Chapter 1
Mathematical Practices

1. In the square viewing window, the graph is steeper.

2. In the square viewing window, the graph is narrower.

3. In the square viewing window, the graph is wider.

4. In the standard viewing window, the graph is flatter.

5. In the square viewing window, the graph is wider.

6. In the square viewing window, the graph is wider.

1.1 Lesson Monitoring Progress

3. The graph is 2 units to the right of the parent quadratic function.

4. The graph is a reflection in the x-axis of the parent absolute value function.

6. The graph of \( h \) is a vertical stretch of the parent quadratic function.

7. The graph of \( c \) is a vertical shrink of the parent absolute value function.

8. The graph of \( h \) is a reflection in the x-axis, a vertical shrink followed by a vertical translation 5 units up of the parent linear function.

9. The graph of \( d \) is a vertical stretch followed by a translation 5 units right and 1 unit down of the parent quadratic function.
11. The graph of $f$ is a vertical translation 1 unit down of the parent quadratic function.

12. The graph of $h$ is a horizontal translation 4 units left of the parent quadratic function.

13. The graph of $g$ is a horizontal translation 5 units right of the parent absolute value function.

14. The graph of $f$ is a vertical translation 4 units up of the parent absolute value function.

15. The graph of $h$ is a reflection in the $x$-axis of the parent quadratic function.

16. The graph of $g$ is a reflection in the $x$-axis of the parent linear function.

17. The graph of $f$ is a vertical translation 2 units up of the parent constant function.

18. The graph of $f$ is a vertical translation 3 units down of the parent constant function.

25. The graph of $h$ is a vertical stretch of the parent absolute value function.

26. The graph of $f$ is a vertical shrink of the parent absolute value function.

27. The graph of $f$ is a vertical stretch followed by a translation 2 units up of the parent linear function.

A2 Additional Answers
1. unit right and 1

The graph of \( h \) is a reflection in the \( x \) axis followed by a translation 5 units up of the parent linear function.

29.

The graph of \( h \) is a vertical stretch and a reflection in the \( x \)-axis followed by a translation 1 unit down of the parent absolute value function.

30.

The graph of \( f \) is a vertical shrink followed by a translation 1 unit up of the parent absolute value function.

31.

The graph of \( g \) is a vertical shrink followed by a translation 6 units down of the parent quadratic function.

32.

The graph of \( f \) is a vertical stretch followed by a translation 3 units down of the parent quadratic function.

33.

The graph of \( f \) is a reflection in the \( x \)-axis followed by a translation 3 units left and \( \frac{1}{2} \) unit up of the parent quadratic function.

34.

The graph of \( g \) is a reflection in the \( x \)-axis followed by a translation 1 unit right and \( \frac{1}{2} \) unit down of the parent absolute value function.
1.3 Monitoring Progress and Modeling with Mathematics

26. a. \(-5 - 4\); the decrease in balance per month
b. domain: \(0 \leq x \leq 24\), range: \(0 \leq y \leq 3000\); The domain represents the term of the balance of the loan from start to finish, and the range represents the balance of the loan.

c. \(\$1500\)

27. no; Because \(r\) is close to 0, the points do not lie close to the line.

28. Sample answer: A: \((0, 4)\), B: \((4, 0)\), C: \((0, 0)\); line connecting A and C: \(x = 0\); line connecting B and C: \(y = 0\)

29. It is negative; As \(x\) increases, \(y\) increases, so \(z\) decreases.

30. D

31. about 2.2 mi

1.3 Maintaining Mathematical Proficiency

33. \((16, -41)\)

34. \((-1, 0)\)

35. \(\left(\frac{16}{11}, \frac{5}{11}\right)\)

36. \((6, 2)\)

1.4 Maintaining Mathematical Proficiency

44. \(x^2 - 4x + 4\)

45. \(9m^2 + 6m + 1\)

46. \(4z^2 - 20z + 25\)

47. \(16 - 8y + y^2\)

48. \(g(x) = |x + 2| - 5\)

49. \(g(x) = -|x| + 5\)

50. \(g(x) = |x| - 1\)

51. \(g(x) = 3|x| - 15\)

Chapter 2

2.1 Lesson Monitoring Progress

1. The graph of \(g\) is a translation 3 units right of the graph of \(f\).

2. The graph of \(g\) is a translation 2 units right and 2 units down of the graph of \(f\).

A4 Additional Answers
3. The graph of \( g \) is a translation 5 units left and 1 unit up of the graph of \( f \).

11. The graph of \( g \) is a translation 7 units right and 1 unit up of the graph of \( f \).

2.1 Monitoring Progress and Modeling with Mathematics

8. The graph of \( g \) is a translation 3 units left of the graph of \( f \).

12. The graph of \( g \) is a translation 10 units left and 3 units down of the graph of \( f \).

9. The graph of \( g \) is a translation 6 units left and 2 units down of the graph of \( f \).

13. A; The graph has been translated 1 unit right.

14. D; The graph has been translated 1 unit up.

15. C; The graph has been translated 1 unit right and 1 unit up.

16. B; The graph has been translated 1 unit left and 1 unit down.

17. The graph of \( g \) is a reflection in the \( x \)-axis of the graph of \( f \).

18. The graph of \( g \) is a reflection in the \( y \)-axis of the graph of \( f \).
19. The graph of $g$ is a vertical stretch by a factor of 3 of the graph of $f$.  

![Graph of $g$ is a vertical stretch by a factor of 3 of the graph of $f$.]

20. The graph of $g$ is a vertical shrink by a factor of $\frac{1}{2}$ of the graph of $f$.  

![Graph of $g$ is a vertical shrink by a factor of $\frac{1}{2}$ of the graph of $f$.]

21. The graph of $g$ is a horizontal shrink by a factor of $\frac{1}{2}$ of the graph of $f$.  

![Graph of $g$ is a horizontal shrink by a factor of $\frac{1}{2}$ of the graph of $f$.]

22. The graph of $g$ is a horizontal shrink by a factor of $\frac{1}{2}$ followed by a reflection in the $x$-axis of the graph of $f$.  

![Graph of $g$ is a horizontal shrink by a factor of $\frac{1}{2}$ followed by a reflection in the $x$-axis of the graph of $f$.]

23. The graph of $g$ is a vertical shrink by a factor of $\frac{1}{2}$ followed by a translation 4 units down.  

![Graph of $g$ is a vertical shrink by a factor of $\frac{1}{2}$ followed by a translation 4 units down.]

24. The graph of $g$ is a vertical shrink by a factor of $\frac{1}{4}$ followed by a translation 1 unit right.  

![Graph of $g$ is a vertical shrink by a factor of $\frac{1}{4}$ followed by a translation 1 unit right.]

25. The graph is a reflection in the $x$-axis, not $y$-axis; The graph is a reflection in the $x$-axis and a vertical stretch by a factor of 6, followed by a translation 4 units up of the graph of the parent quadratic function.  

26. The order of the transformations is not correct and the graph is a translation 4 units up, not down; The graph is a vertical stretch by a factor of 6 and a reflection in the $x$-axis followed by a translation 4 units up of the graph of the parent quadratic function.  

27. The graph of $f$ is a vertical stretch by a factor of 3 followed by a translation 2 units left and 1 unit up of the graph of the parent quadratic function; $(-2, 1)$  

28. The graph of $f$ is a vertical stretch by a factor of 4 followed by a reflection in the $x$-axis and a translation 1 unit left and 5 units down of the graph of the parent quadratic function; $(-1, -5)$  

29. The graph of $f$ is a vertical stretch by a factor of 2 followed by a reflection in the $x$-axis and a translation 5 units up of the graph of the parent quadratic function; $(0, 5)$  

30. The graph of $f$ is a vertical shrink by a factor of $\frac{1}{2}$ followed by a translation 1 unit right of the graph of the parent quadratic function; $(1, 0)$  

31. $a(y) = \frac{-5}{1089}(x - 33)^2 + 5$  

   b. The domain is $0 \leq x \leq 66$ and the range is $0 \leq y \leq 5$; The domain represents the horizontal distance and the range represents the height of the fish.  

   c. yes; The value changes to $-\frac{1}{233}$; The vertex has changed but it still goes through the point $(0, 0)$, so there has been a horizontal stretch or shrink which changes the value of $a$.  

32. The graph of $g$ is a translation 4 units left and 2 units down of the graph of $f$.  

33. $a = 2, h = 1, k = 6; g(x) = 2(x - 1)^2 + 6$  

   b. $g(x) = 2f(x - 1) + 6$; For each function, $a$, $h$, and $k$ are the same but the answer in part (b) does not indicate the type of function that is being translated.  

   c. $a = 2, h = 1, k = 3; g(x) = 2(x - 1)^2 + 3; g(x) = 2f(x - 1) + 3$; For each function, $a$, $h$, and $k$ are the same, but the second function does not indicate the type of function that is being translated.  

   d. Sample answer: vertex form; Writing a transformed function using function notation requires an extra step of substituting $f(x)$ into the newly transformed function.  

34. $g(x) = f(2x) + 6 = -0.5(2x - 6)^2 + 24$; You jump higher but not as far.  

35. a vertical shrink by a factor of $\frac{7}{16}$
2.2 Explorations

2. a. \(-\frac{2}{3}, \frac{2}{3}, 3, \frac{5}{3}, 6, \frac{4}{3}, 3\)

2.2 Lesson Monitoring Progress

2. \(x = 2\)

3. \(f(x) = x^2 + 2x - 1\)

4. \(g(x) = 2(x - 2)^2 + 5\)

2.2 Monitoring Progress and Modeling with Mathematics

6. \(f(x) = -(x + 1)(x + 5)\)

7. \(f(x) = \frac{1}{4}(x - 6)(x - 2)\)

2.2 Explorations

6. \(-3, 4\)

7. \((-5, 0), (-1, 0)\)

2.2 Lesson Monitoring Progress

2. \(x = 2\)

3. \(f(x) = x^2 + 2x - 1\)

4. \(g(x) = 2(x - 2)^2 + 5\)

2.2 Monitoring Progress and Modeling with Mathematics

8. \((-1, 3)\)

9. \((1, -5)\)

10. \((-4, 6)\)

11. \((-2, 1)\)
12. $x = 3$
13. $x = -1$
14. $x = 0$
15. C
16. D
17. B
18. A
19. $x = 2$
20. $x = -3$
21. $x = -1$
22. $x = 0$
23. $x = 1$
24. $x = -3$
31. Both functions have an axis of symmetry of \(x = 2\).
32. If the graph is increasing until it reaches \(x = 2\) and then decreasing after, then the vertex must be the highest point.
50. a. 3090 rev/min; 74.68 ft-lbs
   b. The engine torque increases as the speed increases until the engine speed reaches 3.09 thousands of revolutions per minutes then the torque begins to decrease.
51. \(A = w(20 - w) = -w^2 + 20w\); The maximum area is 100 square units.
52. \(A = \frac{1}{2} b(6 - b) = -\frac{1}{2}b^2 + 3b\); The maximum area is 4.5 square units.
59. \(p = 2\), \(q = -6\); The graph is decreasing to the left of \(x = -2\) and increasing to the right of \(x = -2\).
60. \(p = -1\), \(q = 3\); The graph is decreasing to the left of \(x = 1\) and increasing to the right of \(x = 1\).
61. \(p = 2\), \(q = -6\); The graph is decreasing to the left of \(x = -2\) and increasing to the right of \(x = -2\).
62. \(p = -1\), \(q = 3\); The graph is decreasing to the left of \(x = 1\) and increasing to the right of \(x = 1\).
63. \(p = 4\), \(q = 2\); The graph is increasing to the left of \(x = 3\) and decreasing to the right of \(x = 3\).
64. \(p = -5\), \(q = -1\); The graph is increasing to the left of \(x = -3\) and decreasing to the right of \(x = -3\).
65. the second kick; the first kick
66. 160 ft; about 1.5 ft
67. no; Either of the points could be the axis of symmetry, or neither of the points could be the axis of symmetry. You can only determine the axis of symmetry if the \(y\)-coordinates of the two points are the same, because the axis of symmetry would lie halfway between the two points.
68. Sample answer: \(y = 2(x - 2)(x - 4)\) and \(y = 2(x + 1)(x - 7)\)
69. $1.75
70. $300
71. All three graphs are the same; \(f(x) = x^2 + 4x + 3\), \(g(x) = x^2 + 4x + 3\)
72. \( f(x) = (x + 4)(x - 3) \)

73. no; The vertex of the graph is (3.25, 2.1125), which means the mouse cannot jump over a fence that is higher than 2.1125 feet.

74. a. the minimum
b. Instead of representing the minimum, \( f\left(\frac{p + q}{2}\right) \) would represent the maximum.

75. The domain is \( 0 \leq x \leq 126 \) and the range is \( 0 \leq y \leq 50 \); The domain represents the distance from the start of the bridge on one side of the river, and the range represents the height of the bridge.

76. Sample answer:

Of all possible designs, a square garden with sides of 25 feet will have the greatest area; A square has the largest area of all rectangles with the same perimeter.

77. no; The vertex must lie on the axis of symmetry, and \((0, 5)\) does not lie on \(x = -1\).

78. The y-intercept is \( apq \).

79. a. about 14.1%; about 55.5 cm\(^3\)/g
b. about 13.6%; about 44.1 cm\(^3\)/g

c. The domain for hot-air popping is \( 5.52 \leq x \leq 22.6 \), and the range is \( 0 \leq y \leq 55.5 \). The domain for hot-oil popping is \( 5.35 \leq x \leq 21.8 \), and the range is \( 0 \leq y \leq 44.1 \). This means that the moisture content for the kernels can range from 5.52% to 22.6% and 5.35% to 21.8%, while the popping volume can range from 0 to 55.5 cubic centimeters per gram and 0 to 44.1 cubic centimeters per gram.

80. The \( x \)-coordinate of the vertex does not change, and the \( y \)-coordinate moves further from the \( x \)-axis; The \( x \)-coordinate of the vertex does not change, and the \( y \)-coordinate moves closer to the \( x \)-axis.

2.2 Maintaining Mathematical Proficiency

81. 4 82. 8
83. no solution 84. -2
85. 2 86. 6
87. -12 88. -8

2.2 Mini-Assessment

5. Because 90 yards < 100 yards, your shot travels farther. Because 81 yards > 25 yards, your opponent’s shot travels higher.

2.1–2.2 Quiz

9. 2.3 Explorations

1. a. (0,1); This is the best place for the receiver because any ray entering the satellite dish will hit the receiver.

b. All the reflected rays cross through the point (0, 1).

A10 Additional Answers
2. The focus is (4, 0). The directrix is \( x = -4 \). The axis of symmetry is the \( x \)-axis.

17. The focus is (4, 0). The directrix is \( x = -4 \). The axis of symmetry is the \( x \)-axis.

18. The focus is (0, -12). The directrix is \( y = 12 \). The axis of symmetry is the \( y \)-axis.

19. The focus is \( \left(0, \frac{-1}{8}\right) \). The directrix is \( y = \frac{1}{8} \). The axis of symmetry is the \( y \)-axis.

20. The focus is \( \left(0, \frac{1}{32}\right) \). The directrix is \( y = -\frac{1}{32} \). The axis of symmetry is the \( y \)-axis.

All beams leaving the parabola are parallel; yes. With a spotlight, all beams of light should be pointed at the same object.
21. Instead of a vertical axis of symmetry, the graph should have a horizontal axis of symmetry.

22. Because \( p = -0.5 \), the focus is \((-0.5, 0)\), and the directrix is \( x = 0.5 \).

23. 9.5 in.; The receiver should be placed at the focus. The distance from the vertex to the focus is \( p = \frac{19}{4} = 9.5 \) in.

2.4 Maintaining Mathematical Proficiency

38. \((x + 3)(x + 1)\) 39. \((x - 2)(x - 1)\)

40. \(3(x - 4)(x - 1)\) 41. \(5(x + 3)(x - 2)\)

Chapter 2 Review

2. The graph is a translation 7 units right and 2 units up of the parent quadratic function.

7. The maximum value is 35; The function is increasing to the left of \( x = 4 \) and decreasing to the right of \( x = 4 \).
8. The minimum value is $-25$; The function is decreasing to the left of $x = -2$ and increasing to the right of $x = -2$. 

![Graph of a parabola]

**Chapter 3**

Mathematical Practices

7. $y = -7 + 2\sqrt{-4x^2 + 20.75}$

$-9.6 < x < 9.6$ and $-6.4 < y < 6.4$; The graph has a width-height ratio of $3:2$ and shows the graph touching the $x$-axis.

3.1 Monitoring Progress and Modeling with Mathematics

71. you; The function does not cross the $x$-axis.

72. $(x - 2)(x + 2), (x - 3)(x + 3), (x - a)(x + a)$

![Graph of a quadratic function]

73. a. $mn = a^2$ and $m + n = 0$

b. $m = \sqrt{-a^2} = a\sqrt{-1}$, 
   $n = -\sqrt{-a^2} = -a\sqrt{-1}$; 
   $m$ and $n$ are not real numbers

74. Sample answer:

![Graph of a parabola]

$(4 + 2x)(6 + 2x) = 48; x = 1; 6$ ft by $8$ ft

75. 60 ft

3.1 Maintaining Mathematical Proficiency

76. $3x^2 - x + 2$

77. $x^3 + 4x^2 + 6$

78. $3x^2 - 3x + 1$

79. $-3x^3 + 7x^2 - 15x + 9$

80. $x^2 - 4$

81. $10x^3 - 2x^2 + 6x$

82. $-x^3 + 8x - 7$

83. $-4x^3 + 33x^2 + 88x$

3.2 Monitoring Progress and Modeling with Mathematics

67. Powers of $i$

<table>
<thead>
<tr>
<th>Simplified form</th>
<th>$i^1$</th>
<th>$i^2$</th>
<th>$i^3$</th>
<th>$i^4$</th>
<th>$i^5$</th>
<th>$i^6$</th>
<th>$i^7$</th>
<th>$i^8$</th>
<th>$i^9$</th>
<th>$i^{10}$</th>
<th>$i^{11}$</th>
<th>$i^{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i^1$</td>
<td>$-1$</td>
<td>$i$</td>
<td>$-i$</td>
<td>$1$</td>
<td>$i$</td>
<td>$-i$</td>
<td>$1$</td>
<td>$i$</td>
<td>$-i$</td>
<td>$1$</td>
<td>$i$</td>
<td>$-i$</td>
</tr>
</tbody>
</table>

The results of $i^n$ alternate in the pattern $i, -1, -i, 1$.

3.2 Maintaining Mathematical Proficiency

79. yes

80. no

81. no

82. $y = (x - 1)^2 + 2$

83. $y = 2(x + 3)^2 - 3$

84. $y = -(x - 2)^2 - 1$

3.3 Explorations

2. a. 

<table>
<thead>
<tr>
<th>Expression</th>
<th>Value of $c$ needed to complete the square</th>
<th>Expression written as a binomial squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2 + 2x + c$</td>
<td>1</td>
<td>$(x + 1)^2$</td>
</tr>
<tr>
<td>$x^2 + 4x + c$</td>
<td>4</td>
<td>$(x + 2)^2$</td>
</tr>
<tr>
<td>$x^2 + 8x + c$</td>
<td>16</td>
<td>$(x + 4)^2$</td>
</tr>
<tr>
<td>$x^2 + 10x + c$</td>
<td>25</td>
<td>$(x + 5)^2$</td>
</tr>
</tbody>
</table>

3.3 Monitoring Progress and Modeling with Mathematics

72. a. 

![Graph of a parabola]

b. The graph is shifted vertically, but the axis of symmetry does not change.

73. $x = 0.896$ cm
3.4 Monitoring Progress and Modeling with Mathematics

57. $x = \frac{-1 \pm \sqrt{5}}{2}$; Sample answer: Quadratic Formula; $b$ is not an even number, the equation cannot be factored, and it cannot be easily written in the form $a^2 = d$.

58. $x = -2 \pm 2i$; Sample answer: completing the square; Factor out 9, and $a = 1$ and $b$ is an even number.

59. $x = 6

60. $x = 3.51$

61. about 5.67 sec

62. no; yes; For $ax^2 + 5x + c = 0$ to have one real solution, $ac = \frac{25}{4}$; For any nonzero number $a$, $c = \frac{25}{4a}$

A14 Additional Answers
84. [Graph showing a quadratic equation and its solutions]

3.5 Explorations
2. a. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-1$</th>
<th>$-\frac{1}{2}$</th>
<th>$0$</th>
<th>$\frac{1}{2}$</th>
<th>$1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x^2 + 2x + 1$</td>
<td>$0$</td>
<td>$\frac{1}{4}$</td>
<td>$1$</td>
<td>$\frac{9}{4}$</td>
<td>$4$</td>
</tr>
<tr>
<td>$y = -x^2 + x + 2$</td>
<td>$0$</td>
<td>$\frac{5}{4}$</td>
<td>$2$</td>
<td>$\frac{9}{4}$</td>
<td>$2$</td>
</tr>
</tbody>
</table>

3.5 Monitoring Progress and Modeling with Mathematics
53. no solution; $m = 1$; one solution: $m = 0$; two solutions: $m = -1$

54. your friend; $(3,\ -3)$ and $(-3,\ 3)$ are solutions to Equation 1 but not Equation 2.

55. Sample answer: graphing and Quadratic Formula; graphing because it requires less time and steps than using the Quadratic Formula in this case

56. If $(x, y)$ is a solution, then $(-x, -y)$ must be a solution. The solutions will always be reflections in the origin.

57. a. no solution, one solution, two solutions, three solutions, or four solutions
57. b. (continued)

58. (0, 0) and (2, 3); Translate 2 units up.
59. a. circle: $x^2 + y^2 = 1$, Oak Lane: $y = -\frac{1}{3}x + \frac{2}{3}$
   b. $(-0.6, 0.8)$ and $(0.8, 0.6)$
   c. about 1.41 mi
60. (0, 2)

3.5 Maintaining Mathematical Proficiency
61. $x > 3$
   
62. $x \leq -3$
   
63. $x \leq -4$
   
64. $y \geq -x + 1$
65. $y < x - 2$
66. $y > 2x - 3$

3.6 Lesson Monitoring Progress
1. 
2. 
3. 

3.6 Monitoring Progress and Modeling with Mathematics
10. 
11. 
12. 
13. 
14. 
15. $y > f(x)$
16. $y > f(x)$
17. The graph should be solid, not dashed.
18. The wrong side of the parabola is shaded.

19. The solution represents weights that can be supported by shelves with various thicknesses.

20. The solution represents weights that can be supported by wire ropes with various diameters.

21.

22.

23.

24.

25.

26.

49. \(0.00170x^2 + 0.145x + 2.35 > 10, 0 \leq x \leq 40\); after about 37 days; Because \(L(x)\) is a parabola, \(L(x) = 10\) has two solutions. Because the \(x\)-value must be positive, the domain requires that the negative solution be rejected.

50. your friend; Any points with negative \(y\)-coordinates are solutions.

51. a. \(\frac{32}{3} \approx 10.67\) square units
   b. \(\frac{256}{3} \approx 85.33\) square units
52. **Sample answer:**

The intersection of $y \leq \frac{1}{2}x^2 + 3$ and $y \geq \frac{1}{3}x^2$ create a shape similar to a smile, which could be used by a company that sells toothpaste.

53. **a.** yes; The points on the parabola that are exactly 11 feet high are (6, 11) and (14, 11). Because these points are 8 feet apart, there is enough room for a 7-foot wide truck.

   **b.** 8 ft

   **c.** about 11.2 ft

### 3.6 Maintaining Mathematical Proficiency

54. **55.**

56. **57.** The maximum value is $-1$; The function is increasing to the left of $x = -3$ and decreasing to the right of $x = -3$.

58. The minimum value is $-1$; The function is decreasing to the left of $x = -2$ and increasing to the right of $x = -2$.

59. The maximum value is 25; The function is increasing to the left of $x = -2$ and decreasing to the right of $x = -2$.

60. The minimum value is $-20.25$; The function is decreasing to the left of $x = -\frac{3}{2}$ and increasing to the right of $x = -\frac{3}{2}$.

### Chapter 4

#### 4.1 Lesson Monitoring Progress

7. **8.**

9. The maximum value is 25; The function is increasing to the left of $x = -2$ and decreasing to the right of $x = -2$.
10. The degree is even and the leading coefficient is positive.

37. The degree is even and the leading coefficient is positive.

11. a. The degree is odd and the leading coefficient is negative.

38. The degree is odd and the leading coefficient is negative.

From 2001 to 2004, the number of electric vehicles in use increased. Around 2005, the growth in the numbers in use slowed, but continued to increase through 2010.

b. about 4.4 thousand
c. The model has a very sharp decline beginning around the year 2010. While the model may be used for a few years after 2010, in the long run, \( V(t) < 0 \) is not reasonable. \( V(0) \) is negative. Because negative values do not make sense given the context, the model should not be used for years before 2001.

4.1 Monitoring Progress and Modeling with Mathematics

31. b. The function is increasing when \( x > 4 \) and decreasing when \( x < 4 \).

32. a. The function is increasing when \( x < -4 \) and decreasing when \( x > -4 \).

33. a. The function is increasing when \( x < 4 \) and decreasing when \( x > 4 \).

34. a. The function is increasing when \( x < -4 \) and decreasing when \( x > -4 \).

35. a. The function is increasing when \( x < 0 \) and \( x > 2 \) and decreasing when \( 0 < x < 2 \).

36. a. The function is increasing when \( x < -1 \) and \( x > 1 \) and decreasing when \( -1 < x < 1 \).

37. The degree is even and the leading coefficient is positive.

38. The degree is odd and the leading coefficient is negative.

39. The degree is even and the leading coefficient is positive.

40. The degree is odd and the leading coefficient is positive.

41. a. From 1980 to 2007 the number of open drive-in theaters decreased. Around the year 1995, the rate of decrease began to level off.
42. \( w = 8.53 \) carats

44. **Sample answer:** \( y = -x^2 + 4 \)

45. The viewing window is appropriate if it shows the end behavior of the graph as \( x \to \infty \) and \( x \to -\infty \).

46. If the table is showing the end behavior of each function, then your friend is correct; \( f(x) \to \infty \) as \( x \to \infty \) and as \( x \to -\infty \), so \( f \) is even. \( g(x) \to \infty \) as \( x \to -\infty \) and \( g(x) \to -\infty \) as \( x \to \infty \), so \( g \) is odd.

47. a. The graph of \( y = x \), \( y = x^3 \), and \( y = x^5 \) are all symmetric with respect to the origin.

b. The graph of \( y = x^2 \), \( y = x^4 \), and \( y = x^6 \) are all symmetric with respect to the \( y \)-axis.

c. The graph of \( y = x^{10} \) will be symmetric with respect to the \( y \)-axis. The graph of \( y = x^{11} \) will be symmetric with respect to the origin; The exponent is even. The exponent is odd.

48. a. The degree is odd and the leading coefficient is positive.

b. The function is increasing on the intervals \((-\infty, -3)\) and \((-1, \infty)\), and decreasing on the interval \((-3, -1)\).

c. 4

49. \( f(-5) = -480 \); Substituting the two given points into the function results in the system of equations \( 2 + b + c = 5 = 0 \) and \( 16 + 4b + 2c = -5 = 3 \). Solving for \( b \) and \( c \) gives \( f(x) = 2x^3 - 7x^2 + 10x - 5 \).

50. a. \( y = 0.000008452x^3 \)

vertical shrink by a factor of about 0.0278

4.1 Maintaining Mathematical Proficiency

51. \( -2x^2 + 3xy + y^2 \)

52. \( 7h^3g + 7h^2g^2 + 5hg^3 \)

53. \( 12kz - 4kw \)

54. \( -7a^5 + 10m \)

55. \( -x^3y^2 + 3x^2y + 13xy - 12x + 9 \)

56. \( -3x^2v - 8c^2 + 11cv + 6c \)

4.2 Monitoring Progress and Modeling with Mathematics

61. \[
\begin{array}{cccccc}
  & 1 & 1 & 1 & 1 & 1 \\
 1 & 3 & 3 & 3 & 3 & 3 \\
 5 & 5 & 5 & 5 & 5 & 5 \\
 10 & 10 & 10 & 10 & 10 & 10 \\
 20 & 20 & 20 & 20 & 20 & 20 \\
 30 & 30 & 30 & 30 & 30 & 30 \\
 45 & 45 & 45 & 45 & 45 & 45 \\
 60 & 60 & 60 & 60 & 60 & 60 \\
 90 & 90 & 90 & 90 & 90 & 90 \\
 135 & 135 & 135 & 135 & 135 & 135 \\
 200 & 200 & 200 & 200 & 200 & 200 \\
 \end{array}
\]

\( (x + 3)^7 = x^7 + 21x^6 + 189x^5 + 945x^4 + 2835x^3 + 5103x^2 + 5103x + 2187; \)

\( (x - 5)^9 = x^9 - 45x^8 + 900x^7 - 10,500x^6 + 78,750x^5 - 393,750x^4 + 1,312,500x^3 - 2,812,500x^2 + 3,151,625x - 1,953,125 \)

62. When in standard form the function is \( x^4 + (ax^3 + bx^2 + cx + dx) + (abx^2 + bcv^2 + bdx^2 + cdv^2) + (acx + adx + abcx + abdx + acdx + bcdx) + abed \) which can also be written as \( (a + b + c + d)x^3 + (ab + ad + ac + bc + bd + cd) x^2 + (abe + abd + acd + bcd)x + abed \).

63. a. 5

b. 5

c. 9

d. \( g(x) + h(x) \) has degree \( m \). \( g(x) - h(x) \) has degree \( m \).

64. a. 3, 5, 7; The difference increases by 2 for each consecutive pair of square numbers.

b. The first difference is 3, given by 2(1) + 1 = 3. The second difference is 5, given by 2(2) + 1 = 5. The third difference is 7, given by 2(3) + 1.
c. \((n+1)^2 - n^2 = n^2 + 2n + 1 - n^2 = 2n + 1\)

65. a. \((x^2 - y^2)^2 + (2xy)^2 = (x^2 + y^2)^2\)
   \((x^4 - 2x^2y^2 + y^4) + (4x^2y^2) = x^4 + 2x^2y^2 + y^4\)
   \[x^4 + 2x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4\]
   b. The Pythagorean triple is 11, 60, and 61.
   c. 121 + 3600 = 3721
   \[3721 = 3721\]

4.2 Maintaining Mathematical Proficiency

66. 8 + 7i
67. 5 + 11i
68. 21
69. 9 - 2i

4.3 Monitoring Progress and Modeling with Mathematics

37. A
38. a. -4
   b. The remainders are both 0 because \(f(-3) = f(-1) = 0\).
39. 2x + 5
40. 5x^3 - 3x^2 + 21x - 8; Sample answer: Multiply by \(x + 2\).

4.3 Maintaining Mathematical Proficiency

41. \(x = 3\)
42. \(x = -6\) and \(x = 2\)
43. \(x = -7\)
44. \(x = \pm 3i\)

4.4 Monitoring Progress and Modeling with Mathematics

64. \((3k^2 + 1)(3k - 8)\); Factoring by grouping can be used because the expression contains pairs of monomials that have a common factor.
65. a. no; \(7c(2z + 3)(z - 2)\)
   b. no; \(n(2 - n)(n + 6)(3n - 11)\)
   c. yes
66. 1 million
67. 0.7 million

75. b. \((x - 2)^2 + y^2 = 3^2\); The center of the circle is \((2, 0)\) and the radius is 3.
    c. \((x - 4)^2 + (y + 1)^2 = 6^2\); The center of the circle is \((4, -1)\) and the radius is 6.

76. a. If the volume of the missing block is included, the volume of the diagram is \(a^3\) because the length, width, and height are all \(a\). Because the length, width, and height of the missing piece are \(b\), the volume of the missing block is \(b^3\). Subtracting the volume of the missing block from the entire volume gives \(a^3 - b^3\).
   b. I: \(a^2(a - b)\), II: \(ab(a - b)\), III: \(b^2(a - b)\)
   c. \(a^3 - b^3 = a^3(a - b) + ab(a - b) + b^2(a - b) = (a - b)(a^2 + ab + b^2)\)

4.4 Maintaining Mathematical Proficiency

77. \(x = 6\) and \(x = -5\)
78. \(x = 9\) and \(x = -4\)
79. \(x = \frac{5}{3}\) and \(x = 2\)
80. \(x = \frac{5}{3}\) and \(x = 3\)
81. \(x = 18\) and \(x = -6\)
82. \(x = 4 \pm 3\sqrt{3}\)
83. \(x = -3\) and \(x = -7\)
84. \(x = -9 \pm \sqrt{85}\)

4.5 Monitoring Progress and Modeling with Mathematics

16. \(x = -4, x = 0,\) and \(x = 5\)

17. \(x = -3, x = 0,\) and \(x = 5\)

18. \(x = -5, x = 0,\) and \(x = 3\)
19. \( x = -3, x = -1, \) and \( x = 3 \)

![Graph showing three real zeros at \((-3, 0), (-1, 0), (3, 0)\).]

20. \( x = -2, x = 2, \) and \( x = 5 \)

![Graph showing two real zeros at \((-2, 0), (2, 0), (5, 0)\).]

64. Each side of the base is to be 2 feet and the height is to be 3 feet.

65. \( rs; \) Each factor of \( a_n \) can be written as the numerator with each factor of \( a_0 \) as the denominator, creating \( r \times s \) factors.

### 4.5 Maintaining Mathematical Proficiency

66. polynomial function; \( h(x) = 2x^3 - 3x^2 + 2x - 9; \) degree: 3 (cubic), leading coefficient: 2

67. not a polynomial function

68. polynomial function; \( f(x) = -4x^4 + 2x^3 + \frac{1}{3}x^2 - \sqrt{3}; \) degree: 4 (quartic), leading coefficient: \(-4\)

69. not a polynomial function; The term \( \sqrt{x} \) has an exponent that is not a whole number.

70. \( x = \pm i\sqrt{6} \)

71. \( x = \pm 3i \)

72. \( x = \pm 2i\sqrt{2} \)

73. \( x = \pm \frac{\sqrt{2}}{2} \)

### 4.6 Lesson Monitoring Progress

9. | Positive real zeros | Negative real zeros | Imaginary zeros | Total zeros |
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### 4.6 Monitoring Progress and Modeling with Mathematics

33. | Positive real zeros | Negative real zeros | Imaginary zeros | Total zeros |
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36. | Positive real zeros | Negative real zeros | Imaginary zeros | Total zeros |
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A22  Additional Answers
48. Sample answer: \( f(x) = (x - 1)(x - 2)(x - 2)(x - 5)(x - 7) \)

\[
f(x) = x^5 - 17x^4 + 103x^3 - 275x^2 + 328x - 140
\]

50. a. The function has three positive real zeros, one negative real zero, and two imaginary zeros. Or, the function has five positive real zeros, one negative zero, and zero imaginary zeros.

b. \( f(x) \) could change signs five times or three times

51.

a. For all functions, \( f(x) \to \infty \) as \( x \to \infty \). When \( n \) is even, \( f(x) \to \infty \) as \( x \to -\infty \), but when \( n \) is odd, \( f(x) \to -\infty \) as \( x \to -\infty \).

b. As \( n \) increases, the graph becomes more flat near the zero \( x = -3 \).

c. The graph of \( g \) becomes more vertical and straight near \( x = 4 \).

52. \( f(x): 2, 3; g(x): -3, 1, 2; h(x): -3, 1, 2i, -2i; k(x): -3, 2, 0, 2 + \sqrt{3} \approx 3.73, 2 - \sqrt{3} \approx 0.27 \)

a. If the function is of degree \( n \), the sum of the zeros is equal to the opposite of the coefficient of the \( n-1 \) term.

b. The product of the zeros is equal to the constant term of the polynomial function.

53. a. |   | Year 1 | Year 2 | Year 3 | Year 4 |
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b. \( v = 1000g^3 + 1000g^2 + 1000g + 1000 \)

c. about 1.0484; about 4.84%

4.6 Maintaining Mathematical Proficiency

54. The function is a vertical stretch by a factor of 3 followed by a reflection in the \( x \)-axis of the parent quadratic function.

55. The function is a translation 4 units right and 6 units up of the parent quadratic function.
56. The function is a reflection in the x-axis followed by a translation 1 unit right of the parent quadratic function.

57. The function is a vertical stretch by a factor of 5 followed by a translation 4 units left of the parent quadratic function.

58. \( g(x) = -\frac{1}{3}x \)

59. \( g(x) = \left|\frac{1}{3}x + 1\right| - 3 \)

60. \( g(x) = -(x - 2)^2 + 7 \)

4.7 Lesson Monitoring Progress

2. The graph of \( g \) is a vertical stretch by a factor of 4 followed by a translation 2 units left of the graph of \( f \).

4.7 Monitoring Progress and Modeling with Mathematics

12. The graph of \( g \) is a vertical stretch by a factor of 3 followed by a reflection in the x-axis of the graph of \( f \).

13. The graph of \( g \) is a vertical stretch by a factor of 5 followed by a translation 1 unit up of the graph of \( f \).

14. The graph of \( g \) is a vertical shrink by a factor of \( \frac{1}{2} \) followed by a translation 1 unit up of the graph of \( f \).

15. The graph of \( g \) is a vertical shrink by a factor of \( \frac{3}{4} \) followed by a translation 4 units left of the graph of \( f \).

16. The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{2} \) followed by a translation 3 units down of the graph of \( f \).

17. \( g(x) = (x + 2)^4 + 1 \)

The graph of \( g \) is a translation 2 units left of the graph of \( f \).
18. \( g(x) = 3x^3 - 6x + 9 \)

The graph of \( g \) is a vertical stretch by a factor of 3 of the graph of \( f \).

19. \( g(x) = -x^3 + x^2 - 3 \)

The graph of \( g \) is a vertical shrink by a factor of \( \frac{1}{2} \) followed by a reflection in the x-axis of the graph of \( f \).

20. \( g(x) = x^3 - x^2 - 6 \)

The graph of \( g \) is a reflection in the y-axis followed by a translation 5 units down of the graph of \( f \).

21. The graph has been translated horizontally to the right 2 units instead of to the left 2 units.

22. \( g(x) = -x^3 + 9x^2 - 27x + 21 \)

23. \( g(x) = 2x^3 - 32x^2 + 192x^2 - 508x + 508 \)

24. \( g(x) = -27x^3 - 18x^2 + 7 \)

25. \( g(x) = -6x^3 + 3x^2 + 3x^2 + 11 \)

26. \( W(x) = 27x^3 - 12x; W(5) = 3315; \) When \( x \) is 5 yards, the volume of the pyramid is 3315 cubic feet.

27. \( W(x) = 27x^3 - 12x; W(5) = 3315; \) When \( x \) is 5 yards, the volume of the pyramid is 3315 cubic feet.

28. no; When each side is divided in half the new volume is \( V\left(\frac{1}{2}x\right) = \left(\frac{1}{2}x\right)^3 = \frac{1}{8}x^3 \) and is four times less than when the volume is divided in half.

29. Sample answer: If the function is translated up and then reflected in the x-axis, the order is important; If the function is translated left and then reflected in the x-axis, the order is not important; Reflecting a graph in the x-axis does not affect its x-coordinate, but it does affect its y-coordinate. So, the order is only important if the other translation is in the y.

30. Sample answer: \( f(x) = 2x^3 - 6x^2 + 4x - 2 \)

31. a. 0 m, 4 m, and 7 m 
   b. \( g(x) = -\frac{2}{3}(x - 2)(x - 6)(x - 9) \)

32. The real zeros of \( f \) are \(-2, 0, 2\). The real zeros of \( g \) are \(0, 2, \) and \(4\). Because all of the real zeros have been increased two units, the graph of \( g \) is a translation two units right of the graph of \( f \).

33. \( V(t) = 3\pi t^3(x + 3); W(t) = \frac{\pi}{3}t^3 \left(\frac{1}{3}x + 3\right)\); 
   \( W(3) = 12\pi = 37.70 \); When \( x \) is 3 feet, the volume of the cone is about 37.70 cubic yards.

4.7 Maintaining Mathematical Proficiency

34. The minimum value is \(-7\); The domain is all real numbers and the range is \( y \geq -7 \). The function is decreasing to the left of \( x = -5 \) and increasing to the right of \( x = -5 \).

35. The maximum value is 4; The domain is all real numbers and the range is \( y \leq 4 \). The function is increasing to the left of \( x = 0 \) and decreasing to the right of \( x = 0 \).

36. The minimum value is \(-147\); The domain is all real numbers and the range is \( y \geq -147 \). The function is decreasing to the left of \( x = 3 \) and increasing to the right of \( x = 3 \).

37. The maximum value is 9; The domain is all real numbers and the range is \( y \leq 9 \). The function is decreasing to the left of \( x = -5 \) and decreasing to the right of \( x = -5 \).

38. The minimum value is \(-3\); The domain is all real numbers and the range is \( y \geq -3 \). The function is decreasing to the left of \( x = 1 \) and increasing to the right of \( x = 1 \).

39. The maximum value is 1; The domain is all real numbers and the range is \( y \leq 1 \). The function is increasing to the left of \( x = 1 \) and decreasing to the right of \( x = 1 \).

4.7 Mini-Assessment

3. \( g(x) = x^3 - 4x^2 + 2 \)

The graph of \( g \) is a translation 9 units up of the graph of \( f \).
4.8 Monitoring Progress and Modeling with Mathematics

12. [Graph showing a polynomial function with x-intercepts at -10, -6, -2, 2, and 10.]

13. [Graph showing a polynomial function with x-intercepts at -10, -6, -2, 2, and 10.]

14. [Graph showing a polynomial function with x-intercepts at -10, -6, -2, 2, and 10.]

15. The x-intercepts should be -2 and 1.

16. Because 0 is a repeated zero with an even power, the graph should only touch the x-axis at 0, not cross it. Because 3 is a repeated zero with an odd power, the graph should cross the x-axis at 3.

17. -1, 1, and 4

18. -2, 2, and 3

19. -4, -\frac{1}{2}, and 1

20. -\frac{3}{2}, -1, and 3

21. -4, \frac{3}{2}, and 3

22. -3, -\frac{3}{2}, and 5

27. [Graph showing a polynomial function with x-intercepts at -3 and 3.]

The x-intercept of the graph is x = -2.46. The function has a local maximum at (-1.15, 4.04) and a local minimum at (1.15, 0.96); The function is increasing when x < -1.15 and x > 1.15 and is decreasing when -1.15 < x < 1.15.

28. [Graph showing a polynomial function with x-intercepts at -6 and 6.]

The x-intercepts of the graph are x = -1.15, x = 0, x = 1.64, and x = 3.79. The function has a local maximum at (0.87, 2.78) and local minimums at (-0.68, -2.31) and (3.02, -9.30); The function is increasing when -0.68 < x < 0.87 and x > 3.02 and is decreasing when x < -0.68 and 0.87 < x < 3.02.

29. [Graph showing a polynomial function with x-intercepts at -24 and 24.]

The x-intercepts of the graph are x = -2.10, x = -0.23, and x = 1.97. The function has a local maximum at (-1.46, 18.45) and a local minimum at (1.25, -19.07); The function is increasing when x < -1.46 and x > 1.25 and is decreasing when -1.46 < x < 1.25.

30. [Graph showing a polynomial function with x-intercepts at -10 and 10.]

The x-intercepts of the graph are x = -0.77 and x = 4.54. The function has a local maximum at (0.47, -2.56) and local minimums at (-0.16, -3.09) and (3.44, -39.40); The function is increasing when -0.16 < x < 0.47 and x > 3.44 and is decreasing when x < -0.16 and 0.47 < x < 3.44.

31. (-0.29, 0.48) and (0.29, -0.48); (-0.29, 0.48) corresponds to a local maximum and (0.29, -0.48) corresponds to a local minimum; The real zeros are -0.5, 0, and 0.5. The function is of at least degree 3.

32. (-2.91, -1.36) and (0.57, -6.63); (-2.91, -1.36) corresponds to a local maximum and (0.57, -6.63) corresponds to a local minimum; The real zero is 2.5. The function is of at least degree 3.

33. (1, 0), (3, 0), and (2, -2); (1, 0) and (3, 0) correspond to local maximums, and (2, -2) corresponds to a local minimum; The real zeros are 1 and 3. The function is of at least degree 4.

34. (-1.22, 5.07), (1.96, 7.71), (0, 15, -48.35), and (2.79, -3.74); (-1.22, 5.07) and (1.96, 7.71) correspond to local maximums, and (0.15, -48.35) and (2.79, -3.74) correspond to local minimums; The real zeros are -1.4, -1, 1.5, 2.5, and 3. The function is of at least degree 5.

35. (-1.25, -10.65); (1.25, -10.65) corresponds to a local maximum; The real zeros are -2.07 and 1.78. The function is of at least degree 4.

36. (-1.18, -7.57); (-1.18, -7.57) corresponds to a local minimum; The zeros are -2.45 and 2.45. The function is of at least degree 4.

A26 Additional Answers
Enrollment increases until approximately the 12th year to a maximum of almost 45,000 students. Then, enrollment decreases until approximately the 30th year to a minimum of about 40,000 students. Then, enrollment increases again and ends the period with about 48,000 students.

51. no; When multiplying two odd functions, the exponents of each term will be added, creating an even exponent. So, the product will not be an odd function.

52. a. about 2.94 in.
   b. about 420.11 in.³
   c. 2.94 in. by 14.11 in. by 10.11 in.

53. a. \( \frac{1100 - \pi r^2}{\pi} \)
   b. \( V = 550r - \frac{\pi}{2}r^3 \)
   c. about 10.8 ft

54. Sample answer: \( x^3 - 3.5x^2 - 5.25x + 3.375 \)

There is no maximum degree; The function could have repeated zeros or infinitely many imaginary zeros.

55. \( V(h) = \frac{64\pi h - \frac{\pi}{4}h^3}{\pi} \); about 9.24 in.; about 1238.22 in.³

4.8 Maintaining Mathematical Proficiency

56. linear; The first differences are constant.

57. quadratic; The second differences are constant.

4.9 Monitoring Progress and Modeling with Mathematics

19. \( d = \frac{2y^2}{2^2}; 35 \)

20. no; If the first order differences are given, they can be used to find the second order differences, and so on, until the differences are constant.

21. With real-life data sets, the numbers rarely fit a model perfectly. Because of this, the differences are rarely constant.
84. The focus is (1, 0). The directrix is \( x = -1 \). The axis of symmetry is \( y = 0 \).

85. \( g(x) = -x^4 + 3x^2 + 2x \); The graph of \( g \) is a reflection in the \( x \)-axis of the graph of \( f \).

86. \( g(x) = x^3 - x - 3 \); The graph of \( g \) is a translation 3 units down of the graph of \( f \).

87. \( g(x) = (x - 2)^3 - 4 \); The graph of \( g \) is a translation 2 units right of the graph of \( f \).

88. \( g(x) = 16x^4 + 16x^3 - 16x^2 \); The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{2} \) of the graph of \( f \).

5.3 Lesson Monitoring Progress

2. The graph of \( g \) is a reflection in the \( x \)-axis followed by a translation 2 units down of the graph of \( f \).

5.3 Monitoring Progress and Modeling with Mathematics

14. The domain and range are all real numbers.

15. The domain is \( x \geq 0 \). The range is \( y \geq 3 \).
16. The domain and range are all real numbers.

17. The domain is $x \geq 0$. The range is $y \leq 0$.

18. The domain and range are all real numbers.

19. The graph of $g$ is a translation 1 unit left and 8 units up of the graph of $f$.

20. The graph of $g$ is a vertical stretch by a factor of 2 followed by a translation 1 unit right of the graph of $f$.

21. The graph of $g$ is a reflection in the $x$-axis followed by a translation 1 unit down of the graph of $f$.

22. The graph of $g$ is a translation 4 units left and 5 units down of the graph of $f$.

23. The graph of $g$ is a vertical shrink by a factor of $\frac{1}{2}$ followed by a reflection in the $y$-axis of the graph of $f$.

24. The graph of $g$ is a vertical shrink by a factor of $\frac{1}{3}$ followed by a translation 6 units up of the graph of $f$.

25. The graph of $g$ is a vertical stretch by a factor of 2 followed by a translation 5 units left and 4 units down of the graph of $f$. 
26. The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{2} \) followed by a reflection in the \( y \)-axis and a translation 3 units up of the graph of \( f \).

![Graph of \( f \) and \( g \)]

27. The graph was translated 2 units left but it should be translated 2 units right.

![Graph showing translation]

28. The function is a horizontal stretch by a factor of 2, not a horizontal shrink by a factor of \( \frac{1}{2} \); The graph of \( g \) is a horizontal stretch by a factor of 2 and a translation 3 units up of the parent square root function.

![Graph showing horizontal stretch]

55. \[ y = \pm \sqrt{5x + 40} \]

(-8, 0), right

56. \[ y = \pm \frac{1}{2}x + 4 \]

(-8, 0), right

57. \[ y = \pm \sqrt{9 - x^2} \]

The radius is 3 units. The \( x \)-intercepts are \( \pm 3 \). The \( y \)-intercepts are \( \pm 3 \).

58. \[ y = \pm \sqrt{4 - x^2} \]

The radius is 2 units. The \( x \)-intercepts are \( \pm 2 \). The \( y \)-intercepts are \( \pm 2 \).

59. \[ y = \pm \sqrt{1 - x^2} \]

The radius is 1 unit. The \( x \)-intercepts are \( \pm 1 \). The \( y \)-intercepts are \( \pm 1 \).

60. \[ y = \pm \sqrt{64 - x^2} \]

The radius is 8 units. The \( x \)-intercepts are \( \pm 8 \). The \( y \)-intercepts are \( \pm 8 \).

61. \[ y = \pm \sqrt{36 - x^2} \]

The radius is 6 units. The \( x \)-intercepts are \( \pm 6 \). The \( y \)-intercepts are \( \pm 6 \).

62. \[ y = \pm \sqrt{100 - x^2} \]

The radius is 10 units. The \( x \)-intercepts are \( \pm 10 \). The \( y \)-intercepts are \( \pm 10 \).

63. about 3 ft; Sample answer: Locate the \( T \)-value 2 on the graph and estimate the \( \ell \)-value.

64. square root; The domain and range are restricted; The domain is \( x \geq -3 \). The range is \( y \geq 1 \).

65. a. about 2468 hp
   b. about 0.04 mph/hp

66. Sample answer: \( g(x) = \sqrt{-x + 2} + 1 \); \( h(x) = -\sqrt{x - 1} + 2 \); \( f \) can be written as \( f(x) = \sqrt{-x + 4} \) or \( f(x) = -\sqrt{x - 3} + 1 \).

67. a. The 165-lb skydiver
   b. When \( A = 1 \), the diver is most likely vertical. When \( A = 7 \), the diver is most likely horizontal.
68. a. \[ \pi \left( \frac{r^2 + r + 1}{4} \right) = S + \frac{\pi}{4} \]
\[
\left( \frac{r + 1}{2} \right)^2 = \frac{1}{\pi} \left( S + \frac{\pi}{4} \right)
\]
\[ r = \frac{1}{\sqrt{\pi}} \left( S + \frac{\pi}{4} - 1 \right) \]

b. 0.5 unit

5.4 Monitoring Progress and Modeling with Mathematics

53. a. The greatest stopping distance is 450 feet on ice. On wet asphalt and snow, the stopping distance is 225 feet. The least stopping distance is 90 feet on dry asphalt.

b. about 272.2 ft; When \( s = 35 \) and \( f = 0.15 \), \( d = 272.2 \).

54. a. about 0.16 mph; about 10.77 mph

b. 0.16 \( \leq x \leq 80.41 \)

c. \( x = 8 \) with \( x = 2 \) as an extraneous solution. When solving the second equation, the solution is \( x = 2 \) with \( x = 8 \) as an extraneous solution.

55. a. When solving the first equation, the solution is \( x = 8 \) with \( x = 2 \) as an extraneous solution. When solving the second equation, the solution is \( x = 2 \) with \( x = 8 \) as an extraneous solution.

56. no; Sample answer: \( \sqrt{x - 3} = \sqrt{x^2 + x - 4} \) has two extraneous solutions of \( x = \pm 1 \).

5.5 Monitoring Progress and Modeling with Mathematics

17. Because the functions have an even index, the domain is restricted; the domain of \( (f \circ g)(x) \) is \( x \geq 0 \).

18. The domain is incorrect; The domain of \( \left( \frac{f}{g} \right)(x) \) is all real numbers except \( x = 2 \) and \( x = -2 \).

19. a. \( (F + M)(t) = 0.0001t^3 - 0.016t^2 + 0.21t + 7.4 \)

b. the total number of employees from the ages of 16 to 19 in the United States

20. a. \( (W - F)(t) = -18.3333t^4 + 77.72t^2 + 372.5t + 6615 \)

b. the number of cruise ship departures around the world, excluding departures from Florida

21. yes; When adding or multiplying functions, the order in which they appear does not matter.

26. \( r(t) = \frac{2200\sqrt{0.734}}{w} \), \( r(6.5) = 133.7 \), \( r(300) = 48.3 \), \( r(70,000) = 11.3 \)

27. a. \( r(x) = \frac{20 - x}{6.4} \); \( s(x) = \frac{\sqrt{x^2 + 144}}{0.9} \)

b. \( r(x) = \frac{20 - x}{6.4} + \frac{\sqrt{x^2 + 144}}{0.9} \)

c. \( x = 1.7 \); If Elvis runs along the shore until he is about 1.7 meters from point \( C \) then swims to point \( B \), the time taken to get there will be a minimum.

5.5 Maintaining Mathematical Proficiency

28. \( n = \frac{2y + 3}{x} \)

29. \( n = \frac{5z}{7 + 8z} \)

30. \( n = \frac{6z}{5 - 3b} \)

31. \( n = \frac{3}{7b - 4} \)

32. yes; Every input has exactly one output.

33. no; \( -1 \) has two outputs.

34. yes; Every input has exactly one output.

35. no; \( 2 \) has two outputs.

5.6 Explorations

2. b. The graph of \( g \) is a reflection in the line \( y = x \).

c. The graph of \( g \) is a reflection in the line \( y = x \).

d. The graph of \( g \) is a reflection in the line \( y = x \).
5.6 Monitoring Progress and Modeling with Mathematics

17. \( g(x) = -2x + 8; \)

18. \( g(x) = 3x + 3; \)

19. \( g(x) = \frac{3x + 1}{2}; \)

20. \( g(x) = \frac{5x - 1}{4}; \)

21. \( g(x) = \frac{x - 4}{2}; \) Sample answer: switching \( x \) and \( y \); You can graph the inverse to check your answer.

22. a. yes; The \( x \)- and \( y \)-coordinates are switched. 
b. no; The \( x \)- and \( y \)-coordinates were not switched. 
c. no; The \( x \)- and \( y \)-coordinates were not switched.

23. \( g(x) = -\sqrt{x}; \)

24. \( g(x) = -\frac{\sqrt{x}}{3}; \)

25. \( g(x) = \sqrt{x} + 3 \)

26. \( g(x) = \sqrt{x} - 4; \)

64. a. \( h = \left( \frac{A}{0.2195} \right)^{\frac{10.3964}{0.3964}} \); about 150.1 cm 
b. \( \frac{0.2195}{0.3964} = \frac{A}{0.2195} \cdot \frac{10.3964}{0.3964} = A \)

71. The inverse \( y = \frac{1}{m}x - \frac{b}{m} \) has a slope of \( \frac{1}{m} \) and a \( y \)-intercept of \( -\frac{b}{m} \).
72. a. 

When the graph of \( f(x) = -x \) is reflected across the line \( y = x \), the result is the same graph:

\[
\begin{align*}
  f(x) &= -x \\
  y &= -x \\
  x &= -y \\
  -x &= y
\end{align*}
\]

b. Sample answer:

\[
\begin{align*}
  y &= -x + 5 \\
  y &= -x - 2
\end{align*}
\]

\( y = -x + 5 \) and \( y = -x - 2 \) are their own inverses.

c. A linear function of the form \( y = -x + b \) or \( y = x \) is its own inverse.

5.6 Maintaining Mathematical Proficiency

73. \(-\frac{1}{33}\)  
74. \(2^5\)  
75. \(4^2\)  
76. \(\frac{24}{34}\)

77. The function is increasing when \( x > 1 \) and decreasing when \( x < 1 \). The function is positive when \( x < 0 \) and when \( x > 2 \), and negative when \( 0 < x < 2 \).

78. The function is increasing when \( x < -1.15 \) and \( x > 1.15 \) and decreasing when \( -1.15 < x < 1.15 \). The function is positive when \( -2 < x < 0 \) and \( x > 2 \) and negative when \( x < -2 \) and \( 0 < x < 2 \).

79. The function is increasing when \(-2.89 < x < 2.89\) and decreasing when \( x < -2.89 \) and \( x > 2.89 \). The function is positive when \( x < -5 \) and \( 0 < x < 5 \) and negative when \( -5 < x < 0 \) and \( x > 5 \).

Chapter 6

6.1 Lesson Monitoring Progress

1. exponential growth

2. exponential decay

3. exponential decay

4. exponential growth

13. exponential growth

14. exponential decay
45. \(a\) represents the number of referrals it received at the start of the model. \(b\) represents the growth factor of the number of referrals each year; 50%; 1.50 can be rewritten as \((1 + 0.50)\), showing the percent increase of 50%.

46. a. exponential decay
b. domain: all real numbers, range: \(y > 0\); It is an exponential function and any real number can be used as an exponent.

\[f(x) = \frac{y}{b^x} \rightarrow 0 \text{ as } x \rightarrow \infty \text{ and } f(x) \rightarrow \infty \text{ as } x \rightarrow -\infty.\]

47. no; \(f(x) = 2^x\) eventually increases at a faster rate than \(g(x) = x^2\), but not for all \(x \geq 0\).

48. Sample answer: \(y = (1 - b)^x\)

49. about 221.5; The curve contains the points \((0, 6850)\) and \((6, 8179.26)\), \(\frac{8179.26 - 6850}{6 - 0} \approx 221.5\).

50. a. \(f(x + 1) = \frac{ab^{x+1}}{b^x} = b^{x+1-x} = b^1 = b\)
b. The equation shows that when a value of the function is divided by the previous value, the answer is the constant \(b\). Dividing the \(y\)-values in the table by the previous value does not always produce the same number.

51. a. The decay factor is 0.9978. The percent decrease is 0.22%.
b. 

<table>
<thead>
<tr>
<th>Eggs Produced by Leghorn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (weeks)</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>50</td>
</tr>
<tr>
<td>100</td>
</tr>
<tr>
<td>150</td>
</tr>
<tr>
<td>200</td>
</tr>
<tr>
<td>250</td>
</tr>
<tr>
<td>300</td>
</tr>
<tr>
<td>350</td>
</tr>
<tr>
<td>400</td>
</tr>
<tr>
<td>450</td>
</tr>
<tr>
<td>500</td>
</tr>
<tr>
<td>550</td>
</tr>
<tr>
<td>600</td>
</tr>
<tr>
<td>650</td>
</tr>
<tr>
<td>700</td>
</tr>
<tr>
<td>750</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>850</td>
</tr>
<tr>
<td>900</td>
</tr>
<tr>
<td>950</td>
</tr>
<tr>
<td>1000</td>
</tr>
</tbody>
</table>

52. \(V = 1300(0.6782)^t\)

6.1 Maintaining Mathematical Proficiency

53. \(x^{11}\)

54. \(x\)

55. \(24x^2\)

56. \(16x^8\)

57. \(2x\)

58. \(7x\)

59. \(3 + 5x\)

60. \(216x^{18}\)

6.2 Explorations

2. 

<table>
<thead>
<tr>
<th>(x)</th>
<th>(10^1)</th>
<th>(10^2)</th>
<th>(10^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1 + \frac{1}{x})</td>
<td>2.59374</td>
<td>2.70481</td>
<td>2.71692</td>
</tr>
</tbody>
</table>

\[\left(1 + \frac{1}{x}\right)^x \rightarrow \frac{e}{x} \rightarrow \text{as } x \rightarrow \infty\]

<table>
<thead>
<tr>
<th>(x)</th>
<th>(10^1)</th>
<th>(10^2)</th>
<th>(10^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1 + \frac{1}{x})</td>
<td>2.71815</td>
<td>2.71827</td>
<td>2.71828</td>
</tr>
</tbody>
</table>

Sample answer: \(e \approx 2.71828\); This approximation has more decimal points.
19. exponential growth

\[ y \]

\[ \begin{tabular}{|c|c|c|c|c|}
\hline
\( x \) & -2 & -1 & 0 & 1 & 2 \\
\hline
\( y = e^x \) & 0.1353 & 0.3679 & 1 & 2.7183 & 7.3891 \\
\hline
\end{tabular} \]

20. exponential decay

\[ y \]

\[ \begin{tabular}{|c|c|c|c|c|}
\hline
\( x \) & -2 & -1 & 0 & 1 & 2 \\
\hline
\( y = e^{-x} \) & 7.3891 & 2.7183 & 1 & 0.3679 & 0.1353 \\
\hline
\end{tabular} \]

21. exponential decay

\[ y \]

22. exponential growth

\[ y \]

23. D; The graph shows growth and has a \( y \)-intercept of 1.

24. A; The graph shows decay and has a \( y \)-intercept of 1.

25. B; The graph shows decay and has a \( y \)-intercept of 4.

26. C; The graph shows growth and has a \( y \)-intercept of 0.75.

27. \( y = (1 - 0.221)x; 22.1\% \) decay

28. \( y = (1 - 0.528)x; 52.8\% \) decay

29. \( y = 2(1 + 0.492)x; 49.2\% \) growth

30. \( y = 0.5(1 + 1.226)x; 122.6\% \) growth
41. account 1; With account 1, the balance would be
   \[ A = 2500 \left(1 + \frac{0.06}{4}\right)^{4 \cdot 10} \approx 4535.05. \]
   With account 2, the balance would be
   \[ A = 2500e^{0.04 \cdot 10} \approx 3729.56. \]

42. a. \( \infty \)
   b. \(-3\)

43. a. \( N(t) = 30e^{0.166t} \)
   b. 30

44. \( 6 \times 10^{-3} \)
45. \( 5 \times 10^3 \)
46. \( 2.6 \times 10^7 \)
47. \( 4.7 \times 10^{-8} \)

48. \( y = \frac{x - 5}{3} \)

49. \( y = -\sqrt{x + 1} \)

50. \( y = x^2 - 6, x \geq 0 \)

A36 Additional Answers
51. \( y = \sqrt[3]{x} + 2 \)

6.3 Explorations

2. a. \[
\begin{array}{c|ccccc}
  x & -2 & -1 & 0 & 1 & 2 \\
f(x) = 2^x & \frac{1}{4} & \frac{1}{2} & 1 & 2 & 4 \\
g(x) = \log_2 x & -2 & -1 & 0 & 1 & 2 \\
\end{array}
\]

The functions \( f \) and \( g \) are inverses, so the \( x \)- and \( y \)-coordinates can be switched to create the table for \( g(x) \).

b. \[
\begin{array}{c|ccccc}
  x & -2 & -1 & 0 & 1 & 2 \\
f(x) = 10^x & \frac{1}{100} & \frac{1}{10} & 1 & 10 & 100 \\
g(x) = \log_{10} x & -2 & -1 & 0 & 1 & 2 \\
\end{array}
\]

The functions \( f \) and \( g \) are inverses, so the \( x \)- and \( y \)-coordinates can be switched to create the table for \( g(x) \).

6.3 Monitoring Progress and Modeling with Mathematics

59.

60.

61. domain: \( x > -2 \), range: all real numbers, asymptote: \( x = -2 \)

62. domain: \( x > 0 \), range: all real numbers, asymptote: \( x = 0 \)

63. domain: \( x < 0 \), range: all real numbers, asymptote: \( x = 0 \)

64. domain: \( x > 0 \), range: all real numbers, asymptote: \( x = 0 \)
65. no; Any logarithmic function of the form \( g(x) = \log_b x \) will pass through \((1, 0)\), but if the function has been translated or reflected in the \(x\)-axis, it may not pass through \((1, 0)\).
66. b, c, a, d
69. a.

b. 15 species
c. about 3918 m²
d. The number of species of fish increases; Sample answer: This makes sense because in a smaller pool or lake, one species could dominate another more easily and feed on the weaker species until it became extinct.
70. Sample answer: \( y = \log(x) - 5 \)

71. a. \( \frac{3}{2} \)
b. \( \frac{3}{5} \)
c. \( \frac{4}{3} \)
d. \( \frac{7}{2} \)

6.3 Maintaining Mathematical Proficiency
72. \( g(x) = -\sqrt{x} \)
73. \( g(x) = \sqrt[3]{x} \)
74. \( g(x) = \sqrt[-3]{x} + 3 \)
75. \( g(x) = \sqrt[3]{x} + 2 \)
76. constant; The graph is a translation 3 units down of the parent constant function.
77. quadratic; The graph is a translation 2 units left and 1 unit down of the parent quadratic function.
78. absolute value; The graph is a reflection in the \(x\)-axis followed by a translation 1 unit right and 2 units up of the parent absolute value function.

6.4 Lesson Monitoring Progress
1. The graph of \( g \) is a translation 3 units right and 1 unit up of the graph of \( f \).

2. The graph of \( g \) is a translation 5 units down of the graph of \( f \).

3. The graph of \( g \) is a reflection in the \(y\)-axis followed by a horizontal shrink by a factor of \( \frac{1}{2} \) of the graph of \( f \).

4. The graph of \( g \) is a reflection in the \(x\)-axis followed by a translation 6 units left of the graph of \( f \).

10. The graph of \( g \) is a translation 4 units up of the graph of \( f \).

11. The graph of \( g \) is a translation 7 units right of the graph of \( f \).
12. The graph of \( g \) is a translation 1 unit left of the graph of \( f \).

13. The graph of \( g \) is a translation 6 units up of the graph of \( f \).

14. The graph of \( g \) is a translation 9 units down of the graph of \( f \).

15. The graph of \( g \) is a translation 3 units right and 12 units up of the graph of \( f \).

16. The graph of \( g \) is a translation 2 units left and \( \frac{3}{2} \) unit down of the graph of \( f \).

17. The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{2} \) of the graph of \( f \).

18. The graph of \( g \) is a vertical stretch by a factor of \( \frac{1}{3} \) of the graph of \( f \).

19. The graph of \( g \) is reflection in the \( x \)-axis followed by a translation 3 units right of the graph of \( f \).

20. The graph of \( g \) is a horizontal stretch by a factor of 2 followed by a translation 5 units right of the graph of \( f \).

21. The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{6} \) followed by a vertical stretch by a factor of 3 of the graph of \( f \).
22. The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{5} \) followed by a translation 2 units up of the graph of \( f \).

23. The graph of \( g \) is a vertical stretch by a factor of 6 followed by a translation 5 units left and 2 units down of the graph of \( f \).

24. The graph of \( g \) is a reflection in the \( x \)-axis followed by a translation 7 units right and 1 unit up of the graph of \( f \).

25. The graph of the parent function \( f(x) = 2^x \) was translated 3 units left instead of up.

26. The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{5} \) followed by a translation 2 units up of the graph of \( f \). (Diagram)

27. The graph of \( g \) is a vertical stretch by a factor of 6 followed by a translation 5 units left and 2 units down of the graph of \( f \). (Diagram)

28. The graph of \( g \) is a reflection in the \( x \)-axis followed by a translation 7 units right and 1 unit up of the graph of \( f \). (Diagram)

29. The graph of the parent function \( f(x) = 2^x \) was translated 3 units left instead of up. (Diagram)

30. The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{5} \) followed by a translation 2 units up of the graph of \( f \). (Diagram)

31. A: The graph of \( f \) has been translated 2 units right.

32. B: The graph of \( f \) has been translated 2 units left.

33. C: The graph of \( f \) has been stretched vertically by a factor of 2.

34. D: The graph of \( f \) has been shrunk horizontally by a factor of \( \frac{1}{2} \).

35. \( g(x) = 5^x - 2 \)  
36. \( g(x) = -6(\frac{1}{2})^x + 4 \)

37. \( g(x) = e^x + 5 \)  
38. \( g(x) = \frac{1}{3}e^{x-4} - \frac{1}{3} \)

39. \( g(x) = 6 \log_6 x - 5 \)  
40. \( g(x) = -\log_6(x + 9) \)

41. \( g(x) = \log_{10}(x + 3) + 2 \)  
42. \( g(x) = \ln(\frac{1}{x^3} - 3) + 1 \)

43. Multiply the output by \(-1\); Substitute \( \log_5 x \) for \( f(x) \).

44. Multiply output by 4; Substitute \( 8^x \) for \( f(x) \).

45. The graph of \( g \) is a translation 4 units up of the graph of \( f \); \( y = 4 \)

46. The graph of \( g \) is a translation 9 units right of the graph of \( f \); \( y = 0 \)

47. The graph of \( g \) is a translation 6 units left of the graph of \( f \); \( x = -6 \)

48. The graph of \( g \) is a translation 13 units up of the graph of \( f \); \( x = 0 \)

49. The graph of \( S \) is a vertical shrink by a factor of 0.118 followed by a translation 0.159 unit up of the graph of \( f \).

50. a. The graph of \( g \) is a reflection in the \( y \)-axis of the graph of \( f \).

b. No; When \( f(x) = b^x \) is reflected in the \( y \)-axis, it becomes \( b^{-x} \), which is equal to \( g(x) = \left(\frac{1}{b}\right)^x \) for any \( 0 < b < 1 \).

51. Yes; Sample answer: If the graph is reflected in the \( y \)-axis, the graphs will never intersect because there are no values of \( x \) where \( \log x = \log(-x) \).

52. Yes; \( y = \ln x \) and \( y = e^x \) are inverses. So, the graph of \( y = \ln x \) is a reflection in the line \( y = x \) of the graph of \( y = e^x \).

53. a. Never; The asymptote of \( f(x) = \log x \) is a vertical line and would not change by shifting the graph vertically.

b. Always; The asymptote of \( f(x) = e^x \) is a horizontal line and would be changed by shifting the graph vertically.

c. Always; The domain of \( f(x) = \log x \) is \( x > 0 \) and would not be changed by a horizontal shrink.

d. Sometimes; The graph of the parent exponential function does not intersect the \( x \)-axis, but if it is shifted down, the graph would intersect the \( x \)-axis.

54. a. Domain: \( t \geq 0 \); range: \( 0 < P \leq 100 \)

b. 69.77 g

c. The graph would be a vertical stretch by a factor of 5.5 of the graph of the parent function.

d. The translation does not affect the domain but does affect the range; It changes the range to \( 0 < P \leq 550 \) because the initial amount is 550, but in both cases there can never be 0 grams remaining.

55. The graph of \( h \) is a translation 2 units left of the graph of \( f \); The graph of \( h \) is a reflection in the \( y \)-axis followed by a translation 2 units left of the graph of \( g \); \( x \) has been replaced with \((x + 2) \), \( x \) has been replaced with \(-x + 2 \).

56. Sample answer: \( y = 1.5 \cdot 2^{x+1} + 2 \)

6.4 Maintaining Mathematical Proficiency

57. \( (fg)(x) = x^6; (fg)(3) = 729 \)

58. \( \int \left( \frac{1}{g} \right)(x) = 2x^2; \int \left( \frac{1}{g} \right)(5) = 250 \)

59. \( (f + g)(x) = 14x^3; (f + g)(2) = 112 \)

60. \( (f - g)(x) = -x^3; (f - g)(6) = -36 \)

6.5 Monitoring Progress and Modeling with Mathematics

46. It is not true; By the Product Property, \( \log_a M + \log_a N = \log_a (MN) \). If \( \log_a (M + N) = \log_a (MN) \), then \( M + N = M \cdot N \) which is not always true.

47. Rewrite each logarithm in exponential form to obtain \( a = b^x \), \( c = b^t \), and \( a = c^z \). So, \( \log_b a = \log_b c \cdot \log_b a = \log_b c \).
45. The focus is $\left(0, \frac{1}{16}\right)$, the directrix is $y = -\frac{1}{16}$, and the axis of symmetry is $x = 0$. 

46. The asymptote is the line $y = 256$ and represents the maximum height of the sunflower. 

6.6 Monitoring Progress and Modeling with Mathematics 

35. a. 

Sample answer: $y = 0.50(1.47)^x$

b. about 47%; The base is 1.47 which means that the function shows 47% growth.

36. yes; The points $(x, \ln y)$ are linear.
46. The focus is \( (0, \frac{1}{4}) \), the directrix is \( y = -\frac{3}{4} \), and the axis of symmetry is \( x = 0 \).

47. The focus is \((0.1, 0)\), the directrix is \(x = -0.1\), and the axis of symmetry is \(y = 0\).

**Chapter 6 Test**

11. Both equations transform into a linear equation that is solved to find the value of \(x\). To get the linear equation, the first equation requires that exponent rules are used to make the bases the same. In the second equation, the definition of logarithm is used to write the equation in exponential form to get the linear equation; \(x = 0.8\); \(x = -0.2\)

12. The expressions are all equivalent; change-of-base formula

13. 39.329 billion barrels; \(y = 40.473(1.085)^x\); The inverse function gives the number of wells, \(y\), that need to be drilled to obtain \(x\) billions of barrels.

14. a. \(L(x) = 100e^{-0.02x}\)
   b. exponential decay; Natural base functions of the form \(y = ae^x\) show decay when \(r\) is negative.
   c. about 44.9%

15. **Sample answer:** To find an exponential equation in the form \(y = ab^x\), the common ratio, \(b\), can be found by dividing consecutive terms and \(a\) is the \(y\)-value when \(x\) is 0. Two points can be used to create a system of equations in the form \(y = ab^x\) which can be solved for \(a\) and \(b\). Or, use the regression feature of a graphing calculator to find an exponential model which fits the data; \(4200(0.90)^x\); in the 4th year

**Chapter 7**

**7.1 Explorations**

1. b. 

<table>
<thead>
<tr>
<th>Weight, (x) (in kilograms)</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance, (d) (in centimeters)</td>
<td>0</td>
<td>1.5</td>
<td>3</td>
<td>4.5</td>
<td>6</td>
<td>7.5</td>
<td>9</td>
<td>10.5</td>
</tr>
</tbody>
</table>

As the value of \(x\) increases, the value of \(y\) also increases. The points lie on a line.

2. a. 

<table>
<thead>
<tr>
<th>(x)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>64</td>
<td>32</td>
<td>16</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

c.

As the values of \(x\) increase, the values of \(y\) decrease. The points lie on a curve.

**7.1 Monitoring Progress and Modeling with Mathematics**

25. a. 

<table>
<thead>
<tr>
<th>Size</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of songs</td>
<td>5000</td>
<td>4000</td>
<td>3333</td>
<td>2000</td>
</tr>
</tbody>
</table>

b. The number of songs decreases.

31. **Sample answer:** As the speed of your car increases, the number of minutes per mile decreases.

32. \(x\) varies directly with \(z\); \(xy = a\); so \(y = \frac{a}{x}\); \(yz = b\), so \(y = \frac{b}{z}\); \(\frac{a}{x} = \frac{b}{z}\); \(azy = bx, x = \frac{az}{b}\)

33. cat: 4 ft; dog: 2 ft; The inverse equations are \(d = \frac{a}{7}\) and \(6 - d = \frac{a}{14}\). Because the constant is the same, solve the equation \(7d = 14(6 - d)\) for \(d\).
7.1 Maintaining Mathematical Proficiency

34. \(x - 9\)
35. \(x^2 - 6\)

36. [Graph of a line]
   - Domain: all real numbers, range: \(y > 4\)

37. [Graph of a line]
   - Domain: all real numbers, range: \(y > 0\)

38. [Graph of a line]
   - Domain: \(x > 0\), range: all real numbers

39. [Graph of a line]
   - Domain: \(x > -9\), range: all real numbers

7.2 Lesson Monitoring Progress

4. [Graph of a line]
   - Domain: all real numbers except 1; range: all real numbers except 5

6. [Graph of a line]
   - Domain: all real numbers except \(\frac{1}{2}\); range: all real numbers except \(\frac{1}{2}\)

7. [Graph of a line]
   - Domain: all real numbers except \(-1\); range: all real numbers except 3

8. \(g(x) = \frac{1}{x + 1} + 2\)
   - Translation 1 unit left and 2 units up
7.2 Monitoring Progress and Modeling with Mathematics

7. The graph of $g$ lies farther from the axes. Both graphs lie in the first and third quadrants and have the same asymptotes, domain, and range.

8. The graph of $g$ lies farther from the axes and is reflected over the $x$-axis. Both graphs have the same asymptotes, domain, and range.

9. The graph of $g$ lies closer to the axes and is reflected over the $x$-axis. Both graphs have the same asymptotes, domain, and range.

10. The graph of $g$ lies closer to the axes. Both graphs lie in the first and third quadrants and have the same asymptotes, domain, and range.

11. domain: all real numbers except 0; range: all real numbers except 3

12. domain: all real numbers except 0; range: all real numbers except $-3$

13. domain: all real numbers except 1; range: all real numbers except 0

14. domain: all real numbers except $-2$; range: all real numbers except 0
19. The graph should lie in the second and fourth quadrants instead of the first and third quadrants.

domain: all real numbers except \(-2\); range: all real numbers except 0

16. domain: all real numbers except 7; range: all real numbers except 0

17. domain: all real numbers except 4; range: all real numbers except \(-1\)

18. domain: all real numbers except \(-7\); range: all real numbers except \(-5\)

20. The vertical asymptote should be \(x = 1\);

21. A; The asymptotes are \(x = 3\) and \(y = 1\).
22. C; The asymptotes are \(x = -3\) and \(y = 1\).
23. B; The asymptotes are \(x = 3\) and \(y = -1\).
24. D; The asymptotes are \(x = -3\) and \(y = -1\).

29. domain: all real numbers except \(-\frac{3}{2}\); range: all real numbers except \(-\frac{1}{2}\)

30. domain: all real numbers except \(\frac{1}{2}\); range: all real numbers except 2
31. \[ g(x) = \frac{3}{x - 2} - 5 + 2 \]

- Domain: all real numbers except \(-\frac{3}{2}\)
- Range: all real numbers except \(\frac{5}{2}\)

32. \[ g(x) = \frac{8}{x} + 4 \]

- Domain: all real numbers except 10
- Range: all real numbers except 2

33. \[ g(x) = \frac{1}{x + 1} + 5 \]

- Translation 1 unit left and 5 units up

34. \[ g(x) = \frac{25}{x - 3} + 7 \]

- Translation 3 units right and 7 units up

35. \[ g(x) = \frac{6}{x - 5} + 2 \]

- Translation 5 units right and 2 units up

36. \[ g(x) = \frac{-3}{x - 2} + 4 \]

- Translation 2 units right and 4 units up

37. \[ g(x) = \frac{-24}{x - 6} + 1 \]

- Translation 6 units right and 1 unit up
38. \( g(x) = \frac{-10}{x-8} + 1 \)

Sample answer: domain: \(0 < x < 1\); range: \(0 < y < 150\)

translation 8 units right and 1 unit up

39. \( g(x) = \frac{-111}{x+13} + 7 \)

translation 13 units left and 7 units up

40. \( g(x) = \frac{-66}{x+7} + 9 \)

translation 7 units left and 9 units up

41. a. 50 students
b. The average cost per student approaches $20.

42. a. 10 months
b. The average cost approaches $59.

43. B

44. B

45. a. about 23°C
b. −0.005 sec/°C

46. a. 

Sample answer: domain: \(0 < x < 1\); range: \(0 < y < 150\)
b. about $3333; about $8571; $40,000; no; Doubling the percentage does not double the cost because this is not a linear function.

47. even

48. even

50. odd

54.

55. (4, 3); The point (2, 1) is one unit left and one unit down from (3, 2), so a point on the other branch is one unit right and one unit up from (3, 2).

56. a. decreasing: \((-\infty, 0)\), \((0, \infty)\); The graph falls from left to right.
b. increasing: \((-\infty, 0)\), \((0, \infty)\); The graph rises from left to right.

57. The competitor is a better choice for less than 18 months of service; The cost of Internet service is modeled by \( C = \frac{50 + 43x}{x} \).
The competitor’s cost is lesser when \( x = 6 \) and \( x = 12 \), and greater when \( x = 18 \) and \( x = 24 \).
58. \( f(x) = \frac{740(2000)}{740 - r} \cdot f_e = \frac{740(2000)}{740 + r} \)

59. \( 4(x - 5)(x + 4) \)
60. \( 3(x - 2)(x + 1) \)
61. \( 2(x - 3)(x + 2) \)
62. \( (5x - 2)(2x + 7) \)
63. \( 3^6 \)
64. \( \frac{211}{10} \)
65. \( 6^{2/3} \)
66. \( \frac{1}{6^2} \)

7.3 Monitoring Progress and Modeling with Mathematics

42. You are correct; The values -9 and 5 make the denominator equal to zero in the original expression, so both values are not in the domain.
43. \( \frac{4}{7x} \)
44. \( x - 5 \)
45. \( 9(x + 3), x \neq -\frac{1}{3}, x \neq 5, x \neq 7 \)
46. \( x - 6, x \neq -2, x \neq 2, x \neq 6 \)
47. Galapagos: about 0.371; King: about 0.203; King; The King penguin has a smaller surface area to volume ratio, so it is better equipped to live in a colder environment.
48. no; Sample answer: When \( f(x) = \sqrt{x} \) and \( g(x) = \sqrt{x^3} \),
\( (fg)(x) = x^2, x \geq 0 \), and \( \int g(x) = \frac{1}{x}, x \geq 0 \), but both graphs exclude negative values of \( x \).
49. \( f(x) = \frac{x(x - 1)}{x + 2}, g(x) = \frac{x(x + 2)}{x - 1} \)

The graph of \( g \) is a translation 5 units right and 12 units up of the graph of \( f(x) = \frac{60}{x} \).

33. \( g(x) = \frac{-60}{x - 5} + 12 \)

The graph of \( g \) is a translation 13 units left and 8 units up of the graph of \( f(x) = \frac{-104}{x} \).

34. \( g(x) = \frac{-104}{x + 13} + 8 \)

The graph of \( g \) is a translation 2 units up of the graph of \( f(x) = \frac{3}{x} \).
36. \( g(x) = -\frac{6}{x} + 4 \)

The graph of \( g \) is a translation 4 units up of the graph of \( f(x) = -\frac{6}{x} \).

37. \( g(x) = -\frac{20}{x - 3} + 3 \)

The graph of \( g \) is a translation 3 units right and 3 units up of the graph of \( f(x) = \frac{20}{x} \).

38. \( g(x) = \frac{-79}{x + 10} + 7 \)

The graph of \( g \) is a translation 10 units left and 7 units up of the graph of \( f(x) = \frac{-79}{x} \).

53. \( g(x) = \frac{2.3058}{x^2 + 12.2^2} + 0.003; \) translation 12.2 units left and 0.003 unit up of the graph of \( f(x) \).

54. \( \frac{2(2x^3 + 11x^2 + 26x + 5)}{3x(x + 1)} \)

55. a. \( R_1 = \frac{1}{40}, R_2 = \frac{1}{x}, R_3 = \frac{1}{x + 10} \)

b. \( R = \frac{x^2 + 90x + 400}{40x(x + 10)} \)

c. about 0.0758 car/min; about 4.5 cars/h; Multiply the number of cars washed per minute by the rate 60 min/h to obtain an answer in cars per hour.

56. a. \( A = \frac{391(t - 1)^2 + 0.112}{0.218(t - 1)^2 + 0.991(t - 1)^2 + 1} \)

b. \( A = 391(t - 1)^2 + 0.112 \frac{0.218 + 0.991(t - 1)^2 + 1}{0.218(t - 1)^2 + 0.991(t - 1)^2 + 1} \)

57. \[ 1 + \frac{1}{2} + \frac{1}{2}, 1 + \frac{1}{2} + \frac{1}{2}, 1 + \frac{1}{2} + \frac{1}{2}, 1 + \frac{1}{2} + \frac{1}{2} \]

7.4 Maintaining Mathematical Proficiency

58. (1, 7) and (2, 10)

59. \( \left(\frac{1}{2}, -1\right) \) and \( \left(\frac{9}{8}, \frac{27}{8}\right) \)

60. no solution

7.5 Monitoring Progress and Modeling with Mathematics

33. a.

<table>
<thead>
<tr>
<th>Work rate</th>
<th>Time</th>
<th>Work done</th>
</tr>
</thead>
<tbody>
<tr>
<td>You</td>
<td>1 room</td>
<td>8 hours</td>
</tr>
<tr>
<td>Friend</td>
<td>1 room</td>
<td>t hours</td>
</tr>
</tbody>
</table>

34. a.

<table>
<thead>
<tr>
<th>Work rate</th>
<th>Time</th>
<th>Work done</th>
</tr>
</thead>
<tbody>
<tr>
<td>You</td>
<td>1 park</td>
<td>2 hours</td>
</tr>
<tr>
<td>Friend</td>
<td>1 park</td>
<td>t hours</td>
</tr>
</tbody>
</table>

56. a. yes; Sample answer: \( \frac{1}{x} = \frac{1}{x + 1} \)

b. yes; Sample answer: See Example 2.

c. yes; Sample answer: See Example 3(b).

d. yes; Sample answer: \( \frac{1}{x + 1} = \frac{3}{3x + 3} \)

57. a. always true; When \( x = a \), the denominators of the fractions are both zero.

b. sometimes true; The equation will have exactly one solution except when \( a = 3 \).

c. always true; \( x = a \) is an extraneous solution, so the equation has no solution.

58. yes; Extraneous solutions make the denominator in one of the terms of the equation equal to zero.

7.5 Maintaining Mathematical Proficiency

59. discrete; The number of quarters in your pocket is an integer.
60. continuous; The weight of the broccoli can be any non-negative number.

8.1 Lesson Monitoring Progress
6. \((-2)^0, (-2)^1, (-2)^2, (-2)^3; a_5 = (-2)^4 = 16; a_n = (-2)^{n-1}\)

7. \(1^2 + 1, 2^2 + 1, 3^2 + 1, 4^2 + 1; a_n = 5^2 + 1 = 26; a_n = n^2 + 1\)

8.1 Monitoring Progress and Modeling with Mathematics
59. a. true;
\[
\sum_{i=1}^{n} ca_i = ca_1 + ca_2 + ca_3 + \cdots + ca_n \\
= c(a_1 + a_2 + a_3 + \cdots + a_n) \\
= c\sum_{i=1}^{n} a_i
\]
b. true;
\[
\sum_{i=1}^{n} (a_i + b_i) = (a_1 + b_1) + (a_2 + b_2) + \cdots + (a_n + b_n) \\
= a_1 + a_2 + \cdots + a_n + b_1 + b_2 + \cdots + b_n \\
= \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i
\]
c. false; \(\sum_{j=1}^{3} (2j) = 30, \left(\sum_{j=1}^{3} 2\right) \left(\sum_{j=1}^{3} 3\right) = 54\)
d. false; \(\sum_{j=1}^{3} (2j)^2 = 20, \left(\sum_{j=1}^{3} 2\right)^2 = 36\)

8.2 Monitoring Progress and Modeling with Mathematics
25. \(a_n = -2n + 13\)
26. \(a_n = -3n + 10\)
27. \(a_n = -\frac{1}{2}n + \frac{7}{2}\)
28. \(a_n = -\frac{3}{2}n + \frac{13}{2}\)

29. C

8.2 Maintaining Mathematical Proficiency
70. exponential growth
71. exponential decay
72. exponential decay
73. exponential growth

8.3 Monitoring Progress and Modeling with Mathematics

25.\[ a_n = 60 \left( \frac{1}{2} \right)^{n-1} \]

26.\[ a_n = 256 \left( \frac{1}{2} \right)^{n-1} \]

28.\[ a_n = -3(4)^n - 1 \]

30.\[ a_n = 625 \left( \frac{1}{5} \right)^{n-1} \]

31. The formula should be \( a_n = a_1 r^{n-1} \); \( a_n = 8(6)^n - 1 \)

32. The formula should be \( a_n = a_1 r^{n-1} \); \( a_n = 8(6)^n - 1 \)

33. \( a_n = 7(4)^n - 1 \)

34. \( a_n = 11(2)^n - 1 \)

35. \( a_n = -6(3)^n - 1 \) or \( a_n = -6(-3)^n - 1 \)

36. \( a_n = 2(5)^n - 1 \) or \( a_n = 2(-5)^n - 1 \)

37. \( a_n = 512 \left( \frac{1}{2} \right)^{n-1} \) or \( a_n = -512 \left( -\frac{1}{2} \right)^{n-1} \)

38. \( a_n = (49)^n - 1 \)

39. \( a_n = -432 \left( \frac{1}{6} \right)^{n-1} \) or \( a_n = 432 \left( -\frac{1}{6} \right)^{n-1} \)

40. \( a_n = 192 \left( \frac{1}{3} \right)^{n-1} \)

66. \[ L = \frac{M}{1+i} \quad \text{and} \quad M = \frac{L}{1+i} \]

67. domain: all real numbers except 3; range: all real numbers except 0

68. domain: all real numbers except 0; range: all real numbers except 3

69. domain: all real numbers except 2; range: all real numbers except 1

70. domain: all real numbers except –1; range: all real numbers except –2
8.5 Maintaining Mathematical Proficiency

74. \( x = 64 \) 
75. \( y = \frac{18}{x}; y = \frac{9}{2} \) 
76. \( y = \frac{-12}{x}; y = -3 \) 
77. \( y = \frac{320}{x}; y = 80 \)

Chapter 9

9.1 Monitoring Progress and Modeling with Mathematics

16. \( \sin \theta = \frac{8}{15}, \cos \theta = \frac{\sqrt{161}}{15}, \tan \theta = \frac{8\sqrt{161}}{161}, \sec \theta = \frac{15\sqrt{161}}{161}, \cot \theta = \frac{\sqrt{161}}{8} \)

17. \( \sin \theta = \frac{\sqrt{115}}{14}, \cos \theta = \frac{9}{14}, \tan \theta = \frac{\sqrt{115}}{9}, \csc \theta = \frac{14\sqrt{115}}{115}, \cot \theta = \frac{9\sqrt{115}}{115} \)

18. \( \sin \theta = \frac{11\sqrt{377}}{377}, \cos \theta = \frac{16\sqrt{377}}{377}, \tan \theta = \frac{11}{16}, \csc \theta = \frac{\sqrt{377}}{11}, \sec \theta = \frac{\sqrt{377}}{16} \)

19. The adjacent side was used instead of the opposite; \( \sin \theta = \frac{\text{opp.}}{\text{hyp.}} = \frac{8}{17} \)

49. a. about 59.155 ft 
   b. about 53.613 ft 
   c. about 39.688 ft; Use the tangent function to find the horizontal distance, \( x + y \), from the airplane to the second town to be about 93,301 ft. Subtract 53,613 ft to find the distance between the two towns.

50. about 59 m 

51. yes; The triangle must be a 45-45-90 triangle because both acute angles would be the same and have the same cosine value.

52. \( \sin \theta = \frac{1}{2}, \cos \theta = \frac{\sqrt{3}}{2}, \tan \theta = \frac{\sqrt{3}}{3}, \csc \theta = 2, \sec \theta = 2\sqrt{3} \)
   \( \cot \theta = \sqrt{3}; \) The triangle on the right is equilateral, so all of its angles must be 60°. Using geometry, it can be shown that \( \theta = 30° \) and that the two triangles form a larger 30-60-90 triangle. Because the large triangle is a right triangle, the 6 trig ratios can be found. The legs of the triangle are 1 and \( \sqrt{3} \) and the hypotenuse is 2.

53. a. \( x = 0.5; 6 \) units  
   b. Sample answer: Each side is part of two right triangles, with opposing angles \( \left( \frac{180°}{n} \right) \). So, each side length is \( 2 \sin \left( \frac{180°}{n} \right) \) and there are \( n \) sides.
   c. \( n \cdot \sin \left( \frac{180°}{n} \right) \); about 3.14

9.2 Lesson Monitoring Progress

2.

A52 Additional Answers
9. $-\frac{\sqrt{3}}{2}$
10. $-\sqrt{2}$

11. about 14.6 ft

9.3 Monitoring Progress and Modeling with Mathematics

16. $y = \sin x$;
   $30^\circ$,
   $150^\circ$,
   $-60^\circ$,
   $300^\circ$,

17. $y = \cos x$;
   $40^\circ$,
   $320^\circ$,
   $-40^\circ$,
   $280^\circ$,

18. $y = \tan x$;
   $10^\circ$,
   $-370^\circ$,
   $-20^\circ$,
   $-350^\circ$,

19. $y = \cot x$;
   $\frac{\pi}{4}$,
   $\frac{3\pi}{4}$,
   $-\frac{\pi}{4}$,
   $\frac{5\pi}{4}$,

20. $y = \sec x$;
   $\frac{\pi}{3}$,
   $\frac{2\pi}{3}$,
   $-\frac{\pi}{3}$,
   $-\frac{2\pi}{3}$,

21. $y = \csc x$;
   $\frac{\pi}{6}$,
   $-\frac{5\pi}{6}$,
   $\frac{5\pi}{6}$,
   $\frac{7\pi}{6}$,

22. $y = \tan x$;
   $\frac{\pi}{6}$,
   $-\frac{13\pi}{6}$,
   $\frac{13\pi}{6}$,
   $\frac{25\pi}{6}$,

23. The equation for tangent is $\tan \theta = \frac{y}{x}$; $\tan \theta = \frac{y}{x} = -\frac{2}{3}$

24. The angle found is the angle between the terminal side and the $y$-axis instead of the $x$-axis; $\theta$ is coterminal with $290^\circ$, whose terminal side lies in Quadrant IV. So, $\theta' = 360^\circ - 290^\circ = 70^\circ$

25. $-\sqrt{2}$

26. $\sqrt{3}$

27. $-\frac{1}{2}$

28. $-\frac{2\sqrt{3}}{3}$

29. 1

30. $\frac{\sqrt{3}}{3}$

31. $\frac{\sqrt{2}}{2}$

32. $\frac{2\sqrt{3}}{3}$

41. $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$,
   $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$,
   $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$,
   $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$,

46. no; $\theta = 240^\circ$ is also a solution; any angles coterminal with $60^\circ$ and $240^\circ$ are also solutions.

47. a. $(-58, 1, 114)$
   b. about 218 pm

48. $60^\circ$

9.3 Maintaining Mathematical Proficiency

49. $x = -3$ and $x = 1$

50. $x = -6$, $x = -1$ and $x = 3$
2. a.

\[ x = -\frac{2\pi}{4}, -\frac{7\pi}{4}, -\frac{3\pi}{4}, -\frac{5\pi}{4}, -\pi \]

\[ y = \cos x \]

\[ x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4} \]

\[ y = \cos x \]

\[ x = \frac{\sqrt{2}}{2}, 0, -\sqrt{2}, -1 \]

\[ y = \cos x \]

\[ x = \frac{3\pi}{4}, \frac{2\pi}{4}, \frac{7\pi}{4}, \frac{5\pi}{4} \]

\[ y = \cos x \]

\[ x = 0, \sqrt{2}, 1, \sqrt{2} \]

\[ y = \cos x \]

3. 2, 2: The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{\pi} \)

and a vertical stretch by a factor of 2 of the graph of \( f(x) = \sin x \).

4. \( \frac{1}{3}, 4\pi \): The graph of \( g \) is a horizontal stretch by a factor of 2

followed by a vertical shrink by a factor of \( \frac{1}{3} \) of the graph of

\( f(x) = \cos x \).

9.4 Monitoring Progress and Modeling with Mathematics

16. \( \frac{\pi}{2} \): The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{4} \) of the

graph of \( f(x) = \cos x \).
17. 1, 1: The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{2\pi} \) of the graph of \( f(x) = \sin x \).

18. 3, \( \pi \): The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{3} \) and a vertical stretch by a factor of 3 of the graph of \( f(x) = \sin x \).

19. \( \frac{1}{3}, \frac{\pi}{2} \): The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{4} \) and a vertical shrink by a factor of \( \frac{1}{3} \) of the graph of \( f(x) = \cos x \).

20. \( \frac{1}{2}, \frac{\pi}{2} \): The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{4\pi} \) and a vertical shrink by a factor of \( \frac{1}{2} \) of the graph of \( f(x) = \cos x \).

21. B, D

22. a. \( y = \sin \frac{2\pi x}{5} \)  
   b. \( y = 10 \sin \frac{\pi}{2} x \)  
   c. \( y = 2 \sin x \)  
   d. \( y = \frac{1}{2} \sin \frac{2}{3} x \)

23. The period is \( \frac{1}{4} \) and represents the amount of time, in seconds, that it takes for the pendulum to go back and forth and return to the same position. The amplitude is 4 and represents the maximum distance, in inches, the pendulum will be from its resting position.

25. \( \frac{\pi}{2} \) should be added to the \( x \)-coordinate;  
   Maximum: \( \left( \frac{1}{4} \cdot \frac{2\pi}{5} + \frac{\pi}{2}, 2 \right) = \left( \frac{\pi}{2} + \frac{\pi}{2}, 2 \right) = \left( \pi, 2 \right) \)

27. The graph of \( g \) is a vertical stretch by a factor of 2 followed by a translation \( \frac{\pi}{2} \) units right and 1 unit up of the graph of \( f \).

28. The graph of \( g \) is a vertical stretch by a factor of 3 followed by a translation \( \frac{\pi}{4} \) units left and 2 units down of the graph of \( f \).

29. The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{3} \) followed by a translation 3 units left and 5 units down of the graph of \( f \).

30. The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{2} \) followed by a translation \( \pi \) units right and 9 units up of the graph of \( f \).
64. No; The value of $a$ indicates a vertical stretch or a vertical shrink and changes the amplitude of the graph. It does not affect the $x$-intercepts of the function. The value of $b$ indicates a horizontal stretch or a horizontal shrink and changes the period of the graph, which is the horizontal length of each cycle. So, only the value of $b$ affects the $x$-intercepts of the function.

65. The graph of $g(x) = \cos x$ is a translation $\frac{\pi}{2}$ units to the right of the graph of $f(x) = \sin x$.

68. a. 0.2 ft, $\frac{\pi}{3}$; $t$

b. $t$; $y$

69. $x - 2$, $x \neq -3$

70. $x, x \neq -4, 6$

71. $(x - 5)(x + 1)$

72. $\frac{x + 4}{x + 5}; x \neq 4$

9.4 Maintaining Mathematical Proficiency

69. $x - 2$, $x \neq -3$

70. $x, x \neq -4, 6$

71. $(x - 5)(x + 1)$

72. $\frac{x + 4}{x + 5}; x \neq 4$

9.1–9.4 Quiz

15. $1, \frac{2}{5}$; The graph of $g$ is a horizontal shrink by a factor of $\frac{1}{5\pi}$ followed by a translation 3 units up of the graph of $f$.

16. a. About 380 ft

b. About 170 ft

17. Seated at a window table, you will have traveled about 569 feet in that time. A person at a table 5 feet from the window will have traveled about 509 feet in that time.
9.5 Explorations

1. a. 

<table>
<thead>
<tr>
<th>x</th>
<th>$\frac{-\pi}{2}$</th>
<th>$\frac{-\pi}{3}$</th>
<th>$\frac{-\pi}{4}$</th>
<th>$\frac{-\pi}{6}$</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>y = tan x</td>
<td>undefined</td>
<td>$-\sqrt{3}$</td>
<td>-1</td>
<td>$-\frac{\sqrt{3}}{3}$</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>$\frac{2\pi}{3}$</th>
<th>$\frac{3\pi}{4}$</th>
<th>$\frac{5\pi}{6}$</th>
<th>$\pi$</th>
<th>$\frac{7\pi}{6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>y = tan x</td>
<td>$-\sqrt{3}$</td>
<td>-1</td>
<td>$-\frac{\sqrt{3}}{3}$</td>
<td>0</td>
<td>$\frac{\sqrt{3}}{3}$</td>
</tr>
</tbody>
</table>

9.5 Lesson Monitoring Progress

4. 
The graph of $g$ is a vertical stretch by a factor of 5 and a horizontal shrink by a factor of $\frac{1}{\pi}$ of the graph of $f(x) = \tan x$.

5. 
The graph of $g$ is a horizontal shrink by a factor of $\frac{1}{3}$ of the graph of $f(x) = \csc x$.

6. 
The graph of $g$ is a vertical shrink by a factor of $\frac{1}{2}$ of the graph of $f(x) = \sec x$.

7. 
The graph of $g$ is a horizontal shrink by a factor of $\frac{1}{2}$ and a vertical stretch by a factor of 2 of the graph of $f(x) = \csc x$.

8. 
The graph of $g$ is a horizontal shrink by a factor of $\frac{1}{3}$ and a vertical stretch by a factor of 2 of the graph of $f(x) = \sec x$.

9.5 Monitoring Progress and Modeling with Mathematics

9. 
The graph of $g$ is a horizontal stretch by a factor of 4 and a vertical stretch by a factor of 3 of the graph of $f(x) = \cot x$.

10. 
The graph of $g$ is a horizontal stretch by a factor of 2 and a vertical stretch by a factor of 4 of the graph of $f(x) = \cot x$.

11. 
The graph of $g$ is a horizontal shrink by a factor of $\frac{1}{2}$ and a vertical shrink by a factor of $\frac{1}{2}$ of the graph of $f(x) = \tan x$. 
12. The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{2\pi} \) and a shrink by a factor of \( \frac{1}{3} \) of the graph of \( f(x) = \tan x \).

13. To find the period, use the expression \( \frac{\pi}{|b|} \). Period: \( \frac{\pi}{|b|} = \frac{\pi}{3} \)

14. The horizontal and vertical shrink factors are switched; A vertical stretch by a factor of 2 and a horizontal shrink by a factor of \( \frac{1}{2} \).

15. a. The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{2\pi} \) and a shrink by a factor of \( \frac{1}{3} \) of the graph of \( f(x) = \sec x \).

b. The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{2} \) of the graph of \( f(x) = \cos x \).

16. A

17. The graph of \( g \) is a vertical stretch by a factor of 3 of the graph of \( f(x) = \csc x \).

18. The graph of \( g \) is a vertical stretch by a factor of 2 of the graph of \( f(x) = \csc x \).

19. The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{4} \) of the graph of \( f(x) = \sec x \).

20. The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{3} \) of the graph of \( f(x) = \sec x \).

21. The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{\pi} \) and a vertical shrink by a factor of \( \frac{1}{2} \) of the graph of \( f(x) = \sec x \).

22. The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{2\pi} \) and a vertical shrink by a factor of \( \frac{1}{4} \) of the graph of \( f(x) = \sec x \).
The graph of $g$ is a horizontal stretch by a factor of $\frac{2}{\pi}$ of the graph of $f(x) = \csc x$.

The graph of $g$ is a horizontal stretch by a factor of $\frac{4}{\pi}$ of the graph of $f(x) = \csc x$.

$g(x) = \cot \left( \frac{2x}{\pi} \right) + 3$

$g(x) = 2 \tan(3x - \pi)$

$g(x) = -5 \sec(x - \pi) + 2$

$g(x) = -8 \csc x$

Function $B$ has a local maximum value of $-5$ so Function $A$'s local maximum value of $-\frac{1}{2}$ is greater. Function $A$ has a local minimum of $\frac{1}{2}$ so Function $B$'s local minimum value of $5$ is greater.

$A, B, C, A$

As $d$ increases, $\theta$ increases because, as the car gets farther away, the angle required to see the car gets larger.

$0 < t < 15$ and $0 < h < 320$; The Statue of Liberty is approximately 305 feet tall so it would take almost 15 seconds to span the statue.

$\theta$

$\frac{d}{\pi}$

$\cot \left( \frac{2x}{\pi} \right)$

$\frac{x}{2} + \cot \left( \frac{x}{2} \right)$

Graphing the function produces the same graph as the cosecant function with asymptotes at $0, \pm \pi, \pm 2\pi, \ldots$

Sample answer: $y = 5 \tan \left( \frac{1}{2}x - \frac{3\pi}{4} \right)$

$9.5$ Maintaining Mathematical Proficiency

$y = x^3 - 3x^2 - x + 3$

$-x^3 + 2x^2 + 5x - 6$

$-\frac{1}{3}x^3 + 2x^2 - \frac{1}{3}x - 3$

$\frac{1}{3}x^3 + \frac{1}{3}x^2 - \frac{9}{3}x - \frac{9}{3}$

$5, 2\pi$

$3, \pi$

$1, 8\pi$

$9.5$ Mini-Assessment

The graph of $g$ is a vertical stretch by a factor of $3$ and a horizontal shrink by a factor of $\frac{1}{\pi}$ of the graph of $f$. 
9.6 Monitoring Progress and Modeling with Mathematics

26.

![Graph](image)

The slope of the graph of \( y = \sin x \) is given by the function \( y = \cos x \).

27. a. and b. A cosine function because it does not require determining a horizontal shift.
   
   c. A sine function because it does not require determining a horizontal shift.

28. 2: The graph completes 2 full cycles in 1 unit of \( x \).

29. \( y = 2.5 \sin \left( x - \frac{\pi}{8} \right) + 5.5, y = -2.5 \cos 4x + 5.5 \)

30. no; The period is the reciprocal of the frequency. The reciprocal of \( \frac{1}{2} \) is greater than the reciprocal of 2.

31. a. \( d = -6.5 \cos \frac{\pi t}{6} + 10 \)
   
   b. low tide: 12:00 A.M., 12:00 P.M., high tide: 6:00 A.M., 6:00 P.M.
   
   c. It is a horizontal shift to the left by 3.

9.6 Maintaining Mathematical Proficiency

32. \( \frac{17\sqrt{2}}{2} \)

33. \( \frac{6 + 3\sqrt{6}}{2} \)

34. \(-24 + 8\sqrt{10}\)

35. \( \frac{13\sqrt{11} - 13\sqrt{3}}{8} \)

36. \( \log_8 x - \log_8 7 \)

37. \( \ln 2 + \ln x \)

38. \( \log_2 5 + 3 \log_2 x \)

39. \( \ln 4 + 6 \ln x - \ln y \)

9.7 Explorations

1. d. Sample answer:

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( \sin^2 \theta )</th>
<th>( \cos^2 \theta )</th>
<th>( \sin^2 \theta + \cos^2 \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>QI</td>
<td>( \frac{\pi}{3} )</td>
<td>( \frac{3}{4} )</td>
<td>1</td>
</tr>
<tr>
<td>QII</td>
<td>( \frac{3\pi}{4} )</td>
<td>( \frac{1}{2} )</td>
<td>1</td>
</tr>
<tr>
<td>QIII</td>
<td>( \frac{7\pi}{6} )</td>
<td>( \frac{1}{4} )</td>
<td>( \frac{3}{4} )</td>
</tr>
<tr>
<td>QIV</td>
<td>( \frac{7\pi}{4} )</td>
<td>( \frac{1}{2} )</td>
<td>1</td>
</tr>
</tbody>
</table>

9.7 Monitoring Progress and Modeling with Mathematics

21. \( \sin^2 \theta = 1 - \cos^2 \theta \)

22. \( \tan x = \frac{\sin x}{\cos x} \)
   
   \( \tan x \csc x = \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} \)
   
   \( = \frac{1}{\cos x} \)
   
   \( = \sec x \)

23. \( \sin x \csc x = \sin x \cdot \frac{1}{\sin x} \)
   
   \( = 1 \)

24. \( \tan \theta \csc \theta \cos \theta = \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\sin \theta} \cdot \cos \theta \)
   
   \( = 1 \)

25. \( \cos \left( \frac{\pi}{2} - x \right) \cot x = \sin x \cdot \frac{\cos x}{\sin x} \)
   
   \( = \cos x \)

26. \( \sin \left( \frac{\pi}{2} - x \right) \tan x = \cos x \cdot \frac{\sin x}{\cos x} \)
   
   \( = \sin x \)

27. \( \frac{\cos \left( \frac{\pi}{2} - \theta \right) + 1}{1 - \sin(-\theta)} = \frac{\sin \theta + 1}{1 - (\sin \theta)} \)
   
   \( = \frac{\sin \theta + 1}{1 + \sin \theta} \)
   
   \( = 1 \)

28. \( \sin^2(-x) = \frac{(-\sin x)^2}{\tan^2 x} = \frac{\sin^2 x}{\tan^2 x} \)
   
   \( = \frac{\sin^2 x \cdot \cos^2 x}{\sin^2 x} \)
   
   \( = \cos^2 x \)

29. \( \frac{1 + \cos x}{\sin x} + \frac{\sin x}{1 + \cos x} \)
   
   \( = \frac{1 + \cos x + \sin x(1 - \cos x)}{(1 + \cos x)(1 - \cos x)} \)
   
   \( = \frac{1 + \cos x + \sin x(1 - \cos x)}{1 - \cos^2 x} \)
   
   \( = \frac{1 + \cos x + \sin x(1 - \cos x)}{\sin^2 x + \sin^2 x(1 - \cos x)} \)
   
   \( = \frac{1 + \cos x + \sin x(1 - \cos x)}{\sin^2 x} \)
   
   \( = \frac{\sin x(1 + \cos x + 1 - \cos x)}{\sin^2 x} \)
   
   \( = \frac{\sin x(2)}{\sin^2 x} \)
   
   \( = \frac{2}{\sin x} \)
   
   \( = 2 \sec x \)

30. \( \frac{\sin x}{1 - \cos(-x)} = \frac{\sin x}{1 - \cos x} \)
   
   \( = \frac{\sin x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} \)
   
   \( = \frac{\sin x(1 + \cos x)}{1 - \cos^2 x} \)
   
   \( = \frac{\sin x(1 + \cos x)}{\sin^2 x} \)
   
   \( = \frac{1 + \cos x}{\sin x} \)
   
   \( = \frac{1 + \cos x}{\sin x} \cdot \frac{1 + \cos x}{\sin x} \)
   
   \( = \frac{\csc x + \cot x}{\sin x} \)
31. \( \sin x, \csc x, \tan x, \cot x, \cos x, \sec x; \)
\[
\sin(-\theta) = -\sin \theta \\
\csc(-\theta) = -\csc \theta \\
\tan(-\theta) = -\tan \theta \\
\cot(-\theta) = -\cot \theta
\]

32. Just as \( y = \frac{1}{x} \) decreases as \( x \) increases, \( \sec \theta = \frac{1}{\cos \theta} \) decreases as \( \cos \theta \) increases; this happens on the intervals
\[
\pi + 2n\pi < \theta < \frac{3\pi}{2} + 2n\pi \text{ and } \frac{3\pi}{2} + 2n\pi < \theta < 2\pi + 2n\pi
\]
where \( n \) is an integer.

33. \( y = \frac{1}{x} \) decreases as \( x \) increases.

34. Sample answer: Multiply both sides by \( \frac{1}{\cos x} \), so \( \sin x \cos x = 1 \).

Then \( \tan x = 1 \) because \( \sin x \cos x = \tan x \).

35. \( a. \quad \frac{\frac{n_1}{\text{cot}^2 \theta_1 + 1}}{\frac{n_2}{\text{cot}^2 \theta_2 + 1}} = \frac{n_1}{\text{cot}^2 \theta_1 + 1} \quad \frac{n_2}{\text{cot}^2 \theta_2 + 1} \\
\quad \frac{n_1}{\text{csc} \theta_1 \cdot \text{csc} \theta_2} = \frac{n_2}{\text{csc} \theta_1 \cdot \text{csc} \theta_2} \\
\quad n_1 \sin \theta_1 = n_2 \sin \theta_2 \\
a. \quad n_1 = n_2; \text{ This situation could occur when the mediums have the same composition.}
\]

36. You can obtain the graph of \( y = \cos x \) by reflecting the graph of \( f(x) = \sin x \) in the \( y \)-axis and translating it \( \frac{\pi}{2} \) units right.

37. \( a. \quad u = \tan \theta \\
b. \quad u \) starts at 0 and increases without bound.

38. \( a. \quad \frac{n_1}{\text{cot}^2 \theta_1 + 1} = \frac{n_2}{\text{cot}^2 \theta_2 + 1} \\
\quad \frac{n_1}{\text{csc} \theta_1 \cdot \text{csc} \theta_2} = \frac{n_2}{\text{csc} \theta_1 \cdot \text{csc} \theta_2} \\
\quad n_1 \sin \theta_1 = n_2 \sin \theta_2 \\
b. \quad n_1 = n_2; \text{ This situation could occur when the mediums have the same composition.}
\]

9.7 Maintaining Mathematical Proficiency

41. \( x = 11 \)

42. \( x = \frac{13\sqrt{3}}{2} \)

9.8 Explorations

1. \( b. \quad d = \sqrt{(\cos b - \cos a)^2 + (\sin b + \sin a)^2} \)
\( c. \quad d = \sqrt{(\cos(a-b) - 1)^2 + (\sin(a-b) - 0)^2} \)
\( d. \quad \sqrt{(\cos b - \cos a)^2 + (\sin b - \sin a)^2} \)
\[= \sqrt{(\cos(a-b) - 1)^2 + (\sin(a-b))^2} \]
\[= (\cos(a-b) - 1)^2 + \sin^2(a-b) \]
\[= 2 \cos a \cos b - 2 \sin a \sin b + 2 = \cos^2(a-b) - 2 \cos(a-b) + 1 + \sin^2(a-b) \]
\[= 2 \cos a \cos b - 2 \sin a \sin b + 2 = -2 \cos(a-b) + 2 \]
\[= 2 \cos a \cos b - 2 \sin a \sin b = -2 \cos(a-b) \]
\[= \cos a \cos b + \sin a \sin b = \cos(a-b) \]

9.8 Monitoring Progress and Modeling with Mathematics

35. \( 35 \tan(\theta - 45^\circ) + 35 \tan 45^\circ \)
\[= \frac{35 \tan \theta - 35 \tan 45^\circ + 35 \tan 45^\circ}{h \tan \theta} \]
\[= \frac{35(\tan \theta - 1)}{h \tan \theta} + 35 \]
\[= \frac{35 \tan \theta + 35(1 + \tan \theta)}{h \tan \theta} \]
\[= \frac{35 \tan \theta - 35 + 35 + 35 \tan \theta}{h \tan \theta} \]
\[= \frac{70 \tan \theta}{h \tan \theta} \]
\[= \frac{70}{h(1 + \tan \theta)} \]
36. \( A \cos \left( \frac{2 \pi}{3} - \frac{2 \pi x}{5} \right) + A \cos \left( \frac{2 \pi}{3} - \frac{2 \pi x}{5} \right) \)
   \[ = A \cos \frac{2 \pi}{3} \cos \frac{2 \pi x}{5} + A \sin \frac{2 \pi}{3} \sin \frac{2 \pi x}{5} + A \cos \frac{2 \pi}{3} \cos \frac{2 \pi x}{5} \]
   \[ - A \sin \frac{2 \pi}{3} \sin \frac{2 \pi x}{5} \]
   \[ = A \cos \frac{2 \pi}{3} \cos \frac{2 \pi x}{5} + A \cos \frac{2 \pi}{3} \cos \frac{2 \pi x}{5} \]
   \[ = 2A \cos \frac{2 \pi}{3} \cos \frac{2 \pi x}{5} \]
   \[ = -A \cos \frac{2 \pi x}{5} \]

37. \( y_1 + y_2 = \cos 960 \pi t + \cos 1204 \pi t \)
   \[ = \cos(1100 \pi t - 140 \pi t) + \cos(1100 \pi t + 140 \pi t) \]
   \[ = \cos 1100 \pi t \cos 140 \pi t + \sin 1100 \pi t \sin 140 \pi t \]
   \[ + \cos 1100 \pi t \cos 140 \pi t - \sin 1100 \pi t \sin 140 \pi t \]
   \[ = \cos 1100 \pi t \cos 140 \pi t + \cos 1100 \pi t \cos 140 \pi t \]
   \[ = 2 \cos 1100 \pi t \cos 140 \pi t \]

38. Any point where the two graphs intersect is a solution because if \( f(x) = g(x) \) then \( f(x) - g(x) = 0 \).

39. a. \( \tan(\theta_2 - \theta_1) = \frac{m_2 - m_1}{1 + m_2 m_1} \)
   b. \( 60^\circ \)
   c. \( 3 \sin x - 4 \sin^3 x \)
   d. \( 4 \cos^3 x - 3 \cos x \)
   e. \( 3 \tan x - \tan^3 x \)

9.8 Maintaining Mathematical Proficiency
41. \( x = 4 \)
42. \( x = -\frac{16}{3} \)

Chapter 9 Test
9. The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{\pi} \) and a vertical stretch by a factor of 3 of the graph of \( f(x) = \csc x \).

10. \( \frac{5 \pi}{18}, 310^\circ, -410^\circ \)
11. \( 144^\circ, 14 \frac{\pi}{18}, -6 \frac{\pi}{5} \)
12. \( 480^\circ, \frac{2 \pi}{3}, -\frac{4 \pi}{3} \)
13. \( x = 9.08 \text{ in.}; A = 59.00 \text{ in}^2 \)

14. \( \sin \theta = -\frac{9 \sqrt{5}}{85}, \cos \theta = -\frac{2 \sqrt{85}}{85} \), \( \tan \theta = -\frac{9}{2} \)
   \( \csc \theta = -\frac{\sqrt{85}}{9}, \sec \theta = \frac{\sqrt{85}}{2}, \cot \theta = -\frac{2}{9} \)

15. \( \sin \theta = 0, \cos \theta = -1, \tan \theta = 0, \csc \theta = \text{undefined}, \sec \theta = -1, \cot \theta = \text{undefined} \)

16. Quadrant III; \( \cos \theta < 0, \sin \theta < 0 \), so \( \theta \) must lie in Quadrant II or Quadrant III. Because \( \tan \theta < 0 \) in Quadrant II and \( \tan \theta > 0 \) in Quadrant III, \( \theta \) must lie in Quadrant III.
17. About 178 ft; \( h = 200 \sin 60^\circ + 5 \)
18. \( T = 23.14 \sin(0.495 \nu - 1.95) + 63.7 \); The period represents the amount of time it takes for the weather to complete one cycle, which is about 12.7 months.

Chapter 10
10.1 Monitoring Progress and Modeling with Mathematics
23. a. \( 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 \)
24. \( \pi \), or about 52% \(\frac{\pi}{6} \)
25. \( \frac{\pi}{3} \), or 0.5 \(\frac{\pi}{6} \); about 113; (0.0075)15,000 = 112.5
28. Sample answer: Box A contains three cards numbered 1, 2, and 3. Box B contains 2 cards numbered 1 and 2. One card is removed at random from each box. Find the probability that the product of the two numbers is at least \( 5 \).

10.1 Maintaining Mathematical Proficiency
29. \( \frac{6x^4 y^3}{x^3}, x \neq 0, y \neq 0 \)
30. \( \frac{x^3 - 4x^2 - 15x + 18}{x^4 - 2}, x \neq 0, y \neq 0 \)
31. \( \frac{15x^2}{12x^2 + x - 11}, x \neq 0 \)
32. \( \frac{3x^2 + 2x - 13}{x^6 + 9x^4} \)

10.2 Monitoring Progress and Modeling with Mathematics
25. a. Sample answer: Put 20 pieces of paper with each of the 20 students’ names in a hat and pick one; 5% \(\frac{1}{20} \)
26. a. without \(\frac{1}{20} \)
27. yes; The chance that it will be rescheduled is (0.7)(0.75) = 0.525, which is a greater than a 50% chance.
28. Event A represents rolling at least one 2. Event B represents the dice summing to 5; \( P(A) = \frac{1}{3} \), \( P(A) = \frac{1}{36} \), \( P(B) = \frac{1}{36} \)
29. a. wins: 0%; loses: 1.99%; ties: 98.01% \(\frac{1}{20} \)
30. a. the occurrence of one event does not affect the occurrence of the other, so the probability of each event is the same whether or not the other event has occurred.
31. yes; \( P(A \cap B) = P(A) \cdot P(B) \) and \( P(A) = P(A | B) \).
10.2 Maintaining Mathematical Proficiency

31. \( x = 0.2 \)  
32. \( x = 2 \)  
33. \( x = 0.15 \)

10.3 Explorations

1. 

<table>
<thead>
<tr>
<th>Gender</th>
<th>Play an Instrument</th>
<th>Do Not Play an Instrument</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speak a Foreign Language</td>
<td>16</td>
<td>30</td>
<td>46</td>
</tr>
<tr>
<td>Do Not Speak a Foreign Language</td>
<td>25</td>
<td>9</td>
<td>34</td>
</tr>
<tr>
<td>Total</td>
<td>41</td>
<td>39</td>
<td>80</td>
</tr>
</tbody>
</table>

2. a. 41  b. 46  c. 16  d. 9  e. 25

3. Sample answer:

3. 

<table>
<thead>
<tr>
<th>Play a Sport</th>
<th>In a Club</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>4</td>
</tr>
</tbody>
</table>

6

<table>
<thead>
<tr>
<th>Gender</th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>0.376</td>
<td>0.430</td>
<td>0.806</td>
</tr>
<tr>
<td>No</td>
<td>0.111</td>
<td>0.083</td>
<td>0.194</td>
</tr>
<tr>
<td>Total</td>
<td>0.487</td>
<td>0.513</td>
<td>1</td>
</tr>
</tbody>
</table>

10.3 Monitoring Progress and Modeling with Mathematics

9. 

<table>
<thead>
<tr>
<th>Gender</th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flu</td>
<td>0.1429</td>
<td>0.1518</td>
<td>0.2947</td>
</tr>
<tr>
<td>No Flu</td>
<td>0.2946</td>
<td>0.4107</td>
<td>0.7053</td>
</tr>
<tr>
<td>Total</td>
<td>0.4375</td>
<td>0.5625</td>
<td>1</td>
</tr>
</tbody>
</table>

10. 

<table>
<thead>
<tr>
<th>Gender</th>
<th>Transportation to School</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>Rides Bus</td>
</tr>
<tr>
<td></td>
<td>6</td>
</tr>
<tr>
<td>Female</td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>11</td>
</tr>
</tbody>
</table>

19. Sample answer:

10.3 Lesson Monitoring Progress

3. 

<table>
<thead>
<tr>
<th>Class</th>
<th>Attendance</th>
<th>Not Attending</th>
</tr>
</thead>
<tbody>
<tr>
<td>Junior</td>
<td>0.353</td>
<td>0.634</td>
</tr>
<tr>
<td>Senior</td>
<td>0.647</td>
<td>0.366</td>
</tr>
</tbody>
</table>

Given that a student is attending, the conditional relative frequency that he or she is a junior is about 35.3%. Given that a student is attending, the conditional relative frequency that he or she is a senior is about 64.7%. Given that a student is not attending, the conditional relative frequency that he or she is a junior is about 63.4%. Given that a student is not attending, the conditional relative frequency that he or she is a senior is about 36.6%.

4. 

<table>
<thead>
<tr>
<th>Gender</th>
<th>Response</th>
<th>In Favor</th>
<th>Against</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boy</td>
<td>0.636</td>
<td>0.364</td>
<td></td>
</tr>
<tr>
<td>Girl</td>
<td>0.808</td>
<td>0.192</td>
<td></td>
</tr>
</tbody>
</table>

Given that a student is a boy, the conditional relative frequency that he is in favor is about 63.6%. Given that a student is a girl, the conditional relative frequency that she is in favor is about 80.8%. Given that a student is a boy, the conditional relative frequency that he is against is about 36.4%. Given that a student is a girl, the conditional relative frequency that she is against is about 19.2%.

20. a. the parents surveyed that said no  
   b. the total people that said yes  
   c. the total people surveyed

21. Routine B is the best option, but your friend’s reasoning of why is incorrect; Routine B is the best choice because there is a 66.7% chance of reaching the goal, which is higher than the chances of Routine A (62.5%) and Routine C (63.6%).
22.  

<table>
<thead>
<tr>
<th>Preference</th>
<th>Math</th>
<th>Science</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td>93</td>
<td>57</td>
<td>150</td>
</tr>
<tr>
<td>Female</td>
<td>148</td>
<td>52</td>
<td>200</td>
</tr>
<tr>
<td>Total</td>
<td>241</td>
<td>109</td>
<td>350</td>
</tr>
</tbody>
</table>

23. a. about 0.438  
   b. about 0.387  

24. Sample answer: Venn diagrams show a visual representation of the data, and two-way tables organize the information into rows and columns; An advantage of a Venn diagram is that people who learn visually will easily understand them. A disadvantage is that as more categories are used, the Venn diagram becomes harder to draw and interpret. An advantage of a two-way table is that it is very easy to read and interpret, even with many categories. A disadvantage is that they are not as visual as Venn diagrams.

25. a. More of the current consumers prefer the leader, so they should improve the new snack before marketing it.  
   b. More of the new consumers prefer the leading snack, so there is no need to improve the snack.

26. Sample answer:

<table>
<thead>
<tr>
<th>Owns a Dog</th>
<th>Yes</th>
<th>No</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td>5</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>Female</td>
<td>7</td>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>Total</td>
<td>12</td>
<td>8</td>
<td>20</td>
</tr>
</tbody>
</table>

27. 

28.

29. 

10.4 Monitoring Progress and Modeling with Mathematics

22. a. \( P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) \)
   b. \( P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(A \text{ and } B) - P(A \text{ and } C) - P(B \text{ and } C) + P(A \text{ and } B \text{ and } C) \)

10.5 Monitoring Progress and Modeling with Mathematics

42. a. \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = 1 \)
   b. \( \binom{n}{r} = \frac{n!}{(n-r)!(n-r)!} = \frac{n!}{r!(n-r)!} = \binom{n}{r} \)
   c. \( \binom{n}{r} + \binom{n}{r-1} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!} = \frac{n!(n-r+1) + n!r}{(n-r+1)!} = \frac{n!(n+1)}{(n-r+1)!} = \binom{n+1}{r} \)

10.6 Monitoring Progress and Modeling with Mathematics

5. 

<table>
<thead>
<tr>
<th>w (value)</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcomes</td>
<td>5</td>
<td>21</td>
</tr>
<tr>
<td>( P(w) )</td>
<td>5/26</td>
<td>21/26</td>
</tr>
</tbody>
</table>

6. 

<table>
<thead>
<tr>
<th>n (value)</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcomes</td>
<td>10</td>
<td>90</td>
<td>900</td>
</tr>
<tr>
<td>( P(n) )</td>
<td>1/100</td>
<td>9/100</td>
<td>9/100</td>
</tr>
</tbody>
</table>

A64 Additional Answers
14. a. 

17. a. \( P(0) = 0.099, P(1) = 0.271, P(2) = 0.319, P(3) = 0.208, P(4) = 0.081, P(5) = 0.019, P(6) = 0.0025, P(7) = 0.00014 \)

b. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(x) )</td>
<td>0.099</td>
<td>0.271</td>
<td>0.319</td>
<td>0.208</td>
<td>0.081</td>
</tr>
</tbody>
</table>

c. 

18. 0.2; 0.6

19. no; The data is skewed right, so the probability of failure is greater.

20. no; The probability of not choosing the coin 100 times is \( \binom{99}{100} / 100 \approx 0.366 \).

21. a. The statement is not valid, because having a male and having a female are independent events.

b. 0.03125

c. 

22. \( p > 0.5 \)

10.6 Maintaining Mathematical Proficiency

23. FFF, FFM FMF, FMM, MMM, MMF, MFM, MFF

24. \( D_1C_1, D_1C_2, D_1C_3, D_2C_1, D_2C_2, D_2C_3 \)

Chapter 11

11.1 Monitoring Progress and Modeling with Mathematics

29. a. about \( 4.52 \times 10^{-9} \)

b. yes; The probability that a box contains an amount of cereal significantly less than the mean is very small.

30. 

Sample answer: 0.996

31. one standard deviation above the mean

32. \( \mu = 10, \sigma = 3 \)

33. a. 88th percentile

b. 93rd percentile

c. ACT; Your percentile on the ACT was higher than your percentile on the SAT.

34. Find the \( z \)-score for the ACT. Substitute this value along with the mean and the standard deviation for the SAT in the formula for \( z \)-score, and solve for \( x \). This value is the equivalent SAT score.

35. no; When the mean is greater than the median, the distribution is skewed right.

36. 0.1336
11.2 Monitoring Progress and Modeling with Mathematics

18. Because all the players in a group were used, the mean is a parameter; Because the mean age of 26 is based on every member of a football team, it is a parameter.

21. possibly, but extremely unlikely; The result is unlikely to occur by chance. The sample size of the population is too small to make such a conclusion.

22. population: all the majors of students at a university, sample: majors of students at the university who take chemistry; All the students’ majors at the university make up the entire group, and the majors of students who take chemistry make up a subset of the entire group.

23. Sample answer: population: all American adults, sample: the 801 American adults surveyed; The sample consists of 606 American adults who say the world’s temperature will go up over the next 100 years, 174 American adults who say it will go down, and 21 American adults who have no opinion.

24. yes; The sample mean is an estimate of the population mean, and 7.5 hours is relatively close to 8 hours.

25. simulation 2; Simulation 2 gives a better indication of outcomes that are not likely to occur by chance.

26. a. Sample answer:

b. The maker’s claim is most likely false; The maker’s claim is most likely true.

11.3 Lesson Monitoring Progress

1. cluster sample

2. Sample answer: Students are divided into groups by grade, and a random sample is selected from each grade.

11.3 Monitoring Progress and Modeling with Mathematics

34. a. stratified sample

b. Because the counties (groups) are very different in population (size), selecting the same number of people from each county will underrepresent people living in counties with a large population, and overrepresent people living in counties with a small population.

35. self-selected sample and convenience sample; In a self-selected sample, only people with strong opinions are likely to respond. In a convenience sample, parts of the population have no chance of being selected for the survey.

36. In a blind experiment, the people administering the treatment know which treatment each participant is receiving. In a double-blind experiment, they do not; Sample answer: An advantage of the double-blind experiment is that a person administering the treatment cannot accidently influence the person receiving the treatment.

37. a. to determine the employment rate of graduates in their field of study

b. all graduating seniors of the college

C. Sample answer: Are you employed? If yes, is your job in your field of study?

38. the second sample; In a larger sample, unusual responses will have less of an effect.

39. no; Sample answer: Some groups in the population, like the homeless, are difficult to contact.

40. a. survey

b. Sample answer: Put an optional survey card in each back seat pocket for passengers to complete and mail back to the airline; Passengers with enough leg room are less likely to complete and return the survey.

c. Sample answer: Group passengers by type of plane and class of seat. Select a random sample from each group, and have a flight attendant hand a survey to the selected passengers which they may complete anonymously and return to the box the flight attendant brings around; All segments of the population have a chance of being selected, and passengers are likely to give honest responses on an anonymous survey.

d. Sample answer: biased: Do you feel you do not have enough leg room in a cramped seat area?; unbiased: Do you have enough leg room?

41. a. self-selected sample

b. people who spend a lot of time on the Internet and visit that particular site; The survey is probably biased.
11.3 Maintaining Mathematical Proficiency
42. 32 43. 9
44. −4 45. \(\frac{1}{3}\)
46. 4\(^7\) or 16,384 47. \(\sqrt{\frac{18}{4}}\)
48. 4 49. 3

11.3 Mini-Assessment
5. The question encourages a “yes” response, because the coach is asking the question. One improvement may be to have players go online and complete an anonymous survey about their summer workouts.

11.4 Monitoring Progress and Modeling with Mathematics
11. a. Sample answer: Because the heart rates are monitored for two different types of exercise, the groups cannot be compared. Running on a treadmill may have a different effect on heart rate than lifting weights; Check the heart rates of all the athletes after the same type of exercise.
   b. no potential problems
12. a. no potential problems
   b. Sample answer: Because the subjects volunteer for a particular group, the groups are not similar. Someone who volunteers to read may also have other habits that could raise IQ scores; Randomly assign the volunteers to the treatment group or the control group.
13. Sample answer: The sample size is not large enough to provide valid results; Increase the sample size.
14. It is not possible to control variables of objects in space.
15. no; Your friend would have to perform an observational study, and an observational study can show correlation, but not causality.
16. a. yes; Sample answer: The individuals were randomly assigned to a control group with no ads, and a treatment group with ads.
   b. watching TV with ads for candidate B
   c. no; Sample answer: The increase in preference for candidate B with the ads may be too small to be significant.
17. Sample answer: The placebo effect is response to a dummy treatment that may result from the trust in the researcher or the expectation of a cure; It can be minimized by comparing two groups so the placebo effect has the same effect on both groups.
18. Sample answer: Plants treated with a growth supplement will grow taller than plants that are not treated; Randomly select the same number of plants to be put in each of two groups. Use the growth supplement on the plants in one group, and do not use the growth supplement on the plants in the other group. Keep all other variables constant and record the height of the plants each day.
19. yes; Repetition reduces the effect of unusual results that may occur by chance.

11.4 Maintaining Mathematical Proficiency
20. symmetric
21. skewed right

22. exponential growth
23. exponential decay
24. exponential decay
25. exponential growth

11.4 Mini-Assessment
1. The study is a randomized comparative experiment. The treatment is watching reality TV at least 5 hours a week. The treatment group is the patients who watch reality TV. The control group is patients who do not watch reality TV.
2. Use an experiment. Randomly select the same number of bean seeds to be placed in each of two identical planters. One planter will be set outdoors, while the other remains indoors. Keep all other variables, such as soil and water, the same. Record the growth of the plants every week.
3. The sample size is not large enough to produce valid results. To improve the validity of the experiment, the sample size must be larger and the experiment must be replicated.

11.5 Monitoring Progress and Modeling with Mathematics
22. simulation \(a\); simulation \(c\); Simulation \(a\) has the smallest standard deviation, and simulation \(c\) has the largest standard deviation.
23. about 453 residents
24. 4n; 4 times; The margin of error is inversely proportional to the square root of the sample size, so cutting the margin of error in half requires multiplying the sample size by 2\(^2\), or 4.
25. a. 500 voters  
   b. about \(\pm 4.5\%\)
   c. candidate A: between 42.5% and 51.5%, candidate B: between 48.5% and 57.5%
   d. no; 273 voters
26. a. \(n > 9\)  
   b. \(n > 891\)
   c. The smaller the percentage of a large population that have a certain characteristic, the larger the sample must be to reasonably represent the population.
27. more than 2500; To be confident that sports drink X is preferred, the margin of error would need to be less than 2%.

11.5 Maintaining Mathematical Proficiency
28. \(y = \log x + 3\)  
29. \(y = \log_2(x + 5)\)
30. \(y = e^x - 5\)  
31. \(y = 6x + 1\)
32. arithmetic; \(a_n = -3n + 20\)
33. geometric; \(a_n = 3(2)^n - 1\)
34. geometric; \(a_n = 32\left(\frac{1}{2}\right)^n - 1\)
11.6 Explorations

2. a. Sample answer:

![Graph of Mean Differences From Class Resampling](image)

3. d.

![Graph of Mean Differences From Class Resampling 2](image)

All of the values are below 0.36, so you can reject the hypothesis.

4. \( \bar{x}_{\text{control}} = 0.97; \bar{x}_{\text{treatment}} = 1.33; \bar{x}_{\text{treatment}} - \bar{x}_{\text{control}} = 0.36; \)

Sample answer:

![Graph of Mean Differences From Class Resampling 2](image)

5. The treatment has an effect on the waist circumference of adults. The effect increases the waist circumference; The treatment has no effect on the waist circumference of adults. The effect decreases the waist circumference; The treatment has no effect on the waist circumference of adults.

6. The treatment has an effect on the waist circumference of adults. The effect increases the waist circumference; The treatment has an effect on the waist circumference of adults. The effect decreases the waist circumference; The treatment has no effect on the waist circumference of adults.

7. Sample answer: \(-1.75\)

8. Sample answer: 1.15

9. The hypothesis is most likely false; Music therapy decreases depression scores.

10. The hypothesis is probably true; Music therapy does not have an effect on depression scores.

11. The histogram in Exercise 9 has a roughly normal distribution and shows the mean differences from 200 resamplings. The histogram in Exercise 11 is random and shows the mean differences from 20 resamplings; the histogram in Exercise 9 because it uses a large number of resamplings and the roughly normal distribution suggests music therapy decreases depression scores

12. negative; slightly effective; Most of the treatment values are less than most of the control values.

13. yes; As the number of samplings increase, the individual values should end up in each group approximately the same number of times, so the positive and negative differences in the means should balance out to 0.

14. Sample answer: the mean increase in the height of a plant given a growth supplement; The growth supplement has no effect on the growth of the plants. The growth supplement causes a decrease in the height of the plants. The growth supplement causes a slight increase in the height of the plants. The growth supplement causes a significant increase in the height of the plants.

15. 12,870; The number of combinations of 16 items in groups of 8 amounts to 12,870.

11.6 Maintaining Mathematical Proficiency

16. \(5x^2(x - 3)\)

17. \((y - 2)(y^2 + 2y + 4)\)

18. \((z + 3)(z - 3)(z + 5)\)

19. \((9w^2 + 4)(3w + 2)(3w - 2)\)

20. yes; \(g(x) = \frac{3}{x} - 5\)

21. yes; \(g(x) = \frac{1}{2x + 1}\)

22. yes; \(g(x) = \frac{2}{x + 4}\)

23. no; \(y = \pm \frac{3}{\sqrt{x} - 1}\)

11.6 Mini-Assessment

3. The experimental difference 0.2 falls within the middle 90%. You can conclude that the hypothesis is most likely true. So, hydroponics does not have an effect on the yield of strawberry plants.