two points. A **tangent** is a line in the plane of a circle that intersects the circle in exactly one point, the *point of tangency*. The *tangent ray* $\overrightarrow{AB}$ and the *tangent segment* $\overline{AB}$ are also called tangents.

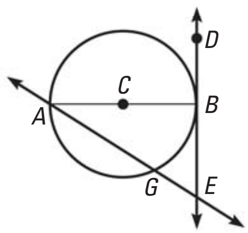


EXAMPLE 1 Identify special segments and lines

Tell whether the line, ray, or segment is best described as a *radius, chord, diameter, secant,* or *tangent* of $\odot C$.

a. $\overline{AC}$ **b.** $\overline{AB}$

c. $\overrightarrow{DE}$ **d.** $\overleftrightarrow{AE}$



Solution

a. $\overline{AC}$ is a radius because C is the center and A is a point on the circle.

b. $\overline{AB}$ is a diameter because it is a chord that contains the center C.

c. $\overrightarrow{DE}$ is a tangent ray because it is contained in a line that intersects the circle at only one point.

d. $\overleftrightarrow{AE}$ is a secant because it is a line that intersects the circle in two points.

✓ GUIDED PRACTICE for Example 1

1. In Example 1, what word best describes $\overline{AG}$? $\overline{CB}$?
2. In Example 1, name a tangent and a tangent segment.

READ VOCABULARY

The plural of radius is *radii.* All radii of a circle are congruent.

RADIUS AND DIAMETER The words *radius* and *diameter* are used for lengths as well as segments. For a given circle, think of *a radius* and *a diameter* as segments and *the radius* and *the diameter* as lengths.

EXAMPLE 2 Find lengths in circles in a coordinate plane

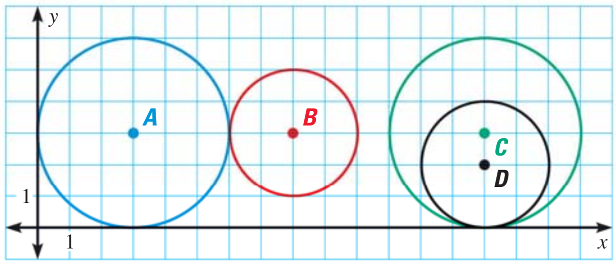
Use the diagram to find the given lengths.

a. Radius of $\odot A$

b. Diameter of $\odot A$

c. Radius of $\odot B$

d. Diameter of $\odot B$



Solution

a. The radius of $\odot A$ is 3 units.

b. The diameter of $\odot A$ is 6 units.

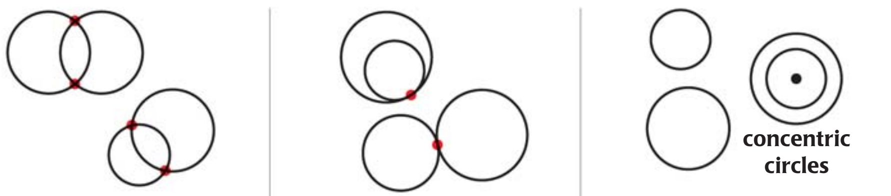
c. The radius of $\odot B$ is 2 units.

d. The diameter of $\odot B$ is 4 units.

✓ GUIDED PRACTICE for Example 2

3. Use the diagram in Example 2 to find the radius and diameter of $\odot C$ and $\odot D$.

COPLANAR CIRCLES Two circles can intersect in two points, one point, or no points. Coplanar circles that intersect in one point are called *tangent circles*. Coplanar circles that have a common center are called *concentric*.

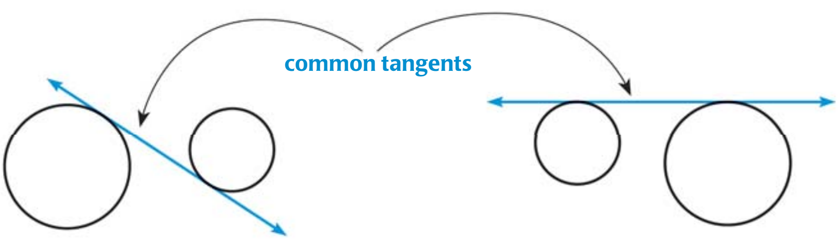


2 points of intersection | **1 point of intersection (tangent circles)** | **no points of intersection**

READ VOCABULARY

A line that intersects a circle in exactly one point is said to be *tangent* to the circle.

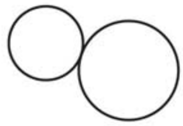
COMMON TANGENTS A line, ray, or segment that is tangent to two coplanar circles is called a *common tangent*.

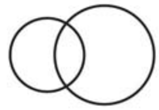


EXAMPLE 3 Draw common tangents

Tell how many common tangents the circles have and draw them.

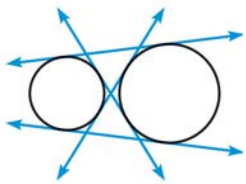
a. 

b. 

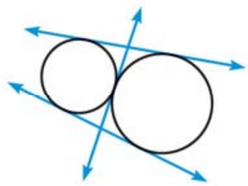
c. 

Solution

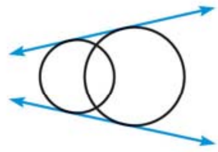
a. 4 common tangents



b. 3 common tangents

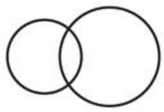


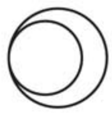
c. 2 common tangents

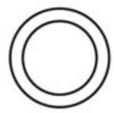


✓ GUIDED PRACTICE for Example 3

Tell how many common tangents the circles have and draw them.

4. 

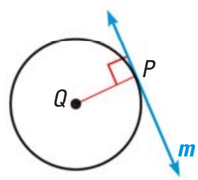
5. 

6. 

THEOREM *For Your Notebook*

THEOREM 10.1

In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.

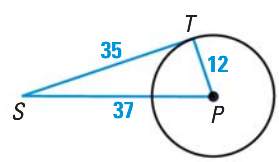


Line m is tangent to $\odot Q$ if and only if $m \perp \overline{QP}$.

Proof: Exs. 39–40, p. 658

EXAMPLE 4 Verify a tangent to a circle

In the diagram, $\overline{PT}$ is a radius of $\odot P$. Is $\overline{ST}$ tangent to $\odot P$?

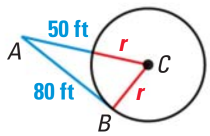


Solution

Use the Converse of the Pythagorean Theorem. Because $12^2 + 35^2 = 37^2$, $\triangle PST$ is a right triangle and $\overline{ST} \perp \overline{PT}$. So, $\overline{ST}$ is perpendicular to a radius of $\odot P$ at its endpoint on $\odot P$. By Theorem 10.1, $\overline{ST}$ is tangent to $\odot P$.

EXAMPLE 5 Find the radius of a circle

In the diagram, *B* is a point of tangency. Find the radius *r* of $\odot C$.



Solution

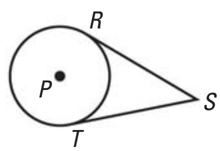
You know from Theorem 10.1 that $\overline{AB} \perp \overline{BC}$, so $\triangle ABC$ is a right triangle. You can use the Pythagorean Theorem.

$AC^2 = BC^2 + AB^2$	**Pythagorean Theorem**
$(r + 50)^2 = r^2 + 80^2$	**Substitute.**
$r^2 + 100r + 2500 = r^2 + 6400$	**Multiply.**
$100r = 3900$	**Subtract from each side.**
$r = 39$ ft	**Divide each side by 100.**

THEOREM *For Your Notebook*

THEOREM 10.2

Tangent segments from a common external point are congruent.

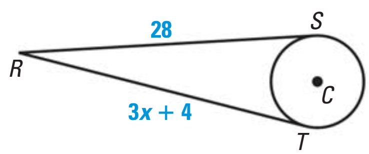


If $\overline{SR}$ and $\overline{ST}$ are tangent segments, then $\overline{SR} \cong \overline{ST}$.

Proof: Ex. 41, p. 658

EXAMPLE 6 Find the radius of a circle

$\overline{RS}$ is tangent to $\odot C$ at *S* and $\overline{RT}$ is tangent to $\odot C$ at *T*. Find the value of *x*.

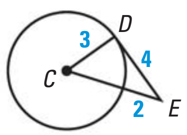


Solution

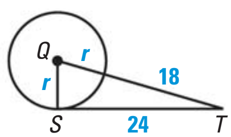
$RS = RT$	**Tangent segments from the same point are $\cong$.**
$28 = 3x + 4$	**Substitute.**
$8 = x$	**Solve for *x*.**

✓ GUIDED PRACTICE for Examples 4, 5, and 6

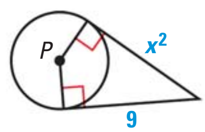
7. Is $\overline{DE}$ tangent to $\odot C$?



8. $\overline{ST}$ is tangent to $\odot Q$. Find the value of *r*.



9. Find the value(s) of *x*.



10.1 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 19, and 37
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 29, 33, and 38

SKILL PRACTICE

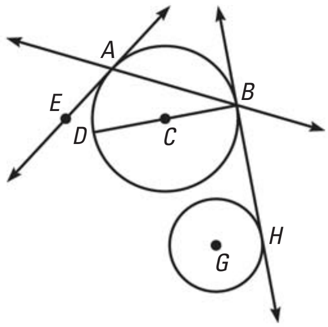
1. **VOCABULARY** Copy and complete: The points A and B are on $\odot C$. If C is a point on $\overline{AB}$, then $\overline{AB}$ is a _?_.

2. ★ **WRITING** Explain how you can determine from the context whether the words *radius* and *diameter* are referring to a segment or a length.

EXAMPLE 1 on p. 651 for Exs. 3–11

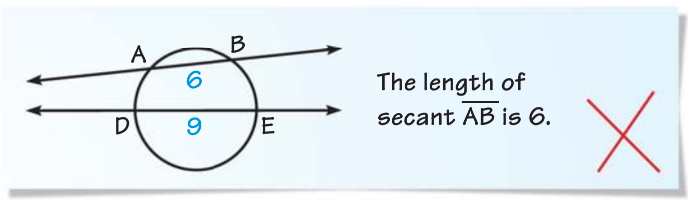
MATCHING TERMS **Match the notation with the term that best describes it.**

3. B — A. Center
4. $\overleftrightarrow{BH}$ — B. Radius
5. $\overline{AB}$ — C. Chord
6. $\overleftrightarrow{AB}$ — D. Diameter
7. $\overleftrightarrow{AE}$ — E. Secant
8. G — F. Tangent
9. $\overline{CD}$ — G. Point of tangency
10. $\overline{BD}$ — H. Common tangent



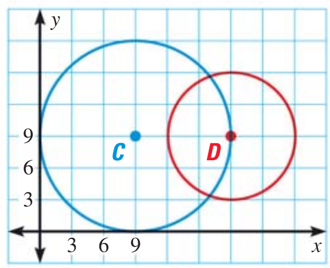
AnimatedGeometry at classzone.com

11. **ERROR ANALYSIS** *Describe* and correct the error in the statement about the diagram.



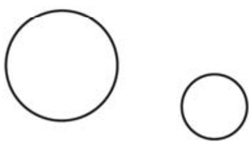
EXAMPLES 2 and 3 on pp. 652–653 for Exs. 12–17

COORDINATE GEOMETRY **Use the diagram at the right.**

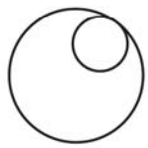


12. What are the radius and diameter of $\odot C$?
13. What are the radius and diameter of $\odot D$?
14. Copy the circles. Then draw all the common tangents of the two circles.

DRAWING TANGENTS **Copy the diagram. Tell how many common tangents the circles have and draw them.**

15. 

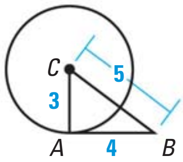
16. 

17. 

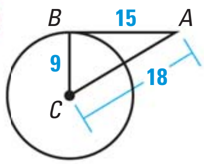
EXAMPLE 4
on p. 653
for Exs. 18–20

DETERMINING TANGENCY **Determine whether $\overline{AB}$ is tangent to $\odot C$. *Explain.***

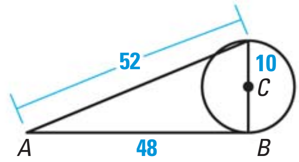
18.



19.



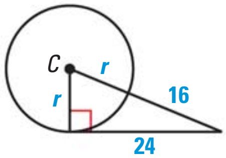
20.



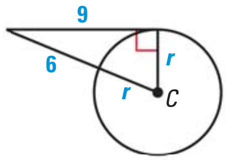
EXAMPLES 5 and 6
on p. 654
for Exs. 21–26

ALGEBRA **Find the value(s) of the variable. In Exercises 24–26, *B* and *D* are points of tangency.**

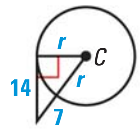
21.



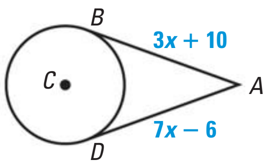
22.



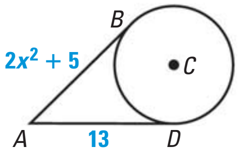
23.



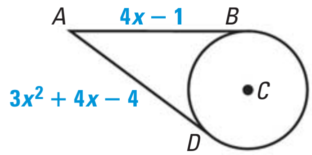
24.



25.

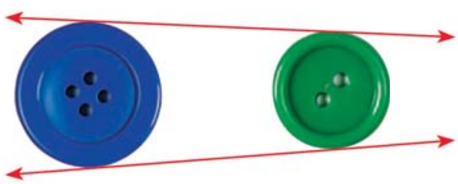


26.

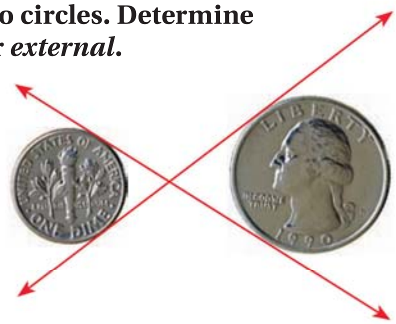


COMMON TANGENTS **A *common internal tangent* intersects the segment that joins the centers of two circles. A *common external* tangent does not intersect the segment that joins the centers of the two circles. Determine whether the common tangents shown are *internal* or *external*.**

27.



28.



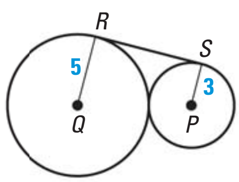
29. ★ **MULTIPLE CHOICE** In the diagram, $\odot P$ and $\odot Q$ are tangent circles. $\overline{RS}$ is a common tangent. Find RS.

Ⓐ $-2\sqrt{15}$

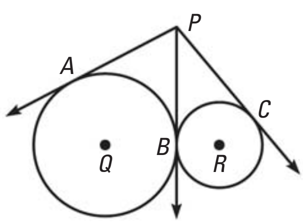
Ⓑ 4

Ⓒ $2\sqrt{15}$

Ⓓ 8

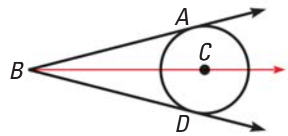


30. **REASONING** In the diagram, $\overrightarrow{PB}$ is tangent to $\odot Q$ and $\odot R$. *Explain* why $\overline{PA} \cong \overline{PB} \cong \overline{PC}$ even though the radius of $\odot Q$ is not equal to the radius of $\odot R$.



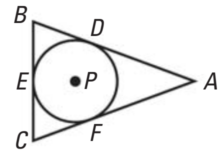
31. **TANGENT LINES** When will two lines tangent to the same circle not intersect? Use Theorem 10.1 to *explain* your answer.

32. **ANGLE BISECTOR** In the diagram at right, A and D are points of tangency on $\odot C$. *Explain* how you know that $\overrightarrow{BC}$ bisects $\angle ABD$. (*Hint*: Use Theorem 5.6, page 310.)



33. ★ **SHORT RESPONSE** For any point outside of a circle, is there ever only one tangent to the circle that passes through the point? Are there ever more than two such tangents? *Explain* your reasoning.

34. **CHALLENGE** In the diagram at the right, $AB = AC = 12$, $BC = 8$, and all three segments are tangent to $\odot P$. What is the radius of $\odot P$?



PROBLEM SOLVING

BICYCLES On modern bicycles, rear wheels usually have *tangential spokes*. Occasionally, front wheels have *radial spokes*. Use the definitions of *tangent* and *radius* to determine if the wheel shown has *tangential spokes* or *radial spokes*.

35.

36.

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EXAMPLE 4
on p. 653
for Ex. 37

37. **GLOBAL POSITIONING SYSTEM (GPS)** GPS satellites orbit about 11,000 miles above Earth. The mean radius of Earth is about 3959 miles. Because GPS signals cannot travel through Earth, a satellite can transmit signals only as far as points A and C from point B, as shown. Find BA and BC to the nearest mile.

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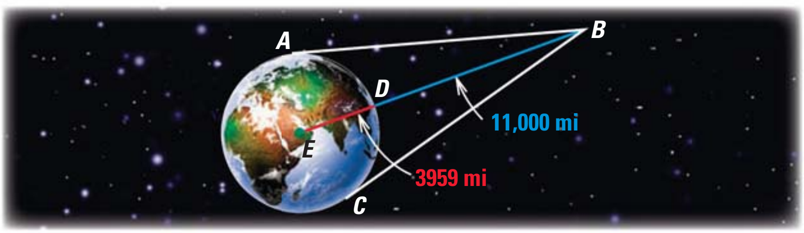
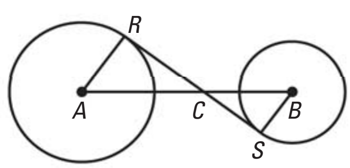
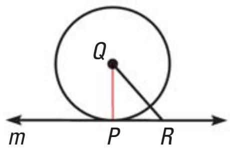


38. ★ **SHORT RESPONSE** In the diagram, $\overline{RS}$ is a common internal tangent (see Exercises 27–28) to $\odot A$ and $\odot B$. Use similar triangles to *explain* why $\frac{AC}{BC} = \frac{RC}{SC}$.



39. **PROVING THEOREM 10.1** Use parts (a)–(c) to prove indirectly that if a line is tangent to a circle, then it is perpendicular to a radius.



GIVEN ▶ Line m is tangent to $\odot Q$ at P.

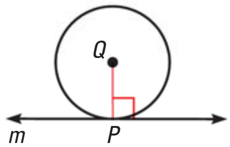
PROVE ▶ $m \perp \overline{QP}$

a. Assume m is not perpendicular to $\overline{QP}$. Then the perpendicular segment from Q to m intersects m at some other point R. Because m is a tangent, R cannot be inside $\odot Q$. *Compare* the length QR to QP.

b. Because $\overline{QR}$ is the perpendicular segment from Q to m, $\overline{QR}$ is the shortest segment from Q to m. Now *compare* QR to QP.

c. Use your results from parts (a) and (b) to complete the indirect proof.

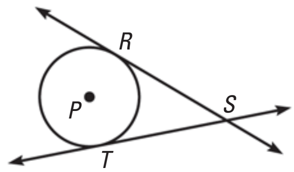
40. **PROVING THEOREM 10.1** Write an indirect proof that if a line is perpendicular to a radius at its endpoint, the line is a tangent.



GIVEN ▶ $m \perp \overline{QP}$

PROVE ▶ Line m is tangent to $\odot Q$.

41. **PROVING THEOREM 10.2** Write a proof that tangent segments from a common external point are congruent.

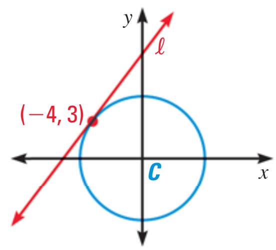


GIVEN ▶ $\overline{SR}$ and $\overline{ST}$ are tangent to $\odot P$.

PROVE ▶ $\overline{SR} \cong \overline{ST}$

Plan for Proof Use the Hypotenuse–Leg Congruence Theorem to show that $\triangle SRP \cong \triangle STP$.

42. **CHALLENGE** Point C is located at the origin. Line ℓ is tangent to $\odot C$ at $(-4, 3)$. Use the diagram at the right to complete the problem.



a. Find the slope of line ℓ.

b. Write the equation for ℓ.

c. Find the radius of $\odot C$.

d. Find the distance from ℓ to $\odot C$ along the y-axis.

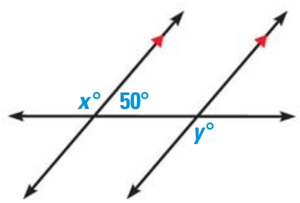
MIXED REVIEW

PREVIEW
Prepare for Lesson 10.2 in Ex. 43.

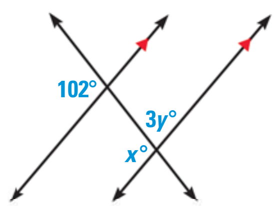
43. D is in the interior of $\angle ABC$. If $m\angle ABD = 25°$ and $m\angle ABC = 70°$, find $m\angle DBC$. *(p. 24)*

Find the values of x and y. *(p. 154)*

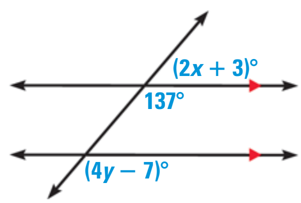
44.



45.



46.



47. A triangle has sides of lengths 8 and 13. Use an inequality to describe the possible length of the third side. What if two sides have lengths 4 and 11? *(p. 328)*

10.2 Find Arc Measures

Before You found angle measures.

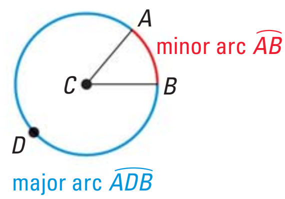
Now You will use angle measures to find arc measures.

Why? So you can describe the arc made by a bridge, as in Ex. 22.

Key Vocabulary
- **central angle**
- **minor arc**
- **major arc**
- **semicircle**
- **measure**
  minor arc, major arc
- **congruent circles**
- **congruent arcs**

A **central angle** of a circle is an angle whose vertex is the center of the circle. In the diagram, $\angle ACB$ is a central angle of $\odot C$.

If $m\angle ACB$ is less than $180°$, then the points on $\odot C$ that lie in the interior of $\angle ACB$ form a **minor arc** with endpoints A and B. The points on $\odot C$ that do not lie on minor arc $\overset{\frown}{AB}$ form a **major arc** with endpoints A and B. A **semicircle** is an arc with endpoints that are the endpoints of a diameter.



