NAME ______________________________ DATE ____________________ PERIOD __________

10-1 Study Guide and Intervention
Trigonometric Identities

Find Trigonometric Values A **trigonometric identity** is an equation involving trigonometric functions that is true for all values for which every expression in the equation is defined.

Basic Trigonometric Identities	**Quotient Identities**	$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$	
	Reciprocal Identities	$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$
	Pythagorean Identities	$\cos^2\theta + \sin^2\theta = 1$	$\tan^2\theta + 1 = \sec^2\theta$	$\cot^2\theta + 1 = \csc^2\theta$

Example: Find the exact value of cot θ if csc $\theta = -\frac{11}{5}$ and $180° < \theta < 270°$.

$\cot^2\theta + 1 = \csc^2\theta$ — Trigonometric identity

$\cot^2\theta + 1 = \left(-\frac{11}{5}\right)^2$ — Substitute $-\frac{11}{5}$ for csc θ.

$\cot^2\theta + 1 = \frac{121}{25}$ — Square $-\frac{11}{5}$.

$\cot^2\theta = \frac{96}{25}$ — Subtract 1 from each side.

$\cot\theta = \pm\frac{4\sqrt{6}}{5}$ — Take the square root of each side.

Since θ is in the third quadrant, cot θ is positive. Thus, $\cot\theta = \frac{4\sqrt{6}}{5}$.

Exercises

Find the exact value of each expression if $0° < \theta < 90°$.

1. If $\cot\theta = 4$, find $\tan\theta$.

2. If $\cos\theta = \frac{\sqrt{3}}{2}$, find $\csc\theta$.

3. If $\sin\theta = \frac{3}{5}$, find $\cos\theta$.

4. If $\sin\theta = \frac{1}{3}$, find $\sec\theta$.

5. If $\tan\theta = \frac{4}{3}$, find $\cos\theta$.

6. If $\sin\theta = \frac{3}{7}$, find $\tan\theta$.

Find the exact value of each expression if $90° < \theta < 180°$.

7. If $\cos\theta = -\frac{7}{8}$, find $\sec\theta$.

8. If $\csc\theta = \frac{12}{5}$, find $\cot\theta$.

Find the exact value of each expression if $270° < \theta < 360°$.

9. If $\cos\theta = \frac{6}{7}$, find $\sin\theta$.

10. If $\csc\theta = -\frac{9}{4}$, find $\sin\theta$.

10-1 Study Guide and Intervention *(continued)*

Trigonometric Identities

Simplify Expressions The simplified form of a trigonometric expression is written as a numerical value or in terms of a single trigonometric function, if possible. Any of the trigonometric identities can be used to simplify expressions containing trigonometric functions.

Example 1: Simplify $(1 - \cos^2\theta)\sec\theta\cot\theta + \tan\theta\sec\theta\cos^2\theta$.

$$\begin{aligned}(1 - \cos^2\theta)\sec\theta\cot\theta + \tan\theta\sec\theta\cos^2\theta &= \sin^2\theta \cdot \frac{1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta} \cdot \cos^2\theta \\ &= \sin\theta + \sin\theta \\ &= 2\sin\theta\end{aligned}$$

Example 2: Simplify $\frac{\sec\theta \cdot \cot\theta}{1 - \sin\theta} - \frac{\csc\theta}{1 + \sin\theta}$.

$$\begin{aligned}\frac{\sec\theta \cdot \cot\theta}{1 - \sin\theta} - \frac{\csc\theta}{1 + \sin\theta} &= \frac{\frac{1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta}}{1 - \sin\theta} - \frac{\frac{1}{\sin\theta}}{1 + \sin\theta} \\ &= \frac{\frac{1}{\sin\theta}(1 + \sin\theta) - \frac{1}{\sin\theta}(1 - \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)} \\ &= \frac{\frac{1}{\sin\theta} + 1 - \frac{1}{\sin\theta} + 1}{1 - \sin^2\theta} \\ &= \frac{2}{\cos^2\theta} \text{ or } 2\sec^2\theta\end{aligned}$$

Exercises

Simplify each expression.

1. $\frac{\tan\theta \cdot \csc\theta}{\sec\theta}$

2. $\frac{\sin\theta \cdot \cot\theta}{\sec^2\theta - \tan^2\theta}$

3. $\frac{\sin^2\theta - \cot\theta \cdot \tan\theta}{\cot\theta \cdot \sin\theta}$

4. $\frac{\cos\theta}{\sec\theta - \tan\theta}$

5. $\frac{\tan\theta \cdot \cos\theta}{\sin\theta} + \cot\theta \cdot \sin\theta \cdot \tan\theta \cdot \csc\theta$

6. $\frac{\csc^2\theta - \cot^2\theta}{\tan\theta \cdot \cos\theta}$

7. $3\tan\theta \cdot \cot\theta + 4\sin\theta \cdot \csc\theta + 2\cos\theta \cdot \sec\theta$

8. $\frac{1 - \cos^2\theta}{\tan\theta \cdot \sin\theta}$