Vocabulary for Working with Trigonometric Graphs

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A **sine wave**, or **sinusoid**, is the graph of the sine function in trigonometry. In addition to mathematics, this function also occurs in other fields of study such as science and engineering.   This wave pattern also occurs in nature as seen in ocean waves, sound waves and light waves. Even average daily temperatures for each day of the year resemble this wave. The term sinusoid was first use by Scotsman Stuart Kenny in 1789 while observing the growth and harvest of soybeans.

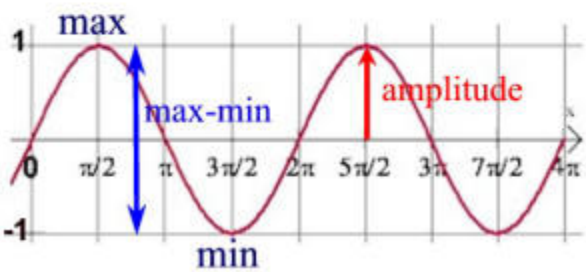
Let's see what vocabulary is needed to discuss sinusoids and other trigonometric graphs.

amplitude:

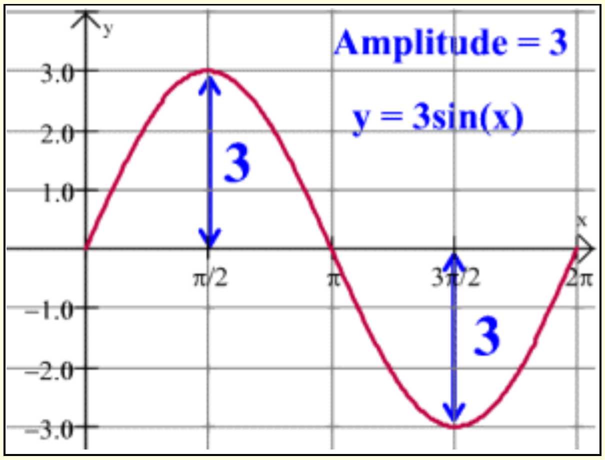
The **amplitude** of a sinusoidal function is one-half of the positive difference between the maximum and minimum values of a function.

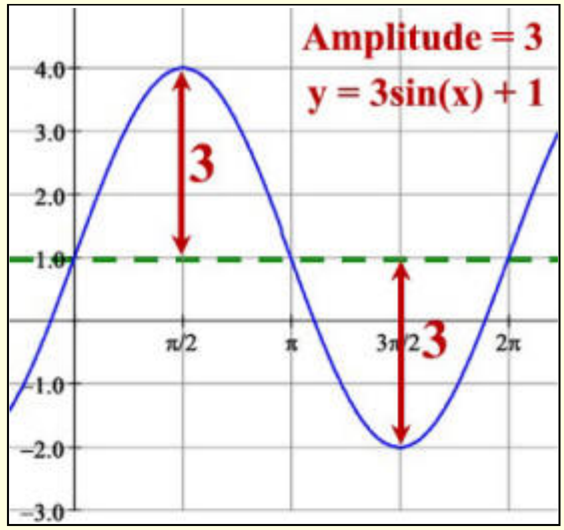
$$\text{amplitude} = \frac{1}{2}\left|\max - \min\right|$$

Amplitude is the magnitude (height) of the oscillation (wave) of a sinusoidal function. Sometimes it is referred to as the "peak from center" of the graph.



The maximum of ***y* = sin *x*** is 1. The minimum is -1. The amplitude is 1/2(1 - (-1)) = 1.

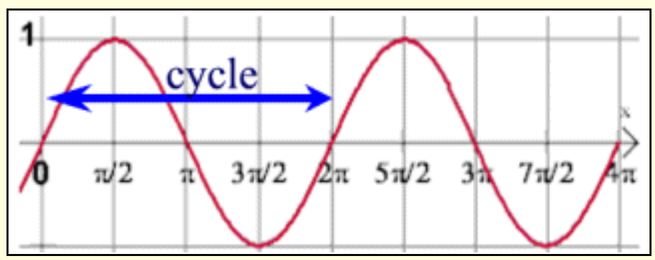




Note: while vertical shifts alter the maximum and minimum values of a function, they do not alter the amplitude. Also horizontal shifts do not affect the amplitude.

period:

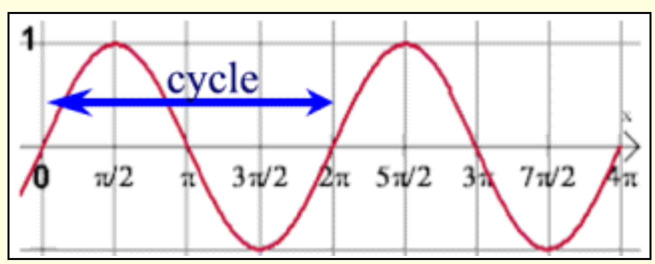
A **periodic function** is an oscillating (wave-like)function which repeats a pattern of *y*-values at regular intervals. One complete repetition of the pattern is called a **cycle**. The **period** of a function is the horizontal length of one complete cycle.



This sine curve, ***y* = sin *x*,** has a period of 2π, the horizontal length of one complete cycle.

frequency:

The **frequency** of a trigonometric function is the number of cycles it completes in a given interval. This interval is generally 2π radians (or 360º) for the sine and cosine curves**.**



This sine curve, ***y* = sin *x*,** completes 1 cycle in the interval from 0 to 2π radians. Its frequency is 1 in the interval 2π.

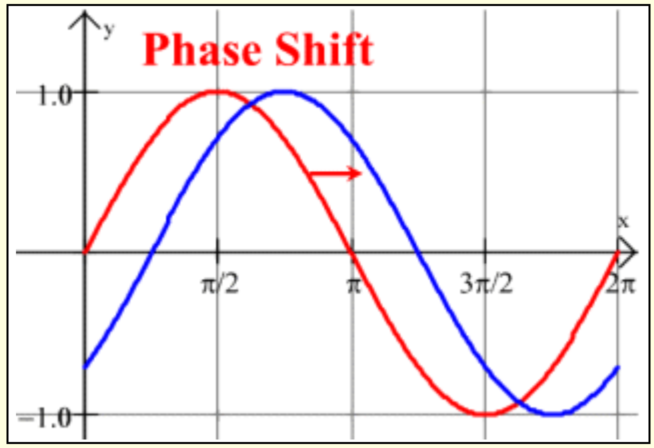
horizontal shift:

From the sinusoidal equation,

$$y = A\sin\left(B(x - C)\right) + D$$

the horizontal shift is obtained by determining the change being made to the *x* value. The horizontal shift is *C*. If the value of *B* is 1, the horizontal shift can also be called a **phase shift.** Remember that the expression (*x* - *C*) from the equation will look like (for example):

- (*x* - 2) where 2 is a positive value being subtracted, when the shift is to the right.
- (*x* + 2) where 2 is a negative value being subtracted, when the shift is to the left.

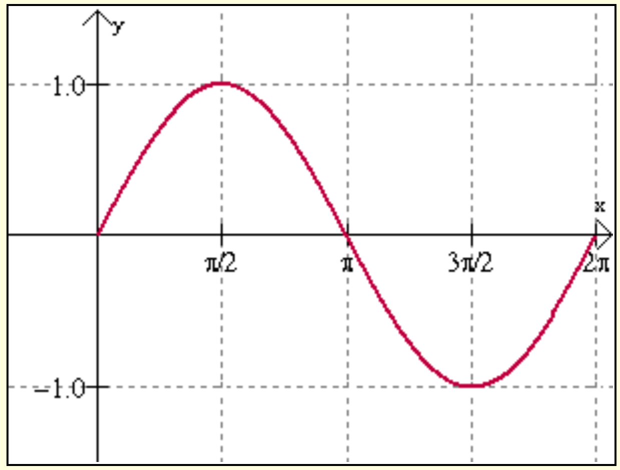


$y = \sin(x)$ $\qquad y = \sin\left(x - \frac{\pi}{4}\right)$

The sine and cosine functions take on y-values between -1 and 1.

Sine Function: $y = \sin x$

$y = \sin x$

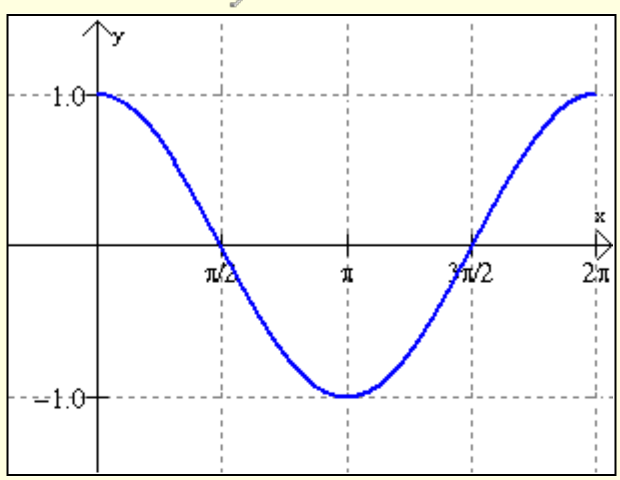


- called a "wave" because of its rolling wave-like appearance (oscillating)
- amplitude: 1 (height)
- period: 2π **or** 360° (length of one cycle)
- frequency: 1 cycle in 2π radians [or $1/(2\pi)$]
- domain: $\{x \mid x \in \mathbb{R}\}$
- range: $\{y \mid -1 \leq y \leq 1\}$

At $x = 0$, the sine wave is on the shoreline!
(meaning the y-value is equal to zero)

Cosine Function: $y = \cos x$

$y = \cos x$



- called a "wave" because of its rolling wave-like appearance
- amplitude: 1
- period: 2π
- frequency: 1 cycle in 2π radians [or $1/(2\pi)$]
- domain: $\{x \mid x \in \mathbb{R}\}$
- range: $\{y \mid -1 \leq y \leq 1\}$

At $x = 0$, the cosine wave breaks off the cliff!
(meaning the y-value is equal to one)

What affect does the value A have on the graph?

$y = A \sin(Bx)$ or $y = A \cos(Bx)$

A number in front of sine or cosine changes the height of the graph.

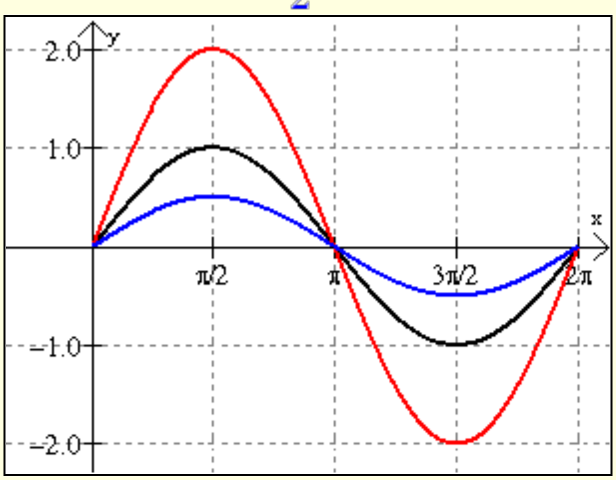
The value A affects the amplitude. The amplitude (half the distance between the maximum and minimum values of the function) will be $|A|$, since distance is always positive.

Increasing or decreasing the value of A will **vertically stretch** or **shrink** the graph. Consider these examples:

$y = \sin x$

$y = 2\sin x$

$y = \frac{1}{2}\sin x$



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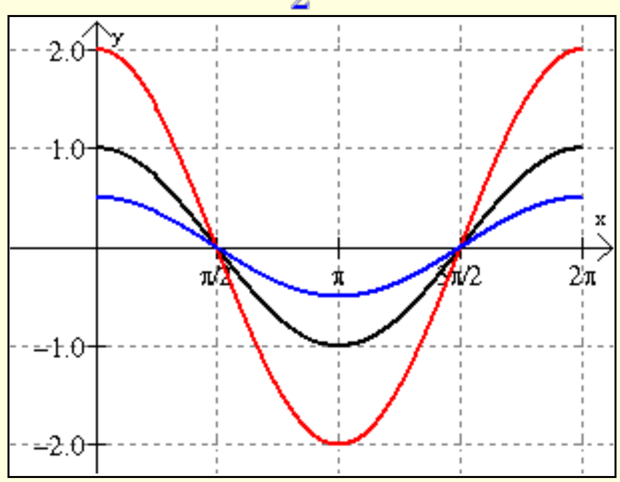


Notice: These graphs change "height" but do not change horizontal width. The graphs are still drawn from 0 to 2π.

What affect does the value B have on the graph?
$y = A\sin(Bx)$ or $y = A\cos(Bx)$

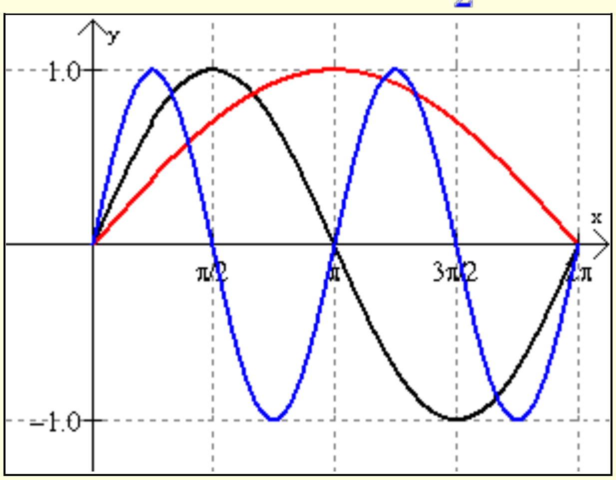
B is the number of cycles you see in a 0 to 2π (360º) interval.

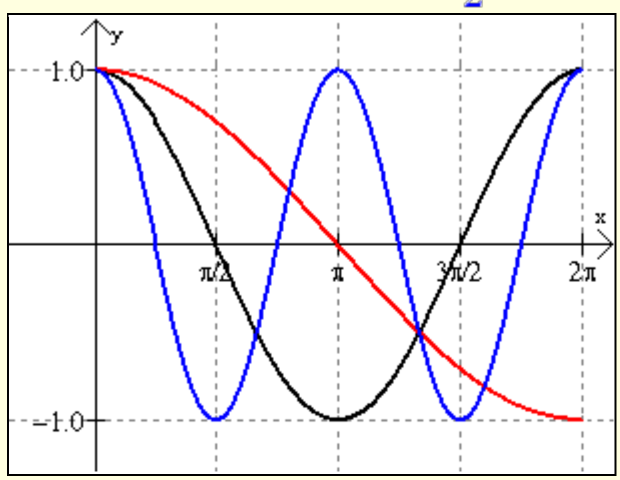
The value ***B*** is the number of cycles it completes in an interval of from 0 to 2π or 360º**.** The value***B*** affects the period. The period of sine and cosine is $\left|\frac{2\pi}{B}\right|$. When 0 < ***B*** < 1, the period of the function will be greater than 2π and the graph will be a **horizontal stretching.** When ***B*** > 1, the period of the function will be less than 2π and the graph will be a **horizontal shrinking.** Consider these examples:

$$y = \sin x;\ \text{period} = \frac{2\pi}{1} = 2\pi \qquad y = \cos x;\ \text{period} = \frac{2\pi}{1} = 2\pi$$

$$y = \sin\left(\frac{1}{2}x\right);\ \text{period} = \frac{2\pi}{1/2} = 4\pi \qquad y = \cos\left(\frac{1}{2}x\right);\ \text{period} = \frac{2\pi}{1/2} = 4\pi$$

$$y = \sin(2x);\ \text{period} = \frac{2\pi}{2} = \pi \qquad y = \cos(2x);\ \text{period} = \frac{2\pi}{2} = \pi$$





Notice: These graphs change horizontal "width" but do not change height. The two red graphs only show us a portion (in this example "half") of the original graphs in their 0 to 2π windows.