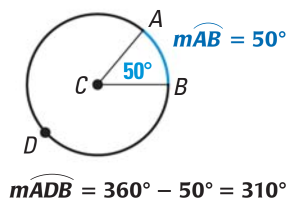
NAMING ARCS Minor arcs are named by their endpoints. The minor arc associated with $\angle ACB$ is named $\overset{\frown}{AB}$. Major arcs and semicircles are named by their endpoints and a point on the arc. The major arc associated with $\angle ACB$ can be named $\overset{\frown}{ADB}$.

KEY CONCEPT *For Your Notebook*

Measuring Arcs

The **measure of a minor arc** is the measure of its central angle. The expression $m\overset{\frown}{AB}$ is read as "the measure of arc AB."

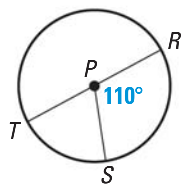
The measure of the entire circle is $360°$. The **measure of a major arc** is the difference between $360°$ and the measure of the related minor arc. The measure of a semicircle is $180°$.



EXAMPLE 1 Find measures of arcs

Find the measure of each arc of $\odot P$, where $\overline{RT}$ is a diameter.

a. $\overset{\frown}{RS}$ **b.** $\overset{\frown}{RTS}$ **c.** $\overset{\frown}{RST}$



Solution

a. $\overset{\frown}{RS}$ is a minor arc, so $m\overset{\frown}{RS} = m\angle RPS = 110°$.

b. $\overset{\frown}{RTS}$ is a major arc, so $m\overset{\frown}{RTS} = 360° - 110° = 250°$.

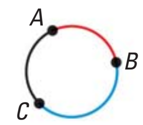
c. $\overline{RT}$ is a diameter, so $\overset{\frown}{RST}$ is a semicircle, and $m\overset{\frown}{RST} = 180°$.

ADJACENT ARCS Two arcs of the same circle are *adjacent* if they have a common endpoint. You can add the measures of two adjacent arcs.

POSTULATE *For Your Notebook*

POSTULATE 23 Arc Addition Postulate

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

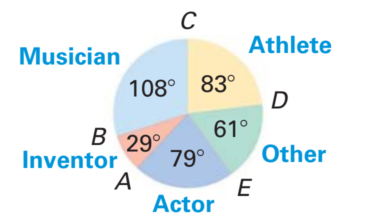


$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$

EXAMPLE 2 Find measures of arcs

SURVEY A recent survey asked teenagers if they would rather meet a famous musician, athlete, actor, inventor, or other person. The results are shown in the circle graph. Find the indicated arc measures.

Whom Would You Rather Meet?



a. $m\overset{\frown}{AC}$ **b.** $m\overset{\frown}{ACD}$

c. $m\overset{\frown}{ADC}$ **d.** $m\overset{\frown}{EBD}$

Solution

a. $m\overset{\frown}{AC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$
$= 29° + 108°$
$= 137°$

b. $m\overset{\frown}{ACD} = m\overset{\frown}{AC} + m\overset{\frown}{CD}$
$= 137° + 83°$
$= 220°$

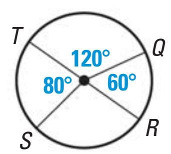
c. $m\overset{\frown}{ADC} = 360° - m\overset{\frown}{AC}$
$= 360° - 137°$
$= 223°$

d. $m\overset{\frown}{EBD} = 360° - m\overset{\frown}{ED}$
$= 360° - 61°$
$= 299°$

ARC MEASURES
The measure of a minor arc is less than 180°. The measure of a major arc is greater than 180°.

GUIDED PRACTICE for Examples 1 and 2

Identify the given arc as a *major arc, minor arc,* or *semicircle,* and find the measure of the arc.



1. $\overset{\frown}{TQ}$ **2.** $\overset{\frown}{QRT}$ **3.** $\overset{\frown}{TQR}$

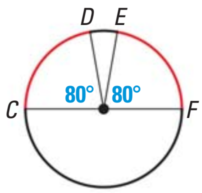
4. $\overset{\frown}{QS}$ **5.** $\overset{\frown}{TS}$ **6.** $\overset{\frown}{RST}$

CONGRUENT CIRCLES AND ARCS Two circles are **congruent circles** if they have the same radius. Two arcs are **congruent arcs** if they have the same measure and they are arcs of the same circle or of congruent circles. If $\odot C$ is congruent to $\odot D$, then you can write $\odot C \cong \odot D$.

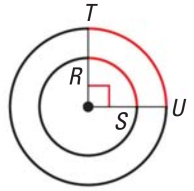
EXAMPLE 3 Identify congruent arcs

Tell whether the red arcs are congruent. Explain why or why not.

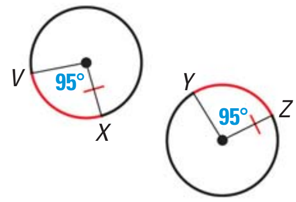
a.



b.



c.



Solution

a. $\overparen{CD} \cong \overparen{EF}$ because they are in the same circle and $m\overparen{CD} = m\overparen{EF}$.

b. $\overparen{RS}$ and $\overparen{TU}$ have the same measure, but are not congruent because they are arcs of circles that are not congruent.

c. $\overparen{VX} \cong \overparen{YZ}$ because they are in congruent circles and $m\overparen{VX} = m\overparen{YZ}$.

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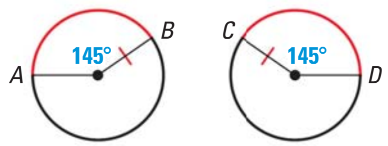
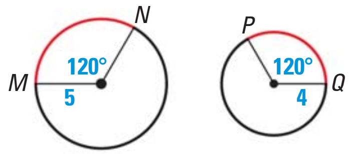
✓ GUIDED PRACTICE for Example 3

Tell whether the red arcs are congruent. *Explain* why or why not.

7.



8.



10.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 13, and 23

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 11, 17, 18, and 24

SKILL PRACTICE

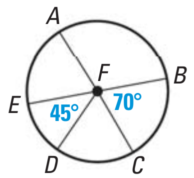
1. **VOCABULARY** Copy and complete: If $\angle ACB$ and $\angle DCE$ are congruent central angles of $\odot C$, then $\overparen{AB}$ and $\overparen{DE}$ are __?__.

2. ★ **WRITING** What do you need to know about two circles to show that they are congruent? *Explain.*

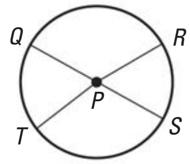
EXAMPLES 1 and 2
on pp. 659–660 for Exs. 3–11

MEASURING ARCS $\overline{AC}$ **and** $\overline{BE}$ **are diameters of** $\odot F$**. Determine whether the arc is a *minor arc*, a *major arc*, or a *semicircle* of** $\odot F$**. Then find the measure of the arc.**

3. $\overparen{BC}$
4. $\overparen{DC}$
5. $\overparen{DB}$
6. $\overparen{AE}$
7. $\overparen{AD}$
8. $\overparen{ABC}$
9. $\overparen{ACD}$
10. $\overparen{EAC}$



11. ★ **MULTIPLE CHOICE** In the diagram, $\overline{QS}$ is a diameter of $\odot P$. Which arc represents a semicircle?



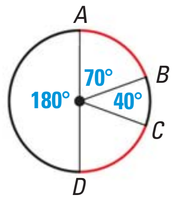
Ⓐ $\overset{\frown}{QR}$ Ⓑ $\overset{\frown}{RQT}$

Ⓒ $\overset{\frown}{QRS}$ Ⓓ $\overset{\frown}{QRT}$

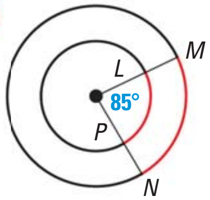
EXAMPLE 3
on p. 661
for Exs. 12–14

CONGRUENT ARCS **Tell whether the red arcs are congruent. *Explain* why or why not.**

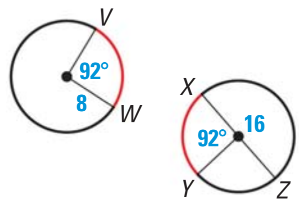
12.



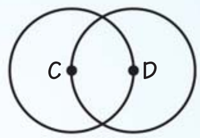
13.



14.



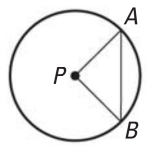
15. **ERROR ANALYSIS** *Explain* what is wrong with the statement.



You cannot tell if $\odot C \cong \odot D$ because the radii are not given.

16. **ARCS** Two diameters of $\odot P$ are $\overline{AB}$ and $\overline{CD}$. If $m\overset{\frown}{AD} = 20°$, find $m\overset{\frown}{ACD}$ and $m\overset{\frown}{AC}$.

17. ★ **MULTIPLE CHOICE** $\odot P$ has a radius of 3 and $\overset{\frown}{AB}$ has a measure of 90°. What is the length of $\overline{AB}$?



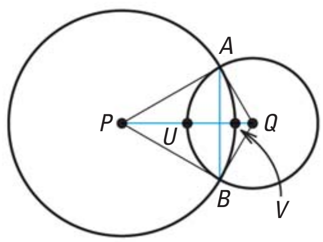
Ⓐ $3\sqrt{2}$ Ⓑ $3\sqrt{3}$

Ⓒ 6 Ⓓ 9

18. ★ **SHORT RESPONSE** On $\odot C$, $m\overset{\frown}{EF} = 100°$, $m\overset{\frown}{FG} = 120°$, and $m\overset{\frown}{EFG} = 220°$. If H is on $\odot C$ so that $m\overset{\frown}{GH} = 150°$, *explain* why H must be on $\overset{\frown}{EF}$.

19. **REASONING** In $\odot R$, $m\overset{\frown}{AB} = 60°$, $m\overset{\frown}{BC} = 25°$, $m\overset{\frown}{CD} = 70°$, and $m\overset{\frown}{DE} = 20°$. Find two possible values for $m\overset{\frown}{AE}$.

20. **CHALLENGE** In the diagram shown, $\overline{PQ} \perp \overline{AB}$, $\overline{QA}$ is tangent to $\odot P$, and $m\overset{\frown}{AVB} = 60°$. What is $m\overset{\frown}{AUB}$?

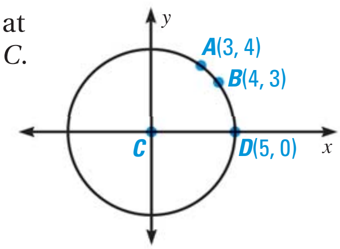


21. **CHALLENGE** In the coordinate plane shown, C is at the origin. Find the following arc measures on $\odot C$.

a. $m\overset{\frown}{BD}$

b. $m\overset{\frown}{AD}$

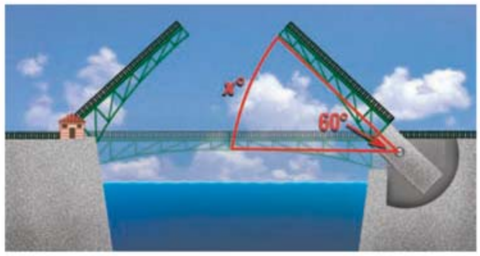
c. $m\overset{\frown}{AB}$



PROBLEM SOLVING

EXAMPLE 1
on p. 659
for Ex. 22

22. BRIDGES The deck of a bascule bridge creates an arc when it is moved from the closed position to the open position. Find the measure of the arc.



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23. DARTS On a regulation dartboard, the outermost circle is divided into twenty congruent sections. What is the measure of each arc in this circle?

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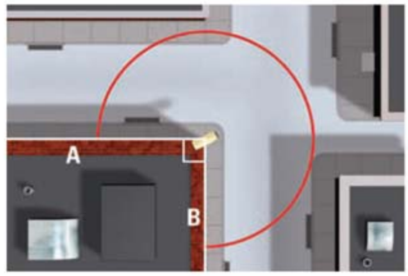
24. ★ **EXTENDED RESPONSE** A surveillance camera is mounted on a corner of a building. It rotates clockwise and counterclockwise continuously between Wall A and Wall B at a rate of 10° per minute.



a. What is the measure of the arc surveyed by the camera?

b. How long does it take the camera to survey the entire area once?

c. If the camera is at an angle of 85° from Wall B while rotating counterclockwise, how long will it take for the camera to return to that same position?

d. The camera is rotating counterclockwise and is 50° from Wall A. Find the location of the camera after 15 minutes.

25. CHALLENGE A clock with hour and minute hands is set to 1:00 P.M.

a. After 20 minutes, what will be the measure of the minor arc formed by the hour and minute hands?

b. At what time before 2:00 P.M., to the nearest minute, will the hour and minute hands form a diameter?

MIXED REVIEW

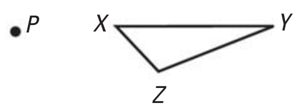
PREVIEW
Prepare for Lesson 10.3 in Exs. 26–27.

Determine if the lines with the given equations are parallel. *(p. 180)*

26. $y = 5x + 2,\ y = 5(1 - x)$

27. $2y + 2x = 5,\ y = 4 - x$

28. Trace $\triangle XYZ$ and point P. Draw a counterclockwise rotation of $\triangle XYZ$ 145° about P. *(p. 598)*



Find the product. *(p. 641)*

29. $(x + 2)(x + 3)$

30. $(2y - 5)(y + 7)$

31. $(x + 6)(x - 6)$

32. $(z - 3)^2$

33. $(3x + 7)(5x + 4)$

34. $(z - 1)(z - 4)$

10.3 Apply Properties of Chords

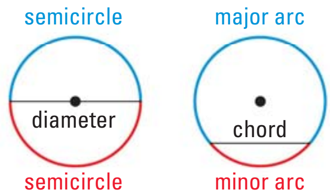
Before You used relationships of central angles and arcs in a circle.

Now You will use relationships of arcs and chords in a circle.

Why? So you can design a logo for a company, as in Ex. 25.

Key Vocabulary
- **chord,** *p. 651*
- **arc,** *p. 659*
- **semicircle,** *p. 659*

Recall that a *chord* is a segment with endpoints on a circle. Because its endpoints lie on the circle, any chord divides the circle into two arcs. A diameter divides a circle into two semicircles. Any other chord divides a circle into a minor arc and a major arc.

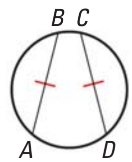


THEOREM — For Your Notebook

THEOREM 10.3

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

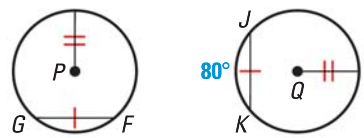
Proof: Exs. 27–28, p. 669



$\overset{\frown}{AB} \cong \overset{\frown}{CD}$ if and only if $\overline{AB} \cong \overline{CD}$.

EXAMPLE 1 Use congruent chords to find an arc measure

In the diagram, $\odot P \cong \odot Q$, $\overline{FG} \cong \overline{JK}$, and $m\overset{\frown}{JK} = 80°$. Find $m\overset{\frown}{FG}$.



Solution

Because $\overline{FG}$ and $\overline{JK}$ are congruent chords in congruent circles, the corresponding minor arcs $\overset{\frown}{FG}$ and $\overset{\frown}{JK}$ are congruent.

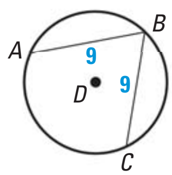
▶ So, $m\overset{\frown}{FG} = m\overset{\frown}{JK} = 80°$.

✓ GUIDED PRACTICE for Example 1

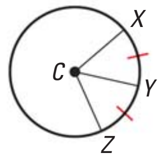
Use the diagram of $\odot D$.

1. If $m\overset{\frown}{AB} = 110°$, find $m\overset{\frown}{BC}$.

2. If $m\overset{\frown}{AC} = 150°$, find $m\overset{\frown}{AB}$.



BISECTING ARCS If $\overset{\frown}{XY} \cong \overset{\frown}{YZ}$, then the point Y, and any line, segment, or ray that contains Y, *bisects* $\overset{\frown}{XYZ}$.

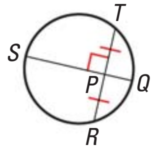


$\overline{CY}$ bisects $\overset{\frown}{XYZ}$.

THEOREMS — *For Your Notebook*

THEOREM 10.4

If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.

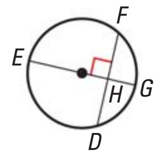


If $\overline{QS}$ is a perpendicular bisector of $\overline{TR}$, then $\overline{QS}$ is a diameter of the circle.

Proof: Ex. 31, p. 670

THEOREM 10.5

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.



If $\overline{EG}$ is a diameter and $\overline{EG} \perp \overline{DF}$, then $\overline{HD} \cong \overline{HF}$ and $\overset{\frown}{GD} \cong \overset{\frown}{GF}$.

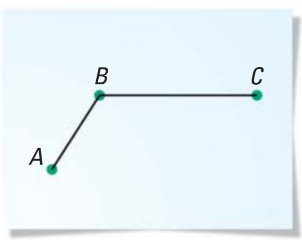
Proof: Ex. 32, p. 670

EXAMPLE 2 Use perpendicular bisectors

GARDENING Three bushes are arranged in a garden as shown. Where should you place a sprinkler so that it is the same distance from each bush?

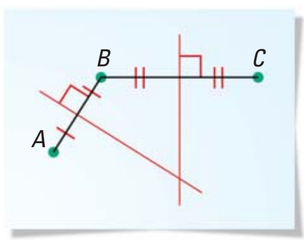
Solution

STEP 1



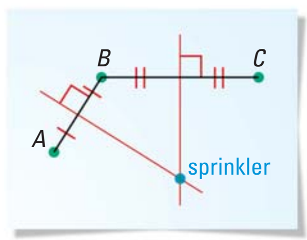
Label the bushes A, B, and C, as shown. Draw segments $\overline{AB}$ and $\overline{BC}$.

STEP 2



Draw the perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$. By Theorem 10.4, these are diameters of the circle containing A, B, and C.

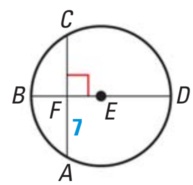
STEP 3



Find the point where these bisectors intersect. This is the center of the circle through A, B, and C, and so it is equidistant from each point.

EXAMPLE 3 Use a diameter

Use the diagram of $\odot E$ to find the length of $\overline{AC}$. Tell what theorem you use.



Solution

Diameter $\overline{BD}$ is perpendicular to $\overline{AC}$. So, by Theorem 10.5, $\overline{BD}$ bisects $\overline{AC}$, and $CF = AF$. Therefore, $AC = 2(AF) = 2(7) = 14$.

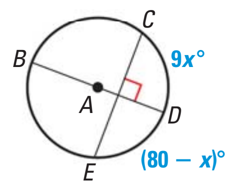
✓ GUIDED PRACTICE for Examples 2 and 3

Find the measure of the indicated arc in the diagram.

3. $\overset{\frown}{CD}$

4. $\overset{\frown}{DE}$

5. $\overset{\frown}{CE}$

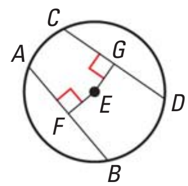


THEOREM *For Your Notebook*

THEOREM 10.6

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

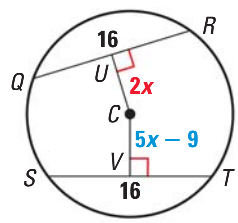
Proof: Ex. 33, p. 670



$\overline{AB} \cong \overline{CD}$ if and only if $EF = EG$.

EXAMPLE 4 Use Theorem 10.6

In the diagram of $\odot C$, $QR = ST = 16$. Find CU.



Solution

Chords $\overline{QR}$ and $\overline{ST}$ are congruent, so by Theorem 10.6 they are equidisant from C. Therefore, $CU = CV$.

$CU = CV$	**Use Theorem 10.6.**
$2x = 5x - 9$	**Substitute.**
$x = 3$	**Solve for *x*.**

▶ So, $CU = 2x = 2(3) = 6$.

✓ GUIDED PRACTICE for Example 4

In the diagram in Example 4, suppose $ST = 32$, and $CU = CV = 12$. Find the given length.

6. QR

7. QU

8. The radius of $\odot C$

10.3 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 9, and 25

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 22, and 26

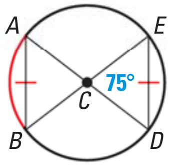
SKILL PRACTICE

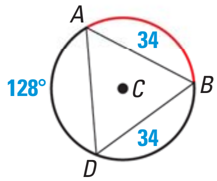
1. **VOCABULARY** *Describe* what it means to *bisect* an arc.

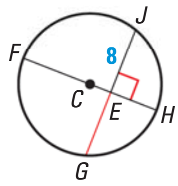
2. ★ **WRITING** Two chords of a circle are perpendicular and congruent. Does one of them have to be a diameter? *Explain* your reasoning.

EXAMPLES 1 and 3 on pp. 664, 666 for Exs. 3–5

FINDING ARC MEASURES **Find the measure of the red arc or chord in ⊙*C*.**

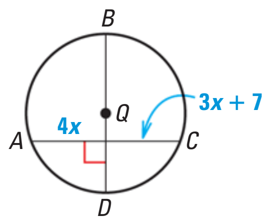
3.


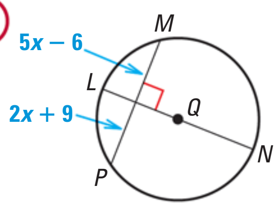
4.


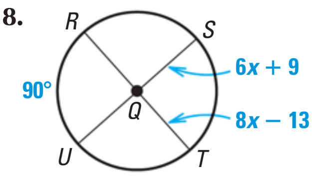
5.


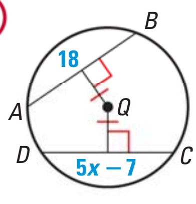
EXAMPLES 3 and 4 on p. 666 for Exs. 6–11

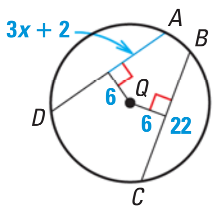
ALGEBRA **Find the value of *x* in ⊙*Q*. *Explain* your reasoning.**

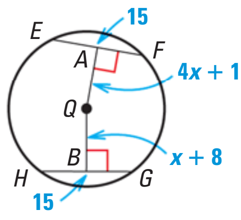
6.


7.


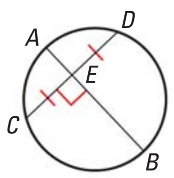
8.


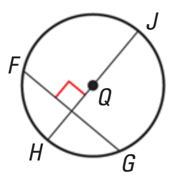
9.


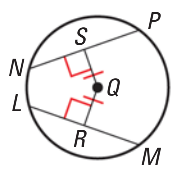
10.


11.


REASONING **In Exercises 12–14, what can you conclude about the diagram shown? State a theorem that justifies your answer.**

12.


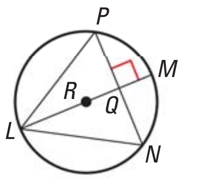
13.


14.


