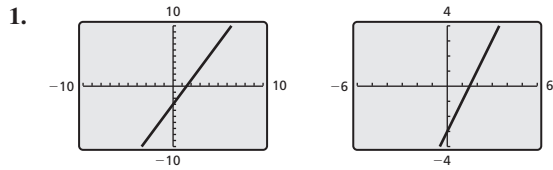


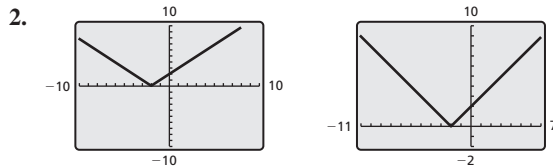
Additional Answers

Chapter 1

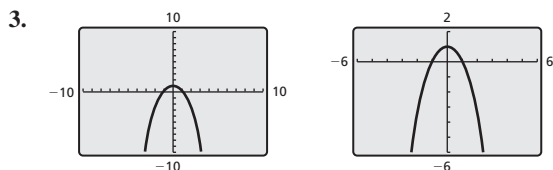
Mathematical Practices



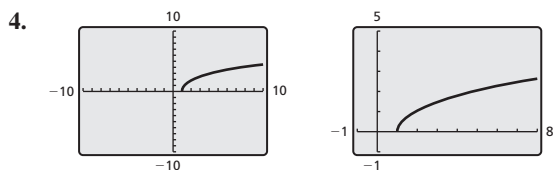
In the square viewing window, the graph is steeper.



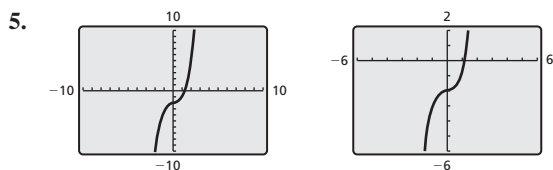
In the square viewing window, the graph is narrower.



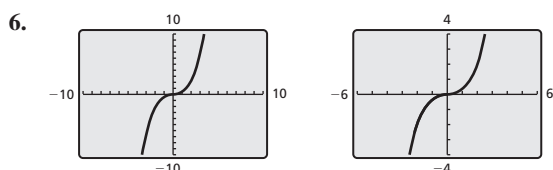
In the square viewing window, the graph is wider.



In the standard viewing window, the graph is flatter.

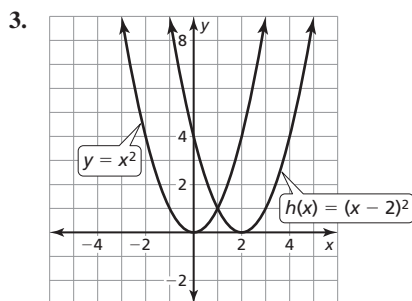


In the square viewing window, the graph is wider.

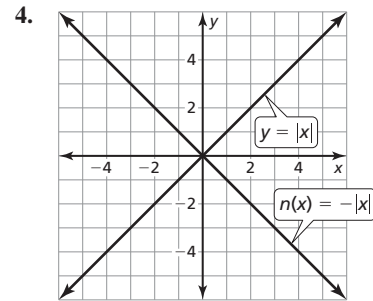


In the square viewing window, the graph is wider.

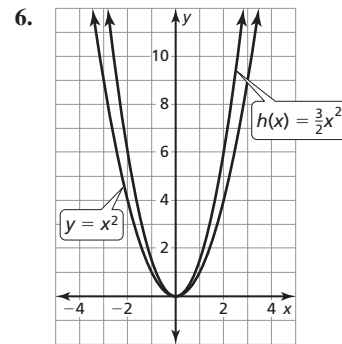
1.1 Lesson Monitoring Progress



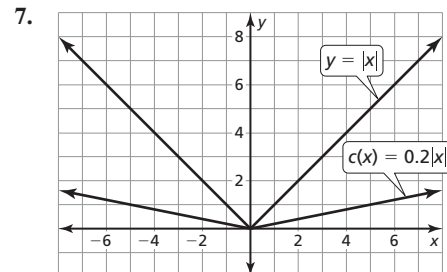
The graph is 2 units to the right of the parent quadratic function.



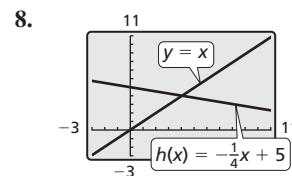
The graph is a reflection in the x -axis of the parent absolute value function.



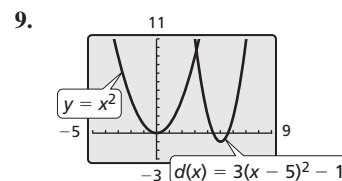
The graph of h is a vertical stretch of the parent quadratic function.



The graph of c is a vertical shrink of the parent absolute value function.

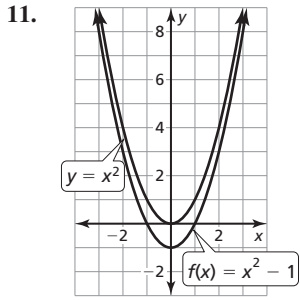


The graph of h is a reflection in the x -axis, a vertical shrink followed by a vertical translation 5 units up of the parent linear function.

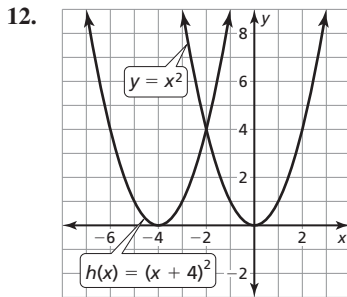


The graph of d is a vertical stretch followed by a translation 5 units right and 1 unit down of the parent quadratic function.

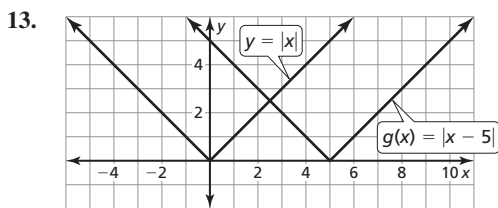
1.1 Monitoring Progress and Modeling with Mathematics



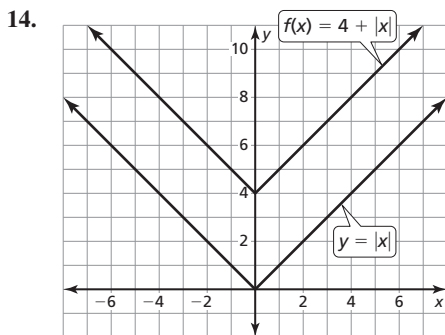
The graph of f is a vertical translation 1 unit down of the parent quadratic function.



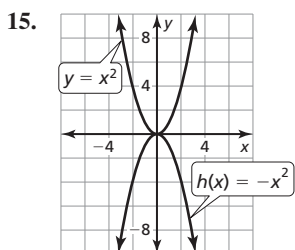
The graph of h is a horizontal translation 4 units left of the parent quadratic function.



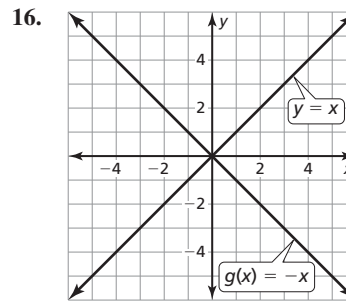
The graph of g is a horizontal translation 5 units right of the parent absolute value function.



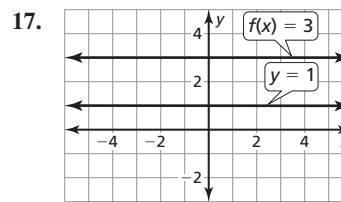
The graph of f is a vertical translation 4 units up of the parent absolute value function.



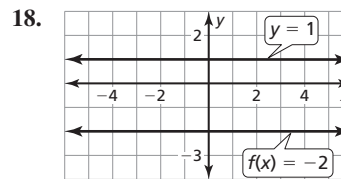
The graph of h is a reflection in the x -axis of the parent quadratic function.



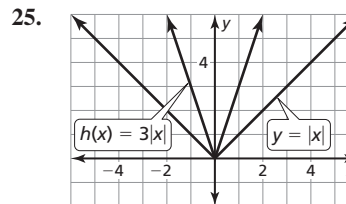
The graph of g is a reflection in the x -axis of the parent linear function.



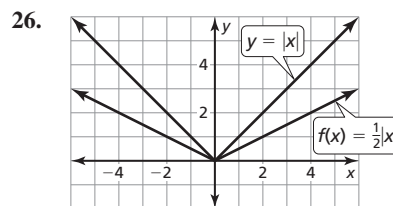
The graph of f is a vertical translation 2 units up of the parent constant function.



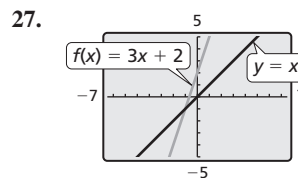
The graph of f is a vertical translation 3 units down of the parent constant function.



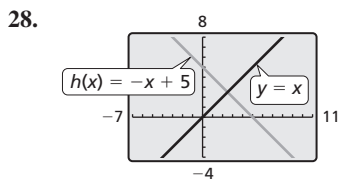
The graph of h is a vertical stretch of the parent absolute value function.



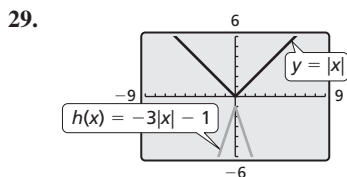
The graph of f is a vertical shrink of the parent absolute value function.



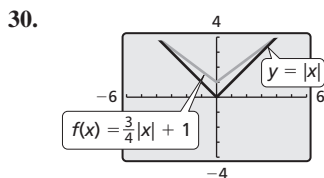
The graph of f is a vertical stretch followed by a translation 2 units up of the parent linear function.



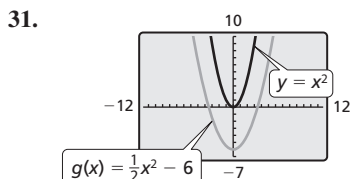
The graph of h is a reflection in the x -axis followed by a translation 5 units up of the parent linear function.



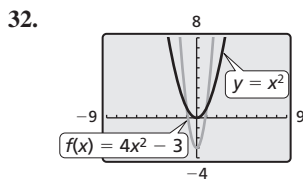
The graph of h is a vertical stretch and a reflection in the x -axis followed by a translation 1 unit down of the parent absolute value function.



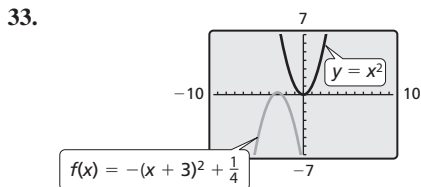
The graph of f is a vertical shrink followed by a translation 1 unit up of the parent absolute value function.



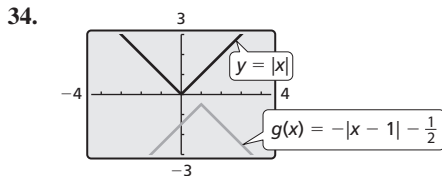
The graph of g is a vertical shrink followed by a translation 6 units down of the parent quadratic function.



The graph of f is a vertical stretch followed by a translation 3 units down of the parent quadratic function.

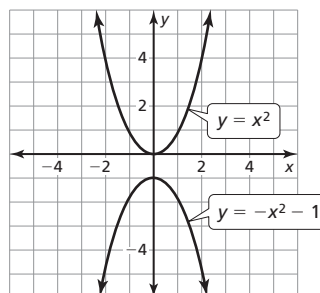


The graph of f is a reflection in the x -axis followed by a translation 3 units left and $\frac{1}{4}$ unit up of the parent quadratic function.



The graph of g is a reflection in the x -axis followed by a translation 1 unit right and $\frac{1}{2}$ unit down of the parent absolute value function.

35. It is a vertical stretch, not shrink. The graph is a reflection in the x -axis followed by a vertical stretch of the parent quadratic function.
36. The function should be $f(x) = |x - 3|$ rather than $f(x) = |x + 3|$. The graph is a translation 3 units right of the parent absolute value function so the function is $f(x) = |x - 3|$.
37. $(2, -1), (-1, -4), (2, -5)$
38. $(-1, -3), (1, -3), (-1, -1), (-3, -1)$
39. absolute value; domain is all real numbers; range is $y \geq -1$
40. absolute value; domain is all real numbers; range is $y \geq 2$
41. linear; domain is all real numbers; range is all real numbers
42. linear; domain is all real numbers; range is all real numbers
43. quadratic; domain is all real numbers; range is $y \geq -2$
44. quadratic; domain is all real numbers; range is $y \leq 6$
45. absolute value; 8 mi/h
46. *Sample answer:*



Reflect the parent function in the x -axis, and translate the parent function 1 unit down.

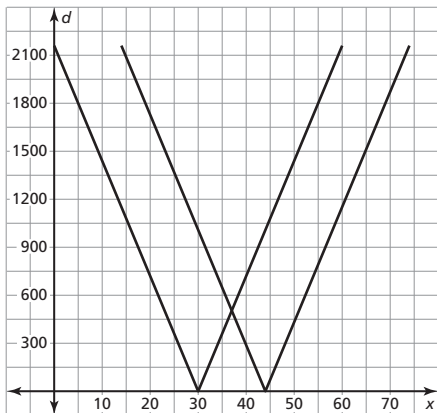
47. no; f is shifted right and g is shifted down.
53. a. vertical translation; The graph will have a vertical stretch and will be shifted 3 units down.
b. horizontal translation; The graph will be shifted 8 units right.
c. both; The graph will be shifted 2 units left and 4 units up.
d. neither; The graph will have a vertical stretch.
54. a. 1; The graph will intersect the x -axis at $x = -\frac{1}{3}$.
b. 2; The graph will intersect the x -axis at $x = 2$ and $x = 4$.
c. -1 ; The graph will intersect the x -axis at $x = 1$ and $x = -1$.
d. 0; The graph will intersect the x -axis everywhere.

1.1 Maintaining Mathematical Proficiency

55. no
56. no
57. yes
58. yes
59. x -intercept: 0; y -intercept: 0
60. x -intercept: -2 ; y -intercept: 2
61. x -intercept: $\frac{1}{3}$; y -intercept: 1
62. x -intercept: 8; y -intercept: -4

1.2 Monitoring Progress and Modeling with Mathematics

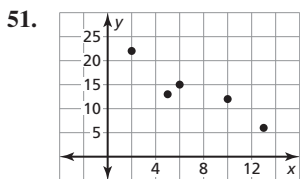
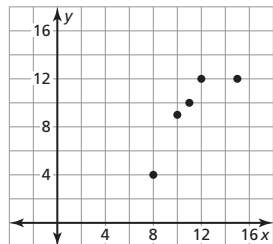
44. *Sample answer:* $d = 72|x - 44|$; You leave on June 15 rather than on June 1.



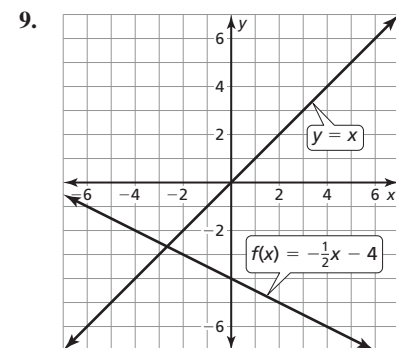
45. $a = -2$, $b = 1$, and $c = 0$; $g(x) = -2|x - 1|$ represents the transformation of $f(x)$.

1.2 Maintaining Mathematical Proficiency

46. 7
48. -2
50.



1.1–1.2 Quiz



The graph of f is a reflection in the x -axis followed by a vertical shrink by a factor of $\frac{1}{2}$ and a translation 4 units down of the parent linear function.

19. The graph of f can be transformed to model the total cost for a senior citizen using a vertical shrink by a factor of $\frac{1}{2}$ followed by a translation 30 units down; \$40

1.3 Monitoring Progress and Modeling with Mathematics

26. a. $-\frac{5}{4}$; the decrease in balance per month
b. domain: $0 \leq x \leq 24$, range: $0 \leq y \leq 3000$; The domain represents the term of the balance of the loan from start to finish, and the range represents the balance of the loan.
c. \$1500
27. no; Because r is close to 0, the points do not lie close to the line.
28. *Sample answer:* A : (0, 4), B : (4, 0), C : (0, 0); line connecting A and C : $x = 0$; line connecting B and C : $y = 0$
29. It is negative; As x increases, y increases, so z decreases.
30. D
31. about 2.2 mi
32. a. yes; If a country has a high number of personal computers per capita, it indicates wealth and a high quality of life which would also indicate that the country would have a health care system and, therefore, a higher life expectancy.
b. no; The high life expectancy is not caused by the computers. Both are benefits of the wealth of a country. This is an example of how correlation does not imply causation.

1.3 Maintaining Mathematical Proficiency

33. (16, -41)
35. $(1, \frac{1}{2})$
37. $(\frac{16}{17}, \frac{15}{17})$
34. $(\frac{1}{2}, 0)$
36. (-1, 0)
38. (6, 2)

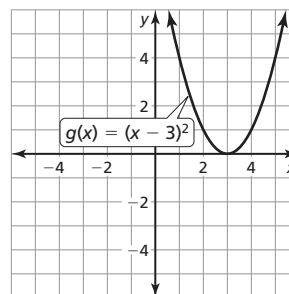
1.4 Maintaining Mathematical Proficiency

44. $x^2 - 4x + 4$
46. $4z^2 - 20z + 25$
48. $g(x) = |x + 2| - 5$
50. $g(x) = |x| - 1$
45. $9m^2 + 6m + 1$
47. $16 - 8y + y^2$
49. $g(x) = -|x| + 5$
51. $g(x) = 3|x| - 15$

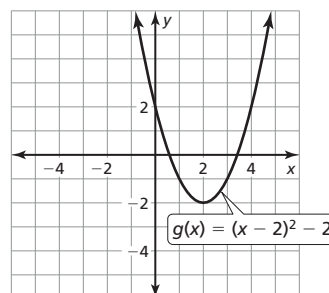
Chapter 2

2.1 Lesson Monitoring Progress

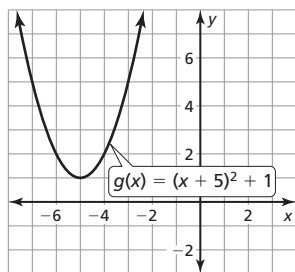
1. The graph of g is a translation 3 units right of the graph of f .



