Reference

Properties

Properties of Exponents

Let a and b be real numbers and let m and n be rational numbers.

Zero Exponent
$a^0 = 1$, where $a \neq 0$

Negative Exponent
$a^{-n} = \frac{1}{a^n}$, where $a \neq 0$

Product of Powers Property
$a^m \cdot a^n = a^{m+n}$

Quotient of Powers Property
$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

Power of a Power Property
$(a^m)^n = a^{mn}$

Power of a Product Property
$(ab)^m = a^m b^m$

Power of a Quotient Property
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $b \neq 0$

Rational Exponents
$a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$

Rational Exponents
$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(a^{1/n})^m} = \frac{1}{(\sqrt[n]{a})^m}$,
where $a \neq 0$

Properties of Radicals

Let a and b be real numbers and let n be an integer greater than 1.

Product Property of Radicals
$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Quotient Property of Radicals
$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, where $a \geq 0$ and $b \neq 0$

Square Root of a Negative Number
1. If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$.
2. By the first property, it follows that $(i\sqrt{r})^2 = -r$.

Properties of Logarithms

Let b, m, and n be positive real numbers with $b \neq 1$.

Product Property
$\log_b mn = \log_b m + \log_b n$

Quotient Property
$\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property
$\log_b m^n = n \log_b m$

Other Properties

Zero-Product Property
If A and B are expressions and $AB = 0$, then $A = 0$ or $B = 0$.

Property of Equality for Exponential Equations
If $b > 0$ and $b \neq 1$, then $b^x = b^y$ if and only if $x = y$.

Property of Equality for Logarithmic Equations
If b, x, and y are positive real numbers with $b \neq 1$, then $\log_b x = \log_b y$ if and only if $x = y$.

Patterns

Square of a Binomial Pattern
$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$

Sum and Difference Pattern
$(a + b)(a - b) = a^2 - b^2$

Cube of a Binomial
$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Completing the Square
$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

Difference of Two Squares Pattern
$a^2 - b^2 = (a + b)(a - b)$

Perfect Square Trinomial Pattern
$a^2 + 2ab + b^2 = (a + b)^2$
$a^2 - 2ab + b^2 = (a - b)^2$

Sum of Two Cubes
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Difference of Two Cubes
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Theorems

The Remainder Theorem
If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

The Factor Theorem
A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

The Rational Root Theorem
If $f(x) = a_nx^n + \cdots + a_1x + a_0$ has *integer* coefficients, then every rational solution of $f(x) = 0$ has the form

$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$.

The Irrational Conjugates Theorem
Let f be a polynomial function with rational coefficients, and let a and b be rational numbers such that $\sqrt{b}$ is irrational.

If $a + \sqrt{b}$ is a zero of f, then $a - \sqrt{b}$ is also a zero of f.

The Fundamental Theorem of Algebra

Theorem If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has at least one solution in the set of complex numbers.

Corollary If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has exactly n solutions provided each solution repeated twice is counted as 2 solutions, each solution repeated three times is counted as 3 solutions, and so on.

The Complex Conjugates Theorem
If f is a polynomial function with real coefficients, and $a + bi$ is an imaginary zero of f, then $a - bi$ is also a zero of f.

Descartes's Rule of Signs
Let $f(x) = a_nx^n + a_nx^{n-1} + \cdots + a_2x^2 + a_1x + a_0$ be a polynomial function with real coefficients.

- The number of positive real zeros of f is equal to the number of changes in sign of the coefficients of $f(x)$ or is less than this by an even number.
- The number of negative real zeros of f is equal to the number of changes in the sign of the coefficients of $f(-x)$ or is less than this by an even number.

Formulas

Algebra

Slope

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Slope-intercept form

$y = mx + b$

Point-slope form

$y - y_1 = m(x - x_1)$

Standard form of a quadratic function

$f(x) = ax^2 + bx + c$, where $a \neq 0$

Vertex form of a quadratic function

$f(x) = a(x - h)^2 + k$, where $a \neq 0$

Intercept form of a quadratic function

$f(x) = a(x - p)(x - q)$, where $a \neq 0$

Quadratic Formula

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a \neq 0$

Standard equation of a circle

$x^2 + y^2 = r^2$

Standard form of a polynomial function

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$

Exponential growth function

$y = ab^x$, where $a \neq 0$ and $b > 1$

Exponential decay function

$y = ab^x$, where $a \neq 0$ and $0 < b < 1$

Logarithm of y with base b

$\log_b y = x$ if and only if $b^x = y$

Change-of-base formula

$\log_c a = \dfrac{\log_b a}{\log_b c}$, where a, b, and c are positive real numbers with $b \neq 1$ and $c \neq 1$.

Sum of n terms of 1

$\displaystyle\sum_{i=1}^{n} 1 = n$

Sum of first n positive numbers

$\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Sum of squares of first n positive integers

$\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

Explicit rule for an arithmetic sequence

$a_n = a_1 + (n - 1)d$

Sum of first n terms of an arithmetic series

$S_n = n\dfrac{(a_1 + a_2)}{2}$

Explicit rule for a geometric sequence

$a_n = a_1 r^{n-1}$

Sum of first n terms of a geometric series

$S_n = a_1\left(\dfrac{1 - r^n}{1 - r}\right)$, where $r \neq 1$

Sum of an infinite geometric series

$S = \dfrac{a_1}{1 - r}$ provided $|r| < 1$

Recursive equation for an arithmetic sequence

$a_n = a_{n-1} + d$

Recursive equation for a geometric sequence

$a_n = r \cdot a_{n-1}$

Statistics

Sample mean

$\bar{x} = \dfrac{\Sigma x}{n}$

Standard deviation

$\sigma = \sqrt{\dfrac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \cdots + (x_n - \mu)^2}{n}}$

***z*-Score**

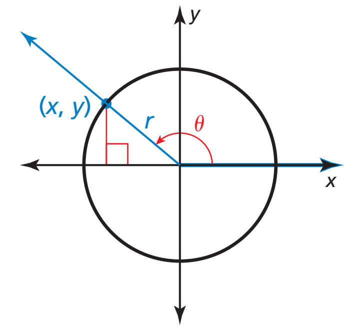
$z = \dfrac{x - \mu}{\sigma}$

Margin of error for sample proportions

$\pm\dfrac{1}{\sqrt{n}}$

Reference

Trigonometry



General definitions of trigonometric functions

Let θ be an angle in standard position, and let (x, y) be the point where the terminal side of θ intersects the circle $x^2 + y^2 = r^2$. The six trigonometric functions of θ are defined as shown.

$\sin \theta = \frac{y}{r}$ $\quad\cos \theta = \frac{x}{r}$ $\quad\tan \theta = \frac{y}{x}, x \neq 0$

$\csc \theta = \frac{r}{y}, y \neq 0$ $\quad\sec \theta = \frac{r}{x}, x \neq 0$ $\quad\cot \theta = \frac{x}{y}, y \neq 0$

Conversion between degrees and radians

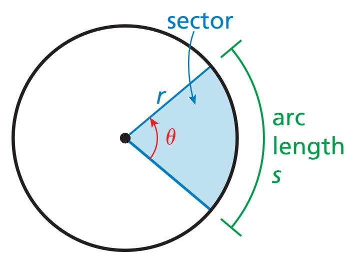
$180° = \pi$ radians

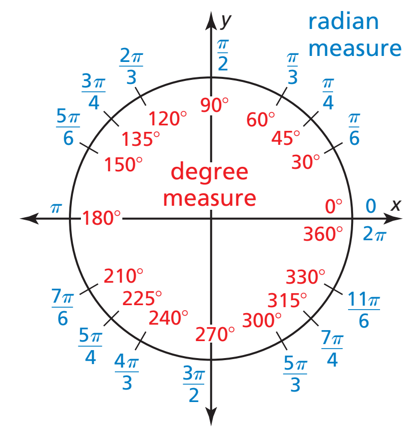
Arc length of a sector

$s = r\theta$

Area of a sector

$A = \frac{1}{2}r^2\theta$





Reciprocal Identities

$\csc \theta = \frac{1}{\sin \theta}$ $\quad\sec \theta = \frac{1}{\cos \theta}$ $\quad\cot \theta = \frac{1}{\tan \theta}$

Tangent and Cotangent Identities

$\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\quad\cot \theta = \frac{\cos \theta}{\sin \theta}$

Pythagorean Identities

$\sin^2 \theta + \cos^2 \theta = 1$

$1 + \tan^2 \theta = \sec^2 \theta$

$1 + \cot^2 \theta = \csc^2 \theta$

Negative Angle Identities

$\sin(-\theta) = -\sin \theta$

$\cos(-\theta) = \cos \theta$

$\tan(-\theta) = -\tan \theta$

Cofunction Identites

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$

$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$

Sum Formulas

$\sin(a + b) = \sin a \cos b + \cos a \sin b$

$\cos(a + b) = \cos a \cos b - \sin a \sin b$

$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$

Difference Formulas

$\sin(a - b) = \sin a \cos b - \cos a \sin b$

$\cos(a - b) = \cos a \cos b + \sin a \sin b$

$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$

Probability and Combinatorics

Theoretical Probability $= \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$

Experimental Probability $= \frac{\text{Number of successes}}{\text{Number of trials}}$

Probability of the complement of an event

$P(\overline{A}) = 1 - P(A)$

Probability of independent events

$P(A \text{ and } B) = P(A) \cdot P(B)$

Probability of dependent events

$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

Probability of compound events

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Permutations

${}_nP_r = \frac{n!}{(n - r)!}$

Combinations

${}_nC_r = \frac{n!}{(n - r)! \cdot r!}$

Binomial experiments

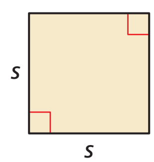
$P(k \text{ successes}) = {}_nC_k p^k (1 - p)^{n - k}$

The Binomial Theorem

$(a + b)^n = {}_nC_0 a^n b^0 + {}_nC_1 a^{n-1} b^1 + {}_nC_2 a^{n-2} b^2 + \cdots + {}_nC_n a^0 b^n$, where n is a positive integer.

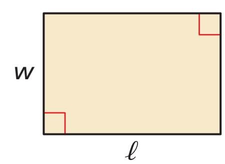
Perimeter, Area, and Volume Formulas

Square



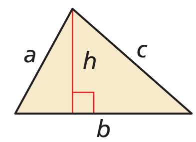
$P = 4s$
$A = s^2$

Rectangle



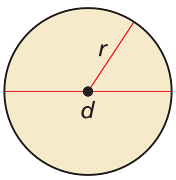
$P = 2\ell + 2w$
$A = \ell w$

Triangle



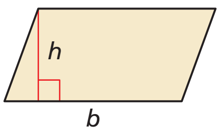
$P = a + b + c$
$A = \frac{1}{2}bh$

Circle



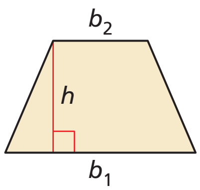
$C = \pi d$ or $C = 2\pi r$
$A = \pi r^2$

Parallelogram



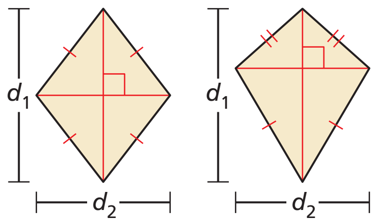
$A = bh$

Trapezoid



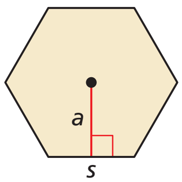
$A = \frac{1}{2}h(b_1 + b_2)$

Rhombus/Kite



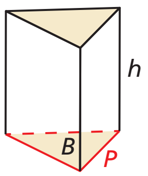
$A = \frac{1}{2}d_1 d_2$

Regular *n*-gon



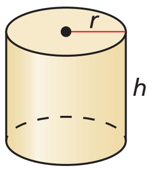
$A = \frac{1}{2}aP$ or $A = \frac{1}{2}a \cdot ns$

Prism



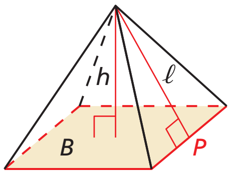
$L = Ph$
$S = 2B + Ph$
$V = Bh$

Cylinder



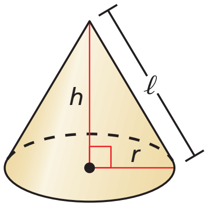
$L = 2\pi rh$
$S = 2\pi r^2 + 2\pi rh$
$V = \pi r^2 h$

Pyramid



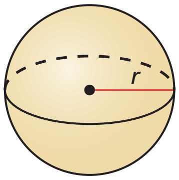
$L = \frac{1}{2}P\ell$
$S = B + \frac{1}{2}P\ell$
$V = \frac{1}{3}Bh$

Cone



$L = \pi r\ell$
$S = \pi r^2 + \pi r\ell$
$V = \frac{1}{3}\pi r^2 h$

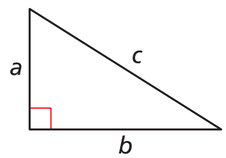
Sphere



$S = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$

Other Formulas

Pythagorean Theorem
$a^2 + b^2 = c^2$



Simple Interest
$I = Prt$

Compound Interest
$A = P\left(1 + \frac{r}{n}\right)^{nt}$

Continuously Compounded Interest
$A = Pe^{rt}$

Distance
$d = rt$

Conversions

U.S. Customary
1 foot = 12 inches
1 yard = 3 feet
1 mile = 5280 feet
1 mile = 1760 yards
1 acre = 43,560 square feet
1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 231 cubic inches
1 pound = 16 ounces
1 ton = 2000 pounds

U.S. Customary to Metric
1 inch = 2.54 centimeters
1 foot ≈ 0.3 meter
1 mile ≈ 1.61 kilometers
1 quart ≈ 0.95 liter
1 gallon ≈ 3.79 liters
1 cup ≈ 237 milliliters
1 pound ≈ 0.45 kilogram
1 ounce ≈ 28.3 grams
1 gallon ≈ 3785 cubic centimeters

Time
1 minute = 60 seconds
1 hour = 60 minutes
1 hour = 3600 seconds
1 year = 52 weeks

Temperature
$C = \frac{5}{9}(F - 32)$
$F = \frac{9}{5}C + 32$

Metric
1 centimeter = 10 millimeters
1 meter = 100 centimeters
1 kilometer = 1000 meters
1 liter = 1000 milliliters
1 kiloliter = 1000 liters
1 milliliter = 1 cubic centimeter
1 liter = 1000 cubic centimeters
1 cubic millimeter = 0.001 milliliter
1 gram = 1000 milligrams
1 kilogram = 1000 grams

Metric to U.S. Customary
1 centimeter ≈ 0.39 inch
1 meter ≈ 3.28 feet
1 meter ≈ 39.37 inches
1 kilometer ≈ 0.62 mile
1 liter ≈ 1.06 quarts
1 liter ≈ 0.26 gallon
1 kilogram ≈ 2.2 pounds
1 gram ≈ 0.035 ounce
1 cubic meter ≈ 264 gallons