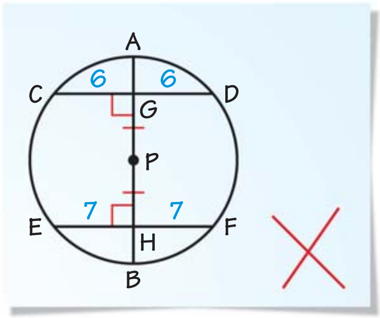
15. ★ **MULTIPLE CHOICE** In the diagram of ⊙*R*, which congruence relation is not necessarily true?

Ⓐ $\overline{PQ} \cong \overline{QN}$ Ⓑ $\overline{NL} \cong \overline{LP}$

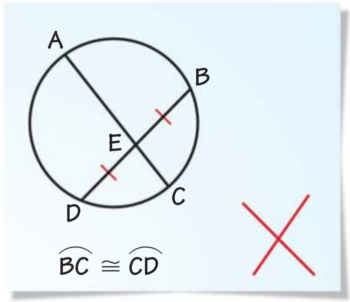
Ⓒ $\overset{\frown}{MN} \cong \overset{\frown}{MP}$ Ⓓ $\overline{PN} \cong \overline{PL}$



16. ERROR ANALYSIS *Explain* what is wrong with the diagram of $\odot P$.

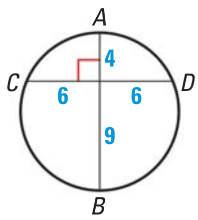


17. ERROR ANALYSIS *Explain* why the congruence statement is wrong.

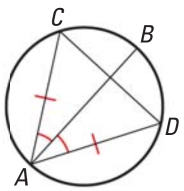


IDENTIFYING DIAMETERS **Determine whether $\overline{AB}$ is a diameter of the circle. *Explain* your reasoning.**

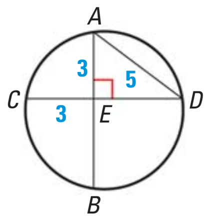
18.



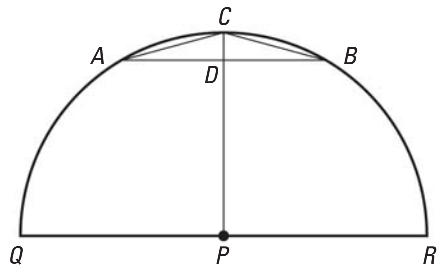
19.



20.



21. REASONING In the diagram of semicircle $\overset{\frown}{QCR}$, $\overline{PC} \cong \overline{AB}$ and $m\overset{\frown}{AC} = 30°$. *Explain* how you can conclude that $\triangle ADC \cong \triangle BDC$.

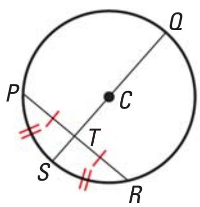


22. ★ WRITING Theorem 10.4 is nearly the converse of Theorem 10.5.

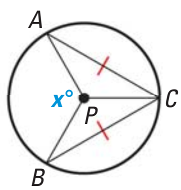
a. Write the converse of Theorem 10.5. *Explain* how it is different from Theorem 10.4.

b. Copy the diagram of $\odot C$ and draw auxiliary segments $\overline{PC}$ and $\overline{RC}$. Use congruent triangles to prove the converse of Theorem 10.5.

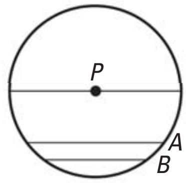
c. Use the converse of Theorem 10.5 to show that $QP = QR$ in the diagram of $\odot C$.



23. xy ALGEBRA In $\odot P$ below, $\overline{AC}$, $\overline{BC}$, and all arcs have integer measures. Show that x must be even.

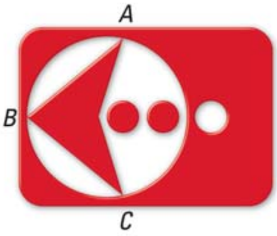


24. CHALLENGE In $\odot P$ below, the lengths of the parallel chords are 20, 16, and 12. Find $m\overset{\frown}{AB}$.



PROBLEM SOLVING

25. **LOGO DESIGN** The owner of a new company would like the company logo to be a picture of an arrow inscribed in a circle, as shown. For symmetry, she wants $\overset{\frown}{AB}$ to be congruent to $\overset{\frown}{BC}$. How should $\overline{AB}$ and $\overline{BC}$ be related in order for the logo to be exactly as desired?



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EXAMPLE 2 on p. 665 for Ex. 26

26. ★ **OPEN-ENDED MATH** In the cross section of the submarine shown, the control panels are parallel and the same length. *Explain* two ways you can find the center of the cross section.

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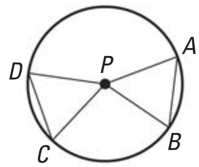
PROVING THEOREM 10.3 **In Exercises 27 and 28, prove Theorem 10.3.**

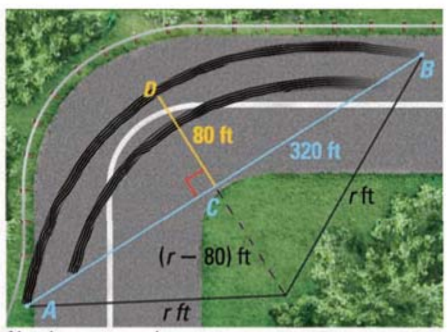
27. **GIVEN** ▶ $\overline{AB}$ and $\overline{CD}$ are congruent chords.
    PROVE ▶ $\overset{\frown}{AB} \cong \overset{\frown}{CD}$

28. **GIVEN** ▶ $\overline{AB}$ and $\overline{CD}$ are chords and $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.
    PROVE ▶ $\overline{AB} \cong \overline{CD}$



29. **CHORD LENGTHS** Make and prove a conjecture about chord lengths.
    a. Sketch a circle with two noncongruent chords. Is the *longer* chord or the *shorter* chord closer to the center of the circle? Repeat this experiment several times.
    b. Form a conjecture related to your experiment in part (a).
    c. Use the Pythagorean Theorem to prove your conjecture.

30. **MULTI-STEP PROBLEM** If a car goes around a turn too quickly, it can leave tracks that form an arc of a circle. By finding the radius of the circle, accident investigators can estimate the speed of the car.
    a. To find the radius, choose points A and B on the tire marks. Then find the midpoint C of $\overline{AB}$. Measure $\overline{CD}$, as shown. Find the radius r of the circle.
    b. The formula $S = 3.86\sqrt{fr}$ can be used to estimate a car's speed in miles per hours, where f is the *coefficient of friction* and r is the radius of the circle in feet. The coefficient of friction measures how slippery a road is. If $f = 0.7$, estimate the car's speed in part (a).



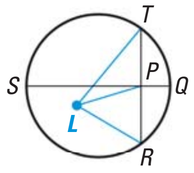
Not drawn to scale

PROVING THEOREMS 10.4 AND 10.5 Write proofs.

31. **GIVEN** ▶ $\overline{QS}$ is the perpendicular bisector of $\overline{RT}$.

PROVE ▶ $\overline{QS}$ is a diameter of $\odot L$.

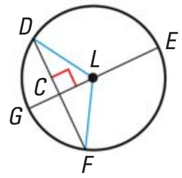
Plan for Proof Use indirect reasoning. Assume center L is not on $\overline{QS}$. Prove that $\triangle RLP \cong \triangle TLP$, so $\overline{PL} \perp \overline{RT}$. Then use the Perpendicular Postulate.



32. **GIVEN** ▶ $\overline{EG}$ is a diameter of $\odot L$. $\overline{EG} \perp \overline{DF}$

PROVE ▶ $\overline{CD} \cong \overline{CF}$, $\overarc{DG} \cong \overarc{FG}$

Plan for Proof Draw $\overline{LD}$ and $\overline{LF}$. Use congruent triangles to show $\overline{CD} \cong \overline{CF}$ and $\angle DLG \cong \angle FLG$. Then show $\overarc{DG} \cong \overarc{FG}$.



33. **PROVING THEOREM 10.6** For Theorem 10.6, prove both cases of the biconditional. Use the diagram shown for the theorem on page 666.

34. **CHALLENGE** A car is designed so that the rear wheel is only partially visible below the body of the car, as shown. The bottom panel is parallel to the ground. Prove that the point where the tire touches the ground bisects $\overarc{AB}$.

MIXED REVIEW

PREVIEW
Prepare for Lesson 10.4 in Exs. 35–37.

35. The measures of the interior angles of a quadrilateral are 100°, 140°, $(x + 20)°$, and $(2x + 10)°$. Find the value of x. *(p. 507)*

Quadrilateral *JKLM* is a parallelogram. Graph $\square JKLM$. Decide whether it is best described as a *rectangle*, a *rhombus*, or a *square*. *(p. 552)*

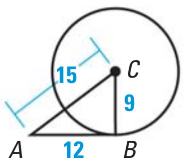
36. $J(-3, 5)$, $K(2, 5)$, $L(2, -1)$, $M(-3, -1)$

37. $J(-5, 2)$, $K(1, 1)$, $L(2, -5)$, $M(-4, -4)$

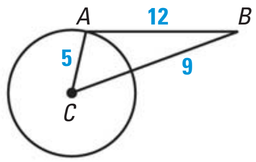
QUIZ *for Lessons 10.1–10.3*

Determine whether $\overline{AB}$ is tangent to $\odot C$. *Explain* your reasoning. *(p. 651)*

1.



2.



3. If $m\overarc{EFG} = 195°$, and $m\overarc{EF} = 80°$, find $m\overarc{FG}$ and $m\overarc{EG}$. *(p. 659)*

4. The points A, B, and D are on $\odot C$, $\overline{AB} \cong \overline{BD}$, and $m\overarc{ABD} = 194°$. What is the measure of $\overarc{AB}$? *(p. 664)*

10.4 Explore Inscribed Angles

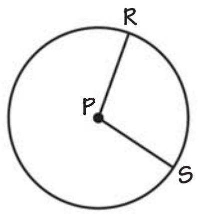
MATERIALS • compass • straightedge • protractor

QUESTION How are inscribed angles related to central angles?

The vertex of a central angle is at the center of the circle. The vertex of an *inscribed angle* is on the circle, and its sides form chords of the circle.

EXPLORE Construct inscribed angles of a circle

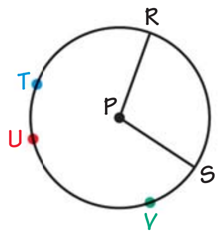
STEP 1



Draw a central angle Use a compass to draw a circle. Label the center *P*. Use a straightedge to draw a central angle. Label it $\angle RPS$.

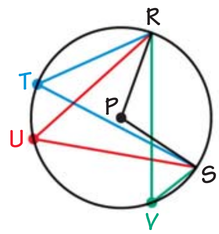
Animated Geometry at classzone.com

STEP 2



Draw points Locate three points on $\odot P$ in the exterior of $\angle RPS$ and label them *T*, *U*, and *V*.

STEP 3



Measure angles Draw $\angle RTS$, $\angle RUS$, and $\angle RVS$. These are called *inscribed angles*. Measure each angle.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Copy and complete the table.

	Central angle	Inscribed angle 1	Inscribed angle 2	Inscribed angle 3
Name	$\angle RPS$	$\angle RTS$	$\angle RUS$	$\angle RVS$
Measure	?	?	?	?

2. Draw two more circles. Repeat Steps 1–3 using different central angles. Record the measures in a table similar to the one above.

3. Use your results to make a conjecture about how the measure of an inscribed angle is related to the measure of the corresponding central angle.

10.4 Use Inscribed Angles and Polygons

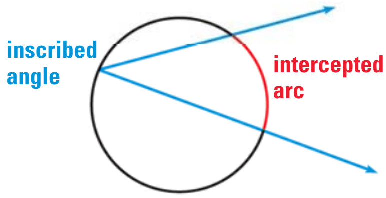
Before You used central angles of circles.

Now You will use inscribed angles of circles.

Why? So you can take a picture from multiple angles, as in Example 4.

Key Vocabulary
- **inscribed angle**
- **intercepted arc**
- **inscribed polygon**
- **circumscribed circle**

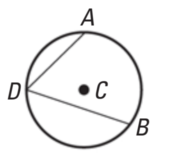
An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the **intercepted arc** of the angle.



THEOREM *For Your Notebook*

THEOREM 10.7 Measure of an Inscribed Angle Theorem

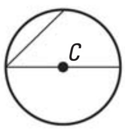
The measure of an inscribed angle is one half the measure of its intercepted arc.



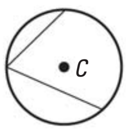
$m\angle ADB = \frac{1}{2}m\overset{\frown}{AB}$

Proof: Exs. 31–33, p. 678

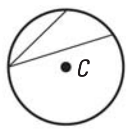
The proof of Theorem 10.7 in Exercises 31–33 involves three cases.



Case 1 Center C is on a side of the inscribed angle.



Case 2 Center C is inside the inscribed angle.

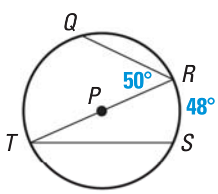


Case 3 Center C is outside the inscribed angle.

EXAMPLE 1 Use inscribed angles

Find the indicated measure in $\odot P$.

a. $m\angle T$ **b.** $m\overset{\frown}{QR}$



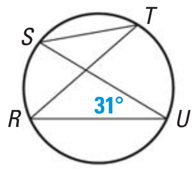
Solution

a. $m\angle T = \frac{1}{2}m\overset{\frown}{RS} = \frac{1}{2}(48°) = 24°$

b. $m\overset{\frown}{TQ} = 2m\angle R = 2 \cdot 50° = 100°$. Because $\overset{\frown}{TQR}$ is a semicircle, $m\overset{\frown}{QR} = 180° - m\overset{\frown}{TQ} = 180° - 100° = 80°$. So, $m\overset{\frown}{QR} = 80°$.

EXAMPLE 2 Find the measure of an intercepted arc

Find $m\overset{\frown}{RS}$ and $m\angle STR$. What do you notice about $\angle STR$ and $\angle RUS$?



Solution

From Theorem 10.7, you know that $m\overset{\frown}{RS} = 2m\angle RUS = 2(31°) = 62°$.

Also, $m\angle STR = \frac{1}{2}m\overset{\frown}{RS} = \frac{1}{2}(62°) = 31°$. So, $\angle STR \cong \angle RUS$.

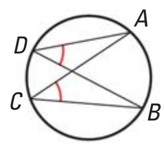
INTERCEPTING THE SAME ARC Example 2 suggests Theorem 10.8.

THEOREM — *For Your Notebook*

THEOREM 10.8

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

Proof: Ex. 34, p. 678



$\angle ADB \cong \angle ACB$

★ EXAMPLE 3 Standardized Test Practice

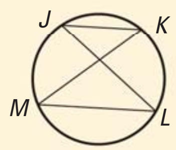
Name two pairs of congruent angles in the figure.

Ⓐ $\angle JKM \cong \angle KJL$, $\angle JLM \cong \angle KML$

Ⓑ $\angle JLM \cong \angle KJL$, $\angle JKM \cong \angle KML$

Ⓒ $\angle JKM \cong \angle JLM$, $\angle KJL \cong \angle KML$

Ⓓ $\angle JLM \cong \angle KJL$, $\angle JLM \cong \angle JKM$



Solution

ELIMINATE CHOICES
You can eliminate choices A and B, because they do not include the pair $\angle JKM \cong \angle JLM$.

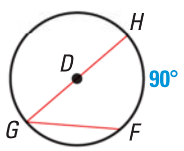
Notice that $\angle JKM$ and $\angle JLM$ intercept the same arc, and so $\angle JKM \cong \angle JLM$ by Theorem 10.8. Also, $\angle KJL$ and $\angle KML$ intercept the same arc, so they must also be congruent. Only choice C contains both pairs of angles.

▶ So, by Theorem 10.8, the correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

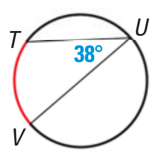
✓ GUIDED PRACTICE for Examples 1, 2, and 3

Find the measure of the red arc or angle.

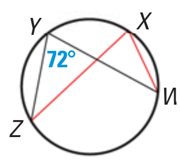
1.



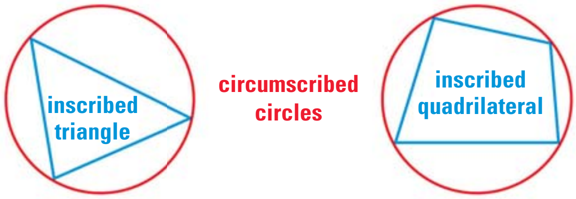
2.



3.



POLYGONS A polygon is an **inscribed polygon** if all of its vertices lie on a circle. The circle that contains the vertices is a **circumscribed circle**.

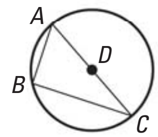


THEOREM — *For Your Notebook*

THEOREM 10.9

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

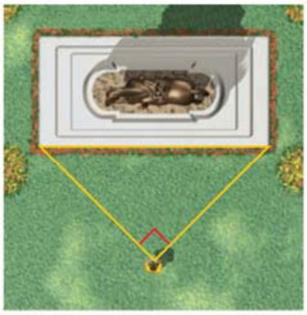
Proof: Ex. 35, p. 678



$m\angle ABC = 90°$ if and only if $\overline{AC}$ is a diameter of the circle.

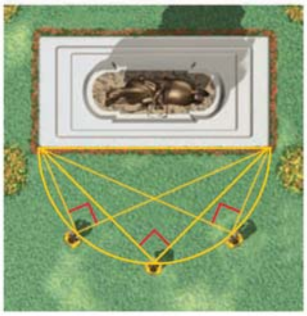
EXAMPLE 4 Use a circumscribed circle

PHOTOGRAPHY Your camera has a 90° field of vision and you want to photograph the front of a statue. You move to a spot where the statue is the only thing captured in your picture, as shown. You want to change your position. Where else can you stand so that the statue is perfectly framed in this way?



Solution

From Theorem 10.9, you know that if a right triangle is inscribed in a circle, then the hypotenuse of the triangle is a diameter of the circle. So, draw the circle that has the front of the statue as a diameter. The statue fits perfectly within your camera's 90° field of vision from any point on the semicircle in front of the statue.



✓ GUIDED PRACTICE for Example 4

4. **WHAT IF?** In Example 4, *explain* how to find locations if you want to frame the front and left side of the statue in your picture.

INSCRIBED QUADRILATERAL Only certain quadrilaterals can be inscribed in a circle. Theorem 10.10 describes these quadrilaterals.

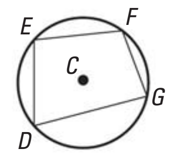
THEOREM — *For Your Notebook*

THEOREM 10.10

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

D, *E*, *F*, and *G* lie on $\odot C$ if and only if $m\angle D + m\angle F = m\angle E + m\angle G = 180°$.

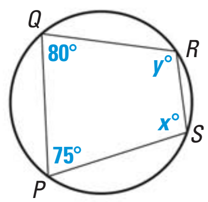
Proof: Ex. 30, p. 678; p. 938



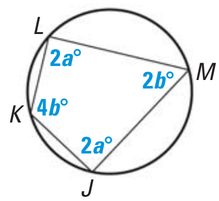
EXAMPLE 5 Use Theorem 10.10

Find the value of each variable.

a.



b.



Solution

a. *PQRS* is inscribed in a circle, so opposite angles are supplementary.

$$m\angle P + m\angle R = 180° \qquad m\angle Q + m\angle S = 180°$$
$$75° + y° = 180° \qquad 80° + x° = 180°$$
$$y = 105 \qquad x = 100$$

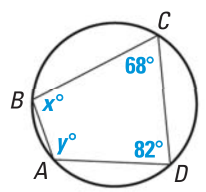
b. *JKLM* is inscribed in a circle, so opposite angles are supplementary.

$$m\angle J + m\angle L = 180° \qquad m\angle K + m\angle M = 180°$$
$$2a° + 2a° = 180° \qquad 4b° + 2b° = 180°$$
$$4a = 180 \qquad 6b = 180$$
$$a = 45 \qquad b = 30$$

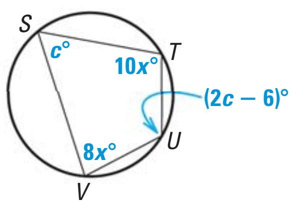
✓ GUIDED PRACTICE for Example 5

Find the value of each variable.

5.



6.



10.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 13, and 29

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 18, 29, and 36

SKILL PRACTICE

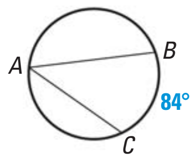
1. **VOCABULARY** Copy and complete: If a circle is circumscribed about a polygon, then the polygon is _?_ in the circle.

2. ★ **WRITING** *Explain* why the diagonals of a rectangle inscribed in a circle are diameters of the circle.

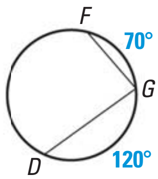
EXAMPLES 1 and 2 on pp. 672–673 for Exs. 3–9

INSCRIBED ANGLES **Find the indicated measure.**

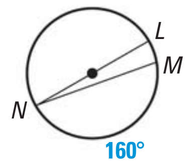
3. $m\angle A$



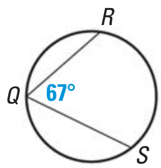
4. $m\angle G$



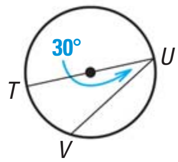
5. $m\angle N$



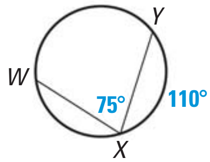
6. $m\widehat{RS}$



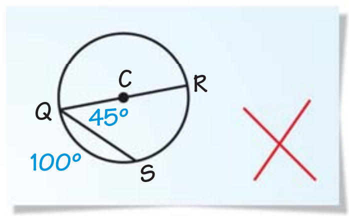
7. $m\widehat{VU}$



8. $m\widehat{WX}$



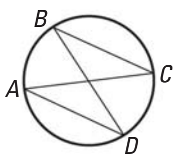
9. **ERROR ANALYSIS** *Describe* the error in the diagram of $\odot C$. Find two ways to correct the error.



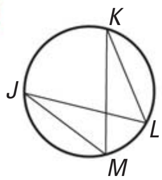
EXAMPLE 3 on p. 673 for Exs. 10–12

CONGRUENT ANGLES **Name two pairs of congruent angles.**

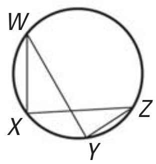
10.



11.



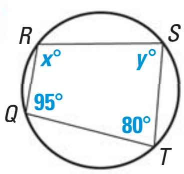
12.



