**2CP Lesson: *Composition of Functions***

**Perform Compositions of Functions** Suppose *f* and *g* are functions such that the range of *g* is a subset of the domain of *f*. Then the composite function *f* ◦ *g* can be described by the equation [*f* ° *g*](*x*) = *f*[*g*(*x*)].

**Example 1:** **For *f* = {(1, 2), (3, 3), (2, 4), (4, 1)} and *g* = {(1, 3), (3, 4), (2, 2), (4, 1)}, find *f* ◦ *g* and *g* ◦ *f* if they exist.**

*f*[*g*(1)] = *f*(3) = 3 *f*[*g*(2)] = *f*(2) = 4 *f*[*g*(3)] = *f*(4) = 1 *f*[*g*(4)] = *f*(1) = 2,

So *f* ◦ *g* = {(1, 3), (2, 4), (3, 1), (4, 2)}

*g*[*f*(1)] = *g*(2) = 2 *g*[*f*(2)] = *g*(4) = 1 *g*[*f*(3)] = *g*(3) = 4 *g*[*f*(4)] = *g*(1) = 3,

So *g* ◦ *f* = {(1, 2), (2, 1), (3, 4), (4, 3)}

**Example 2: Find [*g* ◦ *h*](*x*) and [*h* ◦ *g*](*x*) for *g*(*x*) = 3*x* – 4 and *h*(*x*) =** $x^{2}$ **– 1.**

[*g* ◦ *h*](*x*) = *g*[*h*(*x*)] [*h* ◦ *g*](*x*) = *h*[*g*(*x*)]

 = *g*($x^{2}$ – 1) = *h*(3*x* – 4)

 = 3($x^{2}$ – 1) – 4 = $(3x-4)^{2}$ – 1

 = 3$x^{2}$ – 7 = 9$x^{2}$ – 24*x* + 16 – 1

 = 9$x^{2}$ – 24*x* + 15

**Exercises**

**For each pair of functions, find *f* ◦ *g* and *g* ◦ *f,* if they exist.**

 **1.** *f* = {(–1, 2), (5, 6), (0, 9)}, **2.** *f* = {(5, –2), (9, 8), (–4, 3), (0, 4)},

 *g* = {(6, 0), (2, –1), (9, 5)} *g* = {(3, 7), (–2, 6), (4, –2), (8, 10)}

**Find [*f* ◦ *g*](*x*) and [*g* ◦ *f*](*x*), if they exist.**

 **3.** *f*(*x*) = 2*x* + 7; *g*(*x*) = –5*x* – 1 **4.** *f*(*x*) = $x^{2}$ – 1; *g*(*x*) = –4$x^{2}$

**Apply Compositions of Functions** Composition of functions can be used in real-world situations when functions are applied in sequence.

**Example:** **An appliance store is discounting all new dishwashers by 10%. At the same time, the manufacturer is offering a $100 rebate on all new dishwashers. Danielle is buying a dishwasher that is priced at $850. Will the final price be lower if the discount is applied before the rebate or if the rebate is applied before the discount?**

First, define variables and functions.

Let *x* represent the original price of a new dishwasher.

Let *f*(*x*) represent the price of a dishwasher after the discount.

Let *g*(*x*) represent the price of the dishwasher after the rebate.

Then write equations for *f*(*x*) and *g*(*x*).

If the discount is applied before the rebate, then the final price of the new dishwasher is represented by

If the rebate is applied before the discount, then the final price of the new dishwasher is represented by

[*g* ◦ *f*](850) = \_\_\_\_\_ and [*f* ◦ *g*](850) = \_\_\_\_\_\_. So,

**Exercises**

 **1.** Javier wants to purchase a new television. Electronics Plus offers both an in-store $50 rebate and a 20% discount on a television that normally sells for $1200. Which provides the better price: taking the discount before the rebate or taking the discount after the rebate?

 **2.** Corey wants to purchase a new elliptical. A fitness store offers both an in-store $75 rebate and a 5% discount on an elliptical that normally sells for $2500. Which provides the better price: taking the discount before the rebate or taking the discount after the rebate?