Properties of Exponents

Let $a$ and $b$ be real numbers and let $m$ and $n$ be rational numbers.

Zero Exponent

$$a^0 = 1, \text{ where } a \neq 0$$

Negative Exponent

$$a^{-n} = \frac{1}{a^n}, \text{ where } a \neq 0$$

Product of Powers Property

$$a^m \cdot a^n = a^{m+n}$$

Quotient of Powers Property

$$\frac{a^m}{a^n} = a^{m-n}, \text{ where } a \neq 0$$

Power of a Power Property

$$(a^m)^n = a^{mn}$$

Power of a Quotient Property

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, \text{ where } b \neq 0$$

Rational Exponents

$$a^{m/n} = \left(\sqrt[n]{a}\right)^m = \left(\frac{a}{\sqrt[n]{a}}\right)^m$$

Properties of Radicals

Let $a$ and $b$ be real numbers and let $n$ be an integer greater than 1.

Product Property of Radicals

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

Quotient Property of Radicals

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, \text{ where } a \geq 0 \text{ and } b \neq 0$$

Square Root of a Negative Number

1. If $r$ is a positive real number, then $\sqrt{-r} = i\sqrt{r}$.
2. By the first property, it follows that $(i\sqrt{r})^2 = -r$.

Properties of Logarithms

Let $b$, $m$, and $n$ be positive real numbers with $b \neq 1$.

Product Property

$$\log_b mn = \log_b m + \log_b n$$

Quotient Property

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

Power Property

$$\log_b m^n = n \log_b m$$

Other Properties

Zero-Product Property

If $A$ and $B$ are expressions and $AB = 0$, then $A = 0$ or $B = 0$.

Property of Equality for Exponential Equations

If $b > 0$ and $b \neq 1$, then $b^x = b^y$ if and only if $x = y$.

Property of Equality for Logarithmic Equations

If $b$, $x$, and $y$ are positive real numbers with $b \neq 1$, then $\log_b x = \log_b y$ if and only if $x = y$. 
Patterns

Square of a Binomial Pattern
\((a + b)^2 = a^2 + 2ab + b^2\)
\((a - b)^2 = a^2 - 2ab + b^2\)

Cube of a Binomial
\((a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\)
\((a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3\)

Difference of Two Squares Pattern
\(a^2 - b^2 = (a + b)(a - b)\)

Sum of Two Cubes
\(a^3 + b^3 = (a + b)(a^2 - ab + b^2)\)

Sum and Difference Pattern
\((a + b)(a - b) = a^2 - b^2\)

Completing the Square
\(x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2\)

Perfect Square Trinomial Pattern
\(a^2 + 2ab + b^2 = (a + b)^2\)
\(a^2 - 2ab + b^2 = (a - b)^2\)

Difference of Two Cubes
\(a^3 - b^3 = (a - b)(a^2 + ab + b^2)\)

Theorems

The Remainder Theorem
If a polynomial \(f(x)\) is divided by \(x - k\), then the remainder is \(r = f(k)\).

The Factor Theorem
A polynomial \(f(x)\) has a factor \(x - k\) if and only if \(f(k) = 0\).

The Rational Root Theorem
If \(f(x) = a_nx^n + \cdots + a_1x + a_0\) has integer coefficients, then every rational solution of \(f(x) = 0\) has the form
\[ p = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}. \]

The Irrational Conjugates Theorem
Let \(f\) be a polynomial function with rational coefficients, and let \(a\) and \(b\) be rational numbers such that \(\sqrt{b}\) is irrational.
If \(a + \sqrt{b}\) is a zero of \(f\), then \(a - \sqrt{b}\) is also a zero of \(f\).

The Fundamental Theorem of Algebra
Theorem: If \(f(x)\) is a polynomial of degree \(n\) where \(n > 0\), then the equation \(f(x) = 0\) has at least one solution in the set of complex numbers.

Corollary: If \(f(x)\) is a polynomial of degree \(n\) where \(n > 0\), then the equation \(f(x) = 0\) has exactly \(n\) solutions provided each solution repeated twice is counted as 2 solutions, each solution repeated three times is counted as 3 solutions, and so on.

The Complex Conjugates Theorem
If \(f\) is a polynomial function with real coefficients, and \(a + bi\) is an imaginary zero of \(f\), then \(a - bi\) is also a zero of \(f\).

Descartes’s Rule of Signs
Let \(f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0\) be a polynomial function with real coefficients.
• The number of positive real zeros of \(f\) is equal to the number of changes in sign of the coefficients of \(f(x)\) or is less than this by an even number.
• The number of negative real zeros of \(f\) is equal to the number of changes in the sign of the coefficients of \(f(-x)\) or is less than this by an even number.
### Formulas

#### Algebra

**Slope**
\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

**Slope-intercept form**
\[ y = mx + b \]

**Point-slope form**
\[ y - y_1 = m(x - x_1) \]

**Standard form of a quadratic function**
\[ f(x) = ax^2 + bx + c, \text{ where } a \neq 0 \]

**Intercept form of a quadratic function**
\[ f(x) = a(x - p)(x - q), \text{ where } a \neq 0 \]

**Standard equation of a circle**
\[ x^2 + y^2 = r^2 \]

**Exponential growth function**
\[ y = ab^n, \text{ where } a \neq 0 \text{ and } b > 1 \]

**Logarithm of \( y \) with base \( b \)**
\[ \log_b y = x \text{ if and only if } b^x = y \]

**Vertex form of a quadratic function**
\[ f(x) = a(x - h)^2 + k, \text{ where } a \neq 0 \]

**Quadratic Formula**
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } a \neq 0 \]

**Standard form of a polynomial function**
\[ f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0 \]

**Exponential decay function**
\[ y = ab^n, \text{ where } a \neq 0 \text{ and } 0 < b < 1 \]

**Change-of-base formula**
\[ \log_c a = \frac{\log_b a}{\log_b c}, \text{ where } a, b, \text{ and } c \text{ are positive real numbers with } b \neq 1 \text{ and } c \neq 1. \]

**Sum of first \( n \) positive numbers**
\[ \sum_{i=1}^{n} i = \frac{n(n + 1)}{2} \]

**Explicit rule for an arithmetic sequence**
\[ a_n = a_1 + (n - 1)d \]

**Explicit rule for a geometric sequence**
\[ a_n = a_1r^{n-1} \]

**Sum of an infinite geometric series**
\[ S = \frac{a_1}{1 - r}, \text{ provided } |r| < 1 \]

**Recursive equation for a geometric sequence**
\[ a_n = r \cdot a_{n-1} \]

**Sample mean**
\[ \bar{x} = \frac{\sum x}{n} \]

**z-Score**
\[ z = \frac{x - \mu}{\sigma} \]

**Standard deviation**
\[ \sigma = \sqrt{\frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \cdots + (x_n - \mu)^2}{n}} \]

**Margin of error for sample proportions**
\[ \pm \frac{1}{\sqrt{n}} \]

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**Reference** A99
Trigonometry

General definitions of trigonometric functions
Let \( \theta \) be an angle in standard position, and let \((x, y)\) be the point where the terminal side of \( \theta \) intersects the circle \( x^2 + y^2 = r^2 \). The six trigonometric functions of \( \theta \) are defined as shown.

\[
\begin{align*}
\sin \theta &= \frac{y}{r} \\
\cos \theta &= \frac{x}{r} \\
\tan \theta &= \frac{y}{x}, \quad x \neq 0 \\
csc \theta &= \frac{r}{y}, \quad y \neq 0 \\
\sec \theta &= \frac{r}{x}, \quad x \neq 0 \\
\cot \theta &= \frac{x}{y}, \quad y \neq 0
\end{align*}
\]

Conversion between degrees and radians
\(180^\circ = \pi \) radians

Arc length of a sector \( s = r\theta \)
Area of a sector \( A = \frac{1}{2}r^2\theta \)

Reciprocal Identities
\[
\begin{align*}
csc \theta &= \frac{1}{\sin \theta} \\
\sec \theta &= \frac{1}{\cos \theta} \\
\cot \theta &= \frac{1}{\tan \theta}
\end{align*}
\]

Tangent and Cotangent Identities
\[
\begin{align*}
\tan \theta &= \frac{\sin \theta}{\cos \theta} \\
\cot \theta &= \frac{\cos \theta}{\sin \theta}
\end{align*}
\]

Pythagorean Identities
\[
\begin{align*}
\sin^2 \theta + \cos^2 \theta &= 1 \\
1 + \tan^2 \theta &= \sec^2 \theta \\
1 + \cot^2 \theta &= \csc^2 \theta
\end{align*}
\]

Negative Angle Identities
\[
\begin{align*}
\sin(-\theta) &= -\sin \theta \\
\cos(-\theta) &= \cos \theta \\
\tan(-\theta) &= -\tan \theta
\end{align*}
\]

Cofunction Identities
\[
\begin{align*}
\sin\left(\frac{\pi}{2} - \theta\right) &= \cos \theta \\
\cos\left(\frac{\pi}{2} - \theta\right) &= \sin \theta \\
\tan\left(\frac{\pi}{2} - \theta\right) &= \cot \theta
\end{align*}
\]

Sum Formulas
\[
\begin{align*}
\sin(a + b) &= \sin a \cos b + \cos a \sin b \\
\cos(a + b) &= \cos a \cos b - \sin a \sin b \\
\tan(a + b) &= \frac{\tan a + \tan b}{1 - \tan a \tan b}
\end{align*}
\]

Difference Formulas
\[
\begin{align*}
\sin(a - b) &= \sin a \cos b - \cos a \sin b \\
\cos(a - b) &= \cos a \cos b + \sin a \sin b \\
\tan(a - b) &= \frac{\tan a - \tan b}{1 + \tan a \tan b}
\end{align*}
\]

Probability and Combinatorics

Theoretical Probability = \( \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} \)

Experimental Probability = \( \frac{\text{Number of successes}}{\text{Number of trials}} \)

Probability of the complement of an event
\( P(\overline{A}) = 1 - P(A) \)

Probability of dependent events
\( P(A \text{ and } B) = P(A) \cdot P(B | A) \)

Permutations
\[
\begin{align*}
_nP_r &= \frac{n!}{(n - r)!}
\end{align*}
\]

Combinations
\[
\begin{align*}
_nC_r &= \frac{n!}{(n - r)! \cdot r!}
\end{align*}
\]

Binomial experiments
\[
P(k \text{ successes}) = _nC_k p^k (1 - p)^{n-k}
\]

The Binomial Theorem
\[
(a + b)^n = _nC_0 a^n b^0 + _nC_1 a^{n-1} b^1 + _nC_2 a^{n-2} b^2 + \cdots + _nC_n a^0 b^n,
\]
where \( n \) is a positive integer.

Reference
### Perimeter, Area, and Volume Formulas

#### Square

- **Perimeter** \( P = 4s \)
- **Area** \( A = s^2 \)

#### Circle

- **Circumference** \( C = \pi d \) or \( C = 2\pi r \)
- **Area** \( A = \pi r^2 \)

#### Rectangle

- **Perimeter** \( P = 2l + 2w \)
- **Area** \( A = lw \)

#### Parallelogram

- **Area** \( A = bh \)

#### Trapezoid

- **Area** \( A = \frac{1}{2}h(b_1 + b_2) \)

#### Rhombus/Kite

- **Area** \( A = \frac{1}{2}d_1d_2 \)

#### Regular \( n \)-gon

- **Area** \( A = \frac{1}{2}aP \) or \( A = \frac{1}{2}a \cdot ns \)

#### Prism

- **Lateral Surface Area** \( L = Ph \)
- **Total Surface Area** \( S = 2B + Ph \)
- **Volume** \( V = Bh \)

#### Cylinder

- **Lateral Surface Area** \( L = 2\pi rh \)
- **Total Surface Area** \( S = 2\pi r^2 + 2\pi rh \)
- **Volume** \( V = \pi r^2h \)

#### Cone

- **Lateral Surface Area** \( L = \pi r\ell \)
- **Surface Area** \( S = \pi r^2 + \pi r\ell \)
- **Volume** \( V = \frac{1}{3}\pi r^2h \)

#### Pyramid

- **Lateral Surface Area** \( L = \frac{1}{2}P\ell \)
- **Surface Area** \( S = B + \frac{1}{2}P\ell \)
- **Volume** \( V = \frac{1}{3}Bh \)

#### Sphere

- **Surface Area** \( S = 4\pi r^2 \)
- **Volume** \( V = \frac{4}{3}\pi r^3 \)
Other Formulas

Pythagorean Theorem
\[ a^2 + b^2 = c^2 \]

Simple Interest
\[ I = Prt \]

Distance
\[ d = rt \]

Compound Interest
\[ A = P\left(1 + \frac{r}{n}\right)^{nt} \]

Continuously Compounded Interest
\[ A = Pe^{rt} \]

Conversions

U.S. Customary
1 foot = 12 inches
1 yard = 3 feet
1 mile = 5280 feet
1 mile = 1760 yards
1 acre = 43,560 square feet
1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 pound = 16 ounces
1 ton = 2000 pounds

U.S. Customary to Metric
1 inch = 2.54 centimeters
1 foot ≈ 0.3 meter
1 mile ≈ 1.61 kilometers
1 quart ≈ 0.95 liter
1 gallon ≈ 3.79 liters
1 cup ≈ 237 milliliters
1 pound ≈ 0.45 kilogram
1 ounce ≈ 28.3 grams
1 gallon ≈ 3785 cubic centimeters

Time
1 minute = 60 seconds
1 hour = 60 minutes
1 hour = 3600 seconds
1 year = 52 weeks

Temperature
\[ C = \frac{5}{9}(F - 32) \]
\[ F = \frac{9}{5}C + 32 \]

Metric
1 centimeter = 10 millimeters
1 meter = 100 centimeters
1 kilometer = 1000 meters
1 liter = 1000 milliliters
1 kiloliter = 1000 liters
1 milliliter = 1 cubic centimeter
1 liter = 1000 cubic centimeters
1 cubic millimeter = 0.001 milliliter
1 gram = 1000 milligrams
1 kilogram = 1000 grams

Metric to U.S. Customary
1 centimeter ≈ 0.39 inch
1 meter ≈ 3.28 feet
1 kilometer ≈ 0.62 mile
1 liter ≈ 1.06 quarts
1 liter ≈ 0.26 gallon
1 kilogram ≈ 2.2 pounds
1 gram ≈ 0.035 ounce
1 cubic meter ≈ 264 gallons