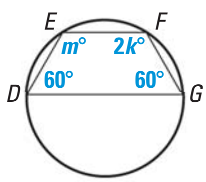
EXAMPLE 5 on p. 675 for Exs. 13–15

ALGEBRA **Find the values of the variables.**

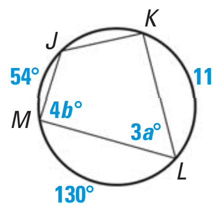
13.



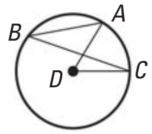
14.



15.



16. ★ **MULTIPLE CHOICE** In the diagram, $\angle ADC$ is a central angle and $m\angle ADC = 60°$. What is $m\angle ABC$?



Ⓐ 15° Ⓑ 30°

Ⓒ 60° Ⓓ 120°

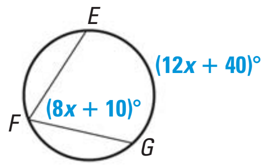
17. **INSCRIBED ANGLES** In each star below, all of the inscribed angles are congruent. Find the measure of an inscribed angle for each star. Then find the sum of all the inscribed angles for each star.

a.

b.

c.

18. ★ **MULTIPLE CHOICE** What is the value of x?



Ⓐ 5 Ⓑ 10

Ⓒ 13 Ⓓ 15

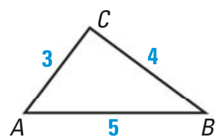
19. **PARALLELOGRAM** Parallelogram $QRST$ is inscribed in $\odot C$. Find $m\angle R$.

REASONING **Determine whether the quadrilateral can always be inscribed in a circle. *Explain* your reasoning.**

20. Square **21.** Rectangle **22.** Parallelogram

23. Kite **24.** Rhombus **25.** Isosceles trapezoid

26. **CHALLENGE** In the diagram, $\angle C$ is a right angle. If you draw the smallest possible circle through C and tangent to $\overline{AB}$, the circle will intersect $\overline{AC}$ at J and $\overline{BC}$ at K. Find the exact length of $\overline{JK}$.



PROBLEM SOLVING

27. **ASTRONOMY** Suppose three moons A, B, and C orbit 100,000 kilometers above the surface of a planet. Suppose $m\angle ABC = 90°$, and the planet is 20,000 kilometers in diameter. Draw a diagram of the situation. How far is moon A from moon C?

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EXAMPLE 4
on p. 674
for Ex. 28

28. **CARPENTER** A *carpenter's square* is an L-shaped tool used to draw right angles. You need to cut a circular piece of wood into two semicircles. How can you use a carpenter's square to draw a diameter on the circular piece of wood?

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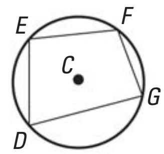
29. ★ **WRITING** A right triangle is inscribed in a circle and the radius of the circle is given. *Explain* how to find the length of the hypotenuse.

30. **PROVING THEOREM 10.10** Copy and complete the proof that opposite angles of an inscribed quadrilateral are supplementary.

GIVEN ▶ $\odot C$ with inscribed quadrilateral $DEFG$

PROVE ▶ $m\angle D + m\angle F = 180°$, $m\angle E + m\angle G = 180°$.



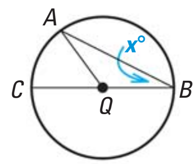
By the Arc Addition Postulate, $m\widehat{EFG}$ + __?__ $= 360°$ and $m\widehat{FGD} + m\widehat{DEF} = 360°$. Using the __?__ Theorem, $m\widehat{EDG} = 2m\angle F$, $m\widehat{EFG} = 2m\angle D$, $m\widehat{DEF} = 2m\angle G$, and $m\widehat{FGD} = 2m\angle E$. By the Substitution Property, $2m\angle D$ + __?__ $= 360°$, so __?__ . Similarly, __?__.

PROVING THEOREM 10.7 If an angle is inscribed in $\odot Q$, the center Q can be on a side of the angle, in the interior of the angle, or in the exterior of the angle. In Exercises 31–33, you will prove Theorem 10.7 for each of these cases.

31. **Case 1** Prove Case 1 of Theorem 10.7.

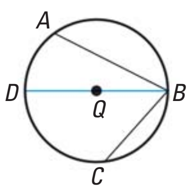
GIVEN ▶ $\angle B$ is inscribed in $\odot Q$. Let $m\angle B = x°$. Point Q lies on $\overline{BC}$.

PROVE ▶ $m\angle B = \frac{1}{2}m\widehat{AC}$

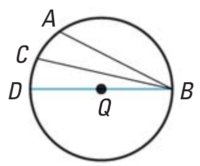


Plan for Proof Show that $\triangle AQB$ is isosceles. Use the Base Angles Theorem and the Exterior Angles Theorem to show that $m\angle AQC = 2x°$. Then, show that $m\widehat{AC} = 2x°$. Solve for x, and show that $m\angle B = \frac{1}{2}m\widehat{AC}$.

32. **Case 2** Use the diagram and auxiliary line to write GIVEN and PROVE statements for Case 2 of Theorem 10.7. Then write a plan for proof.



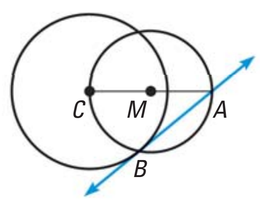
33. **Case 3** Use the diagram and auxiliary line to write GIVEN and PROVE statements for Case 3 of Theorem 10.7. Then write a plan for proof.



34. **PROVING THEOREM 10.8** Write a paragraph proof of Theorem 10.8. First draw a diagram and write GIVEN and PROVE statements.

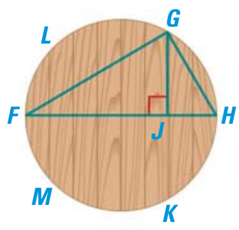
35. **PROVING THEOREM 10.9** Theorem 10.9 is written as a conditional statement and its converse. Write a plan for proof of each statement.

36. ★ **EXTENDED RESPONSE** In the diagram, $\odot C$ and $\odot M$ intersect at B, and $\overline{AC}$ is a diameter of $\odot M$. *Explain* why $\overleftrightarrow{AB}$ is tangent to $\odot C$.



CHALLENGE **In Exercises 37 and 38, use the following information.**

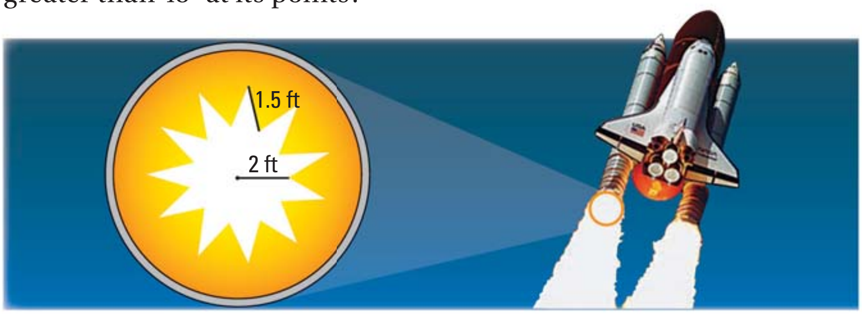
You are making a circular cutting board. To begin, you glue eight 1 inch by 2 inch boards together, as shown at the right. Then you draw and cut a circle with an 8 inch diameter from the boards.



37. $\overline{FH}$ is a diameter of the circular cutting board. Write a proportion relating GJ and JH. State a theorem to justify your answer.

38. Find FJ, JH, and JG. What is the length of the cutting board seam labeled $\overline{GK}$?

39. **SPACE SHUTTLE** To maximize thrust on a NASA space shuttle, engineers drill an 11-point star out of the solid fuel that fills each booster. They begin by drilling a hole with radius 2 feet, and they would like each side of the star to be 1.5 feet. Is this possible if the fuel cannot have angles greater than 45° at its points?



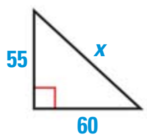
MIXED REVIEW

PREVIEW

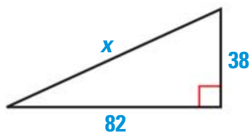
Prepare for Lesson 10.5 in Exs. 40–42.

Find the approximate length of the hypotenuse. Round your answer to the nearest tenth. *(p. 433)*

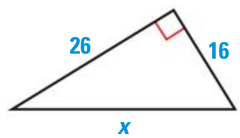
40.



41.

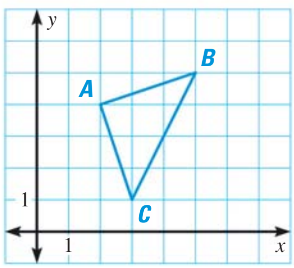


42.

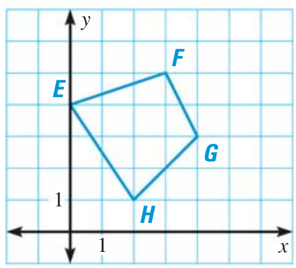


Graph the reflection of the polygon in the given line. *(p. 589)*

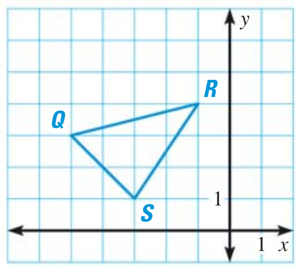
43. y-axis



44. $x = 3$



45. $y = 2$



Sketch the image of $A(3, -4)$ after the described glide reflection. *(p. 608)*

46. **Translation:** $(x, y) \rightarrow (x, y - 2)$
Reflection: in the y-axis

47. **Translation:** $(x, y) \rightarrow (x + 1, y + 4)$
Reflection: in $y = 4x$

10.5 Apply Other Angle Relationships in Circles

Before You found the measures of angles formed on a circle.

Now You will find the measures of angles inside or outside a circle.

Why So you can determine the part of Earth seen from a hot air balloon, as in Ex. 25.

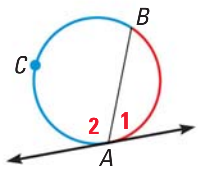
Key Vocabulary
- **chord,** *p. 651*
- **secant,** *p. 651*
- **tangent,** *p. 651*

You know that the measure of an inscribed angle is half the measure of its intercepted arc. This is true even if one side of the angle is tangent to the circle.

THEOREM — *For Your Notebook*

THEOREM 10.11

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.



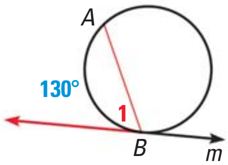
$m\angle 1 = \frac{1}{2}m\overparen{AB}$ $\quad m\angle 2 = \frac{1}{2}m\overparen{BCA}$

Proof: Ex. 27, p. 685

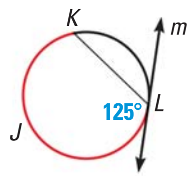
EXAMPLE 1 Find angle and arc measures

Line m is tangent to the circle. Find the measure of the red angle or arc.

a.



b.



Solution

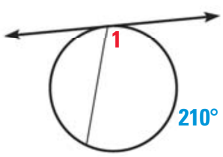
a. $m\angle 1 = \frac{1}{2}(130°) = 65°$

b. $m\overparen{KJL} = 2(125°) = 250°$

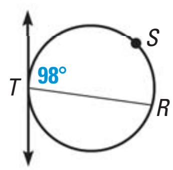
✓ GUIDED PRACTICE for Example 1

Find the indicated measure.

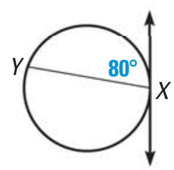
1. $m\angle 1$



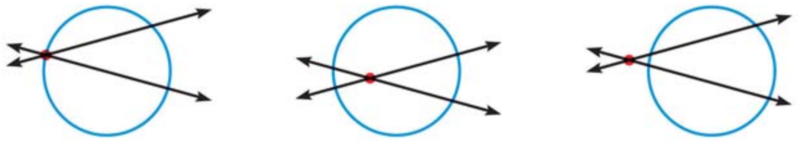
2. $m\overparen{RST}$



3. $m\overparen{XY}$



INTERSECTING LINES AND CIRCLES If two lines intersect a circle, there are three places where the lines can intersect.



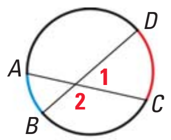
on the circle **inside the circle** **outside the circle**

You can use Theorems 10.12 and 10.13 to find measures when the lines intersect *inside* or *outside* the circle.

THEOREMS *For Your Notebook*

THEOREM 10.12 Angles Inside the Circle Theorem

If two chords intersect *inside* a circle, then the measure of each angle is one half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.



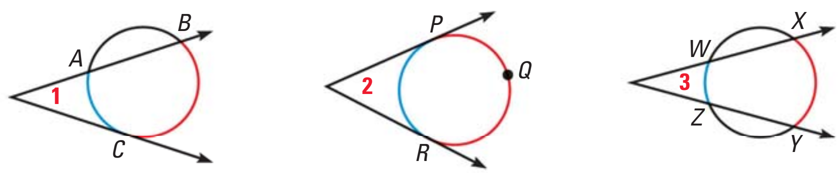
$m\angle 1 = \frac{1}{2}(m\widehat{DC} + m\widehat{AB})$,

$m\angle 2 = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$

Proof: Ex. 28, p. 685

THEOREM 10.13 Angles Outside the Circle Theorem

If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is one half the *difference* of the measures of the intercepted arcs.

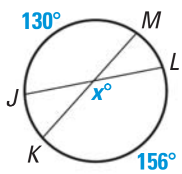


$m\angle 1 = \frac{1}{2}(m\widehat{BC} - m\widehat{AC})$ $m\angle 2 = \frac{1}{2}(m\widehat{PQR} - m\widehat{PR})$ $m\angle 3 = \frac{1}{2}(m\widehat{XY} - m\widehat{WZ})$

Proof: Ex. 29, p. 685

EXAMPLE 2 Find an angle measure inside a circle

Find the value of *x*.



Solution

The chords $\overline{JL}$ and $\overline{KM}$ intersect inside the circle.

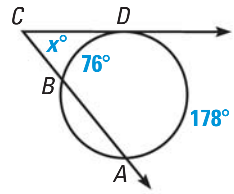
$x° = \frac{1}{2}(m\widehat{JM} + m\widehat{LK})$ **Use Theorem 10.12.**

$x° = \frac{1}{2}(130° + 156°)$ **Substitute.**

$x = 143$ **Simplify.**

EXAMPLE 3 Find an angle measure outside a circle

Find the value of *x*.



Solution

The tangent $\overrightarrow{CD}$ and the secant $\overrightarrow{CB}$ intersect outside the circle.

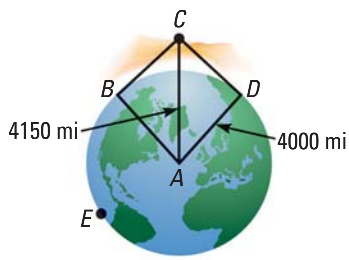
$$m\angle BCD = \frac{1}{2}\left(m\overset{\frown}{AD} - m\overset{\frown}{BD}\right)$$ **Use Theorem 10.13.**

$$x^\circ = \frac{1}{2}(178^\circ - 76^\circ)$$ **Substitute.**

$$x = 51$$ **Simplify.**

EXAMPLE 4 Solve a real-world problem

SCIENCE The Northern Lights are bright flashes of colored light between 50 and 200 miles above Earth. Suppose a flash occurs 150 miles above Earth. What is the measure of arc BD, the portion of Earth from which the flash is visible? (Earth's radius is approximately 4000 miles.)



Not drawn to scale

Solution

Because $\overline{CB}$ and $\overline{CD}$ are tangents, $\overline{CB} \perp \overline{AB}$ and $\overline{CD} \perp \overline{AD}$. Also, $\overline{BC} \cong \overline{DC}$ and $\overline{CA} \cong \overline{CA}$. So, $\triangle ABC \cong \triangle ADC$ by the Hypotenuse-Leg Congruence Theorem, and $\angle BCA \cong \angle DCA$. Solve right $\triangle CBA$ to find that $m\angle BCA \approx 74.5^\circ$. So, $m\angle BCD \approx 2(74.5^\circ) \approx 149^\circ$. Let $m\overset{\frown}{BD} = x^\circ$.

$$m\angle BCD = \frac{1}{2}\left(m\overset{\frown}{DEB} - m\overset{\frown}{BD}\right)$$ **Use Theorem 10.13.**

$$149^\circ \approx \frac{1}{2}[(360^\circ - x^\circ) - x^\circ]$$ **Substitute.**

$$x \approx 31$$ **Solve for *x*.**

▶ The measure of the arc from which the flash is visible is about 31°.

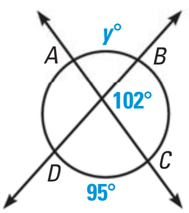
AVOID ERRORS
Because the value for $m\angle BCD$ is an approximation, use the symbol $\approx$ instead of $=$.

Animated Geometry at classzone.com

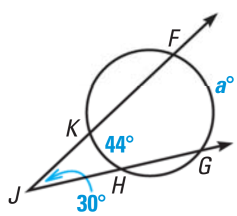
✓ GUIDED PRACTICE for Examples 2, 3, and 4

Find the value of the variable.

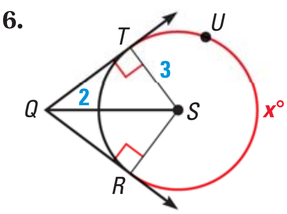
4.



5.



6.



10.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 9, and 23

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 13, 15, 19, and 26

SKILL PRACTICE

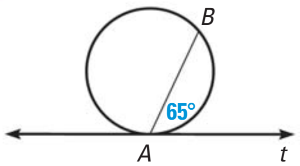
1. **VOCABULARY** Copy and complete: The points A, B, C, and D are on a circle and $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{CD}$ at P. If $m\angle APC = \frac{1}{2}\left(m\widehat{BD} - m\widehat{AC}\right)$, then P is _?_ (*inside*, *on*, or *outside*) the circle.

2. ★ **WRITING** What does it mean in Theorem 10.12 if $m\widehat{AB} = 0°$? Is this consistent with what you learned in Lesson 10.4? *Explain* your answer.

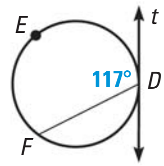
EXAMPLE 1 on p. 680 for Exs. 3–6

FINDING MEASURES **Line t is tangent to the circle. Find the indicated measure.**

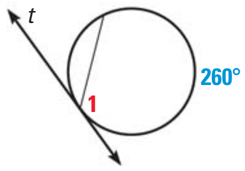
3. $m\widehat{AB}$



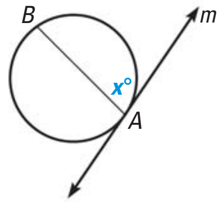
4. $m\widehat{DEF}$



5. $m\angle 1$



6. ★ **MULTIPLE CHOICE** The diagram at the right is not drawn to scale. $\overline{AB}$ is any chord that is not a diameter of the circle. Line m is tangent to the circle at point A. Which statement must be true?



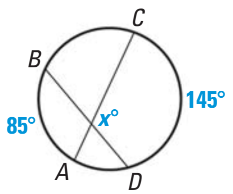
Ⓐ $x \le 90$ Ⓑ $x \ge 90$

Ⓒ $x = 90$ Ⓓ $x \ne 90$

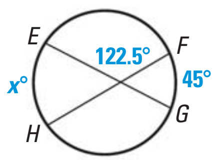
FINDING MEASURES **Find the value of x.**

EXAMPLE 2 on p. 681 for Exs. 7–9

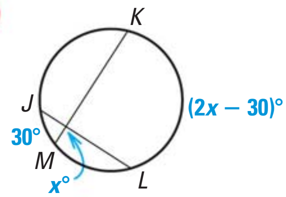
7.



8.

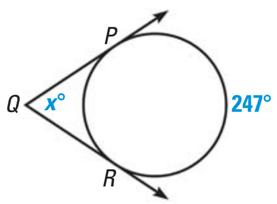


9.

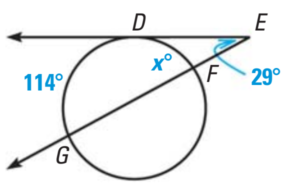


EXAMPLE 3 on p. 682 for Exs. 10–13

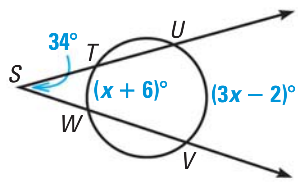
10.



11.



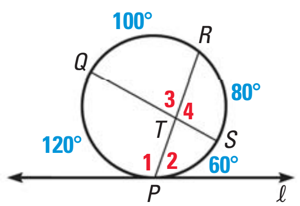
12.



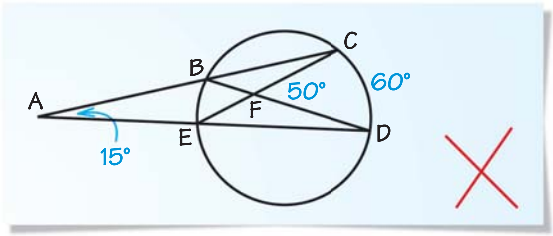
13. ★ **MULTIPLE CHOICE** In the diagram, ℓ is tangent to the circle at P. Which relationship is not true?

Ⓐ $m\angle 1 = 110°$ Ⓑ $m\angle 2 = 70°$

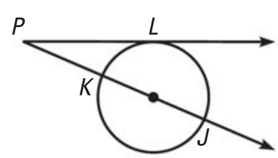
Ⓒ $m\angle 3 = 80°$ Ⓓ $m\angle 4 = 90°$



14. **ERROR ANALYSIS** *Describe* the error in the diagram below.

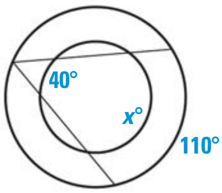


15. ★ **SHORT RESPONSE** In the diagram at the right, $\overrightarrow{PL}$ is tangent to the circle and $\overline{KJ}$ is a diameter. What is the range of possible angle measures of $\angle LPJ$? *Explain.*

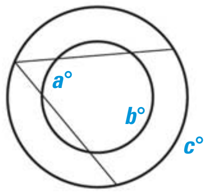


16. **CONCENTRIC CIRCLES** The circles below are concentric.

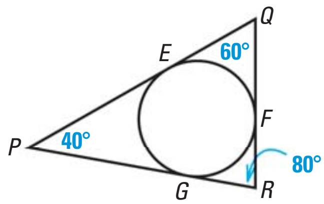
a. Find the value of x.



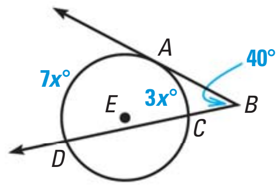
b. Express c in terms of a and b.



17. **INSCRIBED CIRCLE** In the diagram, the circle is inscribed in $\triangle PQR$. Find $m\overset{\frown}{EF}$, $m\overset{\frown}{FG}$, and $m\overset{\frown}{GE}$.