2. The graph of g is a translation 2 units right and 2 units down of the graph of f .

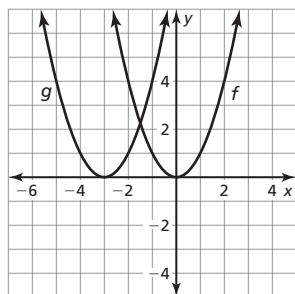


3. The graph of g is a translation 5 units left and 1 unit up of the graph of f .

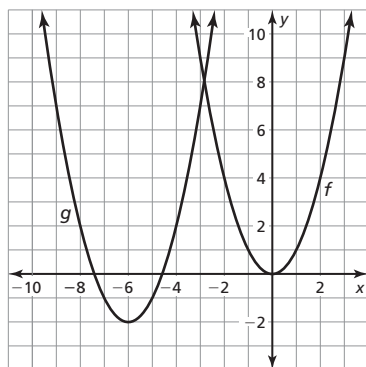


2.1 Monitoring Progress and Modeling with Mathematics

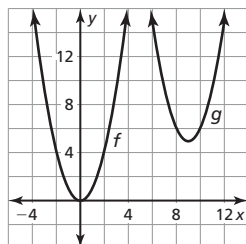
8. The graph of g is a translation 3 units left of the graph of f .



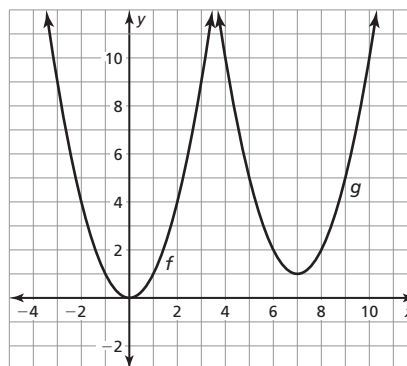
9. The graph of g is a translation 6 units left and 2 units down of the graph of f .



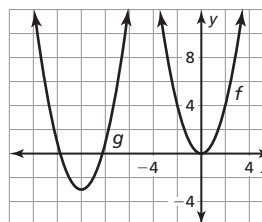
10. The graph of g is a translation 9 units right and 5 units up of the graph of f .



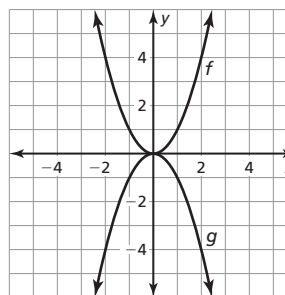
11. The graph of g is a translation 7 units right and 1 unit up of the graph of f .



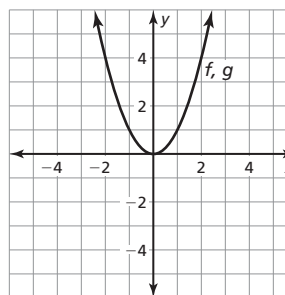
12. The graph of g is a translation 10 units left and 3 units down of the graph of f .



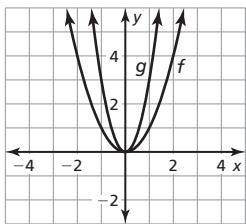
13. A; The graph has been translated 1 unit right.
 14. D; The graph has been translated 1 unit up.
 15. C; The graph has been translated 1 unit right and 1 unit up.
 16. B; The graph has been translated 1 unit left and 1 unit down.
 17. The graph of g is a reflection in the x -axis of the graph of f .



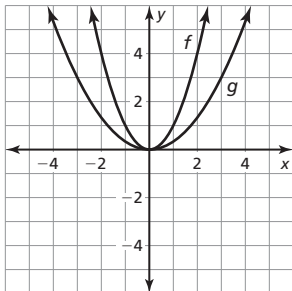
18. The graph of g is a reflection in the y -axis of the graph of f .



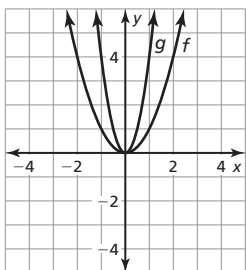
19. The graph of g is a vertical stretch by a factor of 3 of the graph of f .



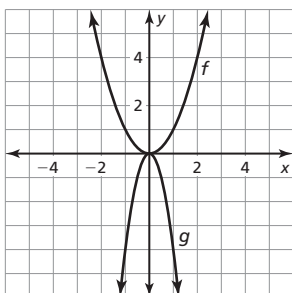
20. The graph of g is a vertical shrink by a factor of $\frac{1}{3}$ of the graph of f .



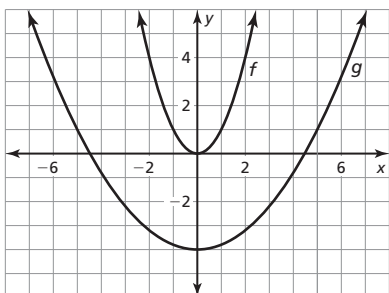
21. The graph of g is a horizontal shrink by a factor of $\frac{1}{2}$ of the graph of f .



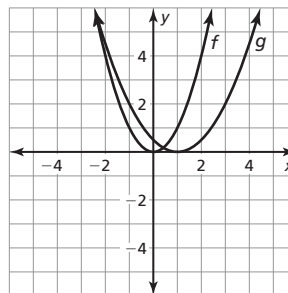
22. The graph of g is a horizontal shrink by a factor of $\frac{1}{2}$ followed by a reflection in the x -axis of the graph of f .



23. The graph of g is a vertical shrink by a factor of $\frac{1}{5}$ followed by a translation 4 units down.



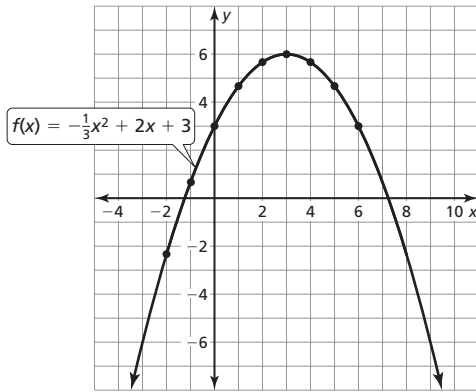
24. The graph of g is a vertical shrink by a factor of $\frac{1}{2}$ followed by a translation 1 unit right.



25. The graph is a reflection in the x -axis, not y -axis; The graph is a reflection in the x -axis and a vertical stretch by a factor of 6, followed by a translation 4 units up of the graph of the parent quadratic function.
26. The order of the transformations is not correct and the graph is a translation 4 units up, not down; The graph is a vertical stretch by a factor of 6 and a reflection in the x -axis followed by a translation 4 units up of the graph of the parent quadratic function.
27. The graph of f is a vertical stretch by a factor of 3 followed by a translation 2 units left and 1 unit up of the graph of the parent quadratic function; $(-2, 1)$
28. The graph of f is a vertical stretch by a factor of 4 followed by a reflection in the x -axis and a translation 1 unit left and 5 units down of the graph of the parent quadratic function; $(-1, -5)$
29. The graph of f is a vertical stretch by a factor of 2 followed by a reflection in the x -axis and a translation 5 units up of the graph of the parent quadratic function; $(0, 5)$
30. The graph of f is a vertical shrink by a factor of $\frac{1}{2}$ followed by a translation 1 unit right of the graph of the parent quadratic function; $(1, 0)$
45. a. $y = \frac{-5}{1089}(x - 33)^2 + 5$
- b. The domain is $0 \leq x \leq 66$ and the range is $0 \leq y \leq 5$; The domain represents the horizontal distance and the range represents the height of the fish.
- c. yes; The value changes to $-\frac{1}{225}$; The vertex has changed but it still goes through the point $(0, 0)$, so there has been a horizontal stretch or shrink which changes the value of a .
46. The graph of g is a translation 4 units left and 2 units down of the graph of f .
47. a. $a = 2, h = 1, k = 6; g(x) = 2(x - 1)^2 + 6$
- b. $g(x) = 2f(x - 1) + 6$; For each function, $a, h,$ and k are the same but the answer in part (b) does not indicate the type of function that is being translated.
- c. $a = 2, h = 1, k = 3; g(x) = 2(x - 1)^2 + 3;$
 $g(x) = 2f(x - 1) + 3$; For each function, $a, h,$ and k are the same, but the second function does not indicate the type of function that is being translated.
- d. *Sample answer:* vertex form; Writing a transformed function using function notation requires an extra step of substituting $f(x)$ into the newly transformed function.
48. *Sample answer:* $g(x) = f(2x) + 6 = -0.5(2x - 6)^2 + 24$; You jump higher but not as far.
49. a vertical shrink by a factor of $\frac{7}{16}$

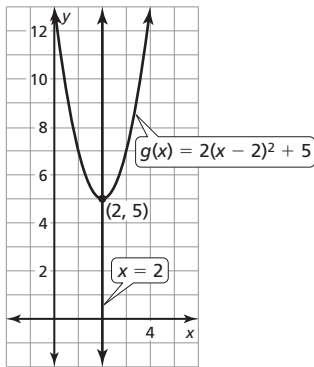
2.2 Explorations

2. a. $-2\frac{1}{3}, 2\frac{2}{3}, 3, 4\frac{2}{3}, 5\frac{2}{3}, 6, 5\frac{2}{3}, 4\frac{2}{3}, 3$

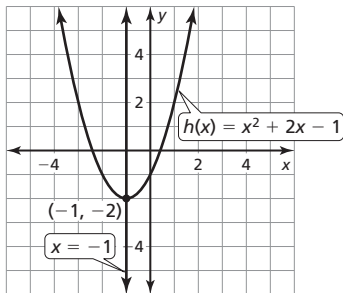


2.2 Lesson Monitoring Progress

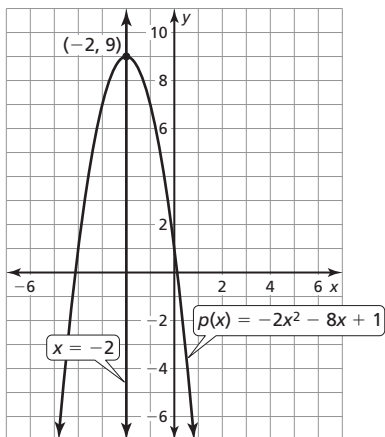
2.



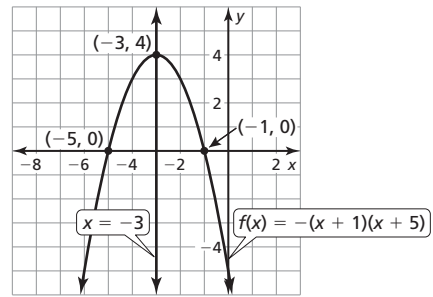
3.



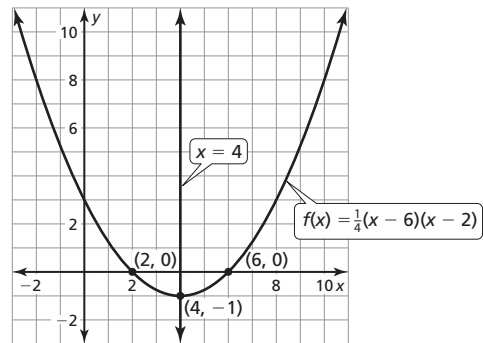
4.



6.

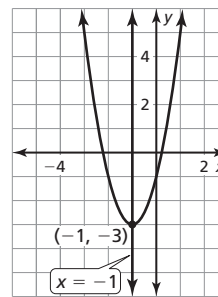


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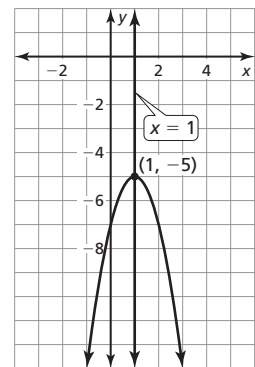


2.2 Monitoring Progress and Modeling with Mathematics

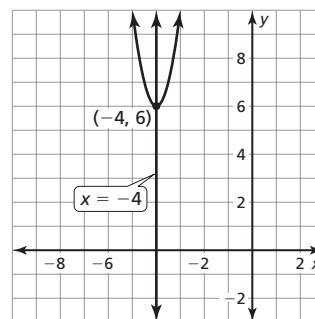
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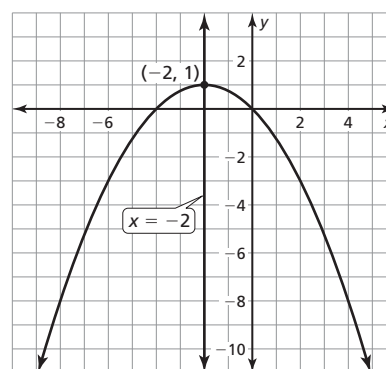
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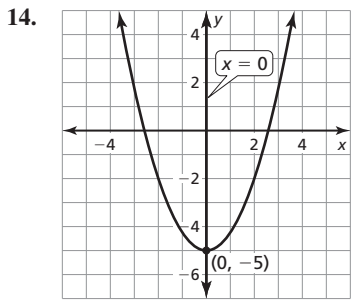
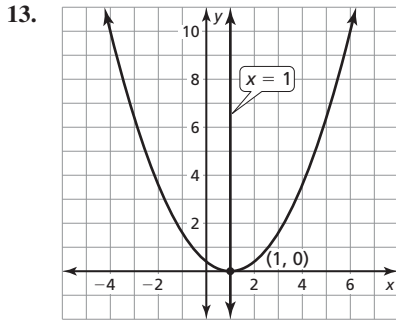
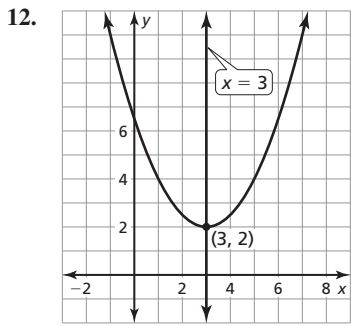


10.

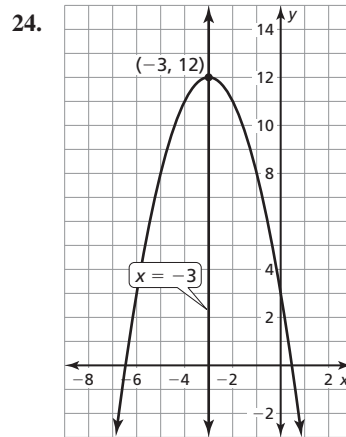
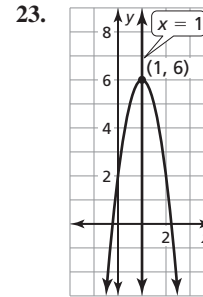
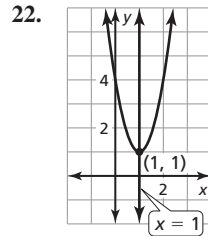
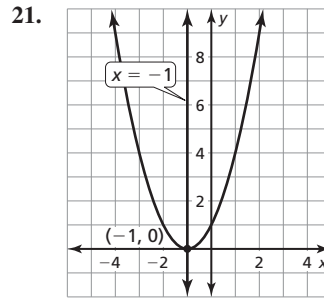
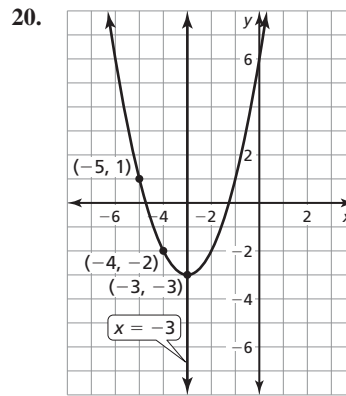
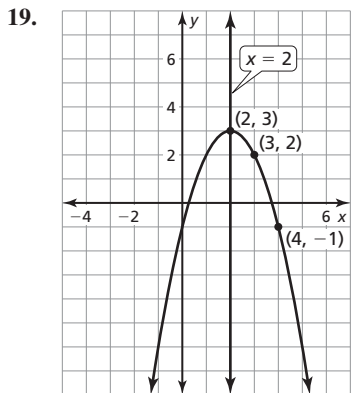


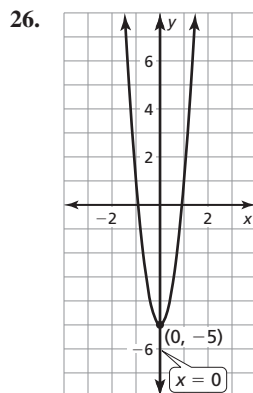
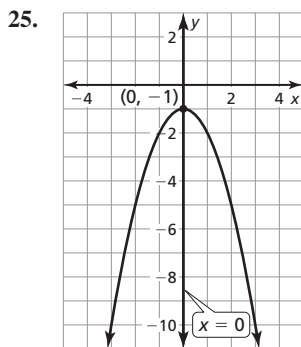
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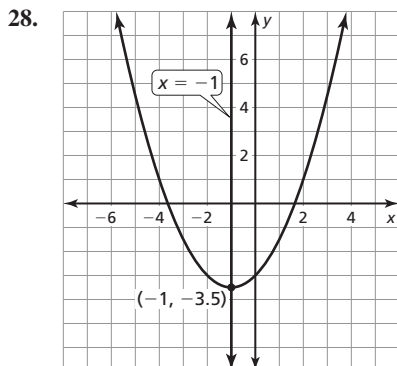
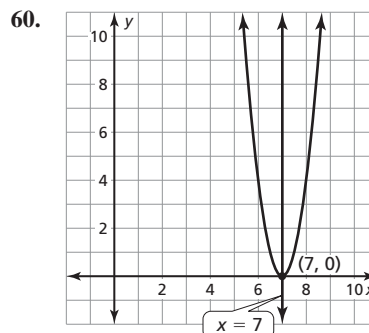
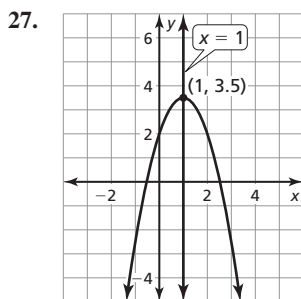
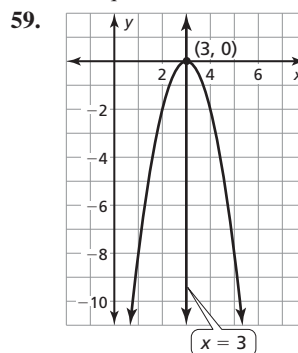


15. C 16. D
17. B 18. A





52. $A = \frac{1}{2}b(6 - b) = -\frac{1}{2}b^2 + 3b$; The maximum area is 4.5 square units.



61. $p = 2, q = -6$; The graph is decreasing to the left of $x = -2$ and increasing to the right of $x = -2$.

62. $p = -1, q = 3$; The graph is decreasing to the left of $x = 1$ and increasing to the right of $x = 1$.

63. $p = 4, q = 2$; The graph is increasing to the left of $x = 3$ and decreasing to the right of $x = 3$.

64. $p = -5, q = -1$; The graph is increasing to the left of $x = -3$ and decreasing to the right of $x = -3$.

