Do all work neatly on a separate sheet of paper.

Translate the verbal sentence into an equation or an inequality. Then use the information to solve for the variable(s).

1. The quotient of $m$ and 7 is greater than or equal to 16.
2. The sum of 4 and the second power of $b$ is equal to 104.
3. The distance $t$ you travel by train is $\frac{2}{3}$ times the distance $d$ you live from the train station. You travel 3 miles to get from your house to the train station.

Evaluate the expression for the given value of $x$.

4. $3 + x + (-4)$, $x = 5$  
5. $-x + 12 - 5$, $x = 9$  
6. $3.5 - (-x)$, $x = 1.5$
7. $(-3)^2(x)$, $x = 7$  
8. $6x(x + 2)$, $x = 2$  
9. $(8x + 1)(-3)$, $x = \frac{1}{2}$
10. $\frac{1}{4}(-x)(-x)(-x)$, $x = 4$  
11. $\frac{x^2 + 4}{6}$, $x = 8$  
12. $(-5)\left(-\frac{3}{4}x\right)$, $x = 6$

Solve the equation. Round the result to the nearest hundredth.

13. $\frac{2}{9}(x - 5) = 12$  
14. $7x - (3x - 2) = 38$  
15. $\frac{1}{3}x + 7 = -7x - 5$
16. $8(x + 3) - 2x = 4(x - 8)$  
17. $11.47 + 6.23x = 7.62 + 5.51x$  
18. $-3(2.98 - 4.1x) = 9.2x + 6.25$

In Exercises 19 and 20, use the graph to the right.

19. Write an equation of a line that is parallel to the line shown.
20. Write an equation of a line that is perpendicular to the line shown.

Graph both lines you created above in the same plane as the original to check your answer.

Decide whether the relation is a function. Then indicate the domain and range of each relation.

21. | Input | Output |
   | -1    | -1    |
   | 1     | -1    |
   | 3     | 1     |
   | 5     | 3     |
   | 7     | 5     |

22. Use the information provided to write an equation of the line in standard form. Then graph each on a separate plane.

25. The slope is $\frac{4}{5}$; the $y$-intercept is -3
26. The line passes through (-1, 2); the slope is $\frac{1}{3}$

Solve the inequality and graph the solution.

27. $-3 < -4x + 9 \leq 14$
28. $|3x + 16| + 2 < 10$
29. $3x - 4 > 5$ or $5x + 1 < 11$
Solve each system of linear equations. Then graph each system on a separate coordinate plane.

30. \[ \begin{align*}
4y &= -8x + 16 \\
2y &= 11x - 7
\end{align*} \]
31. \[ \begin{align*}
-2x + 3y &= 15 \\
10x - 11y &= 9
\end{align*} \]
32. \[ \begin{align*}
y &= 5x - 2 \\
3x + 7y &= 5
\end{align*} \]

Simplify. Then evaluate the expression when \(a=1\) and \(b=2\).

33. \[ \frac{b^8}{b^2} \]
34. \[ 3a^4 \cdot a^{-3} \]
35. \[ (-a^3)(2b^2)^3 \]
36. \[ 4b^3 \cdot (2 + b)^2 \]
37. \[ \frac{4a^{-3}b^3}{ab^{-2}} \]
38. \[ \frac{(5ab^2)^{-2}}{a^{-3}b} \]

Decide how many solutions the equation has. Then solve the equation.

39. \[ 6x^2 + 8 = 34 \]
40. \[ 4x^2 - 9x + 5 = 0 \]
41. \[ 3x^2 + 6x + 3 = 0 \]

Completely factor the expression.

42. \[ x^2 + 6x + 8 \]
43. \[ x^2 - 24x - 112 \]
44. \[ 3x^2 + 17x - 6 \]
45. \[ 4x^2 + 12x + 9 \]
46. \[ x^2 + 10x + 25 \]
47. \[ x^2 - 14x + 49 \]

Solve the equation.

48. \[ (3x + 1)(2x + 7) = 0 \]
49. \[ 6x^2 - x - 7 = 8 \]
50. \[ 4x^2 + 16x + 16 = 0 \]
51. \[ x^3 + 5x^2 - 4x - 20 = 0 \]
52. \[ x^4 + 9x^3 + 18x^2 = 0 \]
53. \[ x^2 - \frac{4}{3}x + \frac{4}{9} = 0 \]

Simplify the expression.

54. \[ \frac{4x}{12x^2} \]
55. \[ \frac{2x + 6}{x^2 - 9} \]
56. \[ \frac{3x}{x^2 - 2x - 24} \cdot \frac{x - 6}{6x^2 + 9x} \]
57. \[ \frac{x^2 - 6x + 8}{x^2 - 2x} \div (3x - 12) \]
58. \[ \frac{4}{x + 2} + \frac{15x}{3x + 6} \]
59. \[ \frac{3x}{x + 4} - \frac{x}{x - 1} \]

Simplify the expression.

60. \[ 4\sqrt{7} + 3\sqrt{7} \]
61. \[ 9\sqrt{2} - 12\sqrt{8} \]
62. \[ \sqrt{6} \left(5\sqrt{3} + 6\right) \]
63. \[ \frac{11}{7 - \sqrt{3}} \]

In Exercises 64 and 65, use the triangles at the right.

64. Find the sine, cosine, and tangent of \(\angle Q\) and \(\angle R\). Round your answer to the nearest hundredth.

65. Find the lengths of the sides of \(\triangle ABC\). Round each answer to the nearest hundredth.

In Exercises 66 and 67, Company A sells a Personal Video Recorder (PVR) for $700 and charges a $5 monthly subscription fee. Company B sells a PVR for $500 and charges a $20 monthly fee.

66. What is the total cost of each PVR after one year? After 3 years?

67. How many months must the PVRs be used in order for the total costs of the two models to be the same?
What Happened to the Peanut Who Went Walking Late at Night?

Express each quotient below in simplest form. Find your answer in the answer column and circle the letter that equals the quotient. Write this letter in each box containing the number of that exercise.

1. \( \frac{12m^3n^5}{m+5} + \frac{3m^3n}{m^2-25} \)

2. \( \frac{n^2-9n+20}{6m^2n^2} - \frac{5n-20}{10mn^2} \)

3. \( \frac{m^2}{m^2-7m} + \frac{1}{m^2-4m-21} \)

4. \( \frac{16-2m}{m^2+2m-24} - \frac{m-8}{3m+13} \)

5. \( \frac{12n-36}{9-n^3} + \frac{8n^5}{n^n+3n} \)

6. \( \frac{m^2-n^2}{m^2+2mn+n^2} + \frac{m^2-n^2}{7m^2} \)

7. \( \frac{n^2-n-12}{2n^2-15n+18} - \frac{3n^2-12n}{2n^2-9n^2} \)

8. \( \frac{17mn^3}{m^2+2m-35} + \frac{34m^8n^4}{m^3+7m} \)

9. \( \frac{4n^3-25n}{3n^2-16n+5} + (10n+25) \)
What Game Do Cannibals Play at Parties?

Express each difference below in simplest form. Find your answer and notice the letter next to it. Write this letter in each box containing the number of that exercise.