18. **ALGEBRA** In the diagram, $\overrightarrow{BA}$ is tangent to $\odot E$. Find $m\overset{\frown}{CD}$.



19. ★ **WRITING** Points A and B are on a circle and t is a tangent line containing A and another point C.

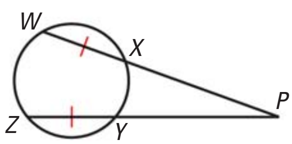
a. Draw two different diagrams that illustrate this situation.

b. Write an equation for $m\overset{\frown}{AB}$ in terms of $m\angle BAC$ for each diagram.

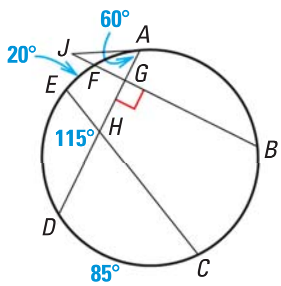
c. When will these equations give the same value for $m\overset{\frown}{AB}$?

CHALLENGE **Find the indicated measure(s).**

20. Find $m\angle P$ if $m\overset{\frown}{WZY} = 200°$.

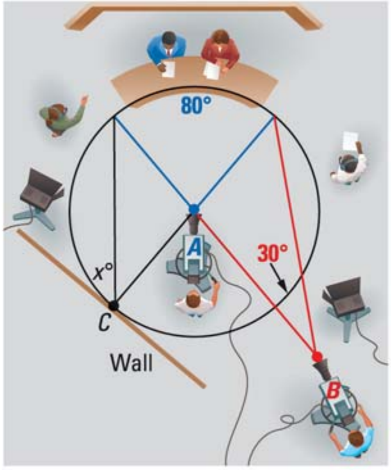


21. Find $m\overset{\frown}{AB}$ and $m\overset{\frown}{ED}$.



PROBLEM SOLVING

VIDEO RECORDING In the diagram at the right, television cameras are positioned at A, B, and C to record what happens on stage. The stage is an arc of $\odot A$. Use the diagram for Exercises 22–24.



22. Find $m\angle A$, $m\angle B$, and $m\angle C$.

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23. The wall is tangent to the circle. Find x without using the measure of $\angle C$.

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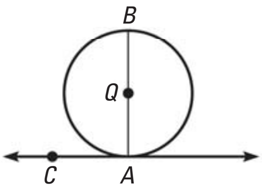
24. You would like Camera B to have a 30° view of the stage. Should you move the camera closer or further away from the stage? *Explain.*

EXAMPLE 4 on p. 682 for Ex. 25

25. HOT AIR BALLOON You are flying in a hot air balloon about 1.2 miles above the ground. Use the method from Example 4 to find the measure of the arc that represents the part of Earth that you can see. The radius of Earth is about 4000 miles.

26. ★ **EXTENDED RESPONSE** A cart is resting on its handle. The angle between the handle and the ground is 14° and the handle connects to the center of the wheel. What are the measures of the arcs of the wheel between the ground and the cart? *Explain.*

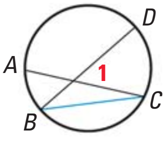
27. PROVING THEOREM 10.11 The proof of Theorem 10.11 can be split into three cases. The diagram at the right shows the case where $\overline{AB}$ contains the center of the circle. Use Theorem 10.1 to write a paragraph proof for this case. What are the other two cases? (*Hint:* See Exercises 31–33 on page 678.) Draw a diagram and write plans for proof for the other cases.



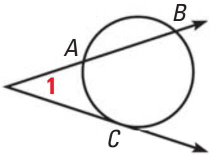
28. PROVING THEOREM 10.12 Write a proof of Theorem 10.12.

GIVEN ▶ Chords $\overline{AC}$ and $\overline{BD}$ intersect.

PROVE ▶ $m\angle 1 = \frac{1}{2}(m\widehat{DC} + m\widehat{AB})$



29. PROVING THEOREM 10.13 Use the diagram at the right to prove Theorem 10.13 for the case of a tangent and a secant. Draw $\overline{BC}$. *Explain* how to use the Exterior Angle Theorem in the proof of this case. Then copy the diagrams for the other two cases from page 681, draw appropriate auxiliary segments, and write plans for proof for these cases.



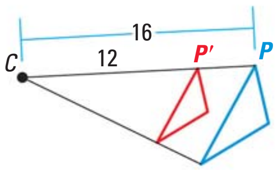
30. **PROOF** Q and R are points on a circle. P is a point outside the circle. $\overline{PQ}$ and $\overline{PR}$ are tangents to the circle. Prove that $\overline{QR}$ is not a diameter.

31. **CHALLENGE** A block and tackle system composed of two pulleys and a rope is shown at the right. The distance between the centers of the pulleys is 113 centimeters and the pulleys each have a radius of 15 centimeters. What percent of the circumference of the bottom pulley is not touching the rope?

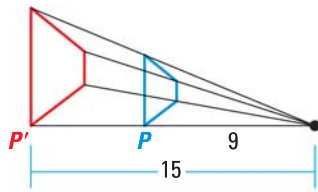
MIXED REVIEW

Classify the dilation and find its scale factor. *(p. 626)*

32.



33.



PREVIEW
Prepare for Lesson 10.6 in Exs. 34–39.

Use the quadratic formula to solve the equation. Round decimal answers to the nearest hundredth. *(pp. 641, 883)*

34. $x^2 + 7x + 6 = 0$

35. $x^2 - x - 12 = 0$

36. $x^2 + 16 = 8x$

37. $x^2 + 6x = 10$

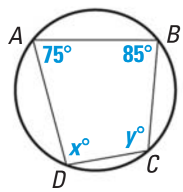
38. $5x + 9 = 2x^2$

39. $4x^2 + 3x - 11 = 0$

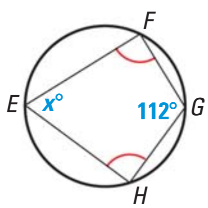
QUIZ *for Lessons 10.4–10.5*

Find the value(s) of the variable(s).

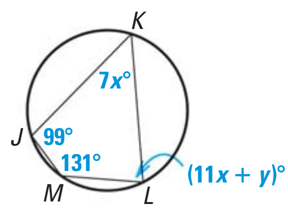
1. $m\overset{\frown}{ABC} = z°$ *(p. 672)*



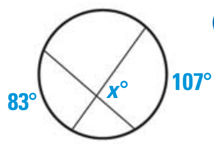
2. $m\overset{\frown}{GHE} = z°$ *(p. 672)*



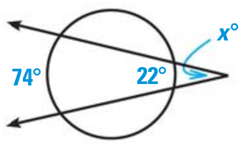
3. $m\overset{\frown}{JKL} = z°$ *(p. 672)*



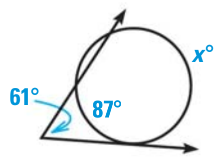
4. *(p. 680)*



5. *(p. 680)*



6. *(p. 680)*



7. **MOUNTAIN** You are on top of a mountain about 1.37 miles above sea level. Find the measure of the arc that represents the part of Earth that you can see. Earth's radius is approximately 4000 miles. *(p. 680)*

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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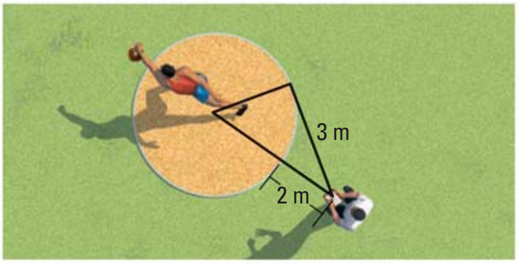
Lessons 10.1–10.5

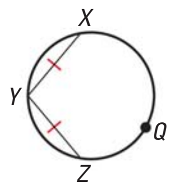
1. **MULTI-STEP PROBLEM** An official stands 2 meters from the edge of a discus circle and 3 meters from a point of tangency.



   a. Find the radius of the discus circle.

   b. How far is the official from the center of the discus circle?

2. **GRIDDED ANSWER** In the diagram, $\overline{XY} \cong \overline{YZ}$ and $m\widehat{XQZ} = 199°$. Find $m\widehat{YZ}$ in degrees.



3. **MULTI-STEP PROBLEM** A wind turbine has three equally spaced blades that are each 131 feet long.

   a. What is the measure of the arc between any two blades?

   b. The highest point reached by a blade is 361 feet above the ground. Find the distance x between the lowest point reached by the blades and the ground.

   c. What is the distance y from the tip of one blade to the tip of another blade? Round your answer to the nearest tenth.

4. **EXTENDED RESPONSE** The Navy Pier Ferris Wheel in Chicago is 150 feet tall and has 40 spokes.

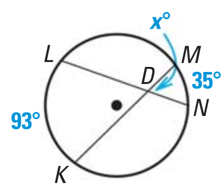
   a. Find the measure of the angle between any two spokes.

   b. Two spokes form a central angle of 72°. How many spokes are between the two spokes?

   c. The bottom of the wheel is 10 feet from the ground. Find the diameter and radius of the wheel. *Explain* your reasoning.

5. **OPEN-ENDED** Draw a quadrilateral inscribed in a circle. Measure two consecutive angles. Then find the measures of the other two angles algebraically.

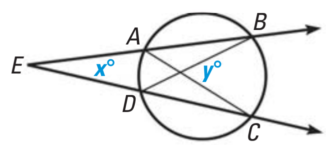
6. **MULTI-STEP PROBLEM** Use the diagram.



   a. Find the value of x.

   b. Find the measures of the other three angles formed by the intersecting chords.

7. **SHORT RESPONSE** Use the diagram to show that $m\widehat{DA} = y° - x°$.



Investigating Geometry ACTIVITY *Use before Lesson 10.6*

@HomeTutor classzone.com Keystrokes

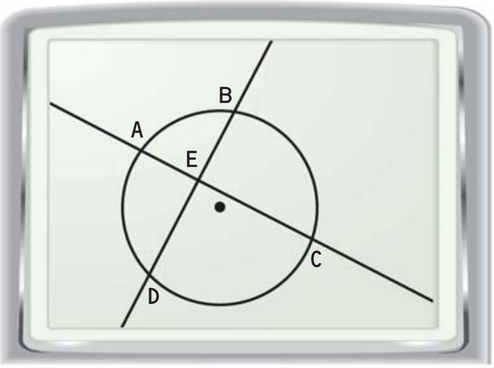
10.6 Investigate Segment Lengths

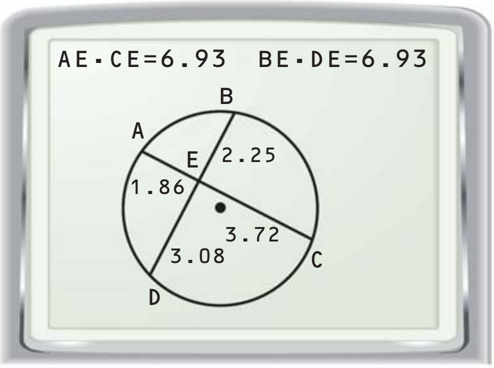
MATERIALS • graphing calculator or computer

QUESTION What is the relationship between the lengths of segments in a circle?

You can use geometry drawing software to find a relationship between the segments formed by two intersecting chords.

EXPLORE Draw a circle with two chords





STEP 1 ***Draw a circle*** Draw a circle and choose four points on the circle. Label them A, B, C, and D.

STEP 2 ***Draw secants*** Draw secants $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ and label the intersection point E.

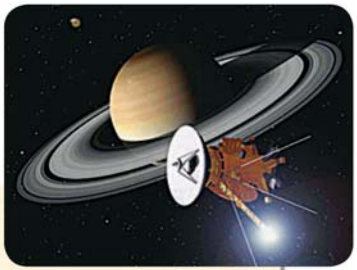
STEP 3 ***Measure segments*** Note that $\overline{AC}$ and $\overline{BD}$ are chords. Measure $\overline{AE}$, $\overline{CE}$, $\overline{BE}$, and $\overline{DE}$ in your diagram.

STEP 4 ***Perform calculations*** Calculate the products $AE \cdot CE$ and $BE \cdot DE$.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. What do you notice about the products you found in Step 4?
2. Drag points A, B, C, and D, keeping point E inside the circle. What do you notice about the new products from Step 4?
3. Make a conjecture about the relationship between the four chord segments.
4. Let $\overline{PQ}$ and $\overline{RS}$ be two chords of a circle that intersect at the point T. If $PT = 9$, $QT = 5$, and $RT = 15$, use your conjecture from Exercise 3 to find ST.

10.6 Find Segment Lengths in Circles



Before You found angle and arc measures in circles.

Now You will find segment lengths in circles.

Why? So you can find distances in astronomy, as in Example 4.

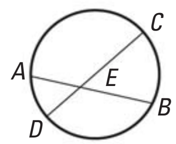
Key Vocabulary
- **segments of a chord**
- **secant segment**
- **external segment**

When two chords intersect in the interior of a circle, each chord is divided into two segments that are called **segments of the chord**.

THEOREM *For Your Notebook*

THEOREM 10.14 Segments of Chords Theorem

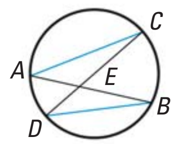
If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the other chord.



$EA \cdot EB = EC \cdot ED$

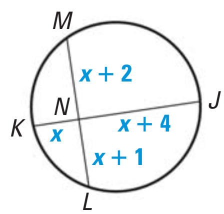
Proof: Ex. 21, p. 694

Plan for Proof To prove Theorem 10.14, construct two similar triangles. The lengths of the corresponding sides are proportional, so $\frac{EA}{ED} = \frac{EC}{EB}$. By the Cross Products Property, $EA \cdot EB = EC \cdot ED$.



EXAMPLE 1 Find lengths using Theorem 10.14

ALGEBRA **Find ML and JK.**



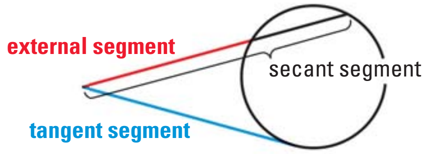
Solution

$NK \cdot NJ = NL \cdot NM$	**Use Theorem 10.14.**
$x \cdot (x + 4) = (x + 1) \cdot (x + 2)$	**Substitute.**
$x^2 + 4x = x^2 + 3x + 2$	**Simplify.**
$4x = 3x + 2$	**Subtract x^2 from each side.**
$x = 2$	**Solve for x.**

Find ML and JK by substitution.

$$ML = (x + 2) + (x + 1) \qquad JK = x + (x + 4)$$
$$= 2 + 2 + 2 + 1 \qquad = 2 + 2 + 4$$
$$= 7 \qquad = 8$$

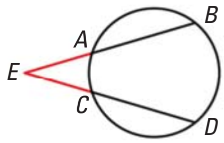
TANGENTS AND SECANTS A **secant segment** is a segment that contains a chord of a circle, and has exactly one endpoint outside the circle. The part of a secant segment that is outside the circle is called an **external segment**.



THEOREM — *For Your Notebook*

THEOREM 10.15 Segments of Secants Theorem

If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.



$EA \cdot EB = EC \cdot ED$

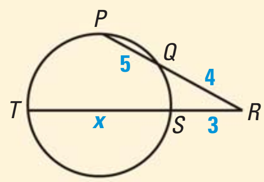
Proof: Ex. 25, p. 694

EXAMPLE 2 Standardized Test Practice

What is the value of x?

Ⓐ 6 Ⓑ $6\frac{2}{3}$

Ⓒ 8 Ⓓ 9



Solution

$$RQ \cdot RP = RS \cdot RT$$ **Use Theorem 10.15.**

$$4 \cdot (5 + 4) = 3 \cdot (x + 3)$$ **Substitute.**

$$36 = 3x + 9$$ **Simplify.**

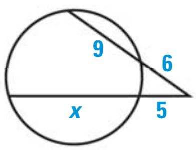
$$9 = x$$ **Solve for x.**

▶ The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

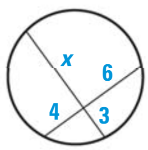
✓ GUIDED PRACTICE for Examples 1 and 2

Find the value(s) of x.

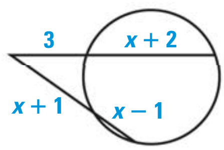
1.



2.



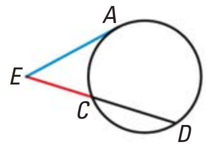
3.



THEOREM — *For Your Notebook*

THEOREM 10.16 Segments of Secants and Tangents Theorem

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.

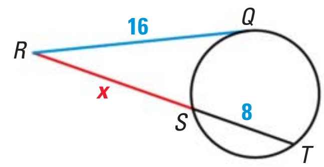


$EA^2 = EC \cdot ED$

Proof: Ex. 26, p. 694

EXAMPLE 3 Find lengths using Theorem 10.16

Use the figure at the right to find *RS*.



Solution

$RQ^2 = RS \cdot RT$	**Use Theorem 10.16.**
$16^2 = x \cdot (x + 8)$	**Substitute.**
$256 = x^2 + 8x$	**Simplify.**
$0 = x^2 + 8x - 256$	**Write in standard form.**
$x = \dfrac{-8 \pm \sqrt{8^2 - 4(1)(-256)}}{2(1)}$	**Use quadratic formula.**
$x = -4 \pm 4\sqrt{17}$	**Simplify.**

Use the positive solution, because lengths cannot be negative.

▶ So, $x = -4 + 4\sqrt{17} \approx 12.49$, and $RS \approx 12.49$.

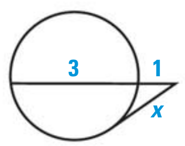
Animated Geometry at classzone.com

ANOTHER WAY
For an alternative method for solving the problem in Example 3, turn to page 696 for the **Problem Solving Workshop**.

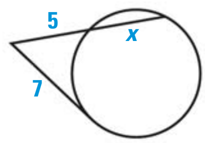
✓ GUIDED PRACTICE for Example 3

Find the value of *x*.

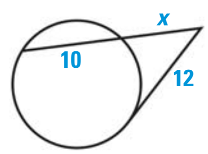
4.



5.

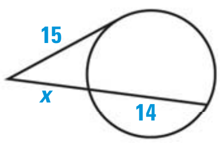


6.



Determine which theorem you would use to find *x*. Then find the value of *x*.

7.



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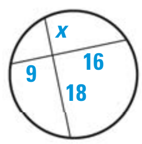

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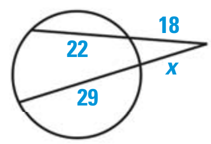
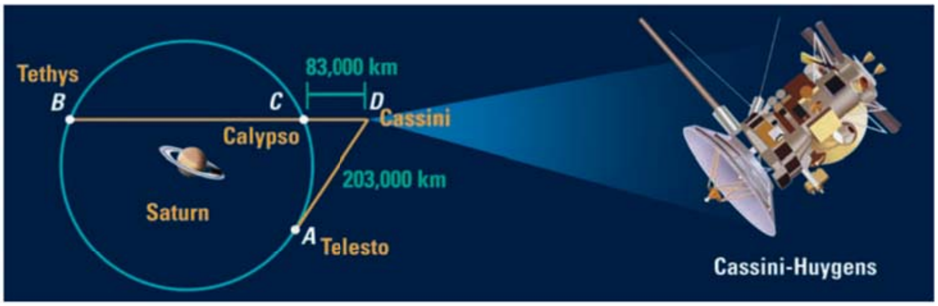


10. In the diagram for Theorem 10.16, what must be true about *EC* compared to *EA*?

EXAMPLE 4 Solve a real-world problem

SCIENCE Tethys, Calypso, and Telesto are three of Saturn's moons. Each has a nearly circular orbit 295,000 kilometers in radius. The Cassini-Huygens spacecraft entered Saturn's orbit in July 2004. Telesto is on a point of tangency. Find the distance DB from Cassini to Tethys.



Solution

$DC \cdot DB = AD^2$	**Use Theorem 10.16.**
$83{,}000 \cdot DB \approx 203{,}000^2$	**Substitute.**
$DB \approx 496{,}494$	**Solve for *DB*.**

▶ Cassini is about 496,494 kilometers from Tethys.

✓ GUIDED PRACTICE for Example 4

11. Why is it appropriate to use the approximation symbol ≈ in the last two steps of the solution to Example 4?

10.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 9, and 21

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 24, and 27

SKILL PRACTICE

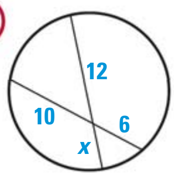
1. **VOCABULARY** Copy and complete: The part of the secant segment that is outside the circle is called a(n) __?__.

2. ★ **WRITING** *Explain* the difference between a tangent segment and a secant segment.

EXAMPLE 1 on p. 689 for Exs. 3–5

FINDING SEGMENT LENGTHS **Find the value of *x*.**

3.



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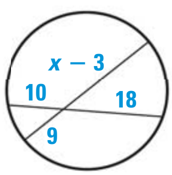

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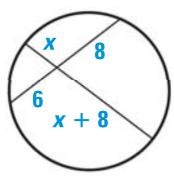
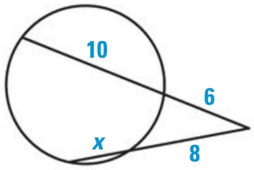


EXAMPLE 2
on p. 690
for Exs. 6–8

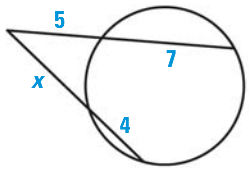
EXAMPLE 3
on p. 691
for Exs. 9–11

FINDING SEGMENT LENGTHS **Find the value of *x*.**

6.



7.



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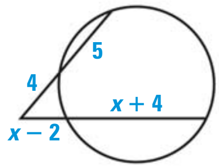

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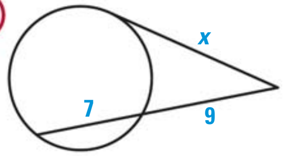

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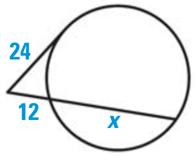
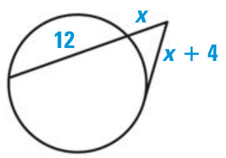
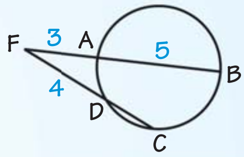


11.



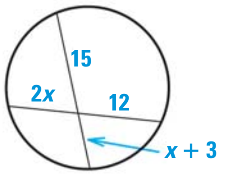
12. **ERROR ANALYSIS** *Describe* and correct the error in finding *CD*.

$CD \cdot DF = AB \cdot AF$
$CD \cdot 4 = 5 \cdot 3$
$CD \cdot 4 = 15$
$CD = 3.75$



FINDING SEGMENT LENGTHS **Find the value of *x*. Round to the nearest tenth.**

13.



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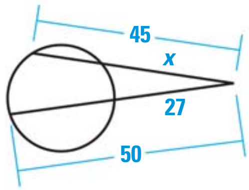

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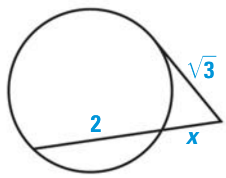
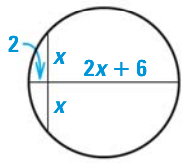


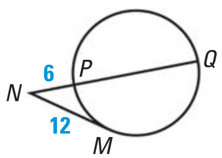
16. ★ **MULTIPLE CHOICE** Which of the following is a possible value of *x*?

Ⓐ −2 Ⓑ 4
Ⓒ 5 Ⓓ 6

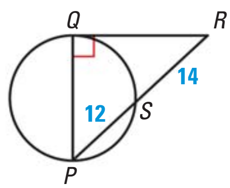


FINDING LENGTHS **Find *PQ*. Round your answers to the nearest tenth.**

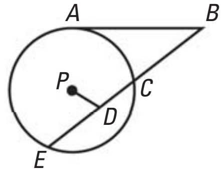
17.



18.



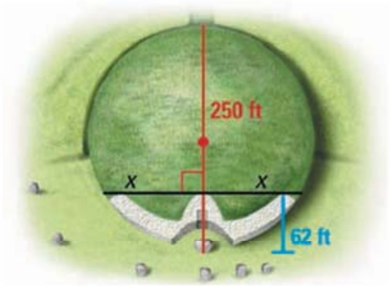
19. **CHALLENGE** In the figure, $AB = 12$, $BC = 8$, $DE = 6$, $PD = 4$, and A is a point of tangency. Find the radius of $\odot P$.



PROBLEM SOLVING

EXAMPLE 4 on p. 692 for Ex. 20

20. **ARCHAEOLOGY** The circular stone mound in Ireland called Newgrange has a diameter of 250 feet. A passage 62 feet long leads toward the center of the mound. Find the perpendicular distance x from the end of the passage to either side of the mound.



@HomeTutor for problem solving help at classzone.com

21. **PROVING THEOREM 10.14** Write a two-column proof of Theorem 10.14. Use similar triangles as outlined in the Plan for Proof on page 689.

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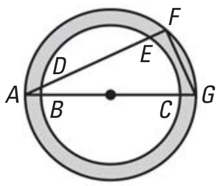
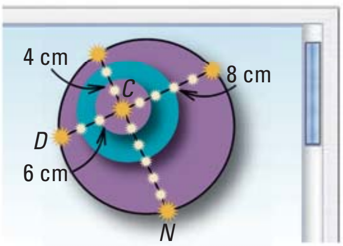
22. **WELLS** In the diagram of the water well, AB, AD, and DE are known. Write an equation for BC using these three measurements.



23. **PROOF** Use Theorem 10.1 to prove Theorem 10.16 for the special case when the secant segment contains the center of the circle.

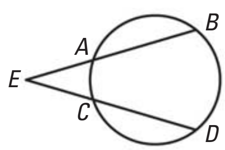
24. ★ **SHORT RESPONSE** You are designing an animated logo for your website. Sparkles leave point C and move to the circle along the segments shown so that all of the sparkles reach the circle at the same time. Sparkles travel from point C to point D at 2 centimeters per second. How fast should sparkles move from point C to point N? *Explain.*



25. **PROVING THEOREM 10.15** Use the plan to prove Theorem 10.15.

GIVEN ▶ $\overline{EB}$ and $\overline{ED}$ are secant segments.

PROVE ▶ $EA \cdot EB = EC \cdot ED$

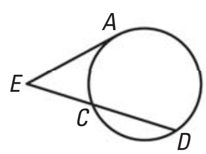


Plan for Proof Draw $\overline{AD}$ and $\overline{BC}$. Show that $\triangle BCE$ and $\triangle DAE$ are similar. Use the fact that corresponding side lengths in similar triangles are proportional.

26. **PROVING THEOREM 10.16** Use the plan to prove Theorem 10.16.

GIVEN ▶ $\overline{EA}$ is a tangent segment.
$\overline{ED}$ is a secant segment.

PROVE ▶ $EA^2 = EC \cdot ED$

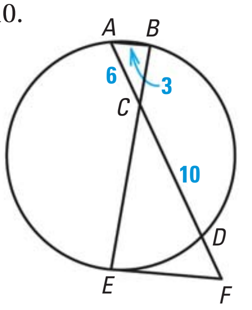


Plan for Proof Draw $\overline{AD}$ and $\overline{AC}$. Use the fact that corresponding side lengths in similar triangles are proportional.

○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

27. ★ **EXTENDED RESPONSE** In the diagram, $\overline{EF}$ is a tangent segment, $m\widehat{AD} = 140°$, $m\widehat{AB} = 20°$, $m\angle EFD = 60°$, $AC = 6$, $AB = 3$, and $DC = 10$.



a. Find $m\angle CAB$.

b. Show that $\triangle ABC \sim \triangle FEC$.

c. Let $EF = y$ and $DF = x$. Use the results of part (b) to write a proportion involving x and y. Solve for y.

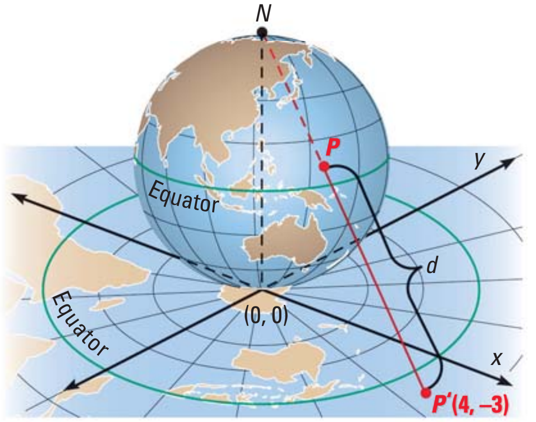
d. Use a theorem from this section to write another equation involving both x and y.

e. Use the results of parts (c) and (d) to solve for x and y.

f. *Explain* how to find CE.

28. **CHALLENGE** Stereographic projection is a map-making technique that takes points on a sphere with radius one unit (Earth) to points on a plane (the map). The plane is tangent to the sphere at the origin.

The map location for each point P on the sphere is found by extending the line that connects N and P. The point's projection is where the line intersects the plane. Find the distance d from the point P to its corresponding point $P'(4, -3)$ on the plane.



Not drawn to scale

MIXED REVIEW

PREVIEW
Prepare for Lesson 10.7 in Exs. 29–32.

Evaluate the expression. *(p. 874)*

29. $\sqrt{(-10)^2 - 8^2}$

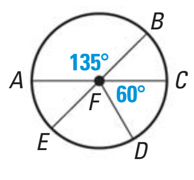
30. $\sqrt{-5 + (-4) + (6 - 1)^2}$

31. $\sqrt{[-2 - (-6)]^2 + (3 - 6)^2}$

32. In right $\triangle PQR$, $PQ = 8$, $m\angle Q = 40°$, and $m\angle R = 50°$. Find QR and PR to the nearest tenth. *(p. 473)*

33. $\overleftrightarrow{EF}$ is tangent to $\odot C$ at E. The radius of $\odot C$ is 5 and $EF = 8$. Find FC. *(p. 651)*

Find the indicated measure. $\overline{AC}$ and $\overline{BE}$ are diameters. *(p. 659)*



34. $m\widehat{AB}$

35. $m\widehat{CD}$

36. $m\widehat{BCA}$

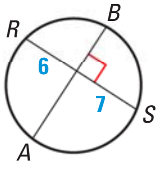
37. $m\widehat{CBD}$

38. $m\widehat{CDA}$

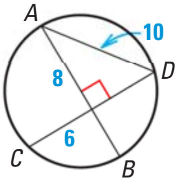
39. $m\widehat{BAE}$

Determine whether $\overline{AB}$ is a diameter of the circle. *Explain.* *(p. 664)*

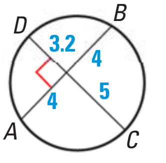
40.



41.



42.



PROBLEM SOLVING WORKSHOP
LESSON 10.6

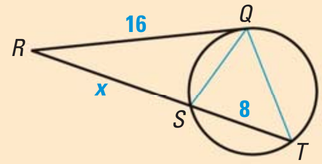
Using ALTERNATIVE METHODS

Another Way to Solve Example 3, page 691

MULTIPLE REPRESENTATIONS You can use similar triangles to find the length of an external secant segment.

PROBLEM

Use the figure at the right to find RS.



METHOD

Using Similar Triangles

STEP 1 **Draw** segments $\overline{QS}$ and $\overline{QT}$, and identify the similar triangles.

Because they both intercept the same arc, $\angle RQS \cong \angle RTQ$.
By the Reflexive Property of Angle Congruence, $\angle QRS \cong \angle TRQ$.
So, $\triangle RSQ \sim \triangle RQT$ by the AA Similarity Postulate.

STEP 2 **Use** a proportion to solve for RS.

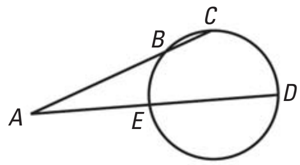
$$\frac{RS}{RQ} = \frac{RQ}{RT} \quad \Rightarrow \quad \frac{x}{16} = \frac{16}{x+8}$$

▶ By the Cross Products Property, $x^2 + 8x = 256$. Use the quadratic formula to find that $x = -4 \pm 4\sqrt{17}$. Taking the positive solution, $x = -4 + 4\sqrt{17}$ and $RS = 12.49$.

PRACTICE

1. **WHAT IF?** Find RQ in the problem above if the known lengths are $RS = 4$ and $ST = 9$.

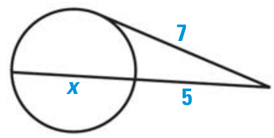
2. **MULTI-STEP PROBLEM** Copy the diagram.



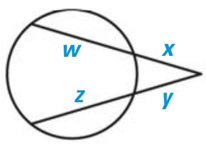
   a. Draw auxiliary segments $\overline{BE}$ and $\overline{CD}$. Name two similar triangles.

   b. If $AB = 15$, $BC = 5$, and $AE = 12$, find DE.

3. **CHORD** Find the value of x.



4. **SEGMENTS OF SECANTS** Use the Segments of Secants Theorem to write an expression for w in terms of x, y, and z.



Extension
Use after Lesson 10.6

Draw a Locus

GOAL Draw the locus of points satisfying certain conditions.

Key Vocabulary
- **locus**

A **locus** in a plane is the set of all points in a plane that satisfy a given condition or a set of given conditions. The word *locus* is derived from the Latin word for "location." The plural of locus is *loci*, pronounced "low-sigh."

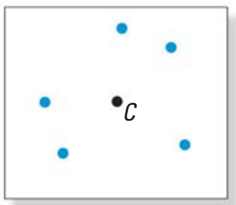
A locus is often described as the path of an object moving in a plane. For example, the reason that many clock faces are circular is that the locus of the end of a clock's minute hand is a circle.

EXAMPLE 1 Find a locus

Draw a point *C* on a piece of paper. Draw and describe the locus of all points on the paper that are 1 centimeter from *C*.

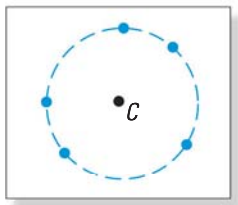
Solution

STEP 1



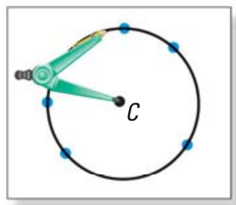
Draw point *C*. Locate several points 1 centimeter from *C*.

STEP 2



Recognize a pattern: the points lie on a circle.

STEP 3



Draw the circle.

▶ The locus of points on the paper that are 1 centimeter from *C* is a circle with center *C* and radius 1 centimeter.

KEY CONCEPT — *For Your Notebook*

How to Find a Locus

To find the locus of points that satisfy a given condition, use the following steps.

STEP 1 **Draw** any figures that are given in the statement of the problem. Locate several points that satisfy the given condition.

STEP 2 **Continue** drawing points until you can recognize the pattern.

STEP 3 **Draw** the locus and describe it in words.

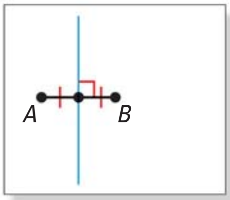
LOCI SATISFYING TWO OR MORE CONDITIONS To find the locus of points that satisfy two or more conditions, first find the locus of points that satisfy each condition alone. Then find the intersection of these loci.

EXAMPLE 2 Draw a locus satisfying two conditions

Points A and B lie in a plane. What is the locus of points in the plane that are equidistant from points A and B and are a distance of AB from B?

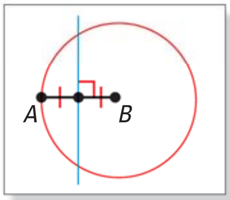
Solution

STEP 1



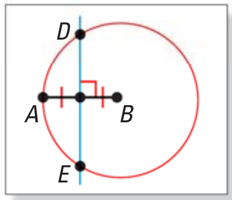
The locus of all points that are equidistant from A and B is the perpendicular bisector of $\overline{AB}$.

STEP 2



The locus of all points that are a distance of AB from B is the circle with center B and radius AB.

STEP 3



These loci intersect at D and E. So D and E form the locus of points that satisfy both conditions.

PRACTICE

EXAMPLE 1
on p. 697
for Exs. 1–4

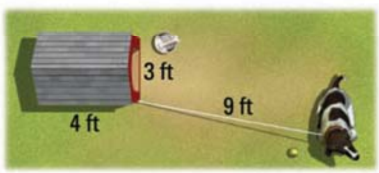
DRAWING A LOCUS **Draw the figure. Then sketch the locus of points on the paper that satisfy the given condition.**

1. Point P, the locus of points that are 1 inch from P
2. Line k, the locus of points that are 1 inch from k
3. Point C, the locus of points that are at least 1 inch from C
4. Line j, the locus of points that are no more than 1 inch from j

EXAMPLE 2
on p. 698
for Exs. 5–9

WRITING **Write a description of the locus. Include a sketch.**

5. Point P lies on line ℓ. What is the locus of points on ℓ and 3 cm from P?
6. Point Q lies on line m. What is the locus of points 5 cm from Q and 3 cm from m?
7. Point R is 10 cm from line k. What is the locus of points that are within 10 cm of R, but further than 10 cm from k?
8. Lines ℓ and m are parallel. Point P is 5 cm from both lines. What is the locus of points between ℓ and m and no more than 8 cm from P?
9. **DOG LEASH** A dog's leash is tied to a stake at the corner of its doghouse, as shown at the right. The leash is 9 feet long. Make a scale drawing of the doghouse and sketch the locus of points that the dog can reach.



10.7 Write and Graph Equations of Circles

Before You wrote equations of lines in the coordinate plane.

Now You will write equations of circles in the coordinate plane.

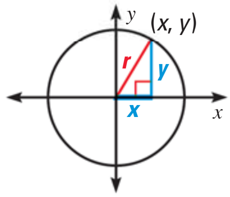
Why? So you can determine zones of a commuter system, as in Ex. 36.

Key Vocabulary
- **standard equation of a circle**

Let (x, y) represent any point on a circle with center at the origin and radius r. By the Pythagorean Theorem,

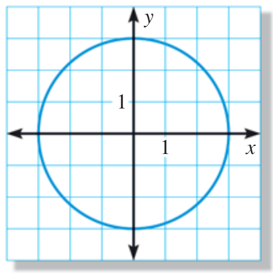
$$x^2 + y^2 = r^2.$$

This is the equation of a circle with radius r and center at the origin.



EXAMPLE 1 Write an equation of a circle

Write the equation of the circle shown.



Solution

The radius is 3 and the center is at the origin.

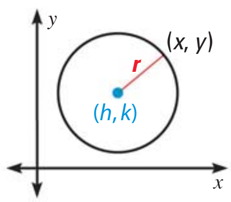
$x^2 + y^2 = r^2$	Equation of circle
$x^2 + y^2 = 3^2$	Substitute.
$x^2 + y^2 = 9$	Simplify.

▶ The equation of the circle is $x^2 + y^2 = 9$.

CIRCLES CENTERED AT (h, k) You can write the equation of *any* circle if you know its radius and the coordinates of its center.

Suppose a circle has radius r and center (h, k). Let (x, y) be a point on the circle. The distance between (x, y) and (h, k) is r, so by the Distance Formula

$$\sqrt{(x - h)^2 + (y - k)^2} = r.$$



Square both sides to find the **standard equation of a circle**.

KEY CONCEPT *For Your Notebook*

Standard Equation of a Circle

The standard equation of a circle with center (h, k) and radius r is:

$$(x - h)^2 + (y - k)^2 = r^2$$

EXAMPLE 2 Write the standard equation of a circle

Write the standard equation of a circle with center (0, −9) and radius 4.2.

Solution

$(x - h)^2 + (y - k)^2 = r^2$ Standard equation of a circle

$(x - 0)^2 + (y - (-9))^2 = 4.2^2$ Substitute.

$x^2 + (y + 9)^2 = 17.64$ Simplify.

✓ GUIDED PRACTICE for Examples 1 and 2

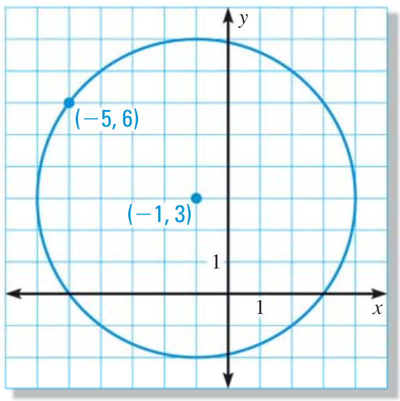
Write the standard equation of the circle with the given center and radius.

1. Center (0, 0), radius 2.5

2. Center (−2, 5), radius 7

EXAMPLE 3 Write the standard equation of a circle

The point (−5, 6) is on a circle with center (−1, 3). Write the standard equation of the circle.



Solution

To write the standard equation, you need to know the values of h, k, and r. To find r, find the distance between the center and the point (−5, 6) on the circle.

$r = \sqrt{[-5 - (-1)]^2 + (6 - 3)^2}$ Distance Formula

$= \sqrt{(-4)^2 + 3^2}$ Simplify.

$= 5$ Simplify.

Substitute $(h, k) = (-1, 3)$ and $r = 5$ into the standard equation of a circle.

$(x - h)^2 + (y - k)^2 = r^2$ Standard equation of a circle

$[x - (-1)]^2 + (y - 3)^2 = 5^2$ Substitute.

$(x + 1)^2 + (y - 3)^2 = 25$ Simplify.

▶ The standard equation of the circle is $(x + 1)^2 + (y - 3)^2 = 25$.

✓ GUIDED PRACTICE for Example 3

3. The point (3, 4) is on a circle whose center is (1, 4). Write the standard equation of the circle.

4. The point (−1, 2) is on a circle whose center is (2, 6). Write the standard equation of the circle.

EXAMPLE 4 Graph a circle

USE EQUATIONS
If you know the equation of a circle, you can graph the circle by identifying its center and radius.

The equation of a circle is $(x-4)^2 + (y+2)^2 = 36$. Graph the circle.

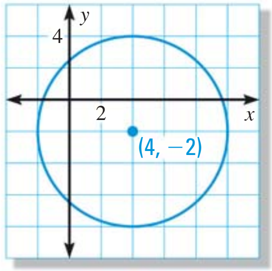
Solution

Rewrite the equation to find the center and radius.

$$(x-4)^2 + (y+2)^2 = 36$$

$$(x-4)^2 + [y-(-2)]^2 = 6^2$$

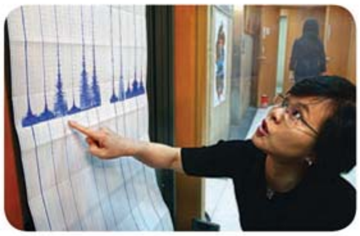
The center is $(4, -2)$ and the radius is 6. Use a compass to graph the circle.



EXAMPLE 5 Use graphs of circles

EARTHQUAKES The epicenter of an earthquake is the point on Earth's surface directly above the earthquake's origin. A seismograph can be used to determine the distance to the epicenter of an earthquake. Seismographs are needed in three different places to locate an earthquake's epicenter.

Use the seismograph readings from locations A, B, and C to find the epicenter of an earthquake.



- The epicenter is 7 miles away from $A(-2, 2.5)$.
- The epicenter is 4 miles away from $B(4, 6)$.
- The epicenter is 5 miles away from $C(3, -2.5)$.

Solution

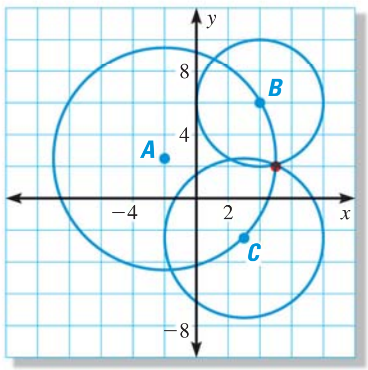
The set of all points equidistant from a given point is a circle, so the epicenter is located on each of the following circles.

$\odot A$ with center $(-2, 2.5)$ and radius 7

$\odot B$ with center $(4, 6)$ and radius 4

$\odot C$ with center $(3, -2.5)$ and radius 5

To find the epicenter, graph the circles on a graph where units are measured in miles. Find the point of intersection of all three circles.



▶ The epicenter is at about $(5, 2)$.

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✓ GUIDED PRACTICE for Examples 4 and 5

5. The equation of a circle is $(x-4)^2 + (y+3)^2 = 16$. Graph the circle.

6. The equation of a circle is $(x+8)^2 + (y+5)^2 = 121$. Graph the circle.

7. Why are three seismographs needed to locate an earthquake's epicenter?

10.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 17, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 26, and 42

SKILL PRACTICE

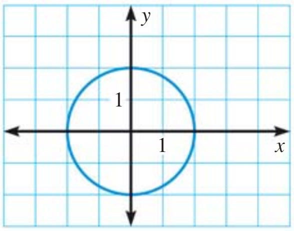
1. VOCABULARY Copy and complete: The standard equation of a circle can be written for any circle with known __?__ and __?__.

2. ★ WRITING *Explain* why the location of the center and one point on a circle is enough information to draw the rest of the circle.

EXAMPLES 1 and 2 on pp. 699–700 for Exs. 3–16

WRITING EQUATIONS **Write the standard equation of the circle.**

3.



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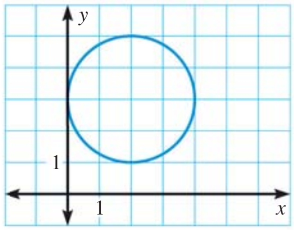

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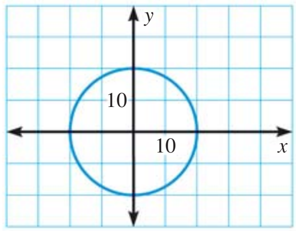

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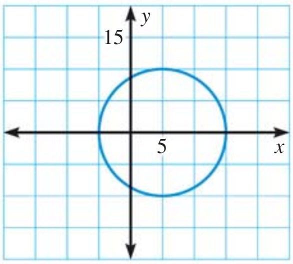

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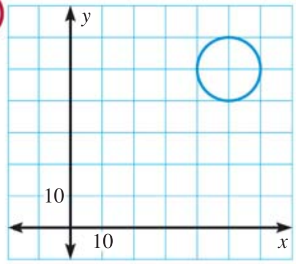

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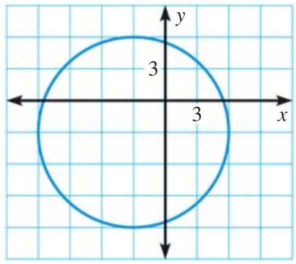


WRITING EQUATIONS **Write the standard equation of the circle with the given center and radius.**

9. Center (0, 0), radius 7 **10.** Center (−4, 1), radius 1 **11.** Center (7, −6), radius 8

12. Center (4, 1), radius 5 **13.** Center (3, −5), radius 7 **14.** Center (−3, 4), radius 5

15. ERROR ANALYSIS *Describe* and correct the error in writing the equation of a circle.

An equation of a circle with center (−3, −5) and radius 3 is $(x - 3)^2 + (y - 5)^2 = 9$.

16. ★ MULTIPLE CHOICE The standard equation of a circle is $(x - 2)^2 + (y + 1)^2 = 16$. What is the diameter of the circle?

Ⓐ 2 Ⓑ 4 Ⓒ 8 Ⓓ 16

EXAMPLE 3 on p. 700 for Exs. 17–19

WRITING EQUATIONS **Use the given information to write the standard equation of the circle.**

17. The center is (0, 0), and a point on the circle is (0, 6).

18. The center is (1, 2), and a point on the circle is (4, 2).

19. The center is (−3, 5), and a point on the circle is (1, 8).

EXAMPLE 4
on p. 701
for Exs. 20–25

GRAPHING CIRCLES **Graph the equation.**

20. $x^2 + y^2 = 49$

21. $(x - 3)^2 + y^2 = 16$

22. $x^2 + (y + 2)^2 = 36$

23. $(x - 4)^2 + (y - 1)^2 = 1$

24. $(x + 5)^2 + (y - 3)^2 = 9$

25. $(x + 2)^2 + (y + 6)^2 = 25$

26. ★ **MULTIPLE CHOICE** Which of the points does not lie on the circle described by the equation $(x + 2)^2 + (y - 4)^2 = 25$?

Ⓐ $(-2, -1)$ Ⓑ $(1, 8)$ Ⓒ $(3, 4)$ Ⓓ $(0, 5)$

xy **ALGEBRA** **Determine whether the given equation defines a circle. If the equation defines a circle, rewrite the equation in standard form.**

27. $x^2 + y^2 - 6y + 9 = 4$

28. $x^2 - 8x + 16 + y^2 + 2y + 4 = 25$

29. $x^2 + y^2 + 4y + 3 = 16$

30. $x^2 - 2x + 5 + y^2 = 81$

IDENTIFYING TYPES OF LINES **Use the given equations of a circle and a line to determine whether the line is a *tangent*, *secant*, *secant that contains a diameter*, or none of these.**

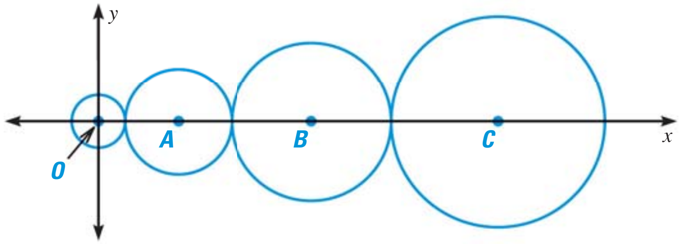
31. Circle: $(x - 4)^2 + (y - 3)^2 = 9$
Line: $y = -3x + 6$

32. Circle: $(x + 2)^2 + (y - 2)^2 = 16$
Line: $y = 2x - 4$

33. Circle: $(x - 5)^2 + (y + 1)^2 = 4$
Line: $y = \frac{1}{5}x - 3$

34. Circle: $(x + 3)^2 + (y - 6)^2 = 25$
Line: $y = -\frac{4}{3}x + 2$

35. **CHALLENGE** Four tangent circles are centered on the x-axis. The radius of $\odot A$ is twice the radius of $\odot O$. The radius of $\odot B$ is three times the radius of $\odot O$. The radius of $\odot C$ is four times the radius of $\odot O$. All circles have integer radii and the point $(63, 16)$ is on $\odot C$. What is the equation of $\odot A$?



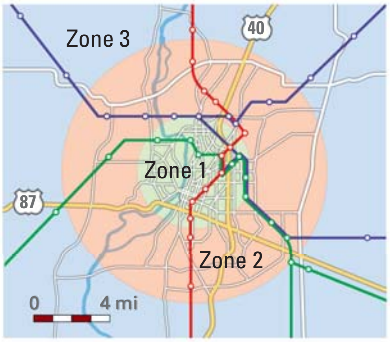
PROBLEM SOLVING

EXAMPLE 5
on p. 701
for Ex. 36

36. **COMMUTER TRAINS** A city's commuter system has three zones covering the regions described. Zone 1 covers people living within three miles of the city center. Zone 2 covers those between three and seven miles from the center, and Zone 3 covers those over seven miles from the center.

a. Graph this situation with the city center at the origin, where units are measured in miles.

b. Find which zone covers people living at $(3, 4)$, $(6, 5)$, $(1, 2)$, $(0, 3)$, and $(1, 6)$.



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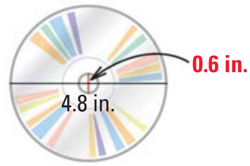
37. **COMPACT DISCS** The diameter of a CD is about 4.8 inches. The diameter of the hole in the center is about 0.6 inches. You place a CD on the coordinate plane with center at (0, 0). Write the equations for the outside edge of the disc and the edge of the hole in the center.



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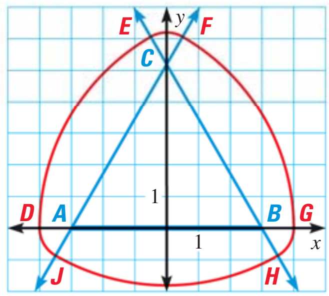
REULEAUX POLYGONS **In Exercises 38–41, use the following information.**

The figure at the right is called a *Reuleaux polygon*. It is not a true polygon because its sides are not straight. $\triangle ABC$ is equilateral.



38. $\overset{\frown}{JD}$ lies on a circle with center A and radius AD. Write an equation of this circle.

39. $\overset{\frown}{DE}$ lies on a circle with center B and radius BD. Write an equation of this circle.

40. **CONSTRUCTION** The remaining arcs of the polygon are constructed in the same way as $\overset{\frown}{JD}$ and $\overset{\frown}{DE}$ in Exercises 38 and 39. Construct a Reuleaux polygon on a piece of cardboard.

41. Cut out the Reuleaux polygon from Exercise 40. Roll it on its edge like a wheel and measure its height when it is in different orientations. *Explain* why a Reuleaux polygon is said to have constant width.

42. ★ **EXTENDED RESPONSE** Telecommunication towers can be used to transmit cellular phone calls. Towers have a range of about 3 km. A graph with units measured in kilometers shows towers at points (0, 0), (0, 5), and (6, 3).

a. Draw the graph and locate the towers. Are there any areas that may receive calls from more than one tower?

b. Suppose your home is located at (2, 6) and your school is at (2.5, 3). Can you use your cell phone at either or both of these locations?

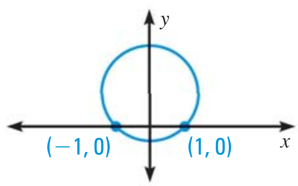
c. City A is located at (−2, 2.5) and City B is at (5, 4). Each city has a radius of 1.5 km. Which city seems to have better cell phone coverage? *Explain*.

43. **REASONING** The lines $y = \frac{3}{4}x + 2$ and $y = -\frac{3}{4}x + 16$ are tangent to $\odot C$ at the points (4, 5) and (4, 13), respectively.

a. Find the coordinates of C and the radius of $\odot C$. *Explain* your steps.

b. Write the standard equation of $\odot C$ and draw its graph.

44. **PROOF** Write a proof.



GIVEN ▶ A circle passing through the points (−1, 0) and (1, 0)

PROVE ▶ The equation of the circle is $x^2 - 2yk + y^2 = 1$ with center at $(0, k)$.

○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

45. CHALLENGE The intersecting lines m and n are tangent to $\odot C$ at the points (8, 6) and (10, 8), respectively.

a. What is the intersection point of m and n if the radius r of $\odot C$ is 2? What is their intersection point if r is 10? What do you notice about the two intersection points and the center C?

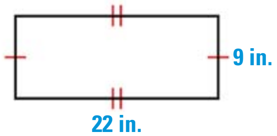
b. Write the equation that describes the locus of intersection points of m and n for all possible values of r.

MIXED REVIEW

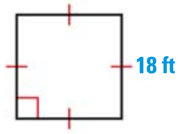
PREVIEW
Prepare for Lesson 11.1 in Exs. 46–48.

Find the perimeter of the figure.

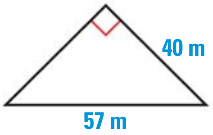
46. *(p. 49)*



47. *(p. 49)*



48. *(p. 433)*



Find the circumference of the circle with given radius *r* or diameter *d*. Use $\pi = 3.14$. *(p. 49)*

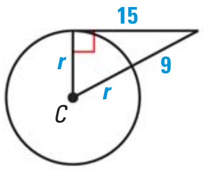
49. $r = 7$ cm

50. $d = 160$ in.

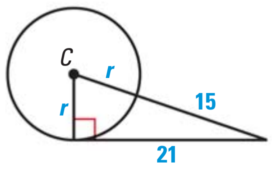
51. $d = 48$ yd

Find the radius *r* of $\odot C$. *(p. 651)*

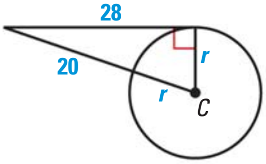
52.



53.



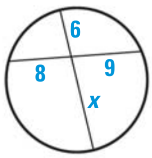
54.



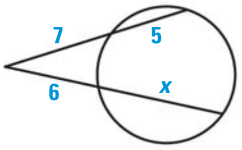
QUIZ *for Lessons 10.6–10.7*

Find the value of *x*. *(p. 689)*

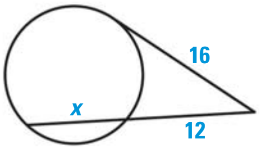
1.



2.



3.



In Exercises 4 and 5, use the given information to write the standard equation of the circle. *(p. 699)*

4. The center is (1, 4), and the radius is 6.

5. The center is (5, −7), and a point on the circle is (5, −3).

6. TIRES The diameter of a certain tire is 24.2 inches. The diameter of the rim in the center is 14 inches. Draw the tire in a coordinate plane with center at (−4, 3). Write the equations for the outer edge of the tire and for the rim where units are measured in inches. *(p. 699)*

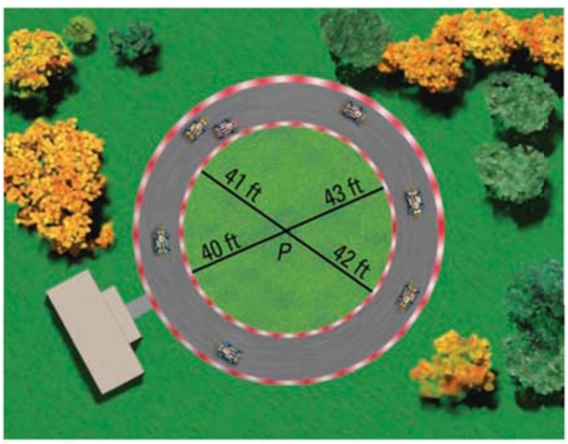
MIXED REVIEW *of Problem Solving*

Lessons 10.6–10.7

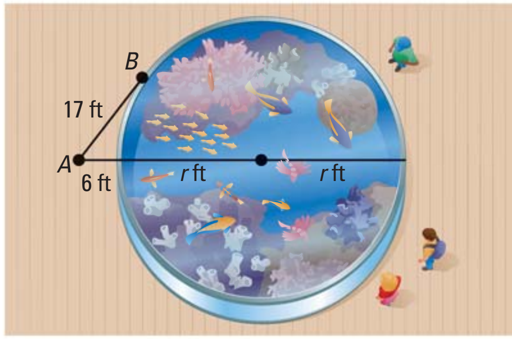
1. **SHORT RESPONSE** A local radio station can broadcast its signal 20 miles. The station is located at the point (20, 30) where units are measured in miles.
   a. Write an inequality that represents the area covered by the radio station.
   b. Determine whether you can receive the radio station's signal when you are located at each of the following points: $E(25, 25)$, $F(10, 10)$, $G(20, 16)$, and $H(35, 30)$.

2. **EXTENDED RESPONSE** Cell phone towers are used to transmit calls. An area has cell phone towers at points (2, 3), (4, 5), and (5, 3) where units are measured in miles. Each tower has a transmission radius of 2 miles.
   a. Draw the area on a graph and locate the three cell phone towers. Are there any areas that can transmit calls using more than one tower?
   b. Suppose you live at (3, 5) and your friend lives at (1, 7). Can you use your cell phone at either or both of your homes?
   c. City A is located at $(-1, 1)$ and City B is located at (4, 7). Each city has a radius of 5 miles. Which city has better coverage from the cell phone towers?

3. **SHORT RESPONSE** You are standing at point P inside a go-kart track. To determine if the track is a circle, you measure the distance to four points on the track, as shown in the diagram. What can you conclude about the shape of the track? *Explain.*



4. **SHORT RESPONSE** You are at point A, about 6 feet from a circular aquarium tank. The distance from you to a point of tangency on the tank is 17 feet.

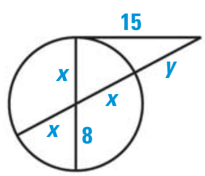


   a. What is the radius of the tank?
   b. Suppose you are standing 4 feet from another aquarium tank that has a diameter of 12 feet. How far, in feet, are you from a point of tangency?

5. **EXTENDED RESPONSE** You are given seismograph readings from three locations.
   - At $A(-2, 3)$, the epicenter is 4 miles away.
   - At $B(5, -1)$, the epicenter is 5 miles away.
   - At $C(2, 5)$, the epicenter is 2 miles away.

   a. Graph circles centered at A, B, and C with radii of 4, 5, and 2 miles, respectively.
   b. Locate the epicenter.
   c. The earthquake could be felt up to 12 miles away. If you live at (14, 16), could you feel the earthquake? *Explain.*

6. **MULTI-STEP PROBLEM** Use the diagram.



   a. Use Theorem 10.16 and the quadratic formula to write an equation for y in terms of x.
   b. Find the value of x.
   c. Find the value of y.

10 CHAPTER SUMMARY

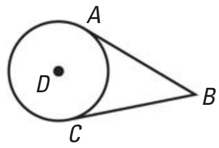
BIG IDEAS — For Your Notebook

Big Idea 1

Using Properties of Segments that Intersect Circles

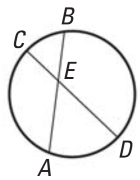
You learned several relationships between tangents, secants, and chords.

Some of these relationships can help you determine that two chords or tangents are congruent. For example, tangent segments from the same exterior point are congruent.



$\overline{AB} \cong \overline{CB}$

Other relationships allow you to find the length of a secant or chord if you know the length of related segments. For example, with the Segments of a Chord Theorem you can find the length of an unknown chord segment.



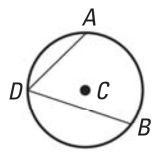
$EA \cdot EB = EC \cdot ED$

Big Idea 2

Applying Angle Relationships in Circles

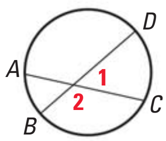
You learned to find the measures of angles formed inside, outside, and on circles.

Angles formed on circles



$m\angle ADB = \frac{1}{2}m\overset{\frown}{AB}$

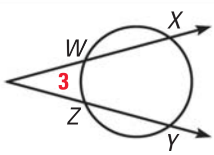
Angles formed inside circles



$m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{AB} + m\overset{\frown}{CD}\right),$

$m\angle 2 = \frac{1}{2}\left(m\overset{\frown}{AD} + m\overset{\frown}{BC}\right)$

Angles formed outside circles



$m\angle 3 = \frac{1}{2}\left(m\overset{\frown}{XY} - m\overset{\frown}{WZ}\right)$

Big Idea 3

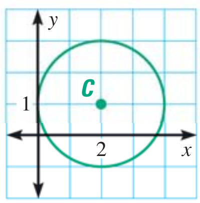
Using Circles in the Coordinate Plane

The standard equation of $\odot C$ is:

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 2)^2 + (y - 1)^2 = 2^2$

$(x - 2)^2 + (y - 1)^2 = 4$



10 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- circle, *p. 651*
  center, radius, diameter
- chord, *p. 651*
- secant, *p. 651*
- tangent, *p. 651*
- central angle, *p. 659*
- minor arc, *p. 659*
- major arc, *p. 659*
- semicircle, *p. 659*
- measure of a minor arc, *p. 659*
- measure of a major arc, *p. 659*
- congruent circles, *p. 660*
- congruent arcs, *p. 660*
- inscribed angle, *p. 672*
- intercepted arc, *p. 672*
- inscribed polygon, *p. 674*
- circumscribed circle, *p. 674*
- segments of a chord, *p. 689*
- secant segment, *p. 690*
- external segment, *p. 690*
- standard equation of a circle, *p. 699*

VOCABULARY EXERCISES

1. Copy and complete: If a chord passes through the center of a circle, then it is called a(n) __?__.

2. Draw and *describe* an inscribed angle and an intercepted arc.

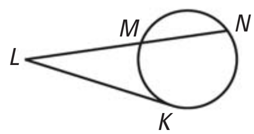
3. **WRITING** *Describe* how the measure of a central angle of a circle relates to the measure of the minor arc and the measure of the major arc created by the angle.

In Exercises 4–6, match the term with the appropriate segment.

4. Tangent segment **A.** $\overline{LM}$

5. Secant segment **B.** $\overline{KL}$

6. External segment **C.** $\overline{LN}$



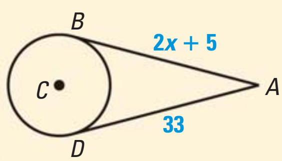
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 10.

10.1 Use Properties of Tangents *pp. 651–658*

EXAMPLE

In the diagram, *B* and *D* are points of tangency on $\odot C$. Find the value of *x*.



Use Theorem 10.2 to find *x*.

$AB = AD$	Tangent segments from the same point are $\cong$.
$2x + 5 = 33$	Substitute.
$x = 14$	Solve for *x*.

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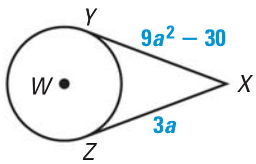
Chapter Review Practice

EXAMPLES 5 and 6 on p. 654 for Exs. 7–9

EXERCISES

Find the value of the variable. Y and Z are points of tangency on $\odot W$.

7.



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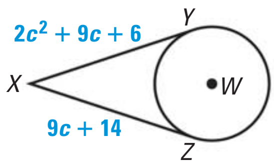
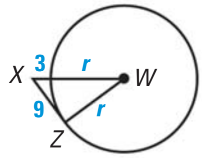


9.

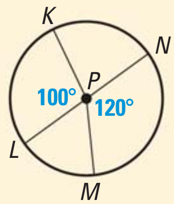


10.2 Find Arc Measures

pp. 659–663

EXAMPLE

Find the measure of the arc of $\odot P$. In the diagram, $\overline{LN}$ is a diameter.



a. $\widehat{MN}$ **b.** $\widehat{NLM}$ **c.** $\widehat{NML}$

a. $\widehat{MN}$ is a minor arc, so $m\widehat{MN} = m\angle MPN = 120°$.

b. $\widehat{NLM}$ is a major arc, so $m\widehat{NLM} = 360° - 120° = 240°$.

c. $\widehat{NML}$ is a semicircle, so $m\widehat{NML} = 180°$.

EXAMPLES 1 and 2 on pp. 659–660 for Exs. 10–13

EXERCISES

Use the diagram above to find the measure of the indicated arc.

10. $\widehat{KL}$ **11.** $\widehat{LM}$ **12.** $\widehat{KM}$ **13.** $\widehat{KN}$

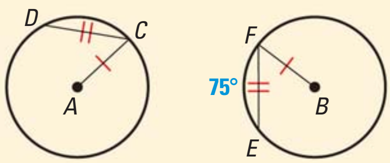
10.3 Apply Properties of Chords

pp. 664–670

EXAMPLE

In the diagram, $\odot A \cong \odot B$, $\overline{CD} \cong \overline{FE}$, and $m\widehat{FE} = 75°$. Find $m\widehat{CD}$.

By Theorem 10.3, $\overline{CD}$ and $\overline{FE}$ are congruent chords in congruent circles, so the corresponding minor arcs $\widehat{FE}$ and $\widehat{CD}$ are congruent. So, $m\widehat{CD} = m\widehat{FE} = 75°$.

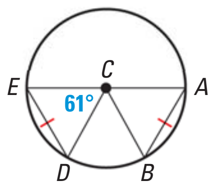


EXAMPLES 1, 3, and 4 on pp. 664, 666 for Exs. 14–16

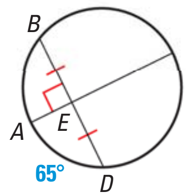
EXERCISES

Find the measure of $\widehat{AB}$.

14.



15.



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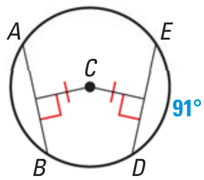


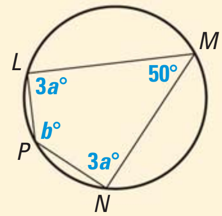
10 CHAPTER REVIEW

10.4 Use Inscribed Angles and Polygons pp. 672–679

EXAMPLE

Find the value of each variable.

LMNP is inscribed in a circle, so by Theorem 10.10, opposite angles are supplementary.



$$m\angle L + m\angle N = 180^\circ \qquad m\angle P + m\angle M = 180^\circ$$

$$3a^\circ + 3a^\circ = 180^\circ \qquad b^\circ + 50^\circ = 180^\circ$$

$$6a = 180 \qquad b = 130$$

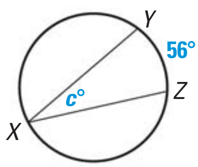
$$a = 30$$

EXERCISES

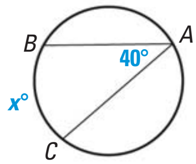
EXAMPLES 1, 2, and 5 on pp. 672–675 for Exs. 17–19

Find the value(s) of the variable(s).

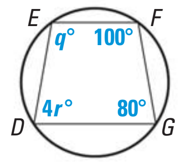
17.



18.



19.

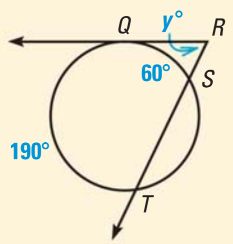


10.5 Apply Other Angle Relationships in Circles pp. 680–686

EXAMPLE

Find the value of *y*.

The tangent $\overrightarrow{RQ}$ and secant $\overrightarrow{RT}$ intersect outside the circle, so you can use Theorem 10.13 to find the value of *y*.



$y^\circ = \frac{1}{2}(m\overset{\frown}{QT} - m\overset{\frown}{SQ})$ **Use Theorem 10.13.**

$y^\circ = \frac{1}{2}(190^\circ - 60^\circ)$ **Substitute.**

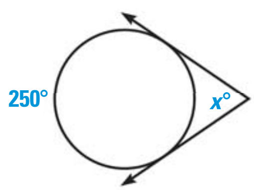
$y = 65$ **Simplify.**

EXERCISES

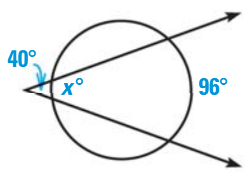
EXAMPLES 2 and 3 on pp. 681–682 for Exs. 20–22

Find the value of *x*.

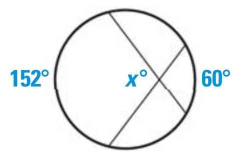
20.



21.



22.



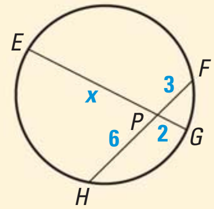
10.6 Find Segment Lengths in Circles

pp. 689–695

EXAMPLE

Find the value of x.

The chords $\overline{EG}$ and $\overline{FH}$ intersect inside the circle, so you can use Theorem 10.14 to find the value of x.



$EP \cdot PG = FP \cdot PH$ **Use Theorem 10.14.**

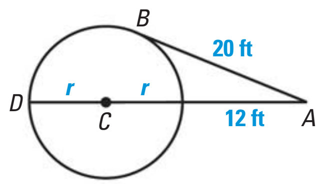
$x \cdot 2 = 3 \cdot 6$ **Substitute.**

$x = 9$ **Solve for x.**

EXERCISE

EXAMPLE 4 on p. 692 for Ex. 23

23. SKATING RINK A local park has a circular ice skating rink. You are standing at point A, about 12 feet from the edge of the rink. The distance from you to a point of tangency on the rink is about 20 feet. Estimate the radius of the rink.



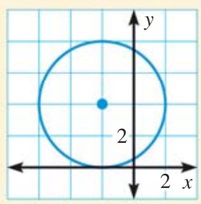
10.7 Write and Graph Equations of Circles

pp. 699–705

EXAMPLE

Write an equation of the circle shown.

The radius is 2 and the center is at $(-2, 4)$.



$(x - h)^2 + (y - k)^2 = r^2$ **Standard equation of a circle**

$(x - (-2))^2 + (y - 4)^2 = 4^2$ **Substitute.**

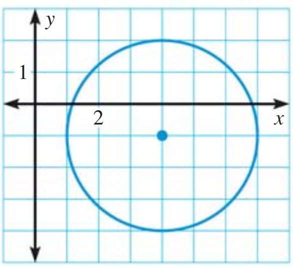
$(x + 2)^2 + (y - 4)^2 = 16$ **Simplify.**

EXERCISES

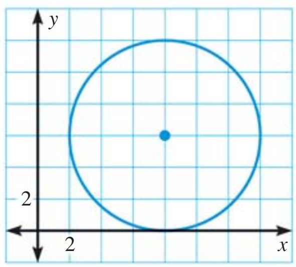
EXAMPLES 1, 2, and 3 on pp. 699–700 for Exs. 24–32

Write an equation of the circle shown.

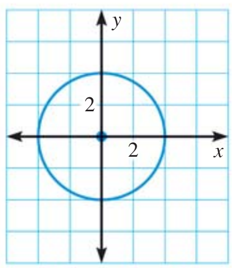
24.



25.



26.



Write the standard equation of the circle with the given center and radius.

27. Center $(0, 0)$, radius 9

28. Center $(-5, 2)$, radius 1.3

29. Center $(6, 21)$, radius 4

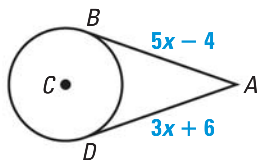
30. Center $(-3, 2)$, radius 16

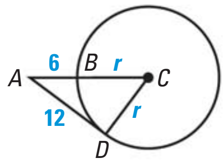
31. Center $(10, 7)$, radius 3.5

32. Center $(0, 0)$, radius 5.2

10 CHAPTER TEST

In $\odot C$, B and D are points of tangency. Find the value of the variable.

1.


2.


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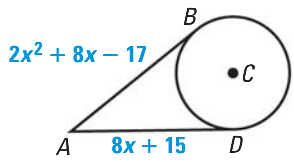
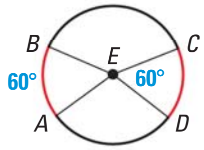


Tell whether the red arcs are congruent. *Explain* why or why not.

4.


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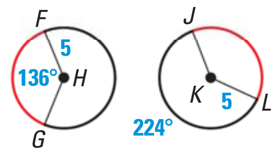
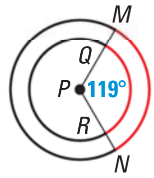
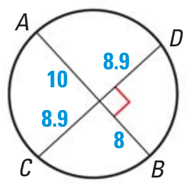
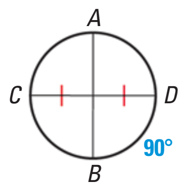
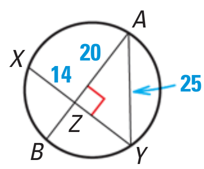


6.


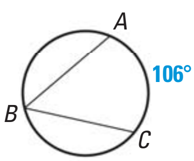
Determine whether $\overline{AB}$ is a diameter of the circle. *Explain* your reasoning.

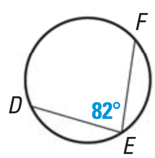
7.


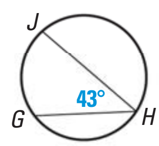
8.


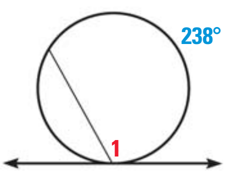
9.


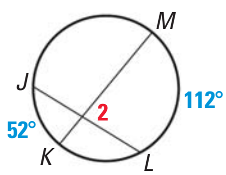
Find the indicated measure.

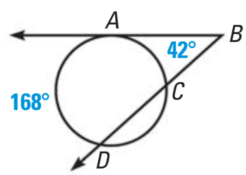
10. $m\angle ABC$


11. $m\overparen{DF}$


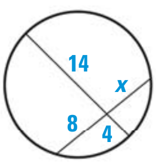
12. $m\overparen{GHJ}$


13. $m\angle 1$


14. $m\angle 2$


15. $m\overparen{AC}$


Find the value of x. Round decimal answers to the nearest tenth.

16.


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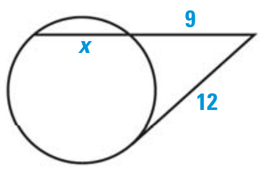

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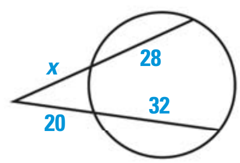


19. Find the center and radius of a circle that has the standard equation $(x + 2)^2 + (y - 5)^2 = 169$.

10 ALGEBRA REVIEW

Animated Algebra
classzone.com

FACTOR BINOMIALS AND TRINOMIALS

EXAMPLE 1 Factor using greatest common factor

Factor $2x^3 + 6x^2$.

Identify the *greatest common factor* of the terms. The greatest common factor (GCF) is the product of all the common factors.

First, factor each term. $2x^3 = 2 \cdot x \cdot x \cdot x$ and $6x^2 = 2 \cdot 3 \cdot x \cdot x$

Then, write the product of the common terms. $\text{GCF} = 2 \cdot x \cdot x = 2x^2$

Finally, use the distributive property with the GCF. $2x^3 + 6x^2 = 2x^2(x + 3)$

EXAMPLE 2 Factor binomials and trinomials

Factor.

a. $2x^2 - 5x + 3$

b. $x^2 - 9$

Solution

a. Make a table of possible factorizations. Because the middle term, $-5x$, is negative, both factors of the third term, 3, must be negative.

Factors of 2	Factors of 3	Possible factorization	Middle term when multiplied	
1, 2	−3, −1	$(x - 3)(2x - 1)$	$-x - 6x = -7x$	✗
1, 2	−1, −3	$(x - 1)(2x - 3)$	$-3x - 2x = -5x$	← Correct

b. Use the special factoring pattern $a^2 - b^2 = (a + b)(a - b)$.

$x^2 - 9 = x^2 - 3^2$ **Write in the form $a^2 - b^2$.**

$= (x + 3)(x - 3)$ **Factor using the pattern.**

EXERCISES

EXAMPLE 1 for Exs. 1–9

EXAMPLE 2 for Exs. 10–24

Factor.

1. $6x^2 + 18x^4$ **2.** $16a^2 - 24b$ **3.** $9r^2 - 15rs$

4. $14x^5 + 27x^3$ **5.** $8t^4 + 6t^2 - 10t$ **6.** $9z^3 + 3z + 21z^2$

7. $5y^6 - 4y^5 + 2y^3$ **8.** $30v^7 - 25v^5 - 10v^4$ **9.** $6x^3y + 15x^2y^3$

10. $x^2 + 6x + 8$ **11.** $y^2 - y - 6$ **12.** $a^2 - 64$

13. $z^2 - 8z + 16$ **14.** $3s^2 + 2s - 1$ **15.** $5b^2 - 16b + 3$

16. $4x^4 - 49$ **17.** $25r^2 - 81$ **18.** $4x^2 + 12x + 9$

19. $x^2 + 10x + 21$ **20.** $z^2 - 121$ **21.** $y^2 + y - 6$

22. $z^2 + 12z + 36$ **23.** $x^2 - 49$ **24.** $2x^2 - 12x - 14$

10 ★ Standardized TEST PREPARATION

MULTIPLE CHOICE QUESTIONS

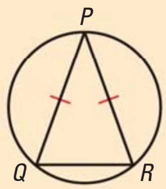
If you have difficulty solving a multiple choice question directly, you may be able to use another approach to eliminate incorrect answer choices and obtain the correct answer.

PROBLEM 1

In the diagram, $\triangle PQR$ is inscribed in a circle. The ratio of the angle measures of $\triangle PQR$ is 4 : 7 : 7. What is $m\overset{\frown}{QR}$?

Ⓐ 20° Ⓑ 40°

Ⓒ 80° Ⓓ 140°



METHOD 1

SOLVE DIRECTLY Use the Interior Angles Theorem to find $m\angle QPR$. Then use the fact that $\angle QPR$ intercepts $\overset{\frown}{QR}$ to find $m\overset{\frown}{QR}$.

STEP 1 **Use** the ratio of the angle measures to write an equation. Because $\triangle EFG$ is isosceles, its base angles are congruent. Let $4x° = m\angle QPR$. Then $m\angle Q = m\angle R = 7x°$. You can write:

$$m\angle QPR + m\angle Q + m\angle R = 180°$$

$$4x° + 7x° + 7x° = 180°$$

STEP 2 **Solve** the equation to find the value of x.

$$4x° + 7x° + 7x° = 180°$$

$$18x° = 180°$$

$$x = 10$$

STEP 3 **Find** $m\angle QPR$. From Step 1, $m\angle QPR = 4x°$, so $m\angle QPR = 4 \cdot 10° = 40°$.

STEP 4 **Find** $m\overset{\frown}{QR}$. Because $\angle QPR$ intercepts $\overset{\frown}{QR}$, $m\overset{\frown}{QR} = 2 \cdot m\angle QPR$. So, $m\overset{\frown}{QR} = 2 \cdot 40° = 80°$.

The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

METHOD 2

ELIMINATE CHOICES Because $\angle QPR$ intercepts $\overset{\frown}{QR}$, $m\angle QPR = \frac{1}{2} \cdot m\overset{\frown}{QR}$. Also, because $\triangle PQR$ is isosceles, its base angles, $\angle Q$ and $\angle R$, are congruent. For each choice, find $m\angle QPR$, $m\angle Q$, and $m\angle R$. Determine whether the ratio of the angle measures is 4 : 7 : 7.

Choice A: If $m\overset{\frown}{QR} = 20°$, $m\angle QPR = 10°$. So, $m\angle Q + m\angle R = 180° - 10° = 170°$, and $m\angle Q = m\angle R = \frac{170}{2} = 85°$. The angle measures 10°, 85°, and 85° are not in the ratio 4 : 7 : 7, so Choice A is not correct.

Choice B: If $m\overset{\frown}{QR} = 40°$, $m\angle QPR = 20°$. So, $m\angle Q + m\angle R = 180° - 20° = 160°$, and $m\angle Q = m\angle R = 80°$. The angle measures 20°, 80°, and 80° are not in the ratio 4 : 7 : 7, so Choice B is not correct.

Choice C: If $m\overset{\frown}{QR} = 80°$, $m\angle QPR = 40°$. So, $m\angle Q + m\angle R = 180° - 40° = 140°$, and $m\angle Q = m\angle R = 70°$. The angle measures 40°, 70°, and 70° are in the ratio 4 : 7 : 7. So, $m\overset{\frown}{QR} = 80°$.

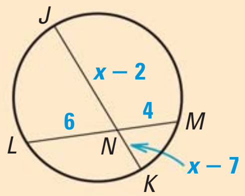
The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

Problem 2

In the circle shown, $\overline{JK}$ intersects $\overline{LM}$ at point N. What is the value of x?

Ⓐ -1 Ⓑ 2

Ⓒ 7 Ⓓ 10



Method 1

SOLVE DIRECTLY Write and solve an equation.

STEP 1 **Write** an equation. By the Segments of a Chord Theorem, $NJ \cdot NK = NL \cdot NM$. You can write $(x - 2)(x - 7) = 6 \cdot 4 = 24$.

STEP 2 **Solve** the equation.

$$(x - 2)(x - 7) = 24$$
$$x^2 - 9x + 14 = 24$$
$$x^2 - 9x - 10 = 0$$
$$(x - 10)(x + 1) = 0$$

So, $x = 10$ or $x = -1$.

STEP 3 **Decide** which value makes sense.
If $x = -1$, then $NJ = -1 - 2 = -3$. But a distance cannot be negative.
If $x = 10$, then $NJ = 10 - 2 = 8$, and $NK = 10 - 7 = 3$. So, $x = 10$.

The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

Method 2

ELIMINATE CHOICES Check to see if any choices do not make sense.

STEP 1 **Check** to see if any choices give impossible values for NJ and NK. Use the fact that $NJ = x - 2$ and $NK = x - 7$.

Choice A: If $x = -1$, then $NJ = -3$ and $NK = -8$. A distance cannot be negative, so you can eliminate Choice A.

Choice B: If $x = 2$, then $NJ = 0$ and $NK = -5$. A distance cannot be negative or 0, so you can eliminate Choice B.

Choice C: If $x = 7$, then $NJ = 5$ and $NK = 0$. A distance cannot be 0, so you can eliminate Choice C.

STEP 2 **Verify** that Choice D is correct. By the Segments of a Chord Theorem, $(x - 7)(x - 2) = 6(4)$. This equation is true when $x = 10$.

The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

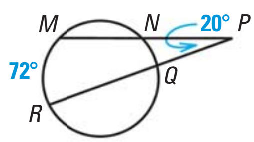
EXERCISES

Explain **why you can eliminate the highlighted answer choice.**

1. In the diagram, what is $m\overset{\frown}{NQ}$?

Ⓐ ✗ 20° Ⓑ 26°

Ⓒ 40° Ⓓ 52°



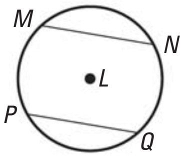
2. Isosceles trapezoid $EFGH$ is inscribed in a circle, $m\angle E = (x + 8)°$, and $m\angle G = (3x + 12)°$. What is the value of x?

Ⓐ ✗ Ⓑ 10 Ⓒ 40 Ⓓ 72

10 ★ Standardized TEST PRACTICE

MULTIPLE CHOICE

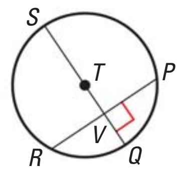
1. In $\odot L$, $\overline{MN} \cong \overline{PQ}$. Which statement is not necessarily true?



Ⓐ $\widehat{MN} \cong \widehat{PQ}$ Ⓑ $\widehat{NQP} \cong \widehat{QNM}$

Ⓒ $\widehat{MP} \cong \widehat{NQ}$ Ⓓ $\widehat{MPQ} \cong \widehat{NMP}$

2. In $\odot T$, $PV = 5x - 2$ and $PR = 4x + 14$. What is the value of x?



Ⓐ -10 Ⓑ 3

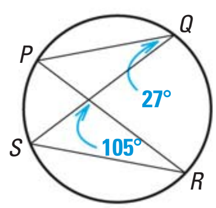
Ⓒ 12 Ⓓ 16

3. What are the coordinates of the center of a circle with equation $(x + 2)^2 + (y - 4)^2 = 9$?

Ⓐ $(-2, -4)$ Ⓑ $(-2, 4)$

Ⓒ $(2, -4)$ Ⓓ $(2, 4)$

4. In the circle shown below, what is $m\widehat{QR}$?



Ⓐ 24° Ⓑ 27°

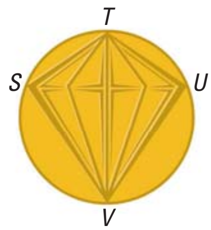
Ⓒ 48° Ⓓ 96°

5. Regular hexagon $FGHJKL$ is inscribed in a circle. What is $m\widehat{KL}$?

Ⓐ 6° Ⓑ 60°

Ⓒ 120° Ⓓ 240°

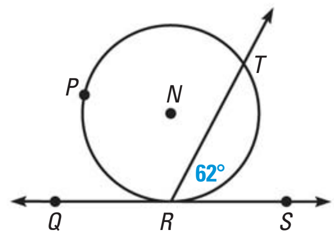
6. In the design for a jewelry store sign, $STUV$ is inscribed inside a circle, $ST = TU =$ 12 inches, and $SV = UV =$ 18 inches. What is the approximate diameter of the circle?



Ⓐ 17 in. Ⓑ 22 in.

Ⓒ 25 in. Ⓓ 30 in.

7. In the diagram shown, $\overleftrightarrow{QS}$ is tangent to $\odot N$ at R. What is $m\widehat{RPT}$?



Ⓐ 62° Ⓑ 118°

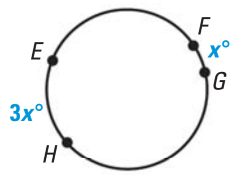
Ⓒ 124° Ⓓ 236°

8. Two distinct circles intersect. What is the maximum number of common tangents?

Ⓐ 1 Ⓑ 2

Ⓒ 3 Ⓓ 4

9. In the circle shown, $m\widehat{EFG} = 146°$ and $m\widehat{FGH} = 172°$. What is the value of x?

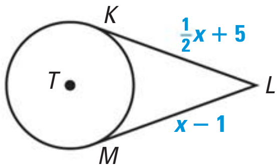


Ⓐ 10.5 Ⓑ 21

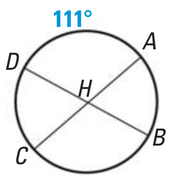
Ⓒ 42 Ⓓ 336

GRIDDED ANSWER

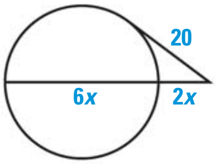
10. $\overline{LK}$ is tangent to $\odot T$ at K. $\overline{LM}$ is tangent to $\odot T$ at M. Find the value of x.



11. In $\odot H$, find $m\angle AHB$ in degrees.

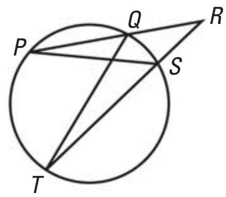


12. Find the value of x.



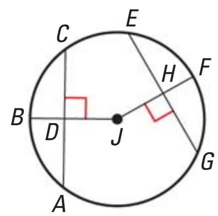
SHORT RESPONSE

13. *Explain* why $\triangle PSR$ is similar to $\triangle TQR$.

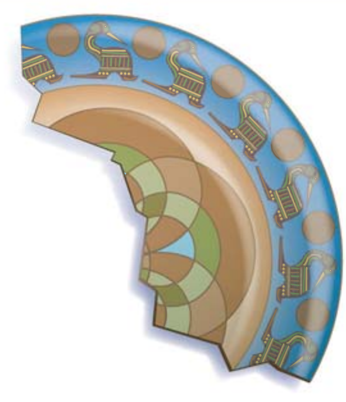


14. Let $x°$ be the measure of an inscribed angle, and let $y°$ be the measure of its intercepted arc. Graph y as a function of x for all possible values of x. Give the slope of the graph.

15. In $\odot J$, $\overline{JD} \cong \overline{JH}$. Write two true statements about congruent arcs and two true statements about congruent segments in $\odot J$. *Justify* each statement.



EXTENDED RESPONSE



16. The diagram shows a piece of broken pottery found by an archaeologist. The archaeologist thinks that the pottery is part of a circular plate and wants to estimate the diameter of the plate.

a. Trace the outermost arc of the diagram on a piece of paper. Draw any two chords whose endpoints lie on the arc.

b. Construct the perpendicular bisector of each chord. Mark the point of intersection of the perpendiculars bisectors. How is this point related to the circular plate?

c. Based on your results, *describe* a method the archaeologist could use to estimate the diameter of the actual plate. *Explain* your reasoning.

17. The point $P(3, -8)$ lies on a circle with center $C(-2, 4)$.

a. Write an equation for $\odot C$.

b. Write an equation for the line that contains radius $\overline{CP}$. *Explain*.

c. Write an equation for the line that is tangent to $\odot C$ at point P. *Explain*.