65. the second kick; the first kick

66. 160 ft; about 1.5 ft

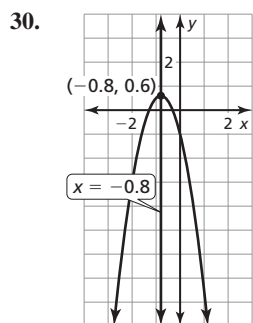
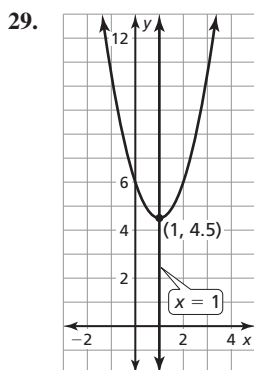
67. no; Either of the points could be the axis of symmetry, or neither of the points could be the axis of symmetry. You can only determine the axis of symmetry if the y-coordinates of the two points are the same, because the axis of symmetry would lie halfway between the two points.

68. *Sample answer:* $y = 2(x - 2)(x - 4)$ and $y = 2(x + 1)(x - 7)$

69. \$1.75

70. \$300

71. All three graphs are the same; $f(x) = x^2 + 4x + 3$, $g(x) = x^2 + 4x + 3$



31. Both functions have an axis of symmetry of $x = 2$.

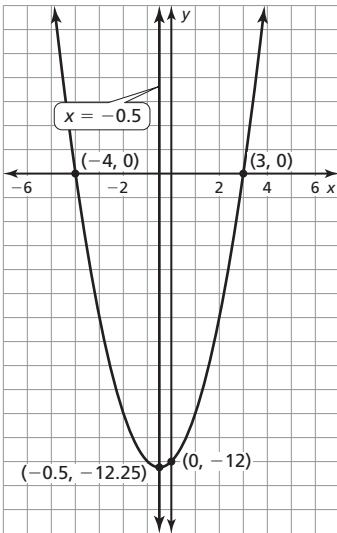
32. highest; If the graph is increasing until it reaches $x = 2$ and then decreasing after, then the vertex must be the highest point.

50. a. 3090 rev/min; 74.68 ft-lbs

b. The engine torque increases as the speed increases until the engine speed reaches 3.09 thousands of revolutions per minutes then the torque begins to decrease.

51. $A = w(20 - w) = -w^2 + 20w$; The maximum area is 100 square units.

72. $f(x) = (x + 4)(x - 3)$

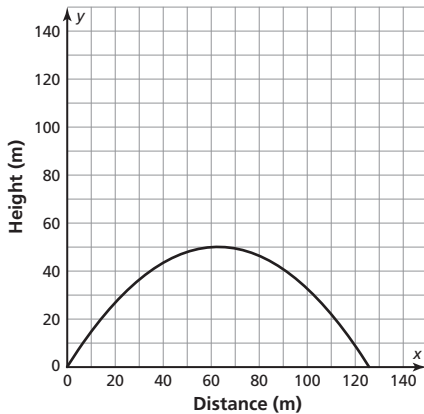


73. no; The vertex of the graph is $(3.25, 2.1125)$, which means the mouse cannot jump over a fence that is higher than 2.1125 feet.

74. a. the minimum

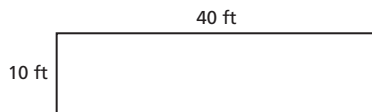
b. Instead of representing the minimum, $f\left(\frac{p+q}{2}\right)$ would represent the maximum.

75.



The domain is $0 \leq x \leq 126$ and the range is $0 \leq y \leq 50$; The domain represents the distance from the start of the bridge on one side of the river, and the range represents the height of the bridge.

76. Sample answer:



Of all possible designs, a square garden with sides of 25 feet will have the greatest area; A square has the largest area of all rectangles with the same perimeter.

77. no; The vertex must lie on the axis of symmetry, and $(0, 5)$ does not lie on $x = -1$.

78. The y -intercept is apq .

79. a. about 14.1%; about $55.5 \text{ cm}^3/\text{g}$

b. about 13.6%; about $44.1 \text{ cm}^3/\text{g}$

c. The domain for hot-air popping is $5.52 \leq x \leq 22.6$, and the range is $0 \leq y \leq 55.5$. The domain for hot-oil popping is $5.35 \leq x \leq 21.8$, and the range is $0 \leq y \leq 44.1$. This means that the moisture content for the kernels can range from 5.52% to 22.6% and 5.35% to 21.8%, while the popping volume can range from 0 to 55.5 cubic centimeters per gram and 0 to 44.11 cubic centimeters per gram.

80. The x -coordinate of the vertex does not change, and the y -coordinate moves further from the x -axis; The x -coordinate of the vertex does not change, and the y -coordinate moves closer to the x -axis.

2.2 Maintaining Mathematical Proficiency

81. 4

82. 8

83. no solution

84. -2

85. 2

86. 6

87. -12

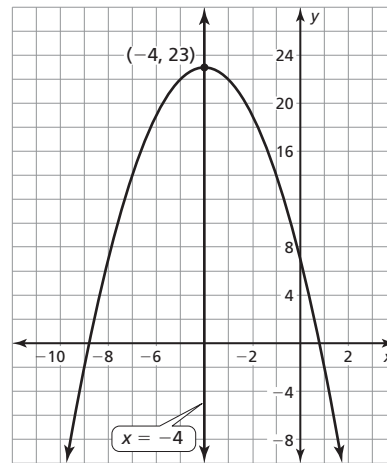
88. -8

2.2 Mini-Assessment

5. Because 90 yards $<$ 100 yards, your shot travels farther. Because 81 yards $>$ 25 yards, your opponent's shot travels higher.

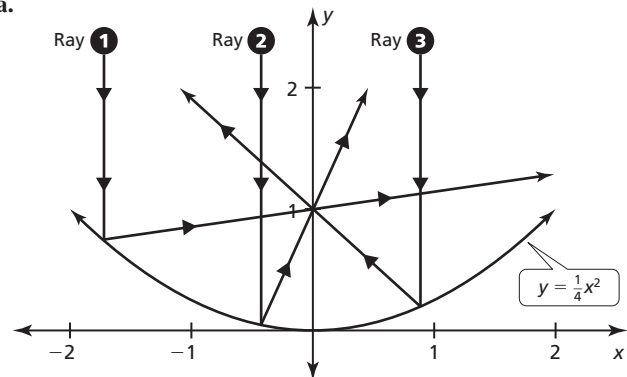
2.1–2.2 Quiz

9.



2.3 Explorations

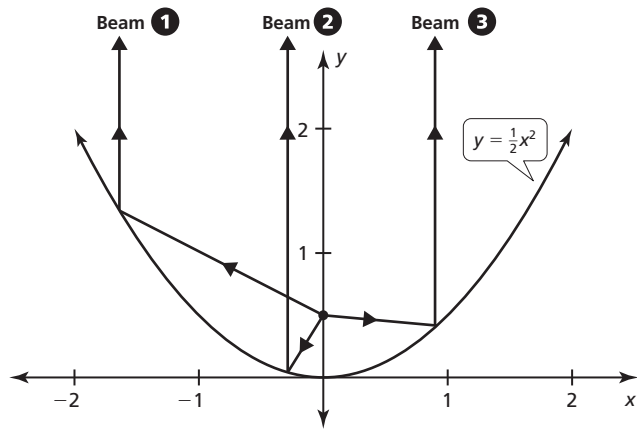
1. a.



b. All the reflected rays cross through the point $(0, 1)$.

c. $(0, 1)$; This is the best place for the receiver because any ray entering the satellite dish will hit the receiver.

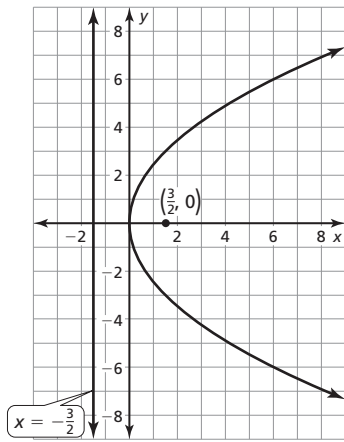
2.



All beams leaving the parabola are parallel; yes; With a spotlight, all beams of light should be pointed at the same object.

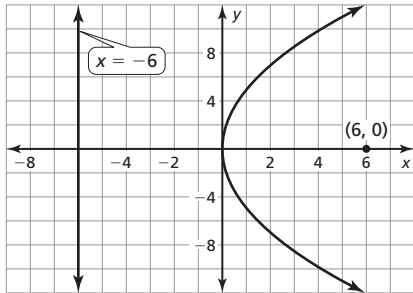
2.3 Lesson Monitoring Progress

4. The focus is $(\frac{3}{2}, 0)$, the directrix is $x = -\frac{3}{2}$, and the axis of symmetry is the x -axis.

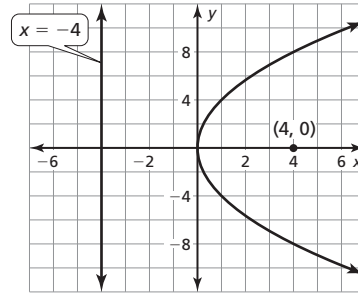


2.3 Monitoring Progress and Modeling with Mathematics

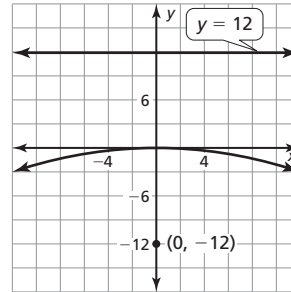
16. The focus is $(6, 0)$. The directrix is $x = -6$. The axis of symmetry is the x -axis.



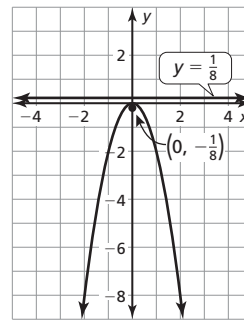
17. The focus is $(4, 0)$. The directrix is $x = -4$. The axis of symmetry is the x -axis.



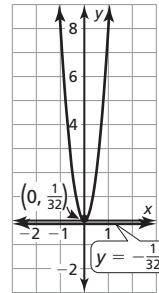
18. The focus is $(0, -12)$. The directrix is $y = 12$. The axis of symmetry is the y -axis.



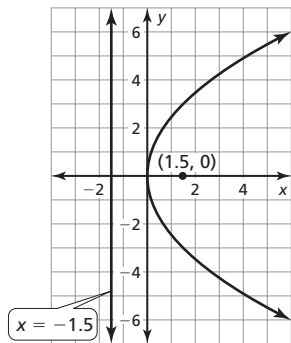
19. The focus is $(0, -\frac{1}{8})$. The directrix is $y = \frac{1}{8}$. The axis of symmetry is the y -axis.



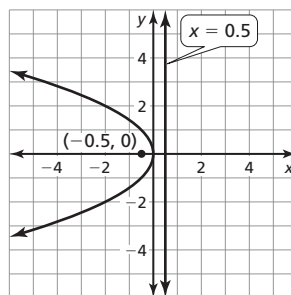
20. The focus is $(0, \frac{1}{32})$. The directrix is $y = -\frac{1}{32}$. The axis of symmetry is the y -axis.



21. Instead of a vertical axis of symmetry, the graph should have a horizontal axis of symmetry.



22. Because $p = -0.5$, the focus is $(-0.5, 0)$, and the directrix is $x = 0.5$.



23. 9.5 in.; The receiver should be placed at the focus. The distance from the vertex to the focus is $p = \frac{38}{4} = 9.5$ in.

2.3 Maintaining Mathematical Proficiency

55. $y = 3x - 7$ 56. $y = -2x + 6$
 57. $y = -\frac{1}{2}x + \frac{5}{2}$ 58. $y = -x + 1$
 59. $y = 3.98x + 0.92$ 60. $y = -1.02x + 18.93$

2.4 Extra Example 2

- b. The function is decreasing over $0 < x < 13.5$ and the average rate of change is -0.675 . The function is increasing over $13.5 < x < 24$ and the average rate of change is 0.525 . Because $|-0.675| > |0.525|$, the average rate at which the temperature decreases from midnight to 1:30 P.M. is greater than the average rate at which it increases from 1:30 P.M. to midnight.

2.4 Monitoring Progress and Modeling with Mathematics

31. neither; The first and second differences are not constant.
 32. quadratic; The second differences are constant;
 $y = -16x^2 - x + 320$
 33. a. The vertex indicates that on the 6th day, 19 people were absent, more than any other day.
 b. $y = -0.5(10 - 6)^2 + 19$; 11 students
 c. From 0 to 6 days, the average rate of change was 3 students per day. From 6 to 11 days, the average rate of change was -2.5 students per day. The rate at which students were missing school was changing more rapidly as more became ill, in comparison to when the students were becoming well.
 34. *Sample answer:* The surfaces of some roads are shaped like parabolas so rain water can run off each side. The height of the road can be modeled by a quadratic equation.
 35. $y = -16x^2 + 6x + 22$; after about 1.24 sec; 1.375 sec

36. a. positive
 b. negative
 c. negative
 d. zero

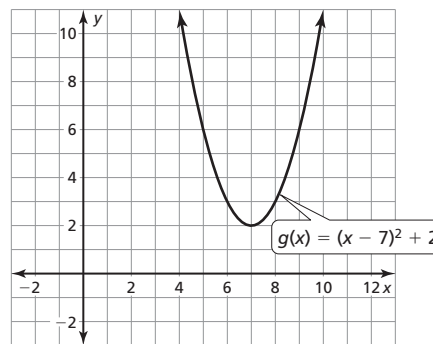
37. 155 tiles

2.4 Maintaining Mathematical Proficiency

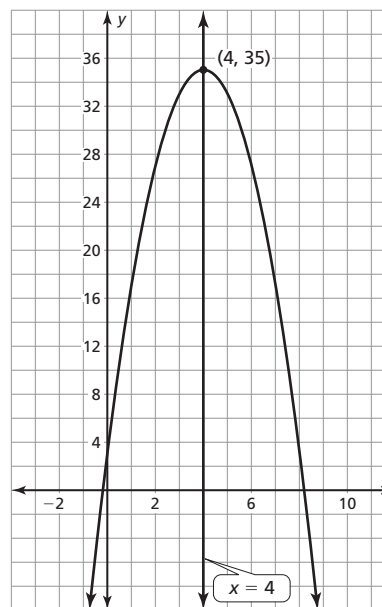
38. $(x + 3)(x + 1)$ 39. $(x - 2)(x - 1)$
 40. $3(x - 4)(x - 1)$ 41. $5(x + 3)(x - 2)$

Chapter 2 Review

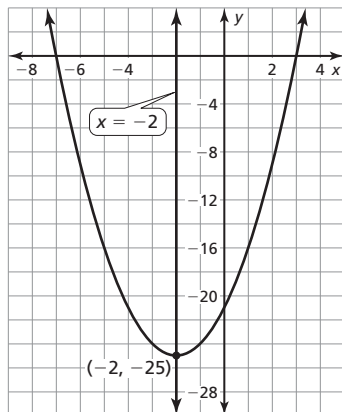
2. The graph is a translation 7 units right and 2 units up of the parent quadratic function.



7. The maximum value is 35; The function is increasing to the left of $x = 4$ and decreasing to the right of $x = 4$.



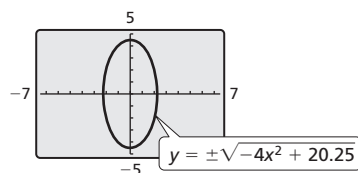
8. The minimum value is -25 ; The function is decreasing to the left of $x = -2$ and increasing to the right of $x = -2$.



Chapter 3

Mathematical Practices

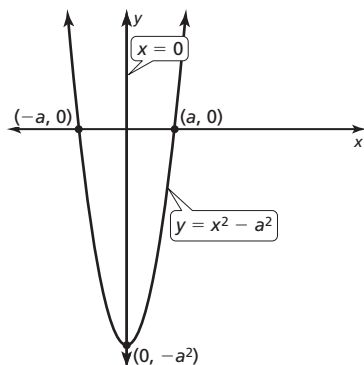
7.



$-9.6 < x < 9.6$ and $-6.4 < y < 6.4$; The graph has a width-height ratio of $3 : 2$ and shows the graph touching the x -axis.

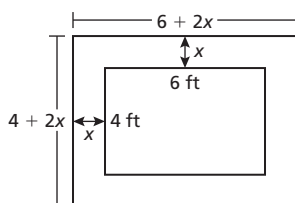
3.1 Monitoring Progress and Modeling with Mathematics

71. you; The function does not cross the x -axis.
72. $(x - 2)(x + 2)$, $(x - 3)(x + 3)$; $(x - a)(x + a)$



73. a. $mn = a^2$ and $m + n = 0$
b. $m = \sqrt{-a^2} = a\sqrt{-1}$,
 $n = -\sqrt{-a^2} = -a\sqrt{-1}$;
 m and n are not real numbers

74. Sample answer:



$$(4 + 2x)(6 + 2x) = 48; x = 1; 6 \text{ ft by } 8 \text{ ft}$$

75. 60 ft

3.1 Maintaining Mathematical Proficiency

76. $3x^2 - x + 2$ 77. $x^3 + 4x^2 + 6$
78. $3x^2 - 3x + 1$ 79. $-3x^3 + 7x^2 - 15x + 9$
80. $x^2 - 4$ 81. $10x^3 - 2x^2 + 6x$
82. $-x^2 + 8x - 7$ 83. $-44x^3 + 33x^2 + 88x$

3.2 Monitoring Progress and Modeling with Mathematics

Powers of i	i^1	i^2	i^3	i^4	i^5	i^6	i^7	i^8	i^9	i^{10}	i^{11}	i^{12}
Simplified form	i	-1	$-i$	1	i	-1	$-i$	1	i	-1	$-i$	1

The results of i^n alternate in the pattern i , -1 , $-i$, and 1 .

3.2 Maintaining Mathematical Proficiency

79. yes 80. no
81. no 82. $y = (x - 1)^2 + 2$
83. $y = 2(x + 3)^2 - 3$ 84. $y = -(x - 2)^2 - 1$

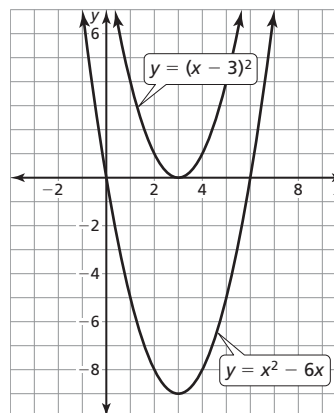
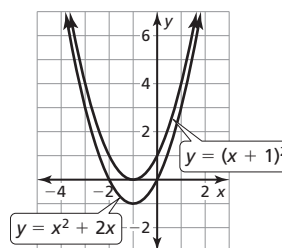
3.3 Explorations

2. a.

