USING A GRAPHING CALCULATOR

Most graphing calculators can calculate permutations.

```
4  nPr  4
             24
4  nPr  2
             12
```

Core Concept

Permutations

Formulas	Examples
The number of permutations of n objects is given by $${}_nP_n = n!.$$	The number of permutations of 4 objects is $${}_4P_4 = 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24.$$
The number of permutations of n objects taken r at a time, where $r \leq n$, is given by $${}_nP_r = \frac{n!}{(n-r)!}.$$	The number of permutations of 4 objects taken 2 at a time is $${}_4P_2 = \frac{4!}{(4-2)!} = \frac{4 \cdot 3 \cdot \cancel{2!}}{\cancel{2!}} = 12.$$

EXAMPLE 2 Using a Permutations Formula

Ten horses are running in a race. In how many different ways can the horses finish first, second, and third? (Assume there are no ties.)

SOLUTION

To find the number of permutations of 3 horses chosen from 10, find ${}_{10}P_3$.

$${}_{10}P_3 = \frac{10!}{(10-3)!} \quad \text{Permutations formula}$$

$$= \frac{10!}{7!} \quad \text{Subtract.}$$

$$= \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}} \quad \text{Expand factorial. Divide out common factor, 7!.}$$

$$= 720 \quad \text{Simplify.}$$

▶ There are 720 ways for the horses to finish first, second, and third.

STUDY TIP

When you divide out common factors, remember that 7! is a factor of 10!.

EXAMPLE 3 Finding a Probability Using Permutations

For a town parade, you will ride on a float with your soccer team. There are 12 floats in the parade, and their order is chosen at random. Find the probability that your float is first and the float with the school chorus is second.

SOLUTION

Step 1 Write the number of possible outcomes as the number of permutations of the 12 floats in the parade. This is ${}_{12}P_{12} = 12!$.

Step 2 Write the number of favorable outcomes as the number of permutations of the other floats, given that the soccer team is first and the chorus is second. This is ${}_{10}P_{10} = 10!$.

Step 3 Find the probability.

$$P(\text{soccer team is 1st, chorus is 2nd}) = \frac{10!}{12!} \quad \text{Form a ratio of favorable to possible outcomes.}$$

$$= \frac{\cancel{10!}}{12 \cdot 11 \cdot \cancel{10!}} \quad \text{Expand factorial. Divide out common factor, 10!.}$$

$$= \frac{1}{132} \quad \text{Simplify.}$$

Monitoring Progress

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3. **WHAT IF?** In Example 2, suppose there are 8 horses in the race. In how many different ways can the horses finish first, second, and third? (Assume there are no ties.)

4. **WHAT IF?** In Example 3, suppose there are 14 floats in the parade. Find the probability that the soccer team is first and the chorus is second.

Combinations

A **combination** is a selection of objects in which order is *not* important. For instance, in a drawing for 3 identical prizes, you would use combinations, because the order of the winners would not matter. If the prizes were different, then you would use permutations, because the order would matter.

EXAMPLE 4 Counting Combinations

Count the possible combinations of 2 letters chosen from the list A, B, C, D.

SOLUTION

List all of the permutations of 2 letters from the list A, B, C, D. Because order is not important in a combination, cross out any duplicate pairs.

AB	AC	AD	~~BA~~	BC	BD
~~CA~~	~~CB~~	CD	~~DA~~	~~DB~~	~~DC~~

BD and DB are the same pair.

▶ There are 6 possible combinations of 2 letters from the list A, B, C, D.

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5. Count the possible combinations of 3 letters chosen from the list A, B, C, D, E.

In Example 4, you found the number of combinations of objects by making an organized list. You can also find the number of combinations using the following formula.

USING A GRAPHING CALCULATOR

Most graphing calculators can calculate combinations.

```
4  nCr  2
            6
```

Core Concept

Combinations

Formula The number of combinations of n objects taken r at a time, where $r \leq n$, is given by

$${}_nC_r = \frac{n!}{(n-r)! \cdot r!}.$$

Example The number of combinations of 4 objects taken 2 at a time is

$${}_4C_2 = \frac{4!}{(4-2)! \cdot 2!} = \frac{4 \cdot 3 \cdot \cancel{2!}}{\cancel{2!} \cdot (2 \cdot 1)} = 6.$$

EXAMPLE 5 Using the Combinations Formula

You order a sandwich at a restaurant. You can choose 2 side dishes from a list of 8. How many combinations of side dishes are possible?

SOLUTION

The order in which you choose the side dishes is not important. So, to find the number of combinations of 8 side dishes taken 2 at a time, find ${}_8C_2$.

$$\begin{aligned} {}_8C_2 &= \frac{8!}{(8-2)! \cdot 2!} && \text{Combinations formula} \\ &= \frac{8!}{6! \cdot 2!} && \text{Subtract.} \\ &= \frac{8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!} \cdot (2 \cdot 1)} && \text{Expand factorials. Divide out common factor, 6!.} \\ &= 28 && \text{Multiply.} \end{aligned}$$

There are 28 different combinations of side dishes you can order.

Check

```
8  nCr  2
              28
```

EXAMPLE 6 Finding a Probability Using Combinations

A yearbook editor has selected 14 photos, including one of you and one of your friend, to use in a collage for the yearbook. The photos are placed at random. There is room for 2 photos at the top of the page. What is the probability that your photo and your friend's photo are the 2 placed at the top of the page?

SOLUTION

Step 1 Write the number of possible outcomes as the number of combinations of 14 photos taken 2 at a time, or ${}_{14}C_2$, because the order in which the photos are chosen is not important.

$$\begin{aligned} {}_{14}C_2 &= \frac{14!}{(14-2)! \cdot 2!} && \text{Combinations formula} \\ &= \frac{14!}{12! \cdot 2!} && \text{Subtract.} \\ &= \frac{14 \cdot 13 \cdot \cancel{12!}}{\cancel{12!} \cdot (2 \cdot 1)} && \text{Expand factorials. Divide out common factor, 12!.} \\ &= 91 && \text{Multiply.} \end{aligned}$$

Step 2 Find the number of favorable outcomes. Only one of the possible combinations includes your photo and your friend's photo.

Step 3 Find the probability.

$$P(\text{your photo and your friend's photos are chosen}) = \frac{1}{91}$$

Monitoring Progress

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6. **WHAT IF?** In Example 5, suppose you can choose 3 side dishes out of the list of 8 side dishes. How many combinations are possible?

7. **WHAT IF?** In Example 6, suppose there are 20 photos in the collage. Find the probability that your photo and your friend's photo are the 2 placed at the top of the page.

Binomial Expansions

In Section 4.2, you used Pascal's Triangle to find binomial expansions. The table shows that the coefficients in the expansion of $(a + b)^n$ correspond to combinations.

	n	Pascal's Triangle as Numbers	Pascal's Triangle as Combinations	Binomial Expansion
0th row	0	1	${}_0C_0$	$(a+b)^0 = 1$
1st row	1	1 1	${}_1C_0$ ${}_1C_1$	$(a+b)^1 = 1a + 1b$
2nd row	2	1 2 1	${}_2C_0$ ${}_2C_1$ ${}_2C_2$	$(a+b)^2 = 1a^2 + 2ab + 1b^2$
3rd row	3	1 3 3 1	${}_3C_0$ ${}_3C_1$ ${}_3C_2$ ${}_3C_3$	$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$

The results in the table are generalized in the **Binomial Theorem**.

Core Concept

The Binomial Theorem

For any positive integer n, the binomial expansion of $(a + b)^n$ is

$$(a+b)^n = {}_nC_0\, a^n b^0 + {}_nC_1\, a^{n-1} b^1 + {}_nC_2\, a^{n-2} b^2 + \cdots + {}_nC_n\, a^0 b^n.$$

Notice that each term in the expansion of $(a + b)^n$ has the form ${}_nC_r\, a^{n-r} b^r$, where r is an integer from 0 to n.

EXAMPLE 7 Using the Binomial Theorem

a. Use the Binomial Theorem to write the expansion of $(x^2 + y)^3$.

b. Find the coefficient of x^4 in the expansion of $(3x + 2)^{10}$.

SOLUTION

a.
$$\begin{aligned}(x^2+y)^3 &= {}_3C_0(x^2)^3y^0 + {}_3C_1(x^2)^2y^1 + {}_3C_2(x^2)^1y^2 + {}_3C_3(x^2)^0y^3 \\ &= (1)(x^6)(1) + (3)(x^4)(y^1) + (3)(x^2)(y^2) + (1)(1)(y^3) \\ &= x^6 + 3x^4y + 3x^2y^2 + y^3\end{aligned}$$

b. From the Binomial Theorem, you know

$$(3x+2)^{10} = {}_{10}C_0(3x)^{10}(2)^0 + {}_{10}C_1(3x)^9(2)^1 + \cdots + {}_{10}C_{10}(3x)^0(2)^{10}.$$

Each term in the expansion has the form ${}_{10}C_r(3x)^{10-r}(2)^r$. The term containing x^4 occurs when $r = 6$.

$${}_{10}C_6(3x)^4(2)^6 = (210)(81x^4)(64) = 1{,}088{,}640x^4$$

▶ The coefficient of x^4 is 1,088,640.

Monitoring Progress

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8. Use the Binomial Theorem to write the expansion of (a) $(x + 3)^5$ and (b) $(2p - q)^4$.

9. Find the coefficient of x^5 in the expansion of $(x - 3)^7$.

10. Find the coefficient of x^3 in the expansion of $(2x + 5)^8$.

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** An arrangement of objects in which order is important is called a(n) __________.

2. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$\dfrac{7!}{2! \cdot 5!}$	${}_7C_5$	${}_7C_2$	$\dfrac{7!}{(7-2)!}$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, find the number of ways you can arrange (a) all of the letters and (b) 2 of the letters in the given word. *(See Example 1.)*

3. AT
4. TRY
5. ROCK
6. WATER
7. FAMILY
8. FLOWERS

In Exercises 9–16, evaluate the expression.

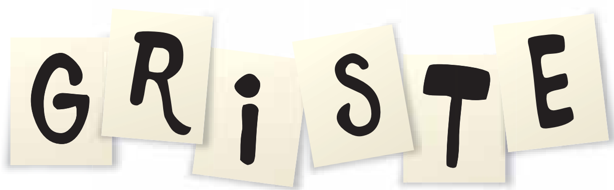
9. ${}_5P_2$
10. ${}_7P_3$
11. ${}_9P_1$
12. ${}_6P_5$
13. ${}_8P_6$
14. ${}_{12}P_0$
15. ${}_{30}P_2$
16. ${}_{25}P_5$

17. **PROBLEM SOLVING** Eleven students are competing in an art contest. In how many different ways can the students finish first, second, and third? *(See Example 2.)*

18. **PROBLEM SOLVING** Six friends go to a movie theater. In how many different ways can they sit together in a row of 6 empty seats?

19. **PROBLEM SOLVING** You and your friend are 2 of 8 servers working a shift in a restaurant. At the beginning of the shift, the manager randomly assigns one section to each server. Find the probability that you are assigned Section 1 and your friend is assigned Section 2. *(See Example 3.)*

20. **PROBLEM SOLVING** You make 6 posters to hold up at a basketball game. Each poster has a letter of the word TIGERS. You and 5 friends sit next to each other in a row. The posters are distributed at random. Find the probability that TIGERS is spelled correctly when you hold up the posters.



In Exercises 21–24, count the possible combinations of *r* letters chosen from the given list. *(See Example 4.)*

21. A, B, C, D; $r = 3$
22. L, M, N, O; $r = 2$
23. U, V, W, X, Y, Z; $r = 3$
24. D, E, F, G, H; $r = 4$

In Exercises 25–32, evaluate the expression.

25. ${}_5C_1$
26. ${}_8C_5$
27. ${}_9C_9$
28. ${}_8C_6$
29. ${}_{12}C_3$
30. ${}_{11}C_4$
31. ${}_{15}C_8$
32. ${}_{20}C_5$

33. **PROBLEM SOLVING** Each year, 64 golfers participate in a golf tournament. The golfers play in groups of 4. How many groups of 4 golfers are possible? *(See Example 5.)*

34. **PROBLEM SOLVING** You want to purchase vegetable dip for a party. A grocery store sells 7 different flavors of vegetable dip. You have enough money to purchase 2 flavors. How many combinations of 2 flavors of vegetable dip are possible?

ERROR ANALYSIS In Exercises 35 and 36, describe and correct the error in evaluating the expression.

35.

$$_{11}P_7 = \frac{11!}{(11-7)} = \frac{11!}{4} = 9{,}979{,}200$$

36.

$$_9C_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = 3024$$

REASONING In Exercises 37–40, tell whether the question can be answered using *permutations* or *combinations*. Explain your reasoning. Then answer the question.

37. To complete an exam, you must answer 8 questions from a list of 10 questions. In how many ways can you complete the exam?

38. Ten students are auditioning for 3 different roles in a play. In how many ways can the 3 roles be filled?

39. Fifty-two athletes are competing in a bicycle race. In how many orders can the bicyclists finish first, second, and third? (Assume there are no ties.)

40. An employee at a pet store needs to catch 5 tetras in an aquarium containing 27 tetras. In how many groupings can the employee capture 5 tetras?

41. **CRITICAL THINKING** Compare the quantities $_{50}C_9$ and $_{50}C_{41}$ without performing any calculations. Explain your reasoning.

42. **CRITICAL THINKING** Show that each identity is true for any whole numbers r and n, where $0 \le r \le n$.

a. $_nC_n = 1$

b. $_nC_r = {}_nC_{n-r}$

c. $_{n+1}C_r = {}_nC_r + {}_nC_{r-1}$

43. **REASONING** Consider a set of 4 objects.

a. Are there more permutations of all 4 of the objects or of 3 of the objects? Explain your reasoning.

b. Are there more combinations of all 4 of the objects or of 3 of the objects? Explain your reasoning.

c. Compare your answers to parts (a) and (b).

44. **OPEN-ENDED** Describe a real-life situation where the number of possibilities is given by $_5P_2$. Then describe a real-life situation that can be modeled by $_5C_2$.

45. **REASONING** Complete the table for each given value of r. Then write an inequality relating $_nP_r$ and $_nC_r$. Explain your reasoning.

	$r = 0$	$r = 1$	$r = 2$	$r = 3$
$_3P_r$				
$_3C_r$				

46. **REASONING** Write an equation that relates $_nP_r$ and $_nC_r$. Then use your equation to find and interpret the value of $\frac{_{182}P_4}{_{182}C_4}$.

47. **PROBLEM SOLVING** You and your friend are in the studio audience on a television game show. From an audience of 300 people, 2 people are randomly selected as contestants. What is the probability that you and your friend are chosen? *(See Example 6.)*

48. **PROBLEM SOLVING** You work 5 evenings each week at a bookstore. Your supervisor assigns you 5 evenings at random from the 7 possibilities. What is the probability that your schedule does not include working on the weekend?

REASONING **In Exercises 49 and 50, find the probability of winning a lottery using the given rules. Assume that lottery numbers are selected at random.**

49. You must correctly select 6 numbers, each an integer from 0 to 49. The order is not important.

50. You must correctly select 4 numbers, each an integer from 0 to 9. The order is important.

In Exercises 51–58, use the Binomial Theorem to write the binomial expansion. *(See Example 7a.)*

51. $(x + 2)^3$

52. $(c - 4)^5$

53. $(a + 3b)^4$

54. $(4p - q)^6$

55. $(w^3 - 3)^4$

56. $(2s^4 + 5)^5$

57. $(3u + v^2)^6$

58. $(x^3 - y^2)^4$

In Exercises 59–66, use the given value of n to find the coefficient of x^n in the expansion of the binomial. *(See Example 7b.)*

59. $(x - 2)^{10}, n = 5$

60. $(x - 3)^7, n = 4$

61. $(x^2 - 3)^8, n = 6$

62. $(3x + 2)^5, n = 3$

63. $(2x + 5)^{12}, n = 7$

64. $(3x - 1)^9, n = 2$

65. $\left(\frac{1}{2}x - 4\right)^{11}, n = 4$

66. $\left(\frac{1}{4}x + 6\right)^6, n = 3$

67. **REASONING** Write the eighth row of Pascal's Triangle as combinations and as numbers.

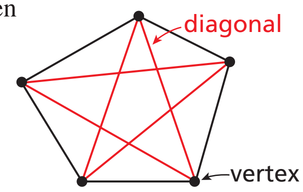
68. **PROBLEM SOLVING** The first four triangular numbers are 1, 3, 6, and 10.

a. Use Pascal's Triangle to write the first four triangular numbers as combinations.

$$\begin{array}{ccccccccccc} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \end{array}$$

b. Use your result from part (a) to write an explicit rule for the nth triangular number T_n.

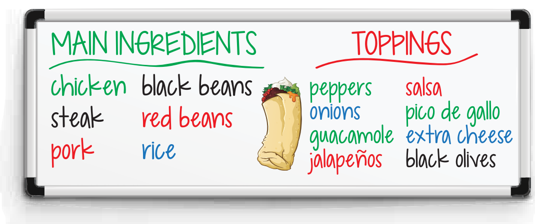
69. **MATHEMATICAL CONNECTIONS** A polygon is convex when no line that contains a side of the polygon contains a point in the interior of the polygon. Consider a convex polygon with n sides.



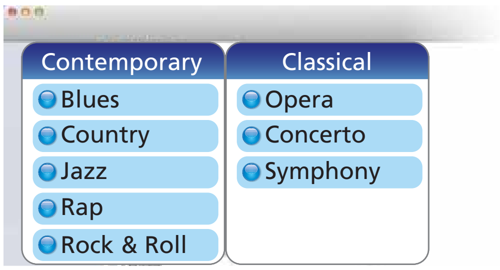
a. Use the combinations formula to write an expression for the number of diagonals in an n-sided polygon.

b. Use your result from part (a) to write a formula for the number of diagonals of an n-sided convex polygon.

70. **PROBLEM SOLVING** You are ordering a burrito with 2 main ingredients and 3 toppings. The menu below shows the possible choices. How many different burritos are possible?



71. **PROBLEM SOLVING** You want to purchase 2 different types of contemporary music CDs and 1 classical music CD from the music collection shown. How many different sets of music types can you choose for your purchase?



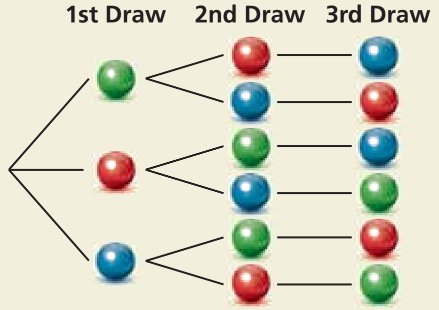
72. **PROBLEM SOLVING** Every student in your history class is required to present a project in front of the class. Each day, 4 students make their presentations in an order chosen at random by the teacher. You make your presentation on the first day.

a. What is the probability that you are chosen to be the first or second presenter on the first day?

b. What is the probability that you are chosen to be the second or third presenter on the first day? Compare your answer with that in part (a).

73. **PROBLEM SOLVING** The organizer of a cast party for a drama club asks each of the 6 cast members to bring 1 food item from a list of 10 items. Assuming each member randomly chooses a food item to bring, what is the probability that at least 2 of the 6 cast members bring the same item?

74. **HOW DO YOU SEE IT?** A bag contains one green marble, one red marble, and one blue marble. The diagram shows the possible outcomes of randomly drawing three marbles from the bag without replacement.



a. How many combinations of three marbles can be drawn from the bag? Explain.

b. How many permutations of three marbles can be drawn from the bag? Explain.

75. **PROBLEM SOLVING** You are one of 10 students performing in a school talent show. The order of the performances is determined at random. The first 5 performers go on stage before the intermission.

a. What is the probability that you are the last performer before the intermission and your rival performs immediately before you?

b. What is the probability that you are *not* the first performer?

76. **THOUGHT PROVOKING** How many integers, greater than 999 but not greater than 4000, can be formed with the digits 0, 1, 2, 3, and 4? Repetition of digits is allowed.

77. **PROBLEM SOLVING** Consider a standard deck of 52 playing cards. The order in which the cards are dealt for a "hand" does not matter.

a. How many different 5-card hands are possible?

b. How many different 5-card hands have all 5 cards of a single suit?

78. **PROBLEM SOLVING** There are 30 students in your class. Your science teacher chooses 5 students at random to complete a group project. Find the probability that you and your 2 best friends in the science class are chosen to work in the group. Explain how you found your answer.

79. **PROBLEM SOLVING** Follow the steps below to explore a famous probability problem called the *birthday problem*. (Assume there are 365 equally likely birthdays possible.)

a. What is the probability that at least 2 people share the same birthday in a group of 6 randomly chosen people? in a group of 10 randomly chosen people?

b. Generalize the results from part (a) by writing a formula for the probability $P(n)$ that at least 2 people in a group of n people share the same birthday. (*Hint*: Use ${}_nP_r$ notation in your formula.)

c. Enter the formula from part (b) into a graphing calculator. Use the *table* feature to make a table of values. For what group size does the probability that at least 2 people share the same birthday first exceed 50%?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

80. A bag contains 12 white marbles and 3 black marbles. You pick 1 marble at random. What is the probability that you pick a black marble? *(Section 10.1)*

81. The table shows the result of flipping two coins 12 times. For what outcome is the experimental probability the same as the theoretical probability? *(Section 10.1)*

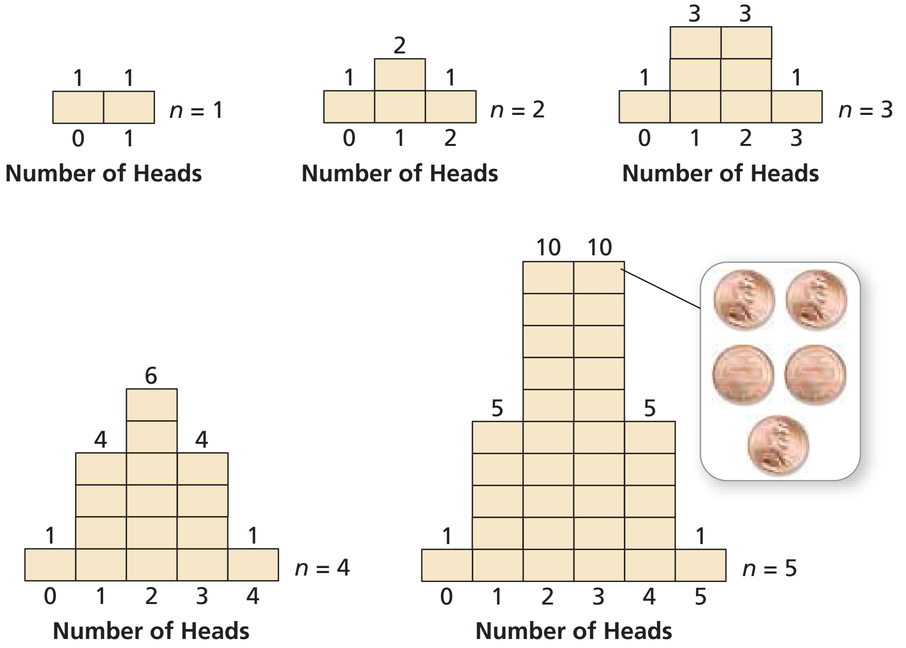
HH	HT	TH	TT
2	6	3	1

10.6 Binomial Distributions

Essential Question How can you determine the frequency of each outcome of an event?

EXPLORATION 1 Analyzing Histograms

Work with a partner. The histograms show the results when n coins are flipped.



STUDY TIP

When 4 coins are flipped ($n = 4$), the possible outcomes are

TTTT TTTH TTHT TTHH

THTT THTH THHT THHH

HTTT HTTH HTHT HTHH

HHTT HHTH HHHT HHHH.

The histogram shows the numbers of outcomes having 0, 1, 2, 3, and 4 heads.

a. In how many ways can 3 heads occur when 5 coins are flipped?

b. Draw a histogram that shows the numbers of heads that can occur when 6 coins are flipped.

c. In how many ways can 3 heads occur when 6 coins are flipped?

EXPLORATION 2 Determining the Number of Occurrences

Work with a partner.

a. Complete the table showing the numbers of ways in which 2 heads can occur when n coins are flipped.

n	3	4	5	6	7
Occurrences of 2 heads					

LOOKING FOR A PATTERN

To be proficient in math, you need to look closely to discern a pattern or structure.

b. Determine the pattern shown in the table. Use your result to find the number of ways in which 2 heads can occur when 8 coins are flipped.

Communicate Your Answer

3. How can you determine the frequency of each outcome of an event?

4. How can you use a histogram to find the probability of an event?

10.6 Lesson

Core Vocabulary

random variable, *p. 580*
probability distribution, *p. 580*
binomial distribution, *p. 581*
binomial experiment, *p. 581*

Previous
histogram

What You Will Learn

- Construct and interpret probability distributions.
- Construct and interpret binomial distributions.

Probability Distributions

A **random variable** is a variable whose value is determined by the outcomes of a probability experiment. For example, when you roll a six-sided die, you can define a random variable x that represents the number showing on the die. So, the possible values of x are 1, 2, 3, 4, 5, and 6. For every random variable, a *probability distribution* can be defined.

Core Concept

Probability Distributions

A **probability distribution** is a function that gives the probability of each possible value of a random variable. The sum of all the probabilities in a probability distribution must equal 1.

Probability Distribution for Rolling a Six-Sided Die

x	1	2	3	4	5	6
$P(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

EXAMPLE 1 Constructing a Probability Distribution

Let x be a random variable that represents the sum when two six-sided dice are rolled. Make a table and draw a histogram showing the probability distribution for x.

SOLUTION

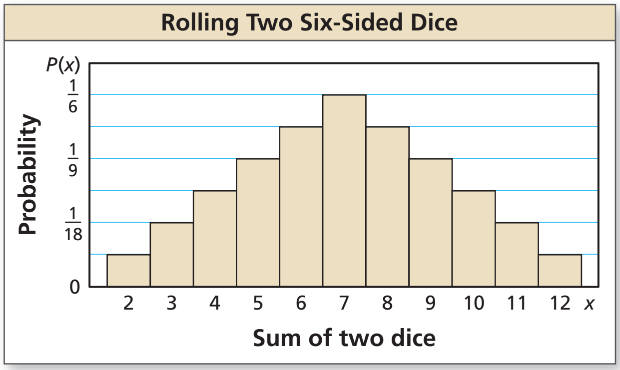
Step 1 Make a table. The possible values of x are the integers from 2 to 12. The table shows how many outcomes of rolling two dice produce each value of x. Divide the number of outcomes for x by 36 to find $P(x)$.

x (sum)	2	3	4	5	6	7	8	9	10	11	12
Outcomes	1	2	3	4	5	6	5	4	3	2	1
$P(x)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

STUDY TIP

Recall that there are 36 possible outcomes when rolling two six-sided dice. These are listed in Example 3 on page 540.

Step 2 Draw a histogram where the intervals are given by x and the frequencies are given by $P(x)$.



EXAMPLE 2 Interpreting a Probability Distribution

Use the probability distribution in Example 1 to answer each question.

a. What is the most likely sum when rolling two six-sided dice?

b. What is the probability that the sum of the two dice is at least 10?

SOLUTION

a. The most likely sum when rolling two six-sided dice is the value of x for which $P(x)$ is greatest. This probability is greatest for $x = 7$. So, when rolling the two dice, the most likely sum is 7.

b. The probability that the sum of the two dice is at least 10 is

$$\begin{aligned} P(x \geq 10) &= P(x = 10) + P(x = 11) + P(x = 12) \\ &= \tfrac{3}{36} + \tfrac{2}{36} + \tfrac{1}{36} \\ &= \tfrac{6}{36} \\ &= \tfrac{1}{6} \\ &\approx 0.167. \end{aligned}$$

▶ The probability is about 16.7%.

Monitoring Progress

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An octahedral die has eight sides numbered 1 through 8. Let x be a random variable that represents the sum when two such dice are rolled.

1. Make a table and draw a histogram showing the probability distribution for x.

2. What is the most likely sum when rolling the two dice?

3. What is the probability that the sum of the two dice is at most 3?

Binomial Distributions

One type of probability distribution is a **binomial distribution**. A binomial distribution shows the probabilities of the outcomes of a *binomial experiment*.

Core Concept

Binomial Experiments

A **binomial experiment** meets the following conditions.

- There are n independent trials.
- Each trial has only two possible outcomes: success and failure.
- The probability of success is the same for each trial. This probability is denoted by p. The probability of failure is $1 - p$.

For a binomial experiment, the probability of exactly k successes in n trials is

$$P(k \text{ successes}) = {}_nC_k p^k (1 - p)^{n - k}.$$

EXAMPLE 3 Constructing a Binomial Distribution

According to a survey, about 33% of people ages 16 and older in the U.S. own an electronic book reading device, or e-reader. You ask 6 randomly chosen people (ages 16 and older) whether they own an e-reader. Draw a histogram of the binomial distribution for your survey.

ATTENDING TO PRECISION

When probabilities are rounded, the sum of the probabilities may differ slightly from 1.

SOLUTION

The probability that a randomly selected person has an e-reader is $p = 0.33$. Because you survey 6 people, $n = 6$.

$P(k = 0) = {}_6C_0(0.33)^0(0.67)^6 \approx 0.090$

$P(k = 1) = {}_6C_1(0.33)^1(0.67)^5 \approx 0.267$

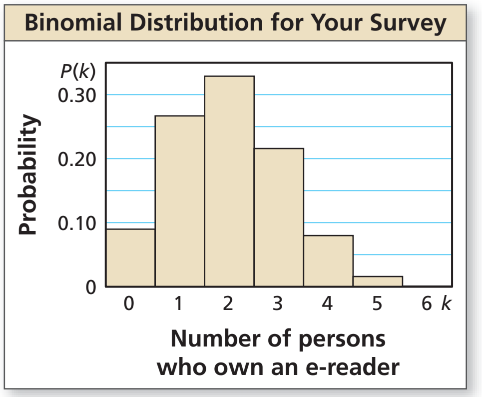
$P(k = 2) = {}_6C_2(0.33)^2(0.67)^4 \approx 0.329$

$P(k = 3) = {}_6C_3(0.33)^3(0.67)^3 \approx 0.216$

$P(k = 4) = {}_6C_4(0.33)^4(0.67)^2 \approx 0.080$

$P(k = 5) = {}_6C_5(0.33)^5(0.67)^1 \approx 0.016$

$P(k = 6) = {}_6C_6(0.33)^6(0.67)^0 \approx 0.001$



A histogram of the distribution is shown.

EXAMPLE 4 Interpreting a Binomial Distribution

Use the binomial distribution in Example 3 to answer each question.

a. What is the most likely outcome of the survey?

b. What is the probability that at most 2 people have an e-reader?

COMMON ERROR

Because a person may not have an e-reader, be sure you include $P(k = 0)$ when finding the probability that at most 2 people have an e-reader.

SOLUTION

a. The most likely outcome of the survey is the value of k for which $P(k)$ is greatest. This probability is greatest for $k = 2$. The most likely outcome is that 2 of the 6 people own an e-reader.

b. The probability that at most 2 people have an e-reader is

$$P(k \leq 2) = P(k = 0) + P(k = 1) + P(k = 2)$$
$$\approx 0.090 + 0.267 + 0.329$$
$$\approx 0.686.$$

▶ The probability is about 68.6%.

Monitoring Progress

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According to a survey, about 85% of people ages 18 and older in the U.S. use the Internet or e-mail. You ask 4 randomly chosen people (ages 18 and older) whether they use the Internet or e-mail.

4. Draw a histogram of the binomial distribution for your survey.

5. What is the most likely outcome of your survey?

6. What is the probability that at most 2 people you survey use the Internet or e-mail?

10.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** What is a random variable?

2. **WRITING** Give an example of a binomial experiment and describe how it meets the conditions of a binomial experiment.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, make a table and draw a histogram showing the probability distribution for the random variable. *(See Example 1.)*

3. x = the number on a table tennis ball randomly chosen from a bag that contains 5 balls labeled "1," 3 balls labeled "2," and 2 balls labeled "3."

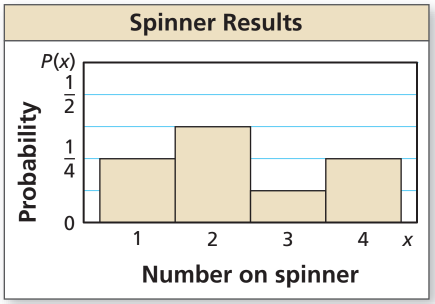
4. $c = 1$ when a randomly chosen card out of a standard deck of 52 playing cards is a heart and $c = 2$ otherwise.

5. $w = 1$ when a randomly chosen letter from the English alphabet is a vowel and $w = 2$ otherwise.

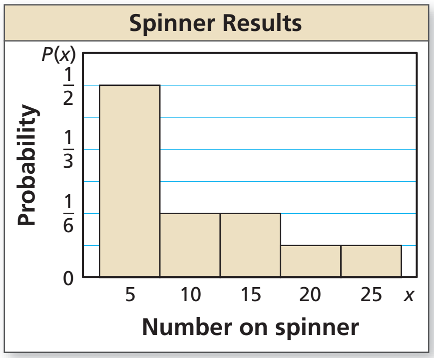
6. n = the number of digits in a random integer from 0 through 999.

In Exercises 7 and 8, use the probability distribution to determine (a) the number that is most likely to be spun on a spinner, and (b) the probability of spinning an even number. *(See Example 2.)*

7.



8.



USING EQUATIONS In Exercises 9–12, calculate the probability of flipping a coin 20 times and getting the given number of heads.

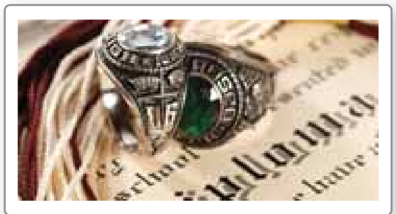
9. 1

10. 4

11. 18

12. 20

13. **MODELING WITH MATHEMATICS** According to a survey, 27% of high school students in the United States buy a class ring. You ask 6 randomly chosen high school students whether they own a class ring. *(See Examples 3 and 4.)*



a. Draw a histogram of the binomial distribution for your survey.

b. What is the most likely outcome of your survey?

c. What is the probability that at most 2 people have a class ring?

14. **MODELING WITH MATHEMATICS** According to a survey, 48% of adults in the United States believe that Unidentified Flying Objects (UFOs) are observing our planet. You ask 8 randomly chosen adults whether they believe UFOs are watching Earth.

a. Draw a histogram of the binomial distribution for your survey.

b. What is the most likely outcome of your survey?

c. What is the probability that at most 3 people believe UFOs are watching Earth?

ERROR ANALYSIS **In Exercises 15 and 16, describe and correct the error in calculating the probability of rolling a 1 exactly 3 times in 5 rolls of a six-sided die.**

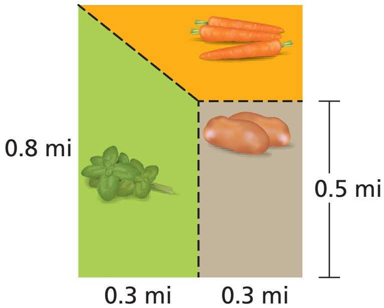
15.

$$P(k = 3) = {}_5C_3\left(\frac{1}{6}\right)^{5-3}\left(\frac{5}{6}\right)^3$$
$$\approx 0.161$$

16.

$$P(k = 3) = \left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{5-3}$$
$$\approx 0.003$$

17. **MATHEMATICAL CONNECTIONS** At most 7 gopher holes appear each week on the farm shown. Let x represent how many of the gopher holes appear in the carrot patch. Assume that a gopher hole has an equal chance of appearing at any point on the farm.



a. Find $P(x)$ for $x = 0, 1, 2, \ldots, 7$.

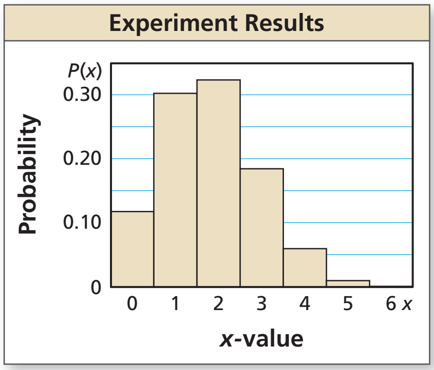
b. Make a table showing the probability distribution for x.

c. Make a histogram showing the probability distribution for x.

18. **HOW DO YOU SEE IT?** Complete the probability distribution for the random variable x. What is the probability the value of x is greater than 2?

x	1	2	3	4
$P(x)$	0.1	0.3	0.4	

19. **MAKING AN ARGUMENT** The binomial distribution shows the results of a binomial experiment. Your friend claims that the probability p of a success must be greater than the probability $1 - p$ of a failure. Is your friend correct? Explain your reasoning.



20. **THOUGHT PROVOKING** There are 100 coins in a bag. Only one of them has a date of 2010. You choose a coin at random, check the date, and then put the coin back in the bag. You repeat this 100 times. Are you certain of choosing the 2010 coin at least once? Explain your reasoning.

21. **MODELING WITH MATHEMATICS** Assume that having a male and having a female child are independent events, and that the probability of each is 0.5.

a. A couple has 4 male children. Evaluate the validity of this statement: "The first 4 kids were all boys, so the next one will probably be a girl."

b. What is the probability of having 4 male children and then a female child?

c. Let x be a random variable that represents the number of children a couple already has when they have their first female child. Draw a histogram of the distribution of $P(x)$ for $0 \le x \le 10$. Describe the shape of the histogram.

22. **CRITICAL THINKING** An entertainment system has n speakers. Each speaker will function properly with probability p, independent of whether the other speakers are functioning. The system will operate effectively when at least 50% of its speakers are functioning. For what values of p is a 5-speaker system more likely to operate than a 3-speaker system?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

List the possible outcomes for the situation. *(Section 10.1)*

23. guessing the gender of three children

24. picking one of two doors and one of three curtains

10.4–10.6 What Did You Learn?

Core Vocabulary

compound event, *p. 564*
overlapping events, *p. 564*
disjoint events, *p. 564*
mutually exclusive events, *p. 564*
permutation, *p. 570*
n factorial, *p. 570*
combination, *p. 572*
Binomial Theorem, *p. 574*
random variable, *p. 580*
probability distribution, *p. 580*
binomial distribution, *p. 581*
binomial experiment, *p. 581*

Core Concepts

Section 10.4

Probability of Compound Events, *p. 564*

Section 10.5

Permutations, *p. 571*
Combinations, *p. 572*
The Binomial Theorem, *p. 574*

Section 10.6

Probability Distributions, *p. 580*
Binomial Experiments, *p. 581*

Mathematical Practices

1. How can you use diagrams to understand the situation in Exercise 22 on page 568?
2. Describe a relationship between the results in part (a) and part (b) in Exercise 74 on page 578.
3. Explain how you were able to break the situation into cases to evaluate the validity of the statement in part (a) of Exercise 21 on page 584.

Performance Task

A New Dartboard

You are a graphic artist working for a company on a new design for the board in the game of darts. You are eager to begin the project, but the team cannot decide on the terms of the game. Everyone agrees that the board should have four colors. But some want the probabilities of hitting each color to be equal, while others want them to be different. You offer to design two boards, one for each group. How do you get started? How creative can you be with your designs?

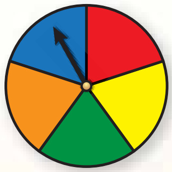
To explore the answers to these questions and more, go to ***BigIdeasMath.com***.

10 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

10.1 Sample Spaces and Probability *(pp. 537–544)*

Each section of the spinner shown has the same area. The spinner was spun 30 times. The table shows the results. For which color is the experimental probability of stopping on the color the same as the theoretical probability?



Spinner Results	
green	4
orange	6
red	9
blue	8
yellow	3

SOLUTION

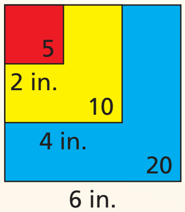
The theoretical probability of stopping on each of the five colors is $\frac{1}{5}$. Use the outcomes in the table to find the experimental probabilities.

$P(\text{green}) = \frac{4}{30} = \frac{2}{15}$ $\quad P(\text{orange}) = \frac{6}{30} = \frac{1}{5}$ $\quad P(\text{red}) = \frac{9}{30} = \frac{3}{10}$ $\quad P(\text{blue}) = \frac{8}{30} = \frac{4}{15}$ $\quad P(\text{yellow}) = \frac{3}{30} = \frac{1}{10}$

▶ The experimental probability of stopping on orange is the same as the theoretical probability.

1. A bag contains 9 tiles, one for each letter in the word HAPPINESS. You choose a tile at random. What is the probability that you choose a tile with the letter S? What is the probability that you choose a tile with a letter other than P?

2. You throw a dart at the board shown. Your dart is equally likely to hit any point inside the square board. Are you most likely to get 5 points, 10 points, or 20 points?



10.2 Independent and Dependent Events *(pp. 545–552)*

You randomly select 2 cards from a standard deck of 52 playing cards. What is the probability that both cards are jacks when (a) you replace the first card before selecting the second, and (b) you do not replace the first card. Compare the probabilities.

SOLUTION

Let event A be "first card is a jack" and event B be "second card is a jack."

a. Because you replace the first card before you select the second card, the events are independent. So, the probability is

$$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{4}{52} \cdot \frac{4}{52} = \frac{16}{2704} = \frac{1}{169} \approx 0.006.$$

b. Because you do not replace the first card before you select the second card, the events are dependent. So, the probability is

$$P(A \text{ and } B) = P(A) \cdot P(B|A) = \frac{4}{52} \cdot \frac{3}{51} = \frac{12}{2652} = \frac{1}{221} \approx 0.005.$$

▶ So, you are $\frac{1}{169} \div \frac{1}{221} \approx 1.3$ times more likely to select 2 jacks when you replace the first card before you select the second card.

Find the probability of randomly selecting the given marbles from a bag of 5 red, 8 green, and 3 blue marbles when (a) you replace the first marble before drawing the second, and (b) you do not replace the first marble. Compare the probabilities.

3. red, then green **4.** blue, then red **5.** green, then green

10.3 Two-Way Tables and Probability (pp. 553–560)

A survey asks residents of the east and west sides of a city whether they support the construction of a bridge. The results, given as joint relative frequencies, are shown in the two-way table. What is the probability that a randomly selected resident from the east side will support the project?

		Location	
		East Side	West Side
Response	Yes	0.47	0.36
	No	0.08	0.09

SOLUTION

Find the joint and marginal relative frequencies. Then use these values to find the conditional probability.

$$P(\text{yes} \mid \text{east side}) = \frac{P(\text{east side and yes})}{P(\text{east side})} = \frac{0.47}{0.47 + 0.08} \approx 0.855$$

▶ So, the probability that a resident of the east side of the city will support the project is about 85.5%.

6. What is the probability that a randomly selected resident who does not support the project in the example above is from the west side?

7. After a conference, 220 men and 270 women respond to a survey. Of those, 200 men and 230 women say the conference was impactful. Organize these results in a two-way table. Then find and interpret the marginal frequencies.

10.4 Probability of Disjoint and Overlapping Events (pp. 563–568)

Let A and B be events such that $P(A) = \frac{2}{3}$, $P(B) = \frac{1}{2}$, and $P(A \text{ and } B) = \frac{1}{3}$. Find $P(A \text{ or } B)$.

SOLUTION

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) && \text{Write general formula.} \\ &= \frac{2}{3} + \frac{1}{2} - \frac{1}{3} && \text{Substitute known probabilities.} \\ &= \frac{5}{6} && \text{Simplify.} \\ &\approx 0.833 && \text{Use a calculator.} \end{aligned}$$

8. Let A and B be events such that $P(A) = 0.32$, $P(B) = 0.48$, and $P(A \text{ and } B) = 0.12$. Find $P(A \text{ or } B)$.

9. Out of 100 employees at a company, 92 employees either work part time or work 5 days each week. There are 14 employees who work part time and 80 employees who work 5 days each week. What is the probability that a randomly selected employee works both part time and 5 days each week?

10.5 Permutations and Combinations (pp. 569–578)

A 5-digit code consists of 5 different integers from 0 to 9. How many different codes are possible?

SOLUTION

To find the number of permutations of 5 integers chosen from 10, find ${}_{10}P_5$.

$${}_{10}P_5 = \frac{10!}{(10-5)!} \quad \text{Permutations formula}$$

$$= \frac{10!}{5!} \quad \text{Subtract.}$$

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot \cancel{5}!}{\cancel{5}!} \quad \text{Expand factorials. Divide out common factor, 5!.}$$

$$= 30{,}240 \quad \text{Simplify.}$$

▶ There are 30,240 possible codes.

Evaluate the expression.

10. ${}_7P_6$ **11.** ${}_{13}P_{10}$ **12.** ${}_6C_2$ **13.** ${}_8C_4$

14. Use the Binomial Theorem to write the expansion of $(2x + y^2)^4$.

15. A random drawing will determine which 3 people in a group of 9 will win concert tickets. What is the probability that you and your 2 friends will win the tickets?

10.6 Binomial Distributions (pp. 579–584)

According to a survey, about 21% of adults in the U.S. visited an art museum last year. You ask 4 randomly chosen adults whether they visited an art museum last year. Draw a histogram of the binomial distribution for your survey.

SOLUTION

The probability that a randomly selected person visited an art museum is $p = 0.21$. Because you survey 4 people, $n = 4$.

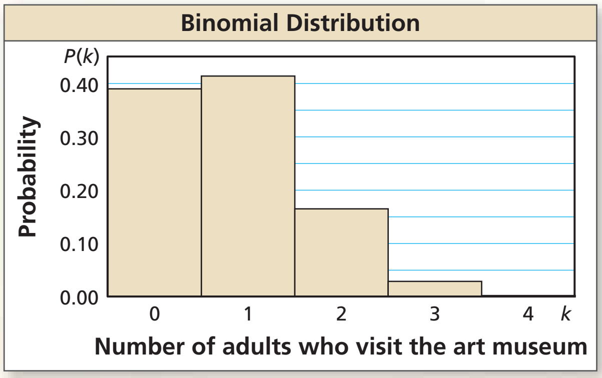
$$P(k = 0) = {}_4C_0(0.21)^0(0.79)^4 \approx 0.390$$

$$P(k = 1) = {}_4C_1(0.21)^1(0.79)^3 \approx 0.414$$

$$P(k = 2) = {}_4C_2(0.21)^2(0.79)^2 \approx 0.165$$

$$P(k = 3) = {}_4C_3(0.21)^3(0.79)^1 \approx 0.029$$

$$P(k = 4) = {}_4C_4(0.21)^4(0.79)^0 \approx 0.002$$



16. Find the probability of flipping a coin 12 times and getting exactly 4 heads.

17. A basketball player makes a free throw 82.6% of the time. The player attempts 5 free throws. Draw a histogram of the binomial distribution of the number of successful free throws. What is the most likely outcome?

10 Chapter Test

You roll a six-sided die. Find the probability of the event described. Explain your reasoning.

1. You roll a number less than 5.

2. You roll a multiple of 3.

Evaluate the expression.

3. ${}_7P_2$

4. ${}_8P_3$

5. ${}_6C_3$

6. ${}_{12}C_7$

7. Use the Binomial Theorem to write the binomial expansion of $(x + y^2)^5$.

8. You find the probability $P(A \text{ or } B)$ by using the equation $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. Describe why it is necessary to subtract $P(A \text{ and } B)$ when the events A and B are overlapping. Then describe why it is *not* necessary to subtract $P(A \text{ and } B)$ when the events A and B are disjoint.

9. Is it possible to use the formula $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$ when events A and B are independent? Explain your reasoning.

10. According to a survey, about 58% of families sit down for a family dinner at least four times per week. You ask 5 randomly chosen families whether they have a family dinner at least four times per week.

a. Draw a histogram of the binomial distribution for the survey.

b. What is the most likely outcome of the survey?

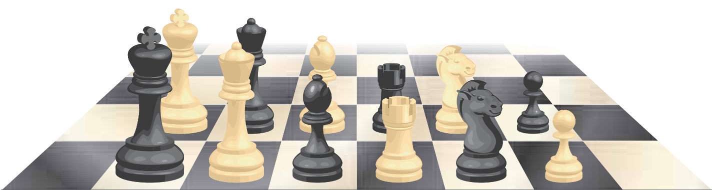
c. What is the probability that at least 3 families have a family dinner four times per week?

11. You are choosing a cell phone company to sign with for the next 2 years. The three plans you consider are equally priced. You ask several of your neighbors whether they are satisfied with their current cell phone company. The table shows the results. According to this survey, which company should you choose?

	Satisfied	Not Satisfied
Company A	IIII	II
Company B	IIII	III
Company C	~~IIII~~ I	~~IIII~~

12. The surface area of Earth is about 196.9 million square miles. The land area is about 57.5 million square miles and the rest is water. What is the probability that a meteorite that reaches the surface of Earth will hit land? What is the probability that it will hit water?

13. Consider a bag that contains all the chess pieces in a set, as shown in the diagram.



	King	Queen	Bishop	Rook	Knight	Pawn	
Black	1	1	2	2	2	8	
White	1	1	2	2	2	8	

a. You choose one piece at random. Find the probability that you choose a black piece or a queen.

b. You choose one piece at random, do not replace it, then choose a second piece at random. Find the probability that you choose a king, then a pawn.

14. Three volunteers are chosen at random from a group of 12 to help at a summer camp.

a. What is the probability that you, your brother, and your friend are chosen?

b. The first person chosen will be a counselor, the second will be a lifeguard, and the third will be a cook. What is the probability that you are the cook, your brother is the lifeguard, and your friend is the counselor?

10 Cumulative Assessment

1. According to a survey, 63% of Americans consider themselves sports fans. You randomly select 14 Americans to survey.

a. Draw a histogram of the binomial distribution of your survey.

b. What is the most likely number of Americans who consider themselves sports fans?

c. What is the probability at least 7 Americans consider themselves sports fans?

2. Order the acute angles from smallest to largest. Explain your reasoning.

$\tan \theta_1 = 1$

$\tan \theta_2 = \frac{1}{2}$

$\tan \theta_3 = \frac{\sqrt{3}}{3}$

$\tan \theta_4 = \frac{23}{4}$

$\tan \theta_5 = \frac{38}{5}$

$\tan \theta_6 = \sqrt{3}$

3. You order a fruit smoothie made with 2 liquid ingredients and 3 fruit ingredients from the menu shown. How many different fruit smoothies can you order?

4. Which statements describe the transformation of the graph of $f(x) = x^3 - x$ represented by $g(x) = 4(x - 2)^3 - 4(x - 2)$?

Ⓐ a vertical stretch by a factor of 4

Ⓑ a vertical shrink by a factor of $\frac{1}{4}$

Ⓒ a horizontal shrink by a factor of $\frac{1}{4}$

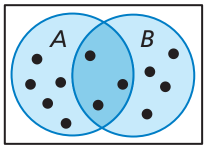
Ⓓ a horizontal stretch by a factor of 4

Ⓔ a horizontal translation 2 units to the right

Ⓕ a horizontal translation 2 units to the left

5. Use the diagram to explain why the equation is true.

$$P(A) + P(B) = P(A \text{ or } B) + P(A \text{ and } B)$$



6. For the sequence $-\frac{1}{2}, -\frac{1}{4}, -\frac{1}{6}, -\frac{1}{8}, \ldots$, describe the pattern, write the next term, graph the first five terms, and write a rule for the *n*th term.

7. A survey asked male and female students about whether they prefer to take gym class or choir. The table shows the results of the survey.

		Class		
		Gym	**Choir**	**Total**
Gender	**Male**			50
	Female	23		
	Total		49	106

a. Complete the two-way table.

b. What is the probability that a randomly selected student is female and prefers choir?

c. What is the probability that a randomly selected male student prefers gym class?

8. The owner of a lawn-mowing business has three mowers. As long as one of the mowers is working, the owner can stay productive. One of the mowers is unusable 10% of the time, one is unusable 8% of the time, and one is unusable 18% of the time.

a. Find the probability that all three mowers are unusable on a given day.

b. Find the probability that at least one of the mowers is unusable on a given day.

c. Suppose the least-reliable mower stops working completely. How does this affect the probability that the lawn-mowing business can be productive on a given day?

9. Write a system of quadratic inequalities whose solution is represented in the graph.