1. \( \frac{8}{x^2 - 4} - \frac{3}{x - 2} \)

2. \( \frac{9}{x^2 - 2x - 15} - \frac{2}{x + 3} \)

3. \( \frac{7x}{x^2 - 9x + 14} - \frac{4}{x - 7} \)

4. \( \frac{3}{x - 4} - \frac{x - 9}{x^2 - 16} \)

5. \( \frac{5}{x + 5} - \frac{2x + 5}{x^2 + 9x + 20} \)

6. \( \frac{3}{d^2 - 7d + 12} - \frac{2}{d^2 - 2d - d} \)

7. \( \frac{d + 2}{4d - 1} - \frac{7}{d + 5} \)

8. \( \frac{d^2 + 3}{d^2 - 4} - \frac{d - 4}{d} \)

9. \( \frac{d^2 - 11}{d^2 - 7d + 12} - \frac{d + 1}{d - 4} \)

Answers:

M. \( \frac{3x}{x + 5} \)

A. \( \frac{-2x + 19}{(x + 3)(x - 5)} \)

N. \( \frac{3d + 8}{d(d + 2)} \)

C. \( \frac{8d - 15}{(5d + 4)(2d - 3)} \)

B. \( \frac{2x + 3}{d - 3} \)

L. \( \frac{2}{d - 3} \)

P. \( \frac{d^2 - 18d + 4}{(4d - 1)(d + 5)} \)

W. \( \frac{6d - 5}{d(d - 2)} \)

H. \( \frac{-3x + 2}{(x + 2)(x - 2)} \)

O. \( \frac{2x + 21}{x + 4}(x - 4) \)

D. \( \frac{d^2 - 21d + 17}{(4d - 1)(d + 5)} \)

U. \( \frac{7x + 11}{(x + 3)(x - 5)} \)

T. \( \frac{11d - 288}{(5d + 4)(2d - 3)} \)

What Sound Did the Sheep Hear When Her Sister Exploded?

Solve each equation and find your answer in the rectangle below. Cross out the box that contains your answer. When you finish, write the letters from the remaining boxes in the spaces at the bottom of the page.

1. \( \frac{2}{x + 3} + \frac{4}{x + 4} = \frac{7}{x + 2} + \frac{1}{x} \)

2. \( \frac{2}{x - 3} + \frac{4}{x - 4} = \frac{7}{x - 5} + \frac{1}{x - 1} \)

3. \( \frac{a - 30}{a + 9} = \frac{2}{a + 21} = \frac{a}{a + 4a} - \frac{2}{x^2 - 3x - 10} \)

4. \( \frac{a}{x + 4} + \frac{1}{x + 2} = \frac{3}{x + 4} + \frac{1}{y + 2} = \frac{4}{x + 4} = \frac{5}{x + 4} \)

OBJECTIVE 1-a: To subtract algebraic fractions.

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Part B- Homework

1) An old copying machine can produce 90 copies of a report in 120 minutes. A new machine can do the same job in 80 minutes. If both machines are used, how long would it take to do the job?

2) A pump was used to empty a flooded basement. After 3 hours, a second pump was added and the basement was emptied in 8 more hours. The first pump could have done the job alone in 20 hours. How long would it take the second pump to do the job alone?

3) Two numbers are in the ratio of 5 to 8. If the first number is decreased by 10 and the second number is increased by 4, the resulting numbers are in the ratio of 1 to 2. Find the original numbers.

4) A barge traveled 7 mph in still water. It travels 6 miles downstream in the same time it travels 2 miles upstream. Find the rate of the current.

5) Twelve fluid ounces of cola contain 145 calories. How many calories are in 16 ounces of cola?

6) The heat loss per hour through a window varies inversely with the thickness of the window. A window with a thickness of 0.3 cm loses 2400 calories per hour. How many calories are lost per hour through a window with a thickness of 0.8 cm?

7) There is a linear relationship between the number of standard bricks required for a wall and the thickness of the mortar joints.

<table>
<thead>
<tr>
<th>Thickness of Mortar Joints (in)</th>
<th>0.25</th>
<th>0.375</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Bricks per ft²</td>
<td>7</td>
<td>6.6</td>
<td>6.2</td>
</tr>
</tbody>
</table>

Write an equation to describe the relationship. Use the equation to find the number of bricks per square foot if the mortar thickness is 0.5625 inches.

8) The diameter in inches (D) of a car cylinder can be approximated using the formula

\[ D = \sqrt{\frac{2.5h}{n}} \]

where \( h \) is the horsepower of the engine and \( n \) is the number of cylinders.

Find \( D \) if \( h = 35 \) and \( n = 6 \).

9) RACQUETBALL You've paid $120 for a membership to a Racquetball Club. Court time is $5 per hour.

a) Write a model that represents your average cost per hour of court time as a function of the number of hours played. Graph the model. What is an equation of the horizontal asymptote and what does it represent?

b) Suppose you can play racquetball at the YMCA for $9 per hour without being a member. How many hours would you have to play at the Racquetball Club before the average cost is below $9 per hour?

10) LIGHTNING Air temperature affects how long it takes sound to travel a given distance. The time it takes for sound to travel one kilometer can be modeled by:

\[ t = \frac{1000}{0.67 + \frac{331}{T}} \]

where \( t \) is the time (in seconds) and \( T \) is the temperature (in degrees Celsius). You are 1 km from the lightning strike and it takes exactly 3 seconds to hear the thunder. Use a graph to find the approximate air temperature.

11) ENERGY EXPENDITURE The total energy expenditure \( E \) (in joules per gram mass per kilometer) of a typical budgerigar parakeet can be modeled by

\[ E = \frac{0.3v^2 - 21.7v + 471.75}{v} \]

where \( v \) is the speed of the bird (in kilometers per hour). Graph the model. What speed minimizes a budgerigar's energy expenditure?

12) OCEANOGRAPHY The mean temperature \( T \) (in degrees Celsius) of the Atlantic Ocean between latitudes 40°N and 40°S can be modeled by:

\[ T = \frac{17,800d + 20,000}{3d^2 + 740d + 1000} \]

where \( d \) is the depth (in meters). Graph the model. Use the graph to estimate the depth at which the mean temperature is 4°C.

13) HOSPITAL COSTS For 1985 to 1995, the average daily cost per patient \( C \) (in dollars) at community hospitals in the United States can be modeled by:

\[ C = \frac{-22,407 + 462,048}{5x^2 - 122x + 1000} \]

where \( x \) is the numbers of years since 1985. Graph the model. Would you use this model to predict patient costs in 2005? Explain.

14) DELIVERY CHARGES You and your friends order pizza and have it delivered to your house. The restaurant charges $8 per pizza plus a $2 delivery fee. Write a model that gives the average cost per pizza as a function of the number of pizzas ordered. Graph the model. Describe what happens to the average cost as the number of pizzas ordered increases.

15) COLLEGE GRADUATES From the 1984-85 school through the 1993-94 school year, the number of female college graduates \( F \) and the total number of college graduates \( G \) in the United States can be modeled by:

\[ F = \frac{-19,600t + 493,000}{-0.0580t + 1} \quad \text{and} \quad G = \frac{7560t^2 + 978,000}{0.00418t + 1} \]

where \( t \) is the number of school years since the 1984-85 school year. Write a model for the number of male college students.

16) DRUG ABSORPTION The amount \( A \) (in milligrams) of an oral drug, such as aspirin, in a person's bloodstream can be modeled by:

\[ A = \frac{-391t^2 + 0.112}{0.218t^4 + 0.999t^2 + 1} \]

where \( t \) is the time (in hours) after one dose is taken.

a) Graph the equation on your graphing calculator.

b) A second dose of the drug is taken one hour after the first dose. Write an equation to model the amount of the second dose in the bloodstream.

c) Write and graph a model for the total amount of the drug in the bloodstream after the second dose is taken.

d) About how long after the second dose has been taken is the greatest amount of the drug in the bloodstream?

17) BASKETBALL STATISTICS So far in the basketball season, you have made 12 free throws out of the 20 free throws you have attempted, for a free throw shooting percentage of 60%. How many consecutive free-throw shots would you have to make to raise your free-throw shooting percentage to 80%?

18) FOOTBALL STATISTICS At the end of the 1998 season, the NFL's all-time leading passer during regular season play was Dan Marino with 4763 completed passes out of 7989 attempts. In his debut 1998 season, Peyton Manning made 326 completed passes out of 473 attempts. How many consecutive passes would Manning have to make to equal Marino's pass completion percentage?

PHONE CARDS A telephone company offers you an opportunity to sell prepaid, 30 minute-long-distance phone cards. You will have to pay the company a one-time setup fee of $200. Each phone card will cost you $5.70. How many cards would you have to sell before your average total cost per card falls to $8?
A matrix (matrices is the plural) is rectangular arrangement of data in rows and columns. You have most likely seen this done in places such as MS Excel.

Matrices can be classified according to its dimensions. Dimensions are always shown as $M_{r \times c}$, where $r$ is the number of rows in the matrix and $c$ is the number of columns. (Think of the word "matrices" with $r$ before $c$)

A row matrix has only one row, where a column matrix has only one column.

We can also refer to a specific entry, called an element, according to its position. Thus, we would refer to the element in row 3, column 2 as $m_{3,2}$.

State the dimensions of each matrix. Then identify the position of the circled element in each matrix.

$$A = \begin{bmatrix} 2 & 9 & -1 \\ 4 & 0 & 7 \\ -3 & -11 & 4 \\ 7 & 6 & 1 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 1 & 0 \\ 6 & 0 & -5 \end{bmatrix} \quad C = [3 \ 1 \ 2 \ 8]$$

Imagine two matrices set equal to one another. This simply means that the corresponding elements must be equal. For example:

$$\begin{bmatrix} x + 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 2x \\ y - 7 \end{bmatrix}$$

This implies that $x + 4 = 2x$ and $5 = y - 7$

Solve each equation to find that $x = 4$ and $y = 12$

Solve each equation:

1. $[4 \ 42] = [24 \ 6y]$
2. $[-2 \ 12 \ -3z] = [6x \ -2y \ 45]$

It is important to note, really really important to note, that you can only add or subtract two matrices if their dimensions are the same. If the matrices have different dimensions, the operation is "impossible."

Otherwise, you add matrices intuitively. Since the matrices have the same dimensions, you must only add or subtract corresponding elements.

Given the matrices, perform the requested operations. If an operation is impossible, write "Impossible."

1. $A + C$
2. $C - A$
3. $C + F$
4. $E + E$
5. $B - D$
6. $D - B$

The last concept of the day is multiplying by a scalar, which is just some constant number. This is also intuitive. If you were asked to find $2E$, what would it look like?

Find the following sum, difference, or product.

7. $2D$
8. $3C$
9. $2A + C$
10. $3B - 2D$
1. Russell collected data at Powder Hill Resort. He found that 86% of the skiers planned to ski the next day and 92% of the snowboarders planned to snowboard the next day.
   a. Draw a transition diagram for Russell’s information.
   b. Write a transition matrix for the same information. Remember that rows indicate the present condition and columns indicate the next condition. List skiers first and snowboarders second.

2. Complete the following transition diagram:

3. Write a transition matrix for the diagram in Exercise 2. Order your information as in Exercise 1b.

4. During a recent softball tournament, information about which side players bat from was recorded in a matrix. Row 1 represents girls and row 2 represents boys. Column 1 represents left-handed batters, column 2 represents right-handed batters, and column 3 represents those who can bat with either hand.

   \[
   [A] = \begin{bmatrix}
   5 & 12 & 2 \\
   4 & 18 & 3 
   \end{bmatrix}
   \]

   a. How many girls and how many boys participated in the tournament?
   b. How many boys batted right-handed?
   c. What is the meaning of the value of \(a_{1x2}\) ?

5. In many countries, more people move into the cities than out of the cities. Suppose that in a certain country, 10% of the rural population moves to the city each year and 1% of the urban population moves out of the city each year.

   a. Draw a transition diagram that displays this information.
   b. Write a transition matrix that represents this same information. List urban dwellers first and rural dwellers second.
   c. If 16 million of the country’s 25 million people live in the city initially, what are the urban and rural populations in millions after 1 yr? After 2 yr? Write your answers as matrices in the form [urban rural].

6. The following matrix represents the number of math, science, and history textbooks sold at the main and branch campus bookstores this week.

   \[
   [A] = \begin{bmatrix}
   83 & 33 \\
   65 & 20 \\
   98 & 50 
   \end{bmatrix}
   \]

   a. Explain the meaning of the value of \(a_{3x2}\).
   b. Explain the meaning of the value of \(a_{2x1}\).
   c. Matrix \([B]\) represents last week’s sales. Compare this week’s sales of math books with last week’s sales.

   \[
   [B] = \begin{bmatrix}
   80 & 25 \\
   65 & 15 \\
   105 & 55 
   \end{bmatrix}
   \]

   d. Write a matrix that represents the total sales during last week and this week.

7. The three largest categories of motor vehicles are sedan, SUV, and minivan. Suppose that of the buyers in a particular community who now own a minivan, 18% will change to an SUV and 20% will change to a sedan. Of the buyers who now own a sedan, 35% will change to a minivan and 20% will change to an SUV, and of those who now own an SUV, 12% will buy a minivan and 32% will buy a sedan.

   a. Draw a transition diagram that displays these changes.
   b. Write a transition matrix that represents this scenario. List the rows and columns in the order minivan, sedan, SUV.
   c. What is the sum of the entries in row 1? Row 2? Row 3? Why does this sum make sense?
Do work on a separate sheet of paper.

1. Find the missing values.
   a. \[
   \begin{bmatrix}
   13 & 23 \\
   -6 & 31
   \end{bmatrix}
   \begin{bmatrix}
   x \\
   y
   \end{bmatrix}
   \]
   = \begin{bmatrix}
   x + y \\
   -6x + 31y
   \end{bmatrix}
   
   b. \[
   \begin{bmatrix}
   .90 & .10 \\
   .05 & .95
   \end{bmatrix}
   \begin{bmatrix}
   a \\
   b
   \end{bmatrix}
   = \begin{bmatrix}
   .90a + .10b \\
   .05a + .95b
   \end{bmatrix}
   
   c. \[
   \begin{bmatrix}
   18 & -23 \\
   5.4 & 32.2
   \end{bmatrix}
   \begin{bmatrix}
   2.4 & 12.2 \\
   5.3 & 10
   \end{bmatrix}
   = \begin{bmatrix}
   18 \cdot 2.4 + (-23) \cdot 5.3 \\
   5.4 \cdot 2.4 + 32.2 \cdot 5.3
   \end{bmatrix}
   = \begin{bmatrix}
   \end{bmatrix}
   
   d. \[
   \begin{bmatrix}
   7 & -4 \\
   18 & 28
   \end{bmatrix}
   + 5 \begin{bmatrix}
   -2.4 & 12.2 \\
   5.3 & 10
   \end{bmatrix}
   = \begin{bmatrix}
   7 & -4 + 5 \cdot (-2.4) \\
   18 & 28 + 5 \cdot 5.3
   \end{bmatrix}
   = \begin{bmatrix}
   \end{bmatrix}
   
   e. \[
   \begin{bmatrix}
   11 & 12 \\
   21 & 22
   \end{bmatrix}
   - \begin{bmatrix}
   74 & 2.4 \\
   5.4 & 32.2
   \end{bmatrix}
   = \begin{bmatrix}
   11 - 74 & 12 - 2.4 \\
   21 - 5.4 & 22 - 32.2
   \end{bmatrix}
   = \begin{bmatrix}
   \end{bmatrix}
   
   f. \[
   \begin{bmatrix}
   12 \\
   3
   \end{bmatrix}
   - \begin{bmatrix}
   23 & 7 \\
   4 & 5
   \end{bmatrix}
   = \begin{bmatrix}
   12 - 23 \\
   3 - 4
   \end{bmatrix}
   = \begin{bmatrix}
   \end{bmatrix}
   
2. Perform matrix arithmetic in 3a–f. If an operation is impossible, explain why.
   a. \[
   \begin{bmatrix}
   1 & 2 \\
   3 & -2 \\
   0 & 1
   \end{bmatrix}
   \begin{bmatrix}
   -1 & -1 \\
   2 & -1
   \end{bmatrix}
   = \begin{bmatrix}
   \end{bmatrix}
   
   b. \[
   \begin{bmatrix}
   1 & -2 \\
   6 & 3
   \end{bmatrix}
   + \begin{bmatrix}
   -3 & 7 \\
   2 & 4
   \end{bmatrix}
   = \begin{bmatrix}
   \end{bmatrix}
   
   c. \[
   \begin{bmatrix}
   5 & -2 \\
   7 & -1 \\
   3 & 2
   \end{bmatrix}
   = \begin{bmatrix}
   \end{bmatrix}
   
   d. \[
   \begin{bmatrix}
   3 & -8 \\
   -1 & 2
   \end{bmatrix}
   + \begin{bmatrix}
   10 & 2 \\
   3 & 4
   \end{bmatrix}
   = \begin{bmatrix}
   \end{bmatrix}
   
   e. \[
   \begin{bmatrix}
   3 & 6 \\
   -4 & 1
   \end{bmatrix}
   \begin{bmatrix}
   -1 & 7 \\
   -8 & 3
   \end{bmatrix}
   = \begin{bmatrix}
   \end{bmatrix}
   
   f. \[
   \begin{bmatrix}
   4 & 1 \\
   7 & 3
   \end{bmatrix}
   \begin{bmatrix}
   -2 & 7 \\
   5 & 2
   \end{bmatrix}
   = \begin{bmatrix}
   \end{bmatrix}
   
3. Find matrix \([B]\) such that \[
\begin{bmatrix}
8 & -5 \\
-6 & 9.5 \\
5 & 5
\end{bmatrix}
- \begin{bmatrix}
5 & -1 \\
-4 & 3.5 \\
2 & 1
\end{bmatrix}
= \begin{bmatrix}
0 & 4.5 \\
1 & 4.5 \\
3 & 1
\end{bmatrix}
\]

4. Find the missing values
   a. \[
\begin{bmatrix}
2 & a \\
\frac{19}{5} & \frac{17}{3}
\end{bmatrix}
= \begin{bmatrix}
\end{bmatrix}
   
   b. \[
\begin{bmatrix}
a & -2 \\
3 & 1
\end{bmatrix}
- \begin{bmatrix}
-3 \\
-5
\end{bmatrix}
= \begin{bmatrix}
\end{bmatrix}
   
Algebra 2H- WS Matrix Day 4
Determinants and Inverses

Do work on a separate sheet of paper and find its inverse.

Find the value of each determinant and find its inverse.

1. \[
\begin{bmatrix}
1 & 6 \\
2 & 7
\end{bmatrix}
\]
   2. \[
\begin{bmatrix}
9 & 6 \\
3 & 2
\end{bmatrix}
\]
   3. \[
\begin{bmatrix}
4 & 1 \\
-2 & -5
\end{bmatrix}
\]
   4. \[
\begin{bmatrix}
-14 & -3 \\
2 & -2
\end{bmatrix}
\]
   5. \[
\begin{bmatrix}
4 & -3 \\
-12 & 4
\end{bmatrix}
\]
   6. \[
\begin{bmatrix}
3 & -4 \\
7 & 9
\end{bmatrix}
\]

Find the value of each determinant using expansion by minors.

7. \[
\begin{bmatrix}
-2 & 3 \\
0 & 4
\end{bmatrix}
\]
   8. \[
\begin{bmatrix}
2 & -4 \\
3 & 0
\end{bmatrix}
\]
   9. \[
\begin{bmatrix}
1 & 1 \\
1 & -1
\end{bmatrix}
\]
   10. \[
\begin{bmatrix}
2 & 1 \\
1 & -1
\end{bmatrix}
\]

Answers:

1. \(-5, -\frac{1}{5}\)
   2. 0, No Inverse Exists
   3. \(-18, -\frac{1}{18}\)
   4. \(34, \frac{1}{34}\)
   5. \(-20, -\frac{1}{20}\)
   6. \(55, \frac{1}{55}\)
   7. \(-48, \) 8. 45
   9. 7
   10. -72
Do work on a separate sheet of paper.

1. Rewrite each system of equations in matrix form.
   a. \[ \begin{align*}
   3x + 4y &= 11 \\
   2x - 5y &= -8
   \end{align*} \]
   b. \[ \begin{align*}
   3x - 4y + 5z &= -11 \\
   -2x - 8y - 3z &= 1
   \end{align*} \]
   c. \[ \begin{align*}
   5.2x + 3.6y &= 7 \\
   -5.2x + 2y &= 8.2
   \end{align*} \]

2. Solve for each variable.
   a. \[ \begin{align*}
   \begin{bmatrix} 1 & 5 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} &= \begin{bmatrix} -7 \\ 33 \end{bmatrix}
   \end{align*} \]
   b. \[ \begin{align*}
   \begin{bmatrix} 1 & 5 & w \\ 6 & 2 & y \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}
   \end{align*} \]

3. Solve each system of equations.
   a. \[ \begin{align*}
   8x + 3y &= 41 \\
   6x + 5y &= 39 \\
   2x + y - 2z &= 1
   \end{align*} \]
   b. \[ \begin{align*}
   11x - 5y &= -38 \\
   9x + 2y &= -25 \\
   4w + x + 2y - 3z &= -16 \\
   -3w + 3x - y + 4z &= 20
   \end{align*} \]
   c. \[ \begin{align*}
   6x + 2y - 4z &= 3 \\
   4x - y + 3z &= 5
   \end{align*} \]
   d. \[ \begin{align*}
   5w + 4x + 3y - z &= -10 \\
   -w + 2x + 5y + z &= -4
   \end{align*} \]

4. At the High Flying Amusement Park there are three kinds of rides: Jolly rides, Adventure rides, and Thrill rides. Admission is free when you buy a book of tickets, which includes ten tickets for each type of ride. Or you can pay $5.00 for admission and then buy tickets for each of the rides individually. Noah, Rita, and Carey decide to pay the admission price and buy individual tickets. Noah pays $19.55 for 7 Jolly rides, 3 Adventure rides, and 9 Thrill rides. Rita pays $13.00 for 9 Jolly rides, 10 Adventure rides, and no Thrill rides. Carey pays $24.95 for 8 Jolly rides, 7 Adventure rides, and 10 Thrill rides. (The prices above do not include the admission price.)

5. A family invested a portion of $5000 in an account at 6% annual interest and the rest in an account at 7.5% annual interest. The total interest they earned in the first year was $340.50. How much did they invest in each account?

6. Being able to solve a system of equations is definitely not “new” mathematics. Mahavira, the best-known Indian mathematician of the 9th century, worked the following problem. See if you can solve it.

   The mixed price of 9 citrons and 7 fragrant wood apples is 107; again, the mixed price of 7 citrons and 9 fragrant wood apples is 101. O you arithmetician, tell me quickly the price of a citron and of a wood apple here, having distinctly separate those prices well.

7. If Alice bought 4 rolls of film, 4 sets of batteries and 3 disposable cameras, her total cost would be $36. She figured if she bought 6 rolls of film, 6 sets of batteries and 1 disposable camera, it would cost $33. If sets of batteries cost $0.50 more than a roll of film; find the cost of each item.

8. One side of a triangle is 3 inches longer than another side of the triangle. The sum of the lengths of those two sides, less 9 inches, equals the length of the third side of the triangle. If the perimeter of the triangle is 29 inches, find the length of each side.

9. The sum of Adam, Betsy and Carol’s ages is 47. Half the sum of Adam and Betsy’s ages, less one, is Carol’s age. If Adam is 6 years older than Betsy, find each person’s age.

10. A vendor sold 3 hot dogs, 2 malts and one soda for $8.30. He sold 1 hot dog and 4 sodas for $7.60. Three malts cost the same as 2 sodas. Find the cost of each item.

11. The perfume, Flora-Scent, may be purchased as a spray for $15, dusting powder for $12, or body lotion for $8. At a fragrance store, 22 Flora-Scent products were sold for a total of $276. Twice as much perfume spray as body lotion was purchased. How many products of each type were sold?
HEALTH CLUB MEMBERSHIP  A health club offers three different membership plans. With Plan A, you can use all club facilities: the pool, fitness center, and racquet club. With Plan B, you can use the pool and fitness center. With Plan C, you can only use the racquet club. The matrices below show the annual cost for a Single and a Family membership for the years 1998-2000.

<table>
<thead>
<tr>
<th>Year</th>
<th>Single (S)</th>
<th>Family (F)</th>
<th>Single (S)</th>
<th>Family (F)</th>
<th>Single (S)</th>
<th>Family (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>336</td>
<td>624</td>
<td>384</td>
<td>720</td>
<td>420</td>
<td>972</td>
</tr>
<tr>
<td>1999</td>
<td>228</td>
<td>528</td>
<td>312</td>
<td>576</td>
<td>360</td>
<td>672</td>
</tr>
<tr>
<td>2000</td>
<td>216</td>
<td>385</td>
<td>240</td>
<td>432</td>
<td>288</td>
<td>528</td>
</tr>
</tbody>
</table>

1) If you purchased a Single Plan A in 1998, a Family Plan B in 1999, and a Family Plan A in 2000, what is the total spent over the three years?

2) You purchased a Family Plan C in 1998 and upgraded your Family Plan to the next highest membership each year. How much did you spend over the three years?

BASKETBALL  A high school basketball coach helps the six seniors on the team to set goals for the season. The single game goals for the seniors are:

Andi: 2 3-pointers, 10 Field Goals, 3 Rebounds
Emily: 1 3-pointer, 6 Field Goals, 3 Rebounds
Sam: 3 3-pointers, 5 Field Goals, 1 Rebound
Mike: 3 3-pointers, 3 Rebounds
Bob: 4 3-pointers, 3 Rebounds
Kyle: 4 3-pointers, 3 Rebounds

3) Write a $6 \times 3$ matrix that represents the game goals for the six seniors.
4) If there are 15 games in a season, write the matrix representing their season goals.

WORLD SERIES  The Yankees won the 1998 World Series in four games. The matrices below show the statistics for runs, hits, and RBIs for each team in each game.

<table>
<thead>
<tr>
<th>Game 1</th>
<th>Game 2</th>
<th>Game 3</th>
<th>Game 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Diego</td>
<td>6 8 5</td>
<td>San Diego</td>
<td>3 10 3</td>
</tr>
<tr>
<td>New York</td>
<td>9 9 9</td>
<td>New York</td>
<td>9 16 8</td>
</tr>
<tr>
<td>R  H  RBI</td>
<td>R  H  RBI</td>
<td>R  H  RBI</td>
<td>R  H  RBI</td>
</tr>
</tbody>
</table>

5) Write a matrix that gives the series statistics for runs, hits, and RBIs for each team.
6) Which team had more hits in the series?

TRIANGLES  A triangle has vertices $(6, 1)$, $(2, 5)$, and $(-3, 2)$. Let $A$ be the $2 \times 3$ matrix whose columns consist of the coordinates of these three points. In the problems, plot each result as a point in a coordinate plane in order to describe, in words, the geometric effect of the operations in the problem on the triangle represented by Matrix $A$.

7) $A + \begin{bmatrix} 0 & 0 & 0 \\ -3 & -3 & -3 \end{bmatrix}$
8) $A + \begin{bmatrix} -1 & -1 & -1 \\ 4 & 4 & 4 \end{bmatrix}$
9) $3A + \begin{bmatrix} 2 & 2 & 2 \\ -5 & -5 & -5 \end{bmatrix}$

Describe the transformation of Matrix $A$ using the changes below.

10) Each entry of the new matrix is the opposite of the corresponding entry of $A$.
11) Each entry in the first row of the new matrix is the opposite of the entry in the second row of $A$, while each entry in the second row of the new matrix is the same as the corresponding entry in the first row of $A$.

SENIOR PLAY  The senior class play was performed on three different evenings. The attendance for each evening is shown in the table below. Adult tickets sold for $3.50 while Student tickets sold for $2.50.

<table>
<thead>
<tr>
<th>Performance</th>
<th>Adults</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening Night</td>
<td>420</td>
<td>300</td>
</tr>
<tr>
<td>Second Night</td>
<td>400</td>
<td>450</td>
</tr>
<tr>
<td>Final Night</td>
<td>510</td>
<td>475</td>
</tr>
</tbody>
</table>

12) Use matrix multiplication to determine how much money was taken in each night.

CLASS ELECTION  Day and Ethan are running for student council president. After attending a debate, some students change their minds about the candidate for whom they will vote. The percent of students who will change their support is shown in the given matrix. Day estimated that prior to the debate she would lose the election 350 votes to 400 votes.

13) After the debate, how many votes will Day and Ethan receive?
Do work on a separate sheet of paper.

1. Use these matrices to do the arithmetic problems 1a–f. If a particular operation is impossible, explain why.
\[ A = \begin{bmatrix} 1 & -2 \\ \end{bmatrix} \quad B = \begin{bmatrix} -3 & 7 \\ 6 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix} \quad D = \begin{bmatrix} -3 & 1 & 2 \\ 2 & 3 & -2 \end{bmatrix} \]

a. \( A + B \)  
   b. \( B - C \)  
   c. \( 4D \)  
   d. \( C[D] \)  
   e. \( D[C] \)  
   f. \( A[D] \)

2. Find the inverse, if it exists, of each matrix.
   a. \( \begin{bmatrix} 2 & -3 \\ 1 & -4 \end{bmatrix} \)  
   b. \( \begin{bmatrix} -2 & 3 \\ 8 & -12 \end{bmatrix} \)  
   c. \( \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix} \)

3. Solve each system of equations.
   a. \( 8x - 5y = -15 \quad 5x + 3y - 7z = 3 \)  
   b. \( y = -1.5x + 7 \quad 10x - 4y + 6z = 5 \)  
   c. \( 6x + 4y = 43 \quad y = -3x + 14 \quad 15x + y - 8z = -2 \)

4. Heather’s water heater needs repair. The plumber says it will cost $300 to fix the unit, which currently costs $75 per year to operate. Or Heather could buy a new energy-saving water heater for $500, including installation, and the new heater would save 60% on annual operating costs. How long would it take for the new unit to pay for itself?

5. A particular color of paint requires a mix of five parts red, six parts yellow, and two parts black. Thomas does not have the pure colors available, but he finds three pre-mixed colors that he can use. The first is two parts red and four parts yellow; the second is one part red and two parts black; the third is three parts red, one part yellow, and one part black.
   a. Write an equation that gives the correct portion of red by using the three available pre-mixed colors. Write an equation that gives the correct portion of yellow. Finally, write another equation that gives the correct portion of black.
   b. Solve the systems of equations in 5a.
   c. Explain the real-world meaning to your solutions to 5b.

6. Interlochen Arts Academy 9th and 10th graders are housed in three dormitories: Picasso, Hemingway, and Mozart. Mozart is an all-female dorm, Hemingway is an all-male dorm, and Picasso is coed. In September, school started with 80 students in Mozart, 60 in Picasso, and 70 in Hemingway. Students are permitted to move from one dorm to another on the first Sunday of each month. This transition graph shows the movements this past year.

   - Write a transition matrix for this situation. List the dorms in the order Mozart, Picasso, Hemingway.
   - What were the populations of the dorms in
     i. October?
     ii. November?
     iii. May?

7. Last semester, all of Ms. Nolte’s students did projects. One-half of the students in her second-period class investigated fractals, one-fourth of the students in that class did research projects, and the remaining students conducted surveys and analyzed their results. In her third-period class, one-third of the students investigated fractals, one-half of the students did research projects, and the remaining students conducted a survey and analyzed their results. In the seventh-period class, one-fourth of the students investigated fractals, one-sixth of the students did research projects, and the remaining students conducted surveys. Overall, 22 students investigated fractals, 18 students did research projects, and 22 students conducted surveys. How many students are in each of Ms. Nolte’s classes?
1. Graph each system of inequalities and label the vertices.

   a. \[
   \begin{align*}
   y' &\leq -0.51x + 5 \\
   y' &\leq -1.6x + 8 \\
   y' &\geq 0.1x + 2 \\
The feasible region is a triangle. \\
   x &\geq 0 \\
   \end{align*}
   \]

   b. \[
   \begin{align*}
   y' &\geq 1.6x - 3 \\
   y' &\leq -(x - 2)^2 + 4 \\
   y' &\geq 0 \\
   x &\geq 0 \\
   \end{align*}
   \]

   c. \[
   \begin{align*}
   4x + 3y &\leq 12 \\
   5x + 2y &\geq 20 \\
   -x + 2y &\geq 0 \\
   \end{align*}
   \]

2. Al just got rid of 40 of his dad’s old records. He sold each classical record for $5 and each jazz record for $2. The rest of the records could not be sold, so he donated them to a thrift shop. Al knows that he sold fewer than 10 jazz records and that he earned more than $100.

   a. Let x represent the number of classical records sold, and let y represent the number of jazz records sold. Write an inequality expressing that Al earned more than $100.

   b. Write an inequality expressing that he sold fewer than 10 jazz records.

   c. Write an inequality expressing that the total number of records sold was no more than 40.

   d. Graph the solution to the system of inequalities, including any commonsense constraints.

   e. Name all the vertices of the feasible region.

3. What system of inequalities produces the graph to the right?

4. Use a combination of the four lines shown on the graph along with the axes to create a system of inequalities whose graph satisfies each description.

   a. The feasible region is a triangle.

   b. The feasible region is a quadrilateral with one side on the y-axis.
Do problems neatly on a separate sheet of paper. For each problem:
- Define the variables you use.
- Write a system of inequalities.
- Draw an accurate graph.
- Lightly shade the solution area (feasible region).
- Find and label the vertices of the feasible region.
- Write the function you want to maximize/minimize.
- Indicate whether it is a maximum or a minimum.
- Make a table showing the function values at the vertices.
- State and explain the meaning of your solution.

A shoe manufacturer makes indoor and outdoor soccer shoes. There is a two-step process for both kinds of shoes. Each pair of outdoor shoes requires 2 hours in Step A and 1 hour in Step B, and produces a profit of $20. Each pair of indoor shoes requires 1 hour in Step A and 3 hours in Step B, and produces a profit of $15. The company has 40 hours of labor available per day in Step A and 60 hours of labor available per day in Step B. How many pairs of each type should be made to maximize the profit?

A farmer can plant up to 8 acres of land with wheat and barley. He can earn $5,000 for every acre he plants with wheat and $3,000 for every acre he plants with barley. His use of a necessary pesticide is limited by federal regulations to 10 gallons for his entire 8 acres. Wheat requires 2 gallons of pesticide per acre planted and barley requires just 1 gallon per acre. How many acres of wheat and how many acres of barley should the farmer plant to maximize his profit?

A painter has exactly 32 units of yellow dye and 54 units of green dye. He plans to mix as many gallons as possible of color A and color B. Each gallon of color A requires 4 units of yellow dye and 1 unit of green dye. Each gallon of color B requires 1 unit of yellow dye and 6 units of green dye. Find the maximum number of gallons he can mix.

Arnetta makes and sells teddy bears to gift shops. For her next shipment, she plans to make at least 90 bears. It costs her $5 to make a large bear and $2 to make a small bear. She plans to spend no more than $450 on materials. She plans to make at least twice as many small bears as large bears. If she makes a profit of $10 on each large bear and $5 on each small bear, how many of each type of bear must she sell to maximize her profit? What is her maximum profit?

1. The junior class has $300 to invest in a candy sale. They can sell no more than 9 cases of suckers and they must sell at least 18 cases of M & M's. Suckers cost $4 a case and M & M's cost $12 a case. If the profit on a case of suckers is $10 and the profit on a case of M & M's is $20, how many each should they buy to maximize their profit?

2. Hi-Fidelity, a small electronics firm, makes CD players and stereo receivers. The plant can make at most 30 CD players and 20 receivers in a week. It takes 2 worker hours to make a CD player and 4 worker hours to make a receiver. If there are 100 worker hours available per week and if the company makes a $150 profit on a receiver and a $50 profit on a CD player, how many of each should it make to maximize its profit?

3. A company makes both regular and deluxe tote bags. A regular tote bag requires 4 yards of canvas and 1 yard of leather. A deluxe tote bag requires 2 yards of canvas and 3 yards of leather. The company has 104 yards of canvas and 56 yards of leather available to make the tote bags. If each regular tote bag has a $20 profit and each deluxe tote bag has a $35 profit, how many of each style of tote bag should the company make to maximize its profit on the tote bags? What is the maximum profit?

4. A skateboard company manufactures both Pro Boards and Specialty Boards. Pro Boards require 1.5 hours of production time and 1 hour of deck finishing/quality control time. Specialty Boards require 2 hours of production time and 0.5 hours of deck finishing/quality control time. The company has 88 worker hours available each day for production and 42 worker hours available each day for deck finishing/quality control. If the profit on a Pro Board is $50 and the profit on a Specialty Board is $85, determine the number of each type of skateboard the company needs to make in order to maximize its profit. What is the maximum profit?

Ans:
1) 9 Suckers, 22 M & M's, $530
2) 10 CD, 20 Receivers, $3500
3) 20 Regular, 12 Deluxe, $820
4) 0 Pro, 44 Specialty, $3740
Analyzing Functions

Challenge

A home owner decides to install a solar home-heating system. The company selling the system offers her two options: Option A is $20,000 to install plus an annual heating cost of $1000; Option B is $15,000 to install plus an annual heating cost of $1500.

1. Write a function, \( A \), to represent the total cost of Option A over \( t \) years. 

2. Graph the function. Write its domain and range in set notation. 

3. Write a function, \( B \), to represent the total cost of Option B over \( t \) years. 

4. Describe \( B \) in terms of a transformation on \( A \).

5. Graph \( B \) on the same coordinate axes as \( A \). Use interval notation to describe the interval where \( A(t) \) is less than \( B(t) \). On what interval is \( B(t) \) less than \( A(t) \)? Which option should the home-owner choose? Describe the results in terms of the problem situation.

6. Find the inverse, \( h(L) \), of the function \( L(h) = 2.5\pi h \), where \( h \) is the height of the can. The total surface area of the can is given by the function \( T(h) = 2.5\pi(h + 1.25) \).

7. Find the inverse, \( h(T) \), of the function \( T(h) \). Explain its meaning.
Absolute-value graphs of the form \( y = |x - h| + k \) have a V shape or an inverted V shape that may have its vertex translated from the origin. The graph shows the following absolute-value function.

\[
y = 5 - |x - 2|
\]

What happens when you introduce the absolute value of an absolute value?

1. Use your graphing calculator to graph the function \( y = |5 - |x - 2|| \).
   a. Sketch the graph on the grid.
   b. What is the domain and range of the function?
   c. Describe how the graph differs from the graph of \( y = 5 - |x - 2| \).

2. Use your graphing calculator to graph the function \( y = |x + 2| - |x - 3| \).
   a. Sketch the graph on the grid.
   b. Notice that the graph is made up of three straight lines. One of the equations is \( y = -5 \) if \( x \leq -2 \). Write the other two equations including the domain for each.

3. Use your graphing calculator to graph the function \( y = ||x + 2| - |x - 3|| \).
   a. Sketch the graph on the grid.
   b. Describe how the graph differs from the graph of \( y = |x + 2| - |x - 3| \).
   c. Write an equation for each straight line section of the graph giving the domain for each.
Quadratic Equations

Challenge

If a quadratic equation with real coefficients has non-real roots, those roots are complex conjugates. But what if the coefficients of the quadratic equation are also complex or imaginary numbers? Consider the factored equation

\[(x + 3i)(x - i) = 0.\]

The solutions of this equation are \(i\) and \(-3i\). The expanded polynomial is

\[x^2 + 2ix + 3 = 0.\]

Notice that the coefficients are not all real numbers. That is why the complex solutions are not conjugates of one another. Equations of this type, where the middle term contains an imaginary number, are factored similarly to those with real coefficients except the sign of the constant term will be different due to the presence of the imaginary numbers.

All Real Coefficients

\[\begin{align*}
  x^2 - 11x + 30 &= 0 \\
  (x - 5)(x - 6) &= 0 \\
  x &= 5 \text{ or } x = 6
\end{align*}\]

Some Imaginary Coefficients

\[\begin{align*}
  x^2 - 11ix - 30 &= 0 \\
  (x - 5i)(x - 6i) &= 0 \\
  x &= 5i \text{ or } x = 6i
\end{align*}\]

Solve each equation by factoring.

1. \(x^2 + 5ix + 14 = 0\)

2. \(x^2 + 14ix - 48 = 0\)

3. \(x^2 + 3ix + 108 = 0\)

4. \(x^2 - 54ix - 245 = 0\)

Look at equations of the form \(2ix^2 + 5x + 12i = 0\). In this case, both the squared term and the constant contain imaginary coefficients. This equation factors into the binomials \((2ix - 3)\) and \((x - 4i)\) and the solutions are \(4i\) and \(\frac{3}{2}i\). Multiply the numerator and denominator of the fraction by \(-2i\) to obtain \(-\frac{3}{2}i\).

Solve each equation by factoring. Write the solutions in bi form.

5. \(3ix^2 - 7x - 4i = 0\)

6. \(5ix^2 + 11x - 2i = 0\)

7. \(2ix^2 - 16x - 30i = 0\)

8. \(4ix^2 + 7x + 65i = 0\)
In general, the graph of a polynomial equation of degree \(n\) contains \((n - 1)\) or fewer turning points. You can explore the factored form of a polynomial equation to note the effect that different constant values have on the shape of the curve.

\[ P(x) = (x - a)(x - b)(x - c)(x - d) \] where \(a, b, c,\) and \(d\) are real numbers.

1. Suppose that \(a, b, c,\) and \(d\) are all different nonzero numbers.
   a. Make a conjecture about the number of turning points. ___________________________
   b. Write a particular function that satisfies the conditions. ___________________________
   c. Use a graphing calculator to obtain a graph of the function you wrote in part b. Sketch the graph on the grid at right.

Follow the steps outlined in Problem 1 to explore different conditions placed on \(a, b, c,\) and \(d.

Summarize your results in the table shown below.

<table>
<thead>
<tr>
<th>Conditions Placed on (a, b, c,) and (d)</th>
<th>Example</th>
<th>Number of Turning Points</th>
<th>Other Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. All of (a, b, c,) and (d) are different and nonzero.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. All of (a, b, c,) and (d) are nonzero and exactly two are the same.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. All of (a, b, c,) and (d) are nonzero and exactly three are the same.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. All of (a, b, c,) and (d) are nonzero and all are equal.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. All of (a, b, c,) and (d) are different and exactly one is 0.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Two of (a, b, c,) and (d) are 0 and the other two are different.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Exactly three of (a, b, c,) and (d) are 0.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. All of (a, b, c,) and (d) are 0.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Synthetic division is an efficient tool for dividing a polynomial by a binomial and also for evaluating a polynomial for a given constant. However, the process works only when the divisor is a binomial of the form \((x - a)\), where the coefficient of \(x\) is 1. How could the process be used to divide the polynomial \(3x^2 + 8x - 12\) by \(2x - 6\)?

Write the division as a fraction and then multiply both the numerator and denominator by \(\frac{1}{2}\) to get the divisor in the form for synthetic division.

\[
\frac{3x^2 - 8x - 12}{2x - 6} \cdot \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{\frac{3}{2}x^2 - 4x - 6}{x - 3}
\]

\[
\begin{array}{c|ccc}
3 & \frac{3}{2} & -4 & -6 \\
\hline
2 & 1 & \frac{9}{2} & 3 \\
3 & 1 & 9 & 2 \\
\end{array}
\]

Now use synthetic division to find the quotient.

\[
\frac{3}{2}x + \frac{1}{2} - \frac{2}{x - 3}, \text{ which simplifies to } \left(\frac{3}{2}x + \frac{1}{2}\right)(2x - 6) - 9
\]

Divide using synthetic division.

1. \((4x^2 + 8x - 10) ÷ (2x + 6)\)
2. \((3x^3 + 12x^2 - 15x + 15) ÷ (3x - 9)\)

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3. \((25x^3 + 30x + 40) ÷ (5x + 10)\)
4. \(\left(x^4 - \frac{1}{16}\right) ÷ (2x - 1)\)

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5. \((2x^2 - 5x + 7) ÷ (2x - 1)\)
6. \((3x^2 + 7x - 13) ÷ (3x - 5)\)

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7. \((4x^5 - 129) ÷ (4x - 8)\)
8. \((x^6 - 729) ÷ [(x + 3)(x - 3)]\)
POLYNOMIAL EQUATIONS

Challenge

A polynomial function may be written in standard form.

\[ P(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x + a_0 \]

Dividing by the leading coefficient, \( a_n \), does not change the zeros of the polynomial. This produces a new polynomial that can also be written in descending order.

\[ Q(x) = x^n + A_1x^{n-1} + A_2x^{n-2} + A_3x^{n-3} + \ldots + A_{n-1}x + A_n \]

The properties below illustrate some relationships that occur between the coefficients in this form and the zeros of the polynomial.

\[ i) \quad \text{The sum of the zeros (roots) is equal to } -A_1. \]
\[ z_1 + z_2 + z_3 + \ldots + z_n = -A_1 \]

\[ ii) \quad \text{The sum of the products of the zeros taken two at a time is equal to } A_2. \]
\[ z_1z_2 + z_1z_3 + z_1z_4 + \ldots + z_{n-2}z_n + z_{n-1}z_n = A_2 \]

\[ iii) \quad \text{The sum of the products of the zeros taken three at a time is equal to } -A_3. \]
\[ z_1z_2z_3 + z_1z_2z_4 + \ldots + z_{n-2}z_{n-1}z_n = -A_3 \]

\[ iv) \quad \text{The product of the zeros is equal to } A_n \text{ or } -A_n. \]
\[ z_1z_2z_3z_4 - \ldots - z_{n-2}z_{n-1}z_n = (-1)^nA_n \]

Solve.

1. Show that the relationships \( i) \) and \( ii) \) hold true for the polynomial function \( P(x) = x^2 + 2x - 15 \), which has zeros at \( z_1 = 3 \) and \( z_2 = -5 \).

2. Show that the relationships \( i) \), \( ii) \), and \( iii) \) hold true for the polynomial function \( P(x) = x^3 - 3x^2 - 6x + 8 \), which has zeros at \( z_1 = 1 \), \( z_2 = 4 \), and \( z_3 = -2 \).

3. Use the relationships to find the final zero of the polynomial \( P(x) = x^3 - 8x^2 - 5x + 84 \) given that two of the zeros are at \( z_1 = 7 \) and \( z_2 = -3 \).

4. Use the relationships and solve a system of equations to find the remaining zeros of the polynomial \( P(x) = x^4 - x^3 - 19x^2 - 11x + 30 \) given that two zeros are at \( z_1 = 5 \) and \( z_2 = -2 \).

5. Prove that the relationships above are true for a polynomial with three zeros by expanding \( (x - z_1)(x - z_2)(x - z_3) \).
Rational Functions

**Challenge**

An asymptote is a line that a graph approaches, but does not reach, as its x- or y-values become very large or very small.

You can find vertical asymptotes of the graph of a rational function by finding values for which the function is not defined.

You can find horizontal asymptotes of the graph of a rational function by comparing the degree of the polynomial function in the numerator to the degree of the polynomial function in the denominator.

- If numerator degree < denominator degree, then \( y = 0 \) is a horizontal asymptote.
- If numerator degree = denominator degree and the leading coefficients in the numerator and denominator are \( a \) and \( b \), respectively, then \( y = \frac{a}{b} \) is a horizontal asymptote.

**Create a rational function that has the given lines as asymptotes.**

1. \( y = 0 \), \( x = -3 \), \( x = 0 \), and \( x = 2 \)
2. \( y = \frac{3}{2} \), \( x = -1 \), and \( x = \frac{1}{2} \)

If the degree of the numerator is greater than that of the denominator, then the graph may have an oblique asymptote, an asymptote that is neither vertical nor horizontal.

**Example**

The function \( y = \frac{x^2 + 7x + 12}{x + 1} \) has its numerator degree greater than its denominator degree. So the function has an oblique asymptote. Divide to find its equation.

\[
\frac{x + 6}{x + 1} + \frac{6}{x + 1}
\]

\[
x + 1\left(x^2 + 7x + 12\right)
\]

As \( x \) becomes very large or very small, \( \frac{6}{x + 1} \) approaches, but never reaches, 0.

Therefore, the function approaches, but never reaches, \( x + 6 \); the line \( y = x + 6 \) is an oblique asymptote. As the graph shows, the function also has a vertical asymptote.

Is the nonvertical asymptote horizontal or oblique? Write its equation.

3. \( y = \frac{2x + 3}{x + 4} \)
4. \( y = \frac{x^2 + 3x + 1}{x - 2} \)

5. \( y = \frac{5x^2 - 2x + 1}{x - 4} \)
6. \( y = \frac{2x^2 - 3x + 6}{2x + 1} \)
The process of multiplying and dividing rational expressions requires an understanding of the process of factoring. The following exercises will challenge some of your factoring skills. Remember to follow all factoring rules. Assume all expressions are defined.

1. Simplify the rational expression. \( \frac{x^3y + 4x^2y - 3x^y - 12}{x^y + 4} \)

2. Simplify the rational expression. \( \frac{5x^3a + 5x^2a y^a}{x^2a - y^2a} \)

3. Simplify the rational expression. \( \frac{j^3k + j^3 - k^4 - k^3}{j^2k + j^2 + jk^2 + jk + k^3 + k^2} \)

4. Multiply. \( \frac{4r^2 + 2rs + s^2}{2r + s} \cdot \frac{4r^2 - s^2}{8r^3 - s^3} \)

5. Multiply. \( \frac{2a^2 - 2a - 12}{a^2 - 49} \cdot \frac{4a^2 - 1}{2a^2 + 5a + 2} \cdot \frac{2a^2 - 13a - 7}{2a^2 - 7a + 3} \)

6. Multiply. \( \frac{p^3 - 4p^2 + p - 4}{2p^3 - 8p^2 + p - 4} \cdot \frac{2p^3 + 2p^2 + p + 1}{p^4 - p^3 + p^2 - p} \cdot \frac{m^3 + n^3}{m^3 - n^3} \cdot \frac{mp - mq - np + nq}{mn - m^2 - n^2} \cdot \frac{mp - mq + np - nq}{mp - mq + np - nq} \cdot \frac{a^{2n} - 1}{a^{2n} + 3a^n + 2} \cdot \frac{a^{2n} + a^n - 12}{a^{2n} - a^n - 6} \cdot \frac{x - y}{4z} \cdot \frac{y - x}{z} \cdot \frac{x - y}{z^2} \)

10. Solve the following equation for \( y \) in terms of \( x \), and write the resulting expression for \( y \) in simplest form. Identify any excluded values of \( x \).
\( x^2(y + 1) = 9(y + 1) + 4x + 12 \)
Radical Functions

Challenge

The first graph shows the function $y = \sqrt{x}$. Observe that both the domain and the range consist of the set of nonnegative numbers. The graph begins at the point (0, 0) and includes points (1, 1), (4, 2), and (9, 3). As $x$ increases from 0 to 1, then from 1 to 4, and then from 4 to 9, the $y$-value increases by 1 each time.

The second graph shows the function $y = 2\sqrt{x}$. In this case, the domain and range are both the set of real numbers. As $x$ increases from 0 to 1, then from 1 to 8, and then from 8 to 27, the $y$-value increases by 1 each time.

Look at the third graph. You can determine the equation from the graph. The square root function has been reflected across the $x$-axis. The starting point is at (2, 6), so both a reflection and a translation are involved. As $x$ increases from the starting point 1 unit to the right, the $y$-value decreases 4 units, so a vertical stretch is also indicated. Putting all these transformations together gives $y = -4\sqrt{x} - 2 + 6$.

Use these observations to write an equation for each graph. Note that the equations may not be unique since many times a vertical stretch or compression can also be written using a horizontal stretch or compression, respectively.
Radical Expressions and Equations

Challenge

Equations may involve more than one radical. In that case, the solution process is repeated to eliminate multiple radicals. For example:

$$\sqrt{x} + \sqrt{x - 5} = 5$$

To solve, isolate one radical and square both sides as shown below.

$$\sqrt{x} = 5 - \sqrt{x - 5}$$

$$\left(\sqrt{x}\right)^2 = (5 - \sqrt{x - 5})^2$$

$$x = 25 - 10\sqrt{x - 5} + (x - 5)$$

Notice that now there is only one radical in the equation. Repeat the process, isolate the radical, square, and solve.

$$x = 25 - 10\sqrt{x - 5} + (x - 5)$$

$$-20 = -10\sqrt{x - 5}$$

$$2 = \sqrt{x - 5}$$

$$2^2 = (\sqrt{x - 5})^2$$

$$4 = x - 5$$

$$9 = x$$

Solve each equation. Check each answer to ensure that it does not include extraneous solutions.

1. $$\sqrt{x - 3} = \sqrt{x + 15} - 2$$

2. $$\sqrt{x + 16} = x - \sqrt{x + 17}$$

3. $$\sqrt{x - 3} - \sqrt{x - 2} = 1$$

4. $$\sqrt{x - 3} = \sqrt{x - 15}$$

5. $$\sqrt{x - 3} = \frac{2}{\sqrt{x - 3}}$$

6. $$\sqrt{x^2 - 7x + 12} - x = x - 6$$

7. $$\sqrt{3x + 1} = \sqrt{50x + 6}$$

8. $$\sqrt{x - 7} = \sqrt{x - 1}$$

9. $$\sqrt{x + 2} = 1 + \sqrt{x - 3}$$

10. $$\frac{2}{\sqrt{x^2} + 5}$$