Expression	Value of c needed to complete the square	Expression written as a binomial squared
$x^2 + 2x + c$	1	$(x + 1)^2$
$x^2 + 4x + c$	4	$(x + 2)^2$
$x^2 + 8x + c$	16	$(x + 4)^2$
$x^2 + 10x + c$	25	$(x + 5)^2$

3.3 Monitoring Progress and Modeling with Mathematics

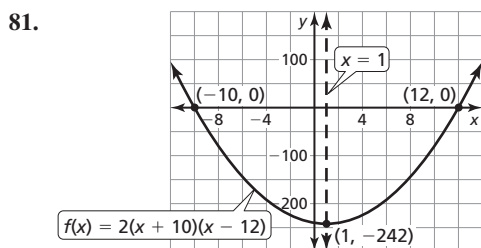
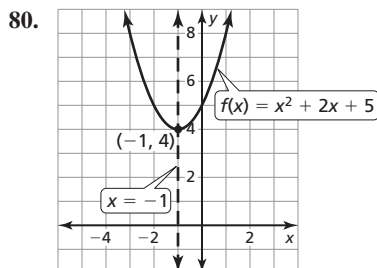
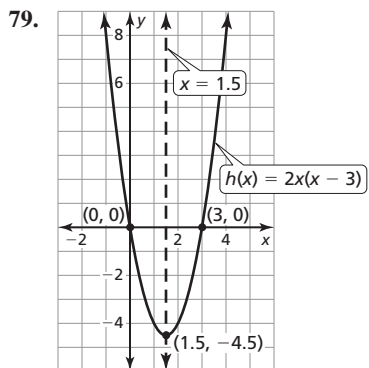
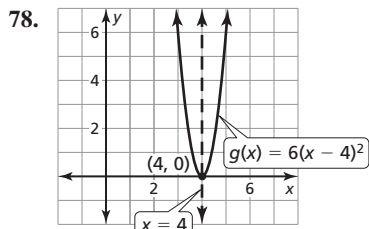
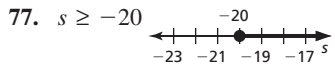
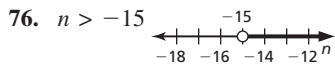
72. a.



b. The graph is shifted vertically, but the axis of symmetry does not change.

73. $x \approx 0.896$ cm

3.3 Maintaining Mathematical Proficiency



3.4 Monitoring Progress and Modeling with Mathematics

57. $x = \frac{-1 \pm \sqrt{5}}{2}$; *Sample answer:* Quadratic Formula; b is not an even number, the equation cannot be factored, and it cannot be easily written in the form $u^2 = d$.

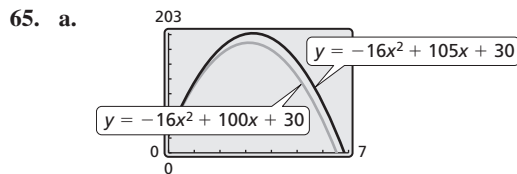
58. $x = -2 \pm 2i$; *Sample answer:* completing the square; Factor out 9, and $a = 1$ and b is an even number.

59. $x = 6$ 60. $x \approx 3.51$

61. about 5.67 sec

62. no; yes; For $ax^2 + 5x + c = 0$ to have one real solution,

$$ac = \frac{25}{4}; \text{ For any nonzero number } a, c = \frac{25}{4a}.$$



Both rockets start from the same height, but your friend's rocket does not go as high and lands about a half of a second earlier.

74. A basketball is pushed downward from a height of 3 feet at a speed of 6 feet per second; $h = -16t^2 - 6t + 3$; about 0.28 second

75. If $x = 3i$ and $x = -2i$ are solutions, then the equation can be written as $a(x - 3i)(x + 2i) = ax^2 - aix + 6a$. a and ai cannot both be real numbers.

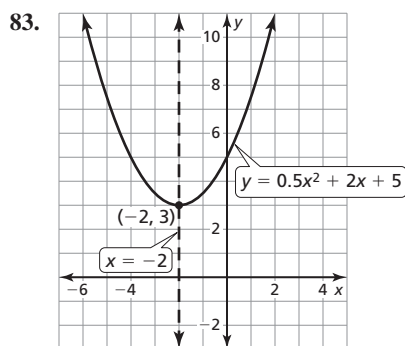
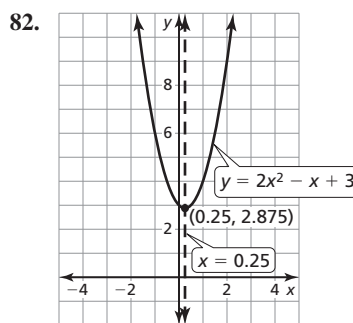
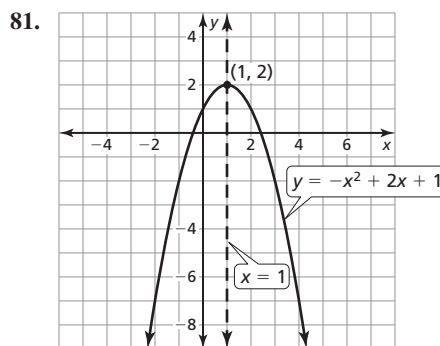
76. a. $v_0 = 32\sqrt{10} \approx 101.19$ ft/sec

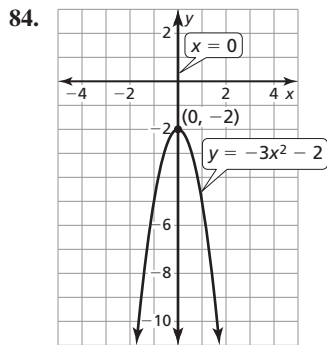
b. about 3.2 sec; At $t = 2$, $h \approx 1059$, so the rider is near the top and the graph is accurate.

3.4 Maintaining Mathematical Proficiency

77. (4, 5) 78. (2, 3)

79. no solution 80. (0, 2)





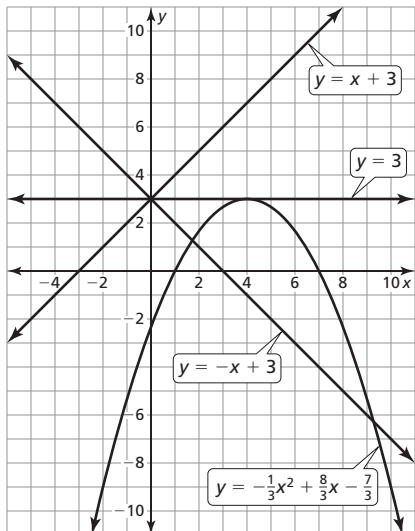
3.5 Explorations

2. a.

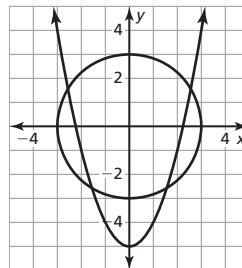
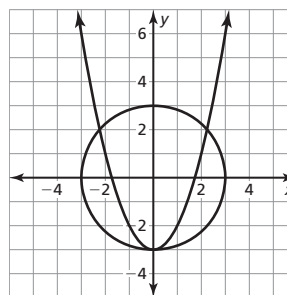
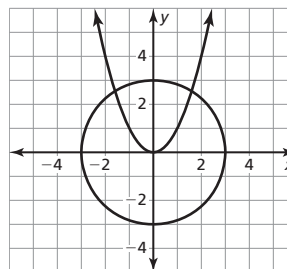
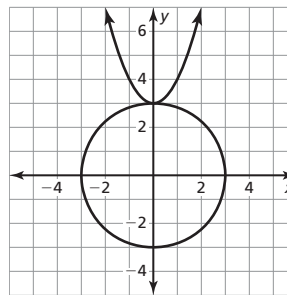
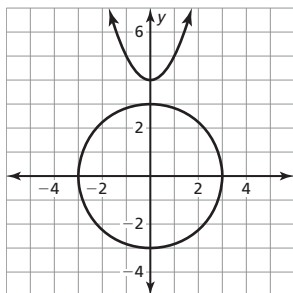
x	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
$y = x^2 + 2x + 1$	0	$\frac{1}{4}$	1	$\frac{9}{4}$	4
$y = -x^2 + x + 2$	0	$\frac{5}{4}$	2	$\frac{9}{4}$	2

3.5 Monitoring Progress and Modeling with Mathematics

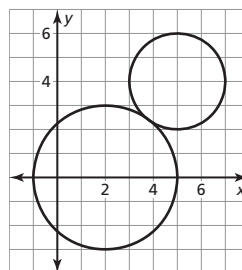
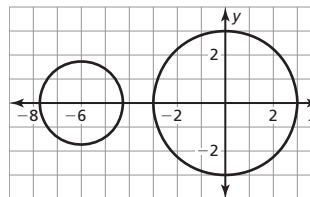
53. no solution: $m = 1$; one solution: $m = 0$; two solutions: $m = -1$



54. your friend; $(3, -3)$ and $(-3, 3)$ are solutions to Equation 1 but not Equation 2.
55. *Sample answer:* graphing and Quadratic Formula; graphing because it requires less time and steps than using the Quadratic Formula in this case
56. If (x, y) is a solution, then $(-x, -y)$ must be a solution. The solutions will always be reflections in the origin.
57. a. no solution, one solution, two solutions, three solutions, or four solutions

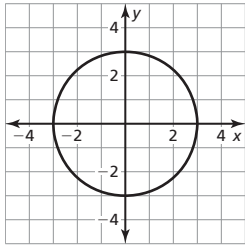
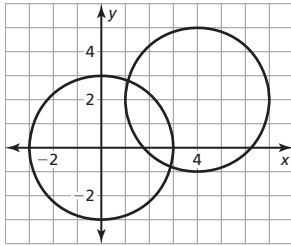


- b. no solution, one solution, two solutions, or infinitely many solutions



(continued)

57. b. (continued)



58. (0, 0) and (2, 3); Translate 2 units up.

59. a. circle: $x^2 + y^2 = 1$, Oak Lane: $y = -\frac{1}{7}x + \frac{5}{7}$

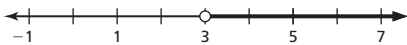
b. (-0.6, 0.8) and (0.8, 0.6)

c. about 1.41 mi

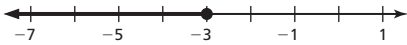
60. (0, 2)

3.5 Maintaining Mathematical Proficiency

61. $x > 3$



62. $x \leq -3$



63. $x \leq -4$

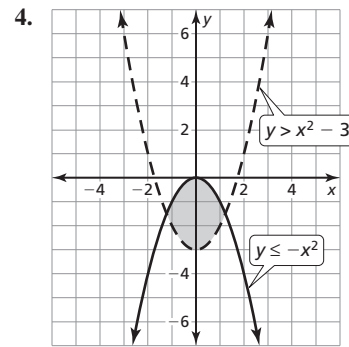
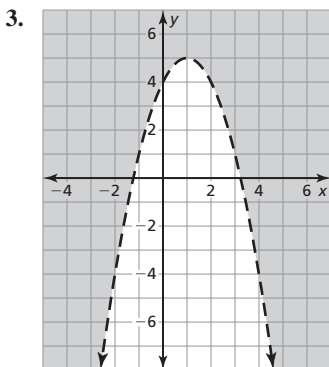
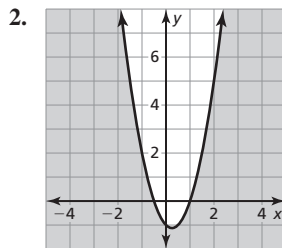
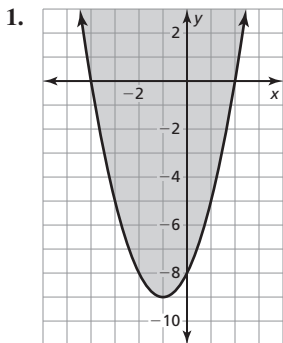


64. $y \geq -x + 1$

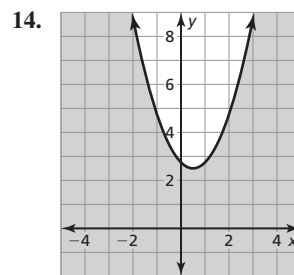
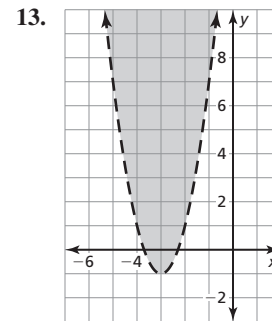
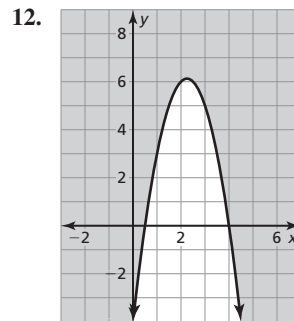
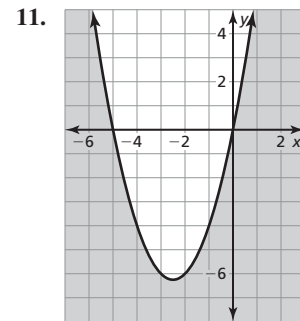
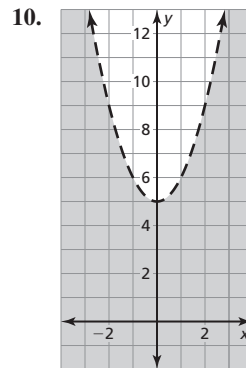
65. $y < x - 2$

66. $y > 2x - 3$

3.6 Lesson Monitoring Progress



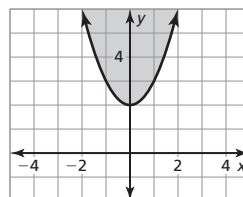
3.6 Monitoring Progress and Modeling with Mathematics



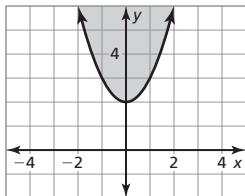
15. $y > f(x)$

16. $y > f(x)$

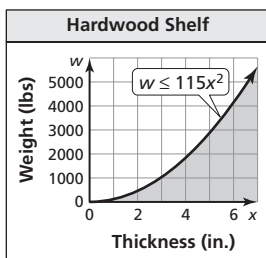
17. The graph should be solid, not dashed.



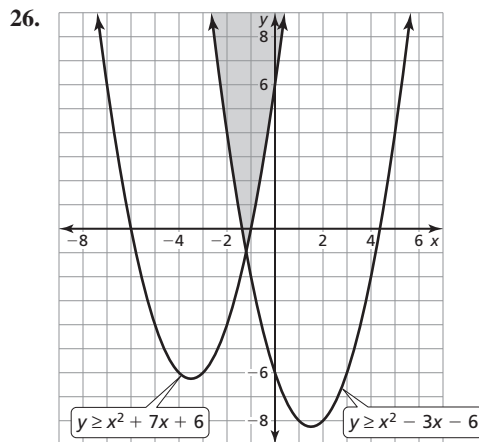
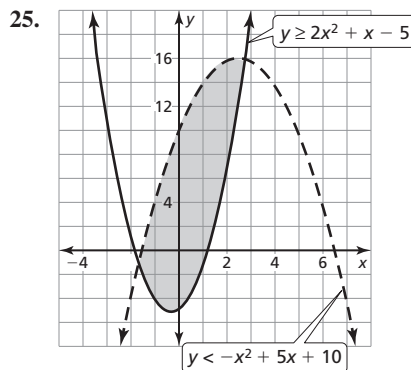
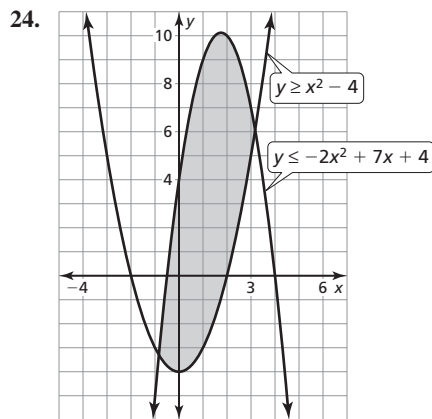
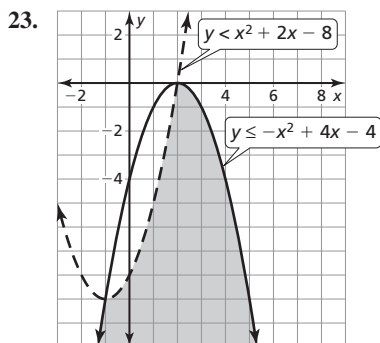
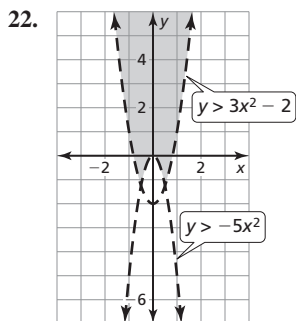
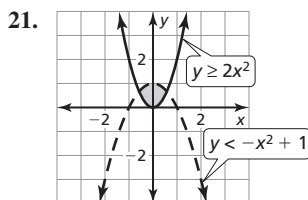
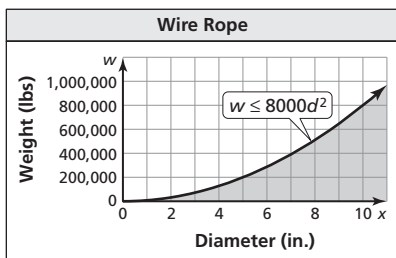
18. The wrong side of the parabola is shaded.



19. The solution represents weights that can be supported by shelves with various thicknesses.

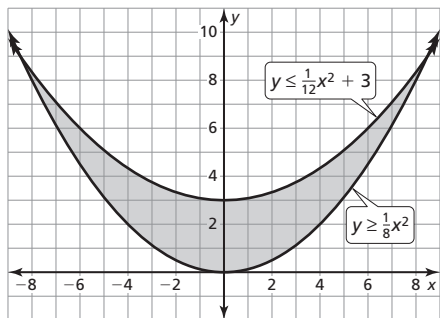


20. The solution represents weights that can be supported by wire ropes with various diameters.



49. $0.00170x^2 + 0.145x + 2.35 > 10, 0 \leq x \leq 40$; after about 37 days; Because $L(x)$ is a parabola, $L(x) = 10$ has two solutions. Because the x -value must be positive, the domain requires that the negative solution be rejected.
50. your friend; Any points with negative y -coordinates are solutions.
51. a. $\frac{32}{3} \approx 10.67$ square units
 b. $\frac{256}{3} \approx 85.33$ square units

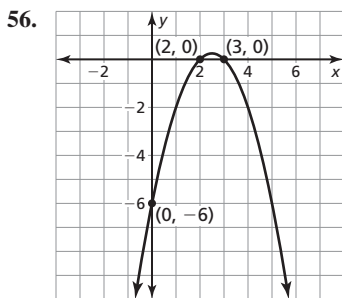
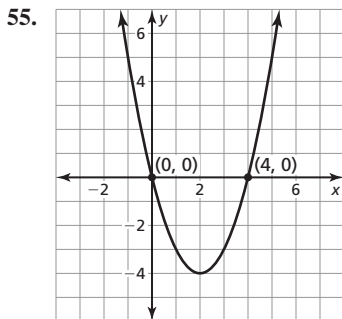
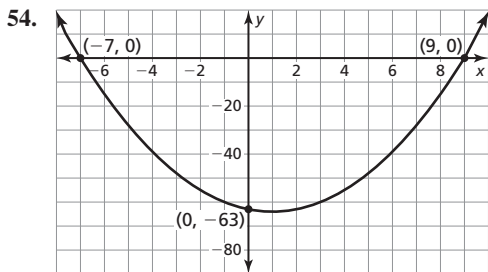
52. Sample answer:



The intersection of $y \leq \frac{1}{12}x^2 + 3$ and $y \geq \frac{1}{8}x^2$ create a shape similar to a smile, which could be used by a company that sells toothpaste.

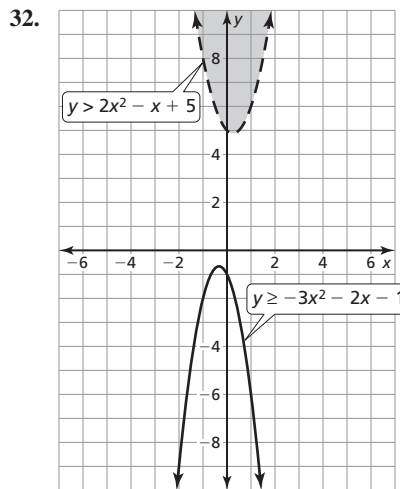
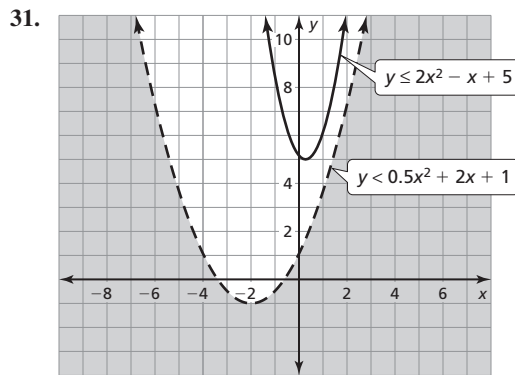
53. a. yes; The points on the parabola that are exactly 11 feet high are (6, 11) and (14, 11). Because these points are 8 feet apart, there is enough room for a 7-foot wide truck.
 b. 8 ft
 c. about 11.2 ft

3.6 Maintaining Mathematical Proficiency



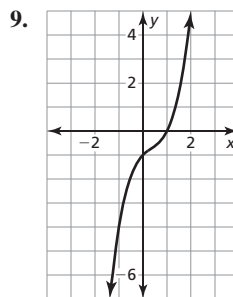
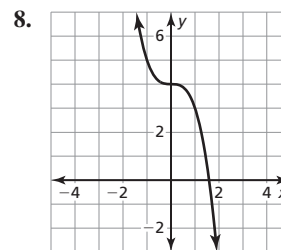
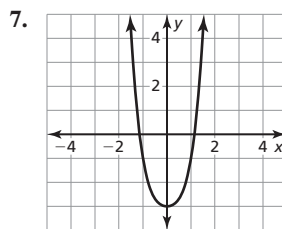
57. The maximum value is -1 ; The function is increasing to the left of $x = -3$ and decreasing to the right of $x = -3$.
 58. The minimum value is -1 ; The function is decreasing to the left of $x = -2$ and increasing to the right of $x = -2$.
 59. The maximum value is 25 ; The function is increasing to the left of $x = -2$ and decreasing to the right of $x = -2$.
 60. The minimum value is -20.25 ; The function is decreasing to the left of $x = -\frac{3}{2}$ and increasing to the right of $x = -\frac{3}{2}$.

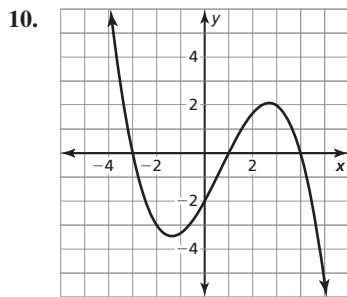
Chapter 3 Review



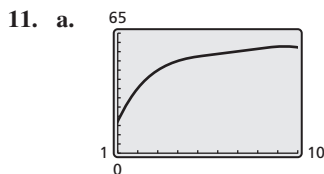
Chapter 4

4.1 Lesson Monitoring Progress





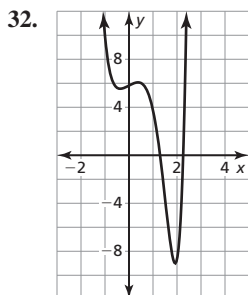
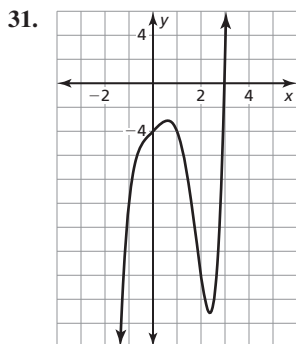
The degree is odd and the leading coefficient is negative.



From 2001 to 2004, the number of electric vehicles in use increased. Around 2005, the growth in the numbers in use slowed, but continued to increase through 2010.

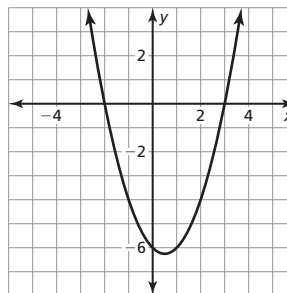
- b. about 4.4 thousand
 c. The model has a very sharp decline beginning around the year 2010. While the model may be used for a few years after 2010, in the long run, $V(t) < 0$ is not reasonable. $V(t)$ is negative. Because negative values do not make sense given the context, the model should not be used for years before 2001.

4.1 Monitoring Progress and Modeling with Mathematics

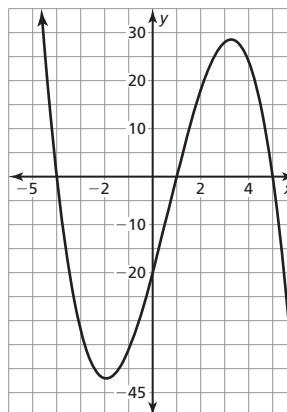


33. a. The function is increasing when $x > 4$ and decreasing when $x < 4$.
 b. $x < 3$ and $x > 5$
 c. $3 < x < 5$
 34. a. The function is increasing when $x < -4$ and decreasing when $x > -4$.
 b. $-6 < x < -2$
 c. $x < -6$ and $x > -2$
 35. a. The function is increasing when $x < 0$ and $x > 2$ and decreasing when $0 < x < 2$.
 b. $-1 < x < 2$ and $x > 2$
 c. $x < -1$
 36. a. The function is increasing when $x < -1$ and $x > 1$ and decreasing when $-1 < x < 1$.
 b. $x > 2$
 c. $x < -1$ and $-1 < x < 2$

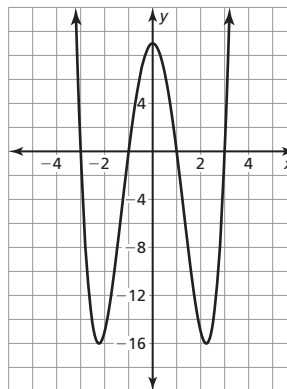
37. The degree is even and the leading coefficient is positive.



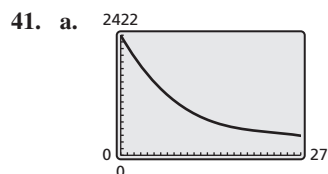
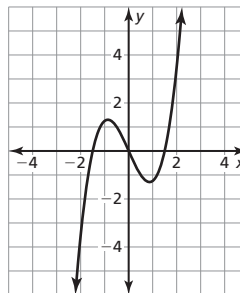
38. The degree is odd and the leading coefficient is negative.



39. The degree is even and the leading coefficient is positive.



40. The degree is odd and the leading coefficient is positive.

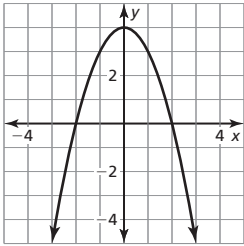


From 1980 to 2007 the number of open drive-in theaters decreased. Around the year 1995, the rate of decrease began to level off.

- b. 1980 to 1995: about -119.6 , 1995 to 2007: about -19.2 ;
About 120 drive-in movie theaters closed each year on average from 1980 to 1995. From 1995 to 2007, drive-in movie theaters were closing at a much lower rate, with about 20 theaters closing each year.
- c. Because the graph declines so sharply in the years leading up to 1980, it is most likely not accurate. The model may be valid for a few years before 1980, but in the long run, decline may not be reasonable. After 2007, the number of drive-in movie theaters declines sharply and soon becomes negative. Because negative values do not make sense given the context, the model cannot be used for years after 2007.

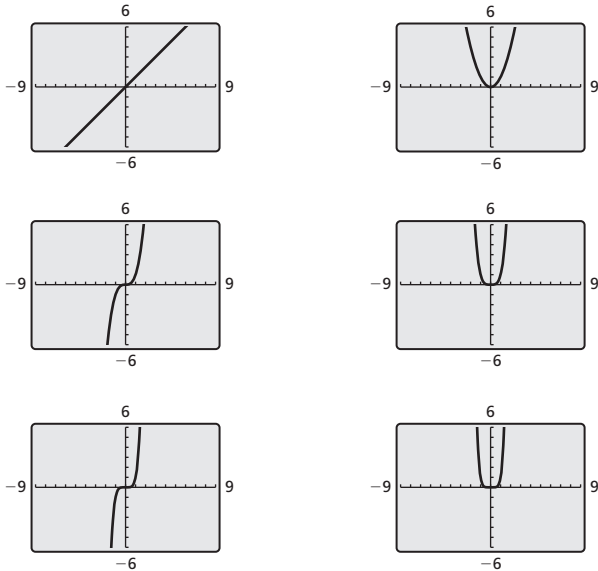
42. $w \approx 8.53$ carats

44. Sample answer: $y = -x^2 + 4$



45. The viewing window is appropriate if it shows the end behavior of the graph as $x \rightarrow \infty$ and $x \rightarrow -\infty$.
46. If the table is showing the end behavior of each function, then your friend is correct; $f(x) \rightarrow \infty$ as $x \rightarrow -\infty$ and as $x \rightarrow \infty$, so f is even. $g(x) \rightarrow \infty$ as $x \rightarrow -\infty$ and $g(x) \rightarrow -\infty$ as $x \rightarrow \infty$, so g is odd.

47. a.



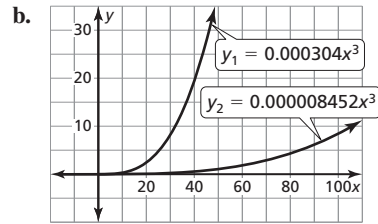
$y = x$, $y = x^3$, and $y = x^5$ are all symmetric with respect to the origin.

$y = x^2$, $y = x^4$, and $y = x^6$ are all symmetric with respect to the y -axis.

- b. The graph of $y = x^{10}$ will be symmetric with respect to the y -axis. The graph of $y = x^{11}$ will be symmetric with respect to the origin; The exponent is even. The exponent is odd.
48. a. The degree is odd and the leading coefficient is positive.
b. The function is increasing on the intervals $(-\infty, -3)$ and $(-1, \infty)$. and decreasing on the interval $(-3, -1)$.
c. 4

49. $f(-5) = -480$; Substituting the two given points into the function results in the system of equations $2 + b + c - 5 = 0$ and $16 + 4b + 2c - 5 = 3$. Solving for b and c gives $f(x) = 2x^3 - 7x^2 + 10x - 5$.

50. a. $y = 0.000008452x^3$



vertical shrink by a factor of about 0.0278

4.1 Maintaining Mathematical Proficiency

51. $-2x^2 + 3xy + y^2$ 52. $7h^3g + 7h^2g^2 + 5hg^3$
53. $12kz - 4kw$ 54. $-7a^5 + 10m$
55. $-x^3y^2 + 3x^2y + 13xy - 12x + 9$
56. $-3c^2v - 8c^2 + 11cv + 6c$

4.2 Monitoring Progress and Modeling with Mathematics

61.

$$\begin{array}{cccccccc} & & & & & & & 1 \\ & & & & & & 1 & 1 \\ & & & & & 1 & 2 & 1 \\ & & & & 1 & 3 & 3 & 1 \\ & & & 1 & 4 & 6 & 4 & 1 \\ & & 1 & 5 & 10 & 10 & 5 & 1 \\ & 1 & 6 & 15 & 20 & 15 & 6 & 1 \\ & 1 & 7 & 21 & 35 & 35 & 21 & 7 & 1 \\ & 1 & 8 & 28 & 56 & 70 & 56 & 28 & 8 & 1 \\ & 1 & 9 & 36 & 84 & 126 & 126 & 84 & 36 & 9 & 1 \\ & 1 & 10 & 45 & 120 & 210 & 252 & 210 & 120 & 45 & 10 & 1 \end{array}$$

$$(x + 3)^7 = x^7 + 21x^6 + 189x^5 + 945x^4 + 2835x^3 + 5103x^2 + 5103x + 2187;$$

$$(x - 5)^9 = x^9 - 45x^8 + 900x^7 - 10,500x^6 + 78,750x^5 - 393,750x^4 + 1,312,500x^3 - 2,812,500x^2 + 3,515,625x - 1,953,125$$

62. When in standard form the function is $x^4 + (ax^3 + bx^3 + cx^3 + dx^3) + (abx^2 + bcx^2 + bdx^2 + cdx^2) + (acx + adx + abcx + abdx + acdx + bcdx) + abcd$ which can also be written as $x^4 + (a + b + c + d)x^3 + (ab + ad + ac + bc + bd + cd)x^2 + (abc + abd + acd + bcd)x + abcd$.
63. a. 5
b. 5
c. 9
d. $g(x) + h(x)$ has degree m . $g(x) - h(x)$ has degree m .
 $g(x) \cdot h(x)$ has degree $(m + n)$.
64. a. 3, 5, 7; The difference increases by 2 for each consecutive pair of square numbers.
b. The first difference is 3, given by $2(1) + 1 = 3$. The second difference is 5, given by $2(2) + 1 = 5$. The third difference is 7, given by $2(3) + 1$.

c. $(n + 1)^2 - n^2 = n^2 + 2n + 1 - n^2 = 2n + 1$

65. a. $(x^2 - y^2)^2 + (2xy)^2 = (x^2 + y^2)^2$
 $(x^4 - 2x^2y^2 + y^4) + (4x^2y^2) = x^4 + 2x^2y^2 + y^4$
 $x^4 + 2x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4$

b. The Pythagorean triple is 11, 60, and 61.

c. $121 + 3600 = 3721$
 $3721 = 3721$

4.2 Maintaining Mathematical Proficiency

66. $8 + 7i$

67. $5 + 11i$

68. 21

69. $9 - 2i$

4.3 Monitoring Progress and Modeling with Mathematics

37. A

38. a. -4

b. The remainders are both 0 because $f(-3) = f(-1) = 0$.

39. $2x + 5$

40. $5x^3 - 3x^2 + 21x - 8$; *Sample answer:* Multiply by $x + 2$.

4.3 Maintaining Mathematical Proficiency

41. $x = 3$

42. $x = -6$ and $x = 2$

43. $x = -7$

44. $x = \pm 3i$

4.4 Monitoring Progress and Modeling with Mathematics

64. $(3k^2 + 1)(3k - 8)$; Factoring by grouping can be used because the expression contains pairs of monomials that have a common factor.

65. a. no; $7z^4(2z + 3)(z - 2)$

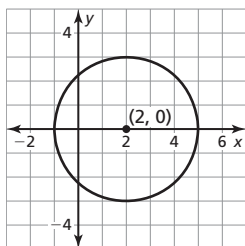
b. no; $n(2 - n)(n + 6)(3n - 11)$

c. yes

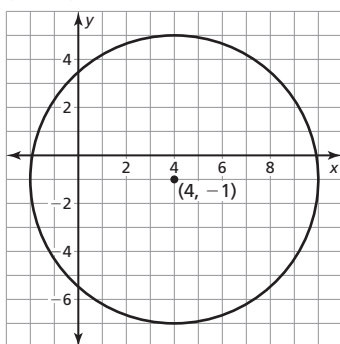
66. 1 million

67. 0.7 million

75. b. $(x - 2)^2 + y^2 = 3^2$; The center of the circle is $(2, 0)$ and the radius is 3.



c. $(x - 4)^2 + (y + 1)^2 = 6^2$; The center of the circle is $(4, -1)$ and the radius is 6.



76. a. If the volume of the missing block is included, the volume of the diagram is a^3 because the length, width, and height are all a . Because the length, width, and height of the missing piece are b , the volume of the missing block is b^3 . Subtracting the volume of the missing block from the entire volume gives $a^3 - b^3$.

b. I: $a^2(a - b)$, II: $ab(a - b)$, III: $b^2(a - b)$

c. $a^3 - b^3 = a^2(a - b) + ab(a - b) + b^2(a - b)$
 $= (a - b)(a^2 + ab + b^2)$

4.4 Maintaining Mathematical Proficiency

77. $x = 6$ and $x = -5$

78. $x = 9$ and $x = -4$

79. $x = \frac{5}{3}$ and $x = 2$

80. $x = \frac{1}{9}$ and $x = 3$

81. $x = 18$ and $x = -6$

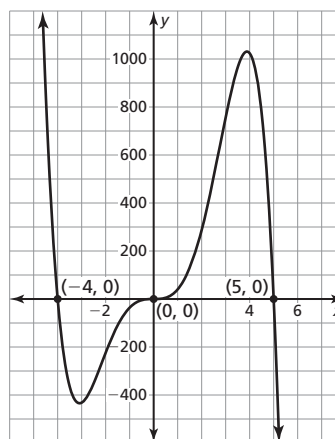
82. $x = 4 \pm 3\sqrt{3}$

83. $x = -3$ and $x = -7$

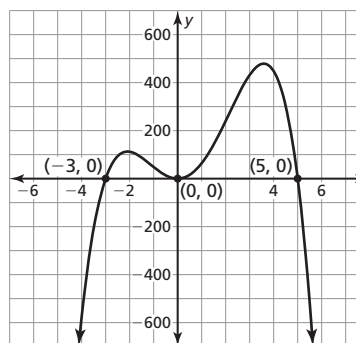
84. $x = \frac{-9 \pm \sqrt{85}}{2}$

4.5 Monitoring Progress and Modeling with Mathematics

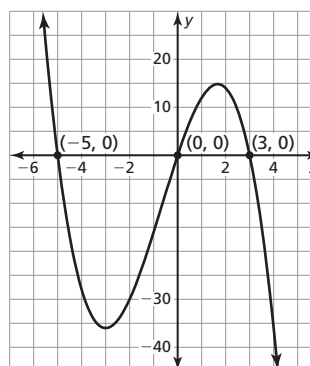
16. $x = -4$, $x = 0$, and $x = 5$



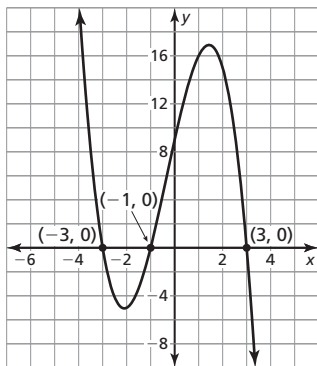
17. $x = -3$, $x = 0$, and $x = 5$



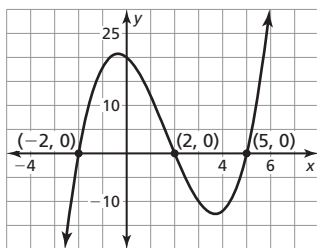
18. $x = -5$, $x = 0$, and $x = 3$



19. $x = -3$, $x = -1$, and $x = 3$



20. $x = -2$, $x = 2$, and $x = 5$



64. Each side of the base is to be 2 feet and the height is to be 3 feet.

65. rs ; Each factor of a_0 can be written as the numerator with each factor of a_n as the denominator, creating $r \times s$ factors.

4.5 Maintaining Mathematical Proficiency

66. polynomial function; $h(x) = 2x^3 - 3x^2 + 2x - 9$; degree: 3 (cubic), leading coefficient: 2

67. not a polynomial function

68. polynomial function; $f(x) = -4x^4 + 2x^3 + \frac{1}{3}x^2 - \sqrt{3}$; degree: 4 (quartic), leading coefficient: -4

69. not a polynomial function; The term $\sqrt[4]{x}$ has an exponent that is not a whole number.

70. $x = \pm i\sqrt{6}$

71. $x = \pm 3i$

72. $x = \pm 2i\sqrt{2}$

73. $x = \pm \frac{\sqrt{2}}{4}$

4.6 Lesson Monitoring Progress

9.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
1	0	2	3

10.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
4	0	0	4
2	0	2	4
0	0	4	4

4.6 Monitoring Progress and Modeling with Mathematics

33.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
1	1	2	4

34.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
1	0	2	3

35.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
2	1	0	3
0	1	2	3

36.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
2	3	0	5
2	1	2	5
0	3	2	5
0	1	4	5

37.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
3	2	0	5
3	0	2	5
1	2	2	5
1	0	4	5

38.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
3	2	0	5
3	0	2	5
1	2	2	5
1	0	4	5

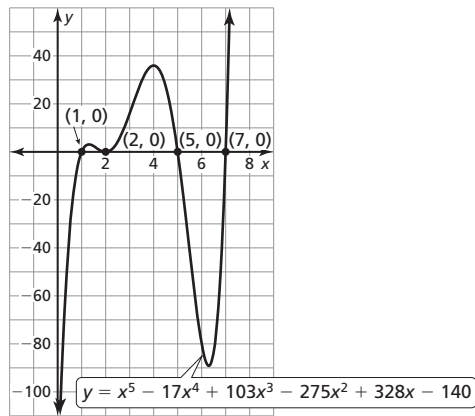
39.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
3	3	0	6
3	1	2	6
1	3	2	6
1	1	4	6

40.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
2	1	4	7
0	1	6	7

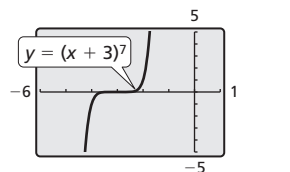
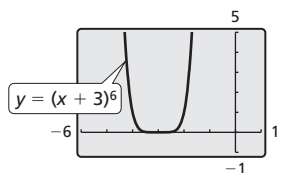
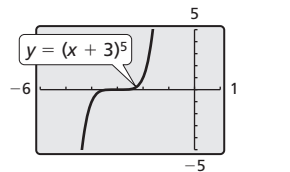
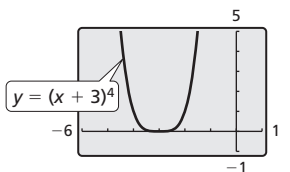
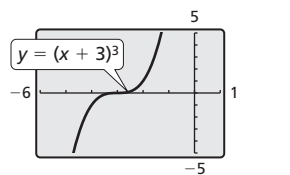
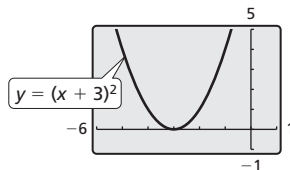
48. Sample answer: $f(x) = (x - 1)(x - 2)(x - 2)(x - 5)(x - 7)$



$$f(x) = x^5 - 17x^4 + 103x^3 - 275x^2 + 328x - 140$$

50. a. The function has three positive real zeros, one negative real zero, and two imaginary zeros. Or, the function has five positive real zeros, one negative zero, and zero imaginary zeros.
- b. $f(x)$ could change signs five times or three times

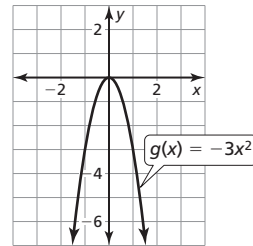
51.



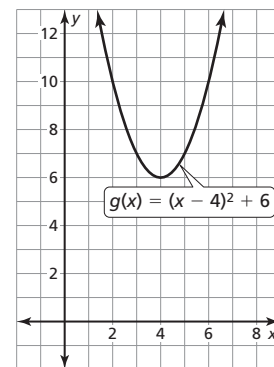
- a. For all functions, $f(x) \rightarrow \infty$ as $x \rightarrow \infty$. When n is even, $f(x) \rightarrow \infty$ as $x \rightarrow -\infty$, but when n is odd, $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$.
- b. As n increases, the graph becomes more flat near the zero $x = -3$.
- c. The graph of g becomes more vertical and straight near $x = 4$.
52. $f(x)$: 2, 3; $g(x)$: $-3, 1, 2$; $h(x)$: $-3, 1, 2i, -2i$;
 $k(x)$: $-3, 2, 0, 2 + \sqrt{3} \approx 3.73, 2 - \sqrt{3} \approx 0.27$
- a. If the function is of degree n , the sum of the zeros is equal to the opposite of the coefficient of the $n-1$ term.
- b. The product of the zeros is equal to the constant term of the polynomial function.
53. a.
- | Deposit | Year 1 | Year 2 | Year 3 | Year 4 |
|-------------|--------|---------|-----------|-----------|
| 1st Deposit | 1000 | $1000g$ | $1000g^2$ | $1000g^3$ |
| 2nd Deposit | | 1000 | $1000g$ | $1000g^2$ |
| 3rd Deposit | | | 1000 | $1000g$ |
| 4th Deposit | | | | 1000 |
- b. $v = 1000g^3 + 1000g^2 + 1000g + 1000$
- c. about 1.0484; about 4.84%

4.6 Maintaining Mathematical Proficiency

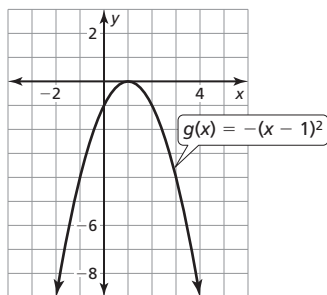
54. The function is a vertical stretch by a factor of 3 followed by a reflection in the x -axis of the parent quadratic function.



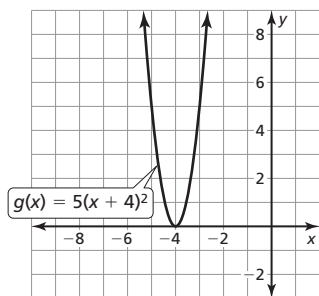
55. The function is a translation 4 units right and 6 units up of the parent quadratic function.



56. The function is a reflection in the x -axis followed by a translation 1 unit right of the parent quadratic function.



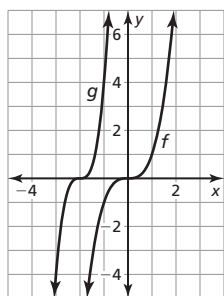
57. The function is a vertical stretch by a factor of 5 followed by a translation 4 units left of the parent quadratic function.



58. $g(x) = -\frac{1}{3}x$
 59. $g(x) = \left|\frac{1}{9}x + 1\right| - 3$
 60. $g(x) = -(x - 2)^2 + 7$

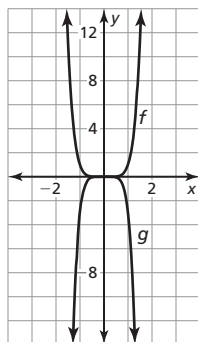
4.7 Lesson Monitoring Progress

2. The graph of g is a vertical stretch by a factor of 4 followed by a translation 2 units left of the graph of f .

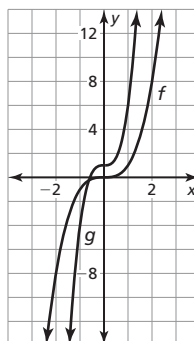


4.7 Monitoring Progress and Modeling with Mathematics

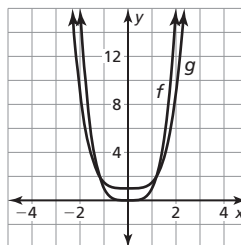
12. The graph of g is a vertical stretch by a factor of 3 followed by a reflection in the x -axis of the graph of f .



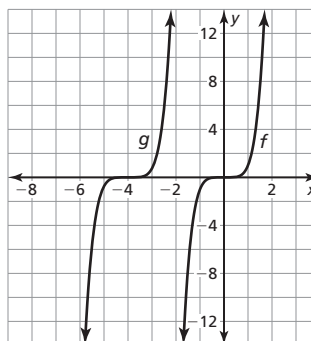
13. The graph of g is a vertical stretch by a factor of 5 followed by a translation 1 unit up of the graph of f .



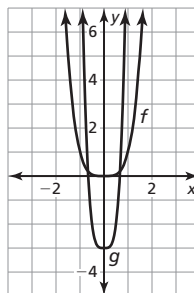
14. The graph of g is a vertical shrink by a factor of $\frac{1}{2}$ followed by a translation 1 unit up of the graph of f .



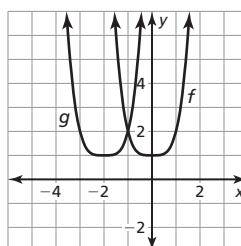
15. The graph of g is a vertical shrink by a factor of $\frac{3}{4}$ followed by a translation 4 units left of the graph of f .



16. The graph of g is a horizontal shrink by a factor of $\frac{1}{2}$ followed by a translation 3 units down of the graph of f .

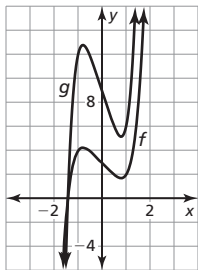


17. $g(x) = (x + 2)^4 + 1$



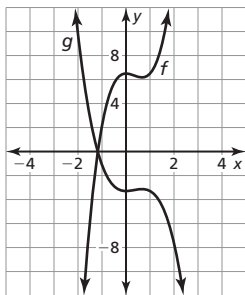
The graph of g is a translation 2 units left of the graph of f .

18. $g(x) = 3x^5 - 6x + 9$



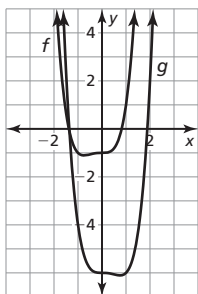
The graph of g is a vertical stretch by a factor of 3 of the graph of f .

19. $g(x) = -x^3 + x^2 - 3$



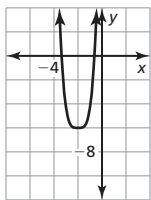
The graph of g is a vertical shrink by a factor of $\frac{1}{2}$ followed by a reflection in the x -axis of the graph of f .

20. $g(x) = x^4 - x^3 - 6$



The graph of g is a reflection in the y -axis followed by a translation 5 units down of the graph of f .

21. The graph has been translated horizontally to the right 2 units instead of to the left 2 units.



23. $g(x) = -x^3 + 9x^2 - 27x + 21$

24. $g(x) = 2x^4 - 32x^3 + 192x^2 - 508x + 508$

25. $g(x) = -27x^3 - 18x^2 + 7$

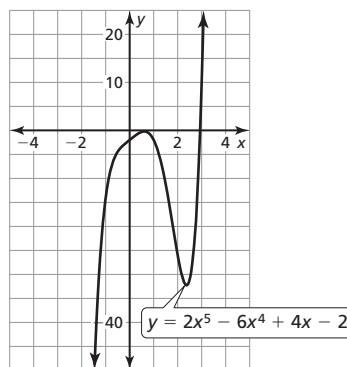
26. $g(x) = -6x^5 + 3x^3 + 3x^2 + 11$

27. $W(x) = 27x^3 - 12x$; $W(5) = 3315$; When x is 5 yards, the volume of the pyramid is 3315 cubic feet.

28. no; When each side is divided in half the new volume is $V\left(\frac{1}{2}x\right) = \left(\frac{1}{2}x\right)^3 = \frac{1}{8}x^3$ and is four times less than when the volume is divided in half.

29. *Sample answer:* If the function is translated up and then reflected in the x -axis, the order is important; If the function is translated left and then reflected in the x -axis, the order is not important; Reflecting a graph in the x -axis does not affect its x -coordinate, but it does affect its y -coordinate. So, the order is only important if the other translation is in the y .

30. *Sample answer:* $f(x) = 2x^5 - 6x^4 + 4x - 2$



31. a. 0 m, 4 m, and 7 m

b. $g(x) = -\frac{2}{5}(x-2)(x-6)^2(x-9)$

32. The real zeros of f are -2 , 0 , and 2 . The real zeros of g are 0 , 2 , and 4 . Because all of the real zeros have been increased two units, the graph of g is a translation two units right of the graph of f .

33. $V(x) = 3\pi x^2(x+3)$; $W(x) = \frac{\pi}{3}x^2\left(\frac{1}{3}x+3\right)$;

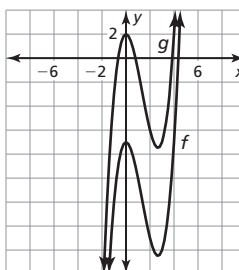
$W(3) = 12\pi \approx 37.70$; When x is 3 feet, the volume of the cone is about 37.70 cubic yards.

4.7 Maintaining Mathematical Proficiency

34. The minimum value is -7 ; The domain is all real numbers and the range is $y \geq -7$. The function is decreasing to the left of $x = -5$ and increasing to the right of $x = -5$.
35. The maximum value is 4 ; The domain is all real numbers and the range is $y \leq 4$. The function is increasing to the left of $x = 0$ and decreasing to the right of $x = 0$.
36. The minimum value is -147 ; The domain is all real numbers and the range is $y \geq -147$. The function is decreasing to the left of $x = 3$ and increasing to the right of $x = 3$.
37. The maximum value is 9 ; The domain is all real numbers and the range is $y \leq 9$. The function is increasing to the left of $x = -5$ and decreasing to the right of $x = -5$.
38. The minimum value is -3 ; The domain is all real numbers and the range is $y \geq -3$. The function is decreasing to the left of $x = 1$ and increasing to the right of $x = 1$.
39. The maximum value is 1 ; The domain is all real numbers and the range is $y \leq 1$. The function is increasing to the left of $x = 1$ and decreasing to the right of $x = 1$.

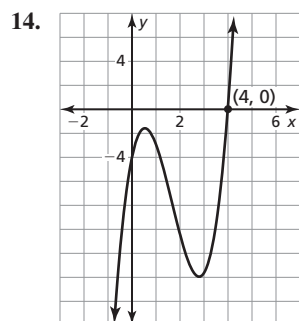
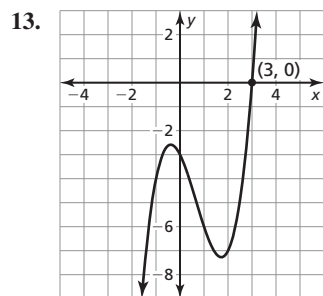
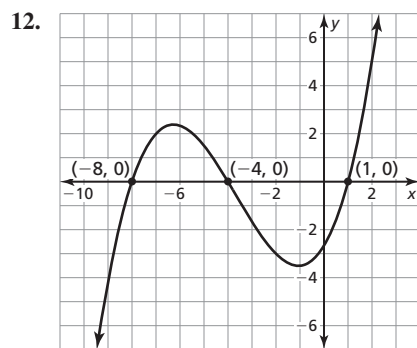
4.7 Mini-Assessment

3. $g(x) = x^3 - 4x^2 + 2$

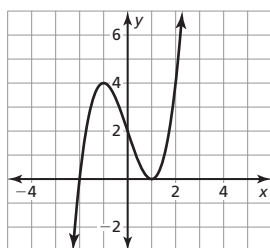


The graph of g is a translation 9 units up of the graph of f .

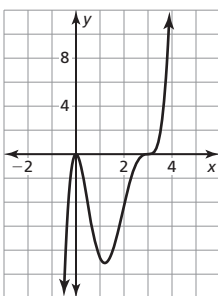
4.8 Monitoring Progress and Modeling with Mathematics



15. The x -intercepts should be -2 and 1 .



16. Because 0 is a repeated zero with an even power, the graph should only touch the x -axis at 0 , not cross it. Because 3 is a repeated zero with an odd power, the graph should cross the x -axis at 3 .



17. -1 , 1 , and 4

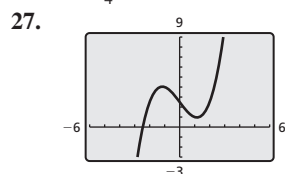
18. -2 , 2 , and 3

19. -4 , $-\frac{1}{2}$, and 1

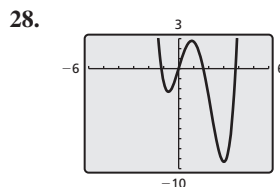
20. $-\frac{3}{2}$, -1 , and 3

21. -4 , $\frac{3}{4}$, and 3

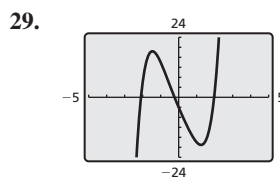
22. -3 , $-\frac{1}{2}$, and 5



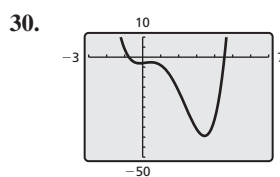
The x -intercept of the graph is $x \approx -2.46$. The function has a local maximum at $(-1.15, 4.04)$ and a local minimum at $(1.15, 0.96)$; The function is increasing when $x < -1.15$ and $x > 1.15$ and is decreasing when $-1.15 < x < 1.15$.



The x -intercepts of the graph are $x \approx -1.15$, $x = 0$, $x \approx 1.64$, and $x \approx 3.79$. The function has a local maximum at $(0.87, 2.78)$ and local minima at $(-0.68, -2.31)$ and $(3.02, -9.30)$; The function is increasing when $-0.68 < x < 0.87$ and $x > 3.02$ and is decreasing when $x < -0.68$ and $0.87 < x < 3.02$.



The x -intercepts of the graph are $x \approx -2.10$, $x \approx -0.23$, and $x \approx 1.97$. The function has a local maximum at $(-1.46, 18.45)$ and a local minimum at $(1.25, -19.07)$; The function is increasing when $x < -1.46$ and $x > 1.25$ and is decreasing when $-1.46 < x < 1.25$.



The x -intercepts of the graph are $x \approx -0.77$ and $x \approx 4.54$. The function has a local maximum at $(0.47, -2.56)$ and local minima at $(-0.16, -3.09)$ and $(3.44, -39.40)$; The function is increasing when $-0.16 < x < 0.47$ and $x > 3.44$ and is decreasing when $x < -0.16$ and $0.47 < x < 3.44$.

31. $(-0.29, 0.48)$ and $(0.29, -0.48)$; $(-0.29, 0.48)$ corresponds to a local maximum and $(0.29, -0.48)$ corresponds to a local minimum; The real zeros are -0.5 , 0 , and 0.5 . The function is of at least degree 3.

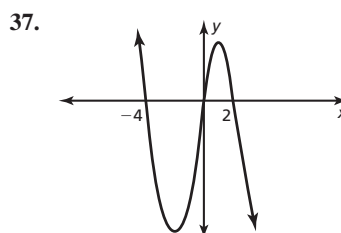
32. $(-2.91, -1.36)$ and $(0.57, -6.63)$; $(-2.91, -1.36)$ corresponds to a local maximum and $(0.57, -6.63)$ corresponds to a local minimum; The real zero is 2.5 . The function is of at least degree 3.

33. $(1, 0)$, $(3, 0)$, and $(2, -2)$; $(1, 0)$ and $(3, 0)$ correspond to local maximums, and $(2, -2)$ corresponds to a local minimum; The real zeros are 1 and 3 . The function is of at least degree 4.

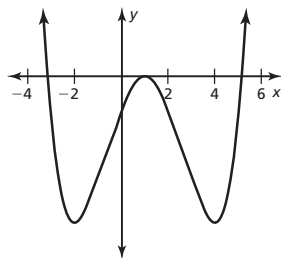
34. $(-1.22, 5.07)$, $(1.96, 7.71)$, $(0.15, -48.35)$, and $(2.79, -3.74)$; $(-1.22, 5.07)$ and $(1.96, 7.71)$ correspond to local maximums, and $(0.15, -48.35)$ and $(2.79, -3.74)$ correspond to local minimums; The real zeros are -1.4 , -1 , 1.5 , 2.5 , and 3 . The function is of at least degree 5.

35. $(-1.25, -10.65)$; $(-1.25, -10.65)$ corresponds to a local minimum; The real zeros are -2.07 and 1.78 . The function is of at least degree 4.

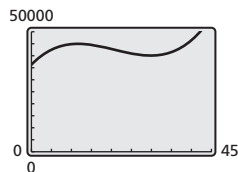
36. $(-1.18, -7.57)$; $(-1.18, -7.57)$ corresponds to a local minimum; The zeros are -2.45 and 2.45 . The function is of at least degree 4.



38.

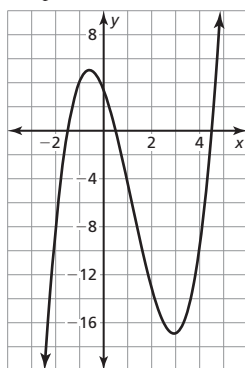


48.



Enrollment increases until approximately the 12th year to a maximum of almost 45,000 students. Then, enrollment decreases until approximately the 30th year to a minimum of about 40,000 students. Then, enrollment increases again and ends the period with about 48,000 students.

51. no; When multiplying two odd functions, the exponents of each term will be added, creating an even exponent. So, the product will not be an odd function.
52. a. about 2.94 in.
b. about 420.11 in.³
c. 2.94 in. by 14.11 in. by 10.11 in.
53. a. $\frac{1100 - \pi r^2}{\pi r}$
b. $V = 550r - \frac{\pi}{2}r^3$
c. about 10.8 ft
54. Sample answer: $x^3 - 3.5x^2 - 5.25x + 3.375$



There is no maximum degree; The function could have repeated zeros or infinitely many imaginary zeros.

55. $V(h) = 64\pi h - \frac{\pi}{4}h^3$; about 9.24 in.; about 1238.22 in.³

4.8 Maintaining Mathematical Proficiency

56. linear; The first differences are constant.
57. quadratic; The second differences are constant.

4.9 Monitoring Progress and Modeling with Mathematics

19. $d = \frac{1}{2}n^2 - \frac{3}{2}n$; 35
20. no; If the first order differences are given, they can be used to find the second order differences, and so on, until the differences are constant.
21. With real-life data sets, the numbers rarely fit a model perfectly. Because of this, the differences are rarely constant.

22. Sample answer: $A = 2, B = 1, C = 5$;

$$f(x) = (x - 1)(x - 2)(x - 5)$$

23. C, A, B, D

24. The third order differences are all $6a$.

4.9 Maintaining Mathematical Proficiency

25. $x = \pm 6$
26. $x = \pm 3\sqrt{5}$
27. $x = 3 \pm 2\sqrt{3}$
28. $x = -5 \pm \sqrt{3}$
29. $x = 1$ and $x = -2.5$
30. $x = 0.5$
31. $x = \frac{-3 \pm \sqrt{29}}{10}$
32. $x = 2 \pm 4i$

Chapter 4 Review

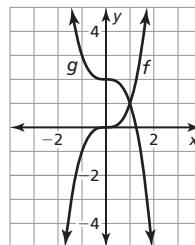
32.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
2	0	2	4
0	0	4	4

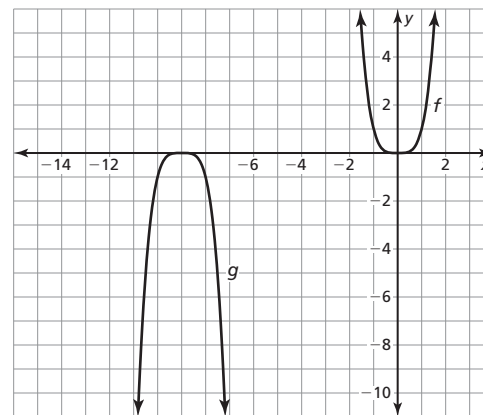
33.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
1	3	0	4
1	1	2	4

34. The graph of g is a reflection in the y -axis followed by a translation 2 units up of the graph of f .



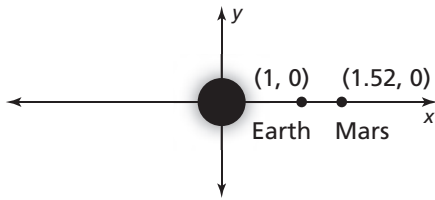
35. The graph of g is a reflection in the x -axis followed by a translation 9 units left of the graph of f .



Chapter 5

5.1 Monitoring Progress and Modeling with Mathematics

48.



Sample answer: $d = 1.88^{2/3} \approx 1.52$, so Mars must be approximately 1.52 astronomical units from the Sun.

49. about 753 ft³/sec

50. 1.12 m/sec; 1.47 m/sec; 2.15 m/sec

5.1 Maintaining Mathematical Proficiency

51. 5^5

52. $\frac{1}{4^5}$

53. $\frac{1}{z^6}$

54. $\frac{81x^4}{16}$

55. 5000

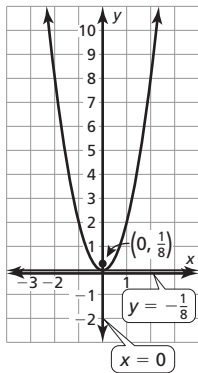
56. 0.04

57. 0.82

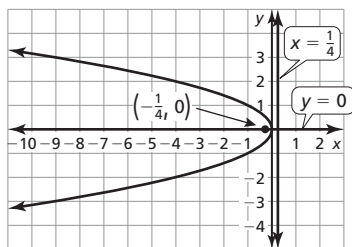
58. 6,930,000

5.2 Maintaining Mathematical Proficiency

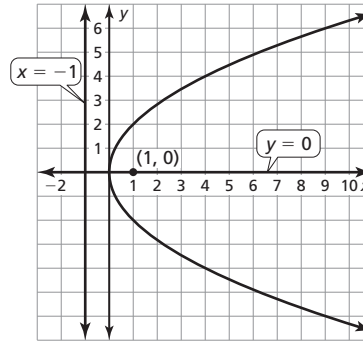
82. The focus is $(0, \frac{1}{8})$. The directrix is $y = -\frac{1}{8}$. The axis of symmetry is $x = 0$.



83. The focus is $(-\frac{1}{4}, 0)$. The directrix is $x = \frac{1}{4}$. The axis of symmetry is $y = 0$.



84. The focus is $(1, 0)$. The directrix is $x = -1$. The axis of symmetry is $y = 0$.



85. $g(x) = -x^4 + 3x^2 + 2x$; The graph of g is a reflection in the x -axis of the graph of f .

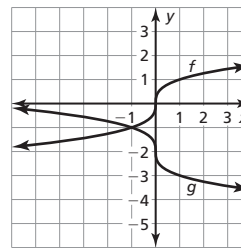
86. $g(x) = x^3 - x - 3$; The graph of g is a translation 3 units down of the graph of f .

87. $g(x) = (x - 2)^3 - 4$; The graph of g is a translation 2 units right of the graph of f .

88. $g(x) = 16x^4 + 16x^3 - 16x^2$; The graph of g is a horizontal shrink by a factor of $\frac{1}{2}$ of the graph of f .

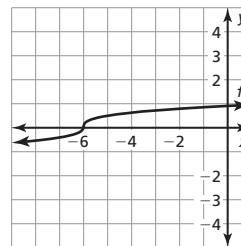
5.3 Lesson Monitoring Progress

2. The graph of g is a reflection in the x -axis followed by a translation 2 units down of the graph of f .



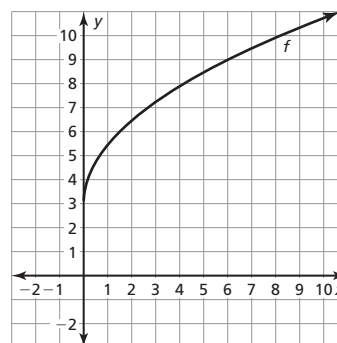
5.3 Monitoring Progress and Modeling with Mathematics

14.

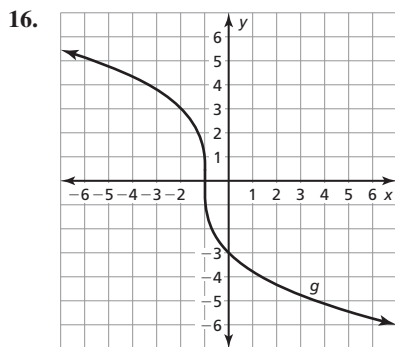


The domain and range are all real numbers.

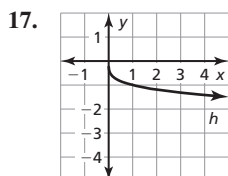
15.



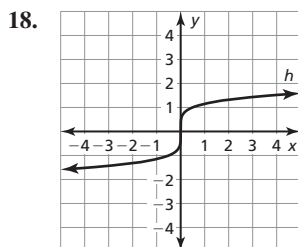
The domain is $x \geq 0$. The range is $y \geq 3$.



The domain and range are all real numbers.

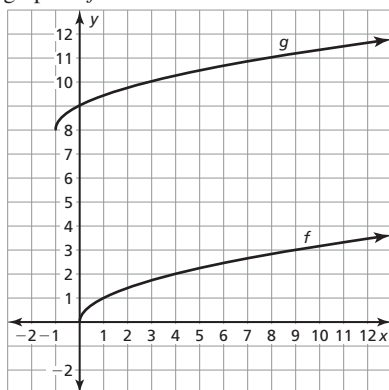


The domain is $x \geq 0$. The range is $y \leq 0$.

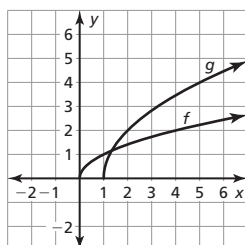


The domain and range are all real numbers.

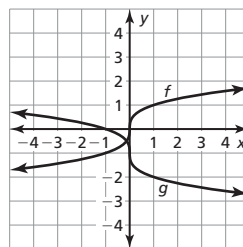
19. The graph of g is a translation 1 unit left and 8 units up of the graph of f .



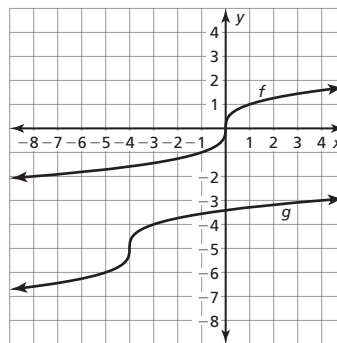
20. The graph of g is a vertical stretch by a factor of 2 followed by a translation 1 unit right of the graph of f .



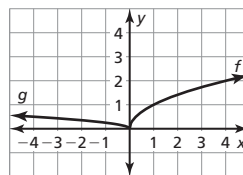
21. The graph of g is a reflection in the x -axis followed by a translation 1 unit down of the graph of f .



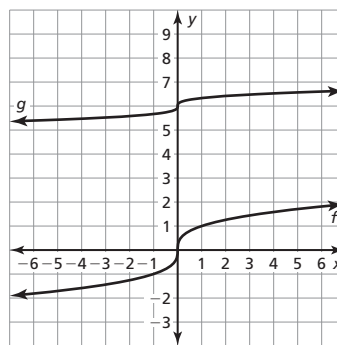
22. The graph of g is a translation 4 units left and 5 units down of the graph of f .



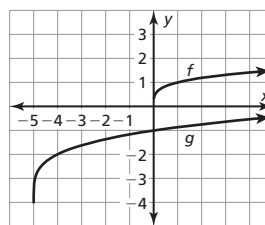
23. The graph of g is a vertical shrink by a factor of $\frac{1}{4}$ followed by a reflection in the y -axis of the graph of f .



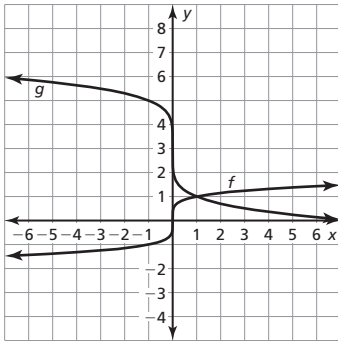
24. The graph of g is a vertical shrink by a factor of $\frac{1}{3}$ followed by a translation 6 units up of the graph of f .



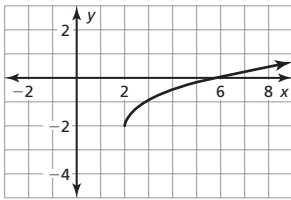
25. The graph of g is a vertical stretch by a factor of 2 followed by a translation 5 units left and 4 units down of the graph of f .



26. The graph of g is a horizontal shrink by a factor of $\frac{1}{32}$ followed by a reflection in the y -axis and a translation 3 units up of the graph of f .

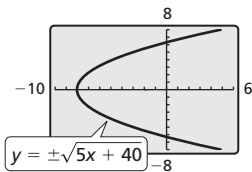


27. The graph was translated 2 units left but it should be translated 2 units right.



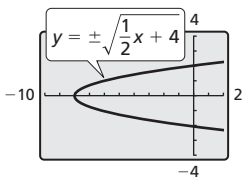
28. The function is a horizontal stretch by a factor of 2, not a horizontal shrink by a factor of $\frac{1}{2}$; The graph of g is a horizontal stretch by a factor of 2 and a translation 3 units up of the parent square root function.

55.



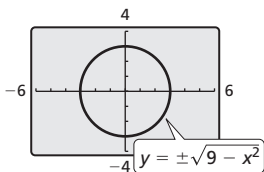
$(-8, 0)$, right

56.



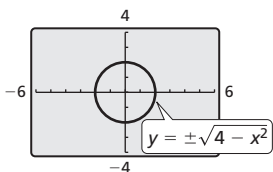
$(-8, 0)$, right

57.



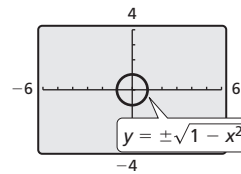
The radius is 3 units. The x -intercepts are ± 3 . The y -intercepts are ± 3 .

58.



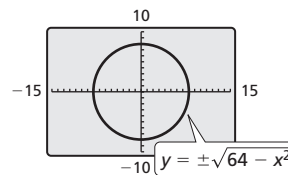
The radius is 2 units. The x -intercepts are ± 2 . The y -intercepts are ± 2 .

59.



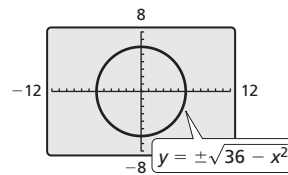
The radius is 1 unit. The x -intercepts are ± 1 . The y -intercepts are ± 1 .

60.



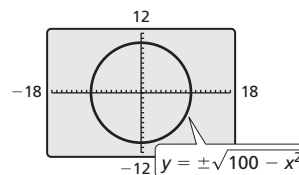
The radius is 8 units. The x -intercepts are ± 8 . The y -intercepts are ± 8 .

61.



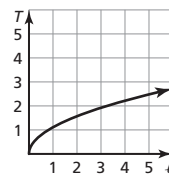
The radius is 6 units. The x -intercepts are ± 6 . The y -intercepts are ± 6 .

62.



The radius is 10 units. The x -intercepts are ± 10 . The y -intercepts are ± 10 .

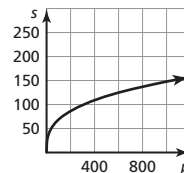
63.



about 3 ft; *Sample answer:* Locate the T -value 2 on the graph and estimate the ℓ -value.

64. square root; The domain and range are restricted; The domain is $x \geq -3$. The range is $y \geq 1$.

65.



a. about 2468 hp

b. about 0.04 mph/hp

66. *Sample answer:* $g(x) = \sqrt{-x+2} + 1$, $h(x) = -\sqrt{x-1} + 2$; f can be written

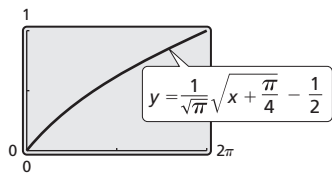
as $f(x) = \sqrt{-x+4}$ or $f(x) = -\sqrt{x-3} + 1$.

67. a. the 165-lb skydiver

b. When $A = 1$, the diver is most likely vertical. When $A = 7$, the diver is most likely horizontal.

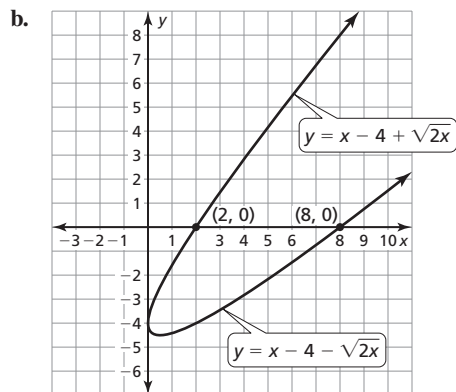
68. a. $\pi\left(r^2 + r + \frac{1}{4}\right) = S + \frac{\pi}{4}$
 $\left(r + \frac{1}{2}\right)^2 = \frac{1}{\pi}\left(S + \frac{\pi}{4}\right)$
 $r = \frac{1}{\sqrt{\pi}}\sqrt{S + \frac{\pi}{4}} - \frac{1}{2}$

b. 0.5 unit



5.4 Monitoring Progress and Modeling with Mathematics

53. a. The greatest stopping distance is 450 feet on ice. On wet asphalt and snow, the stopping distance is 225 feet. The least stopping distance is 90 feet on dry asphalt.
 b. about 272.2 ft; When $s = 35$ and $f = 0.15$, $d \approx 272.2$.
54. a. about 0.16 mph; about 10.77 mph
 b. $0.16 \leq s \leq 80.41$
55. a. When solving the first equation, the solution is $x = 8$ with $x = 2$ as an extraneous solution. When solving the second equation, the solution is $x = 2$ with $x = 8$ as an extraneous solution.



56. no; *Sample answer:* $\sqrt{x-3} = \sqrt{x^2+x-4}$ has two extraneous solutions of $x = \pm 1$.

5.5 Monitoring Progress and Modeling with Mathematics

17. Because the functions have an even index, the domain is restricted; The domain of $(fg)(x)$ is $x \geq 0$.
18. The domain is incorrect; The domain of $\left(\frac{f}{g}\right)(x)$ is all real numbers except $x = 2$ and $x = -2$.
19. a. $(F + M)(t) = 0.0001t^3 - 0.016t^2 + 0.21t + 7.4$
 b. the total number of employees from the ages of 16 to 19 in the United States
20. a. $(W - F)(t) = -18.3333t^3 + 77.72t^2 + 372.5t + 6615$
 b. the number of cruise ship departures around the world, excluding departures from Florida
21. yes; When adding or multiplying functions, the order in which they appear does not matter.

26. $r(w) = \frac{220w^{0.734}}{w}$, $r(6.5) \approx 133.7$, $r(300) \approx 48.3$,
 $r(70,000) \approx 11.3$

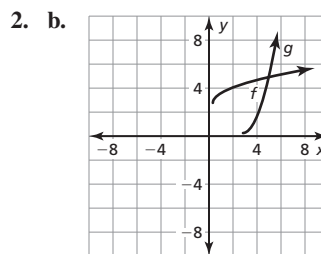
27. a. $r(x) = \frac{20-x}{6.4}$; $s(x) = \frac{\sqrt{x^2+144}}{0.9}$
 b. $t(x) = \frac{20-x}{6.4} + \frac{\sqrt{x^2+144}}{0.9}$

c. $x \approx 1.7$; If Elvis runs along the shore until he is about 1.7 meters from point C then swims to point B , the time taken to get there will be a minimum.

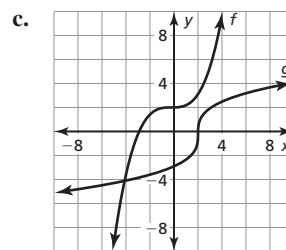
5.5 Maintaining Mathematical Proficiency

28. $n = \frac{2y+3}{x}$ 29. $n = \frac{5z}{7+8z}$
 30. $n = \frac{6z}{5-3b}$ 31. $n = \frac{3}{7b-4}$
 32. yes; Every input has exactly one output.
 33. no; -1 has two outputs.
 34. yes; Every input has exactly one output.
 35. no; 2 has two outputs.

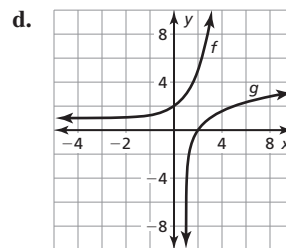
5.6 Explorations



The graph of g is a reflection in the line $y = x$.



The graph of g is a reflection in the line $y = x$.

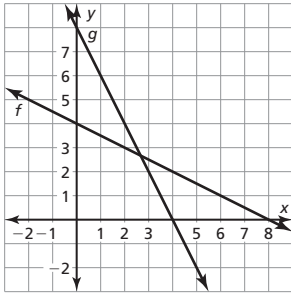


The graph of g is a reflection in the line $y = x$.

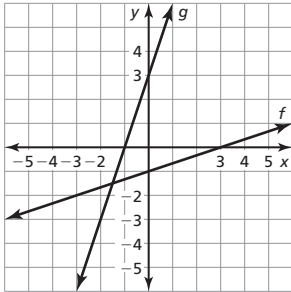
3. Reflect the function in the line $y = x$.
 4. The operations in one equation are the inverses of the operations in the other; $g(x) = \frac{x+3}{2}$

5.6 Monitoring Progress and Modeling with Mathematics

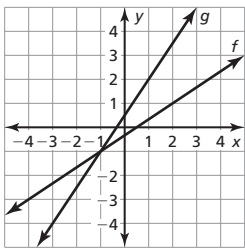
17. $g(x) = -2x + 8$;



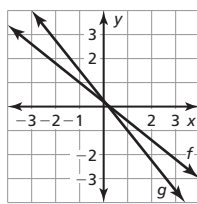
18. $g(x) = 3x + 3$;



19. $g(x) = \frac{3x + 1}{2}$;



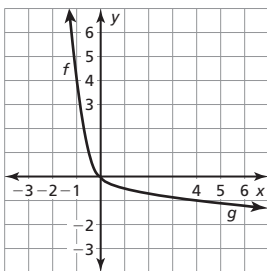
20. $g(x) = \frac{5x - 1}{-4}$;



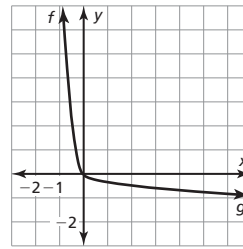
21. $g(x) = \frac{x - 4}{-3}$; *Sample answer:* switching x and y ; You can graph the inverse to check your answer.

22. a. yes; The x - and y -coordinates are switched.
 b. no; The x - and y -coordinates were not switched.
 c. no; The x - and y -coordinates were not switched.

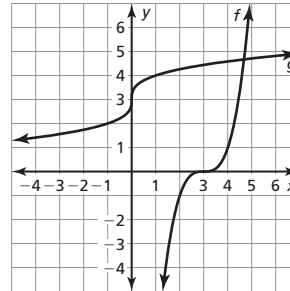
23. $g(x) = -\frac{\sqrt{x}}{2}$;



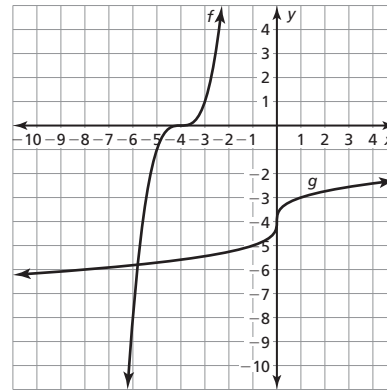
24. $g(x) = -\frac{\sqrt{x}}{3}$;



25. $g(x) = \sqrt[3]{x} + 3$



26. $g(x) = \sqrt[3]{x} - 4$;

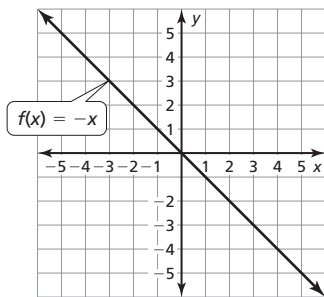


64. a. $h = \left(\frac{A}{0.2195}\right)^{1/0.3964}$; about 150.1 cm

b. $\left(\frac{0.2195h^{0.3964}}{0.2195}\right)^{1/0.3964} = h$;
 $0.2195\left(\left(\frac{A}{0.2195}\right)^{1/0.3964}\right)^{0.3964} = A$

71. The inverse $y = \frac{1}{m}x - \frac{b}{m}$ has a slope of $\frac{1}{m}$ and a y -intercept of $-\frac{b}{m}$.

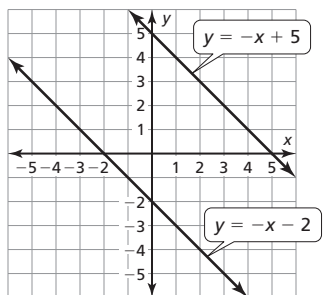
72. a.



When the graph of $f(x) = -x$ is reflected across the line $y = x$, the result is the same graph;

$$\begin{aligned} f(x) &= -x \\ y &= -x \\ x &= -y \\ -x &= y \end{aligned}$$

b. Sample answer:



$y = -x + 5$ and $y = -x - 2$ are their own inverses.

c. A linear function of the form $y = -x + b$ or $y = x$ is its own inverse.

5.6 Maintaining Mathematical Proficiency

73. $\frac{1}{3^3}$

74. 2^5

75. 4^2

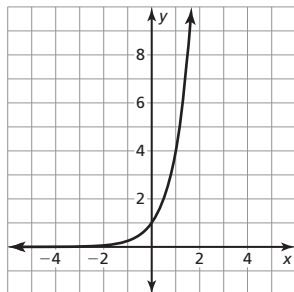
76. $\frac{2^4}{3^4}$

77. The function is increasing when $x > 1$ and decreasing when $x < 1$. The function is positive when $x < 0$ and when $x > 2$, and negative when $0 < x < 2$.
78. The function is increasing when $x < -1.15$ and $x > 1.15$ and decreasing when $-1.15 < x < 1.15$. The function is positive when $-2 < x < 0$ and $x > 2$ and negative when $0 < x < 2$.
79. The function is increasing when $-2.89 < x < 2.89$ and decreasing when $x < -2.89$ and $x > 2.89$. The function is positive when $x < -5$ and $0 < x < 5$ and negative when $-5 < x < 0$ and $x > 5$.

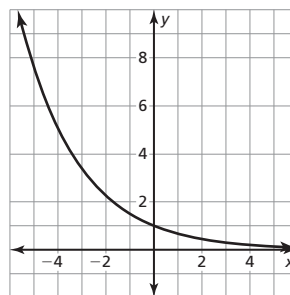
Chapter 6

6.1 Lesson Monitoring Progress

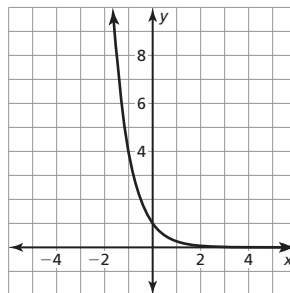
1. exponential growth



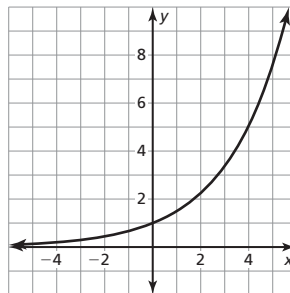
2. exponential decay



3. exponential decay

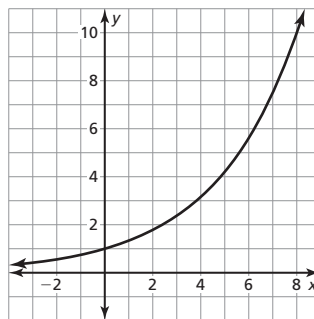


4. exponential growth

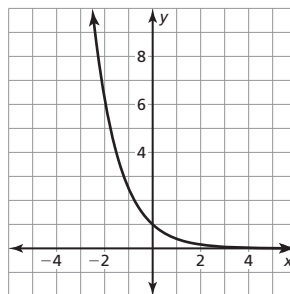


6.1 Monitoring Progress and Modeling with Mathematics

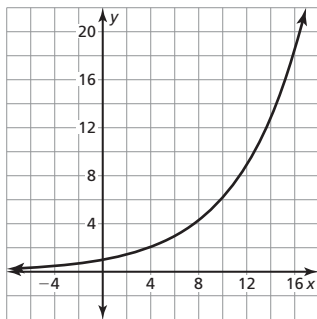
13. exponential growth



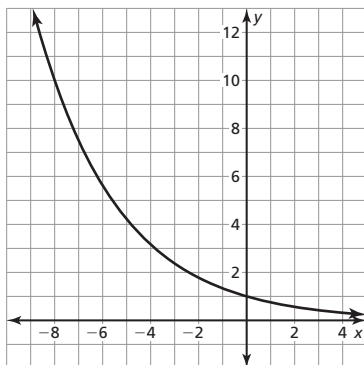
14. exponential decay



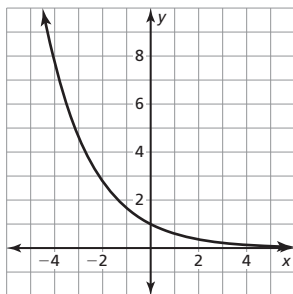
15. exponential growth



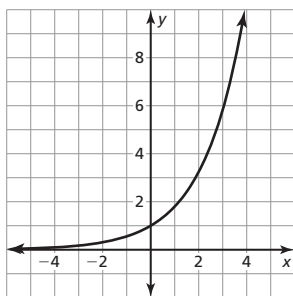
16. exponential decay



17. exponential decay



18. exponential growth



19. $b = 3$

20. $b = 5$

21. a. exponential decay

b. 25% decrease

c. in about 4.8 years

22. a. exponential growth

b. 3% increase

c. about 6 years after the start of the decade

23. a. $y = 233(1.06)^t$; about 261.8 million

b. 2009

45. a represents the number of referrals it received at the start of the model. b represents the growth factor of the number of referrals each year; 50%; 1.50 can be rewritten as $(1 + 0.50)$, showing the percent increase of 50%.

46. a. exponential decay

b. domain: all real numbers, range: $y > 0$; It is an exponential function and any real number can be used as an exponent.

$$f(x) \rightarrow 0 \text{ as } x \rightarrow \infty \text{ and } f(x) \rightarrow \infty \text{ as } x \rightarrow -\infty.$$

47. no; $f(x) = 2^x$ eventually increases at a faster rate than $g(x) = x^2$, but not for all $x \geq 0$.

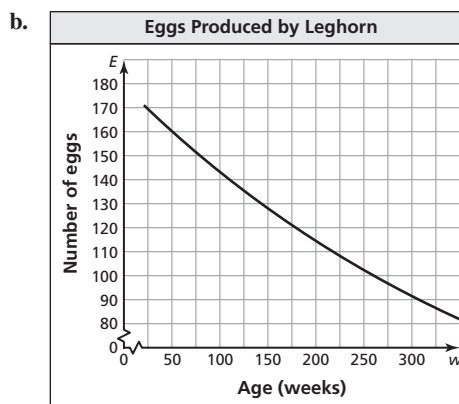
48. Sample answer: $y = (1 - b)^x$

49. about 221.5; The curve contains the points $(0, 6850)$ and $(6, 8179.26)$ and $\frac{8179.26 - 6850}{6 - 0} \approx 221.5$.

50. a. $\frac{f(x+1)}{f(x)} = \frac{ab^{x+1}}{ab^x} = \frac{b^{x+1}}{b^x} = b^{(x+1)-x} = b^1 = b$

b. The equation shows that when a value of the function is divided by the previous value, the answer is the constant b . Dividing the y -values in the table by the previous value does not always produce the same number.

51. a. The decay factor is 0.9978. The percent decrease is 0.22%.



c. about 134 eggs per year

d. Replace $\frac{w}{52}$ with y , where y represents the age of the chicken in years.

52. $V = 1300(0.6782)^t$

6.1 Maintaining Mathematical Proficiency

53. x^{11}

54. x

55. $24x^2$

56. $16x^8$

57. $2x$

58. $7x$

59. $3 + 5x$

60. $216x^{18}$

6.2 Explorations

2.

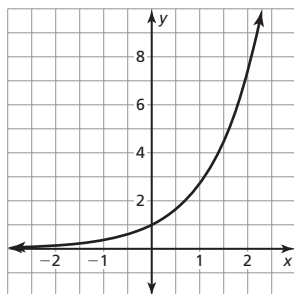
x	10^1	10^2	10^3
$\left(1 + \frac{1}{x}\right)^x$	2.59374	2.70481	2.71692

x	10^4	10^5	10^6
$\left(1 + \frac{1}{x}\right)^x$	2.71815	2.71827	2.71828

Sample answer: $e \approx 2.71828$; This approximation has more decimal points.

3.

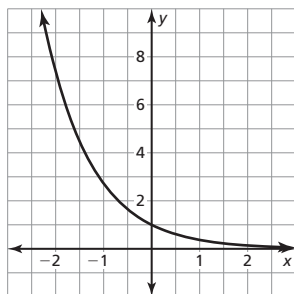
x	-2	-1	0	1	2
$y = e^x$	0.1353	0.3679	1	2.7183	7.3891



domain: all real numbers, range: $y > 0$; It is an exponential function and any real number can be used as an exponent.
 $f(x) \rightarrow 0$ as $x \rightarrow -\infty$ and $f(x) \rightarrow \infty$ as $x \rightarrow \infty$.

5.

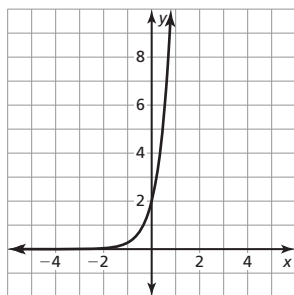
x	-2	-1	0	1	2
$y = e^{-x}$	7.3891	2.7183	1	0.3679	0.1353



domain: all real numbers, range: $y > 0$; The graph of $y = e^{-x}$ is a reflection in the y -axis of the graph of $y = e^x$.
 The domain and range of each graph are the same.

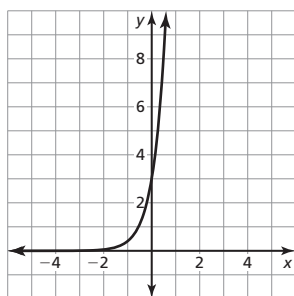
6.2 Lesson Monitoring Progress

6. exponential growth

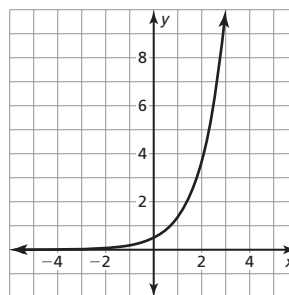


6.2 Monitoring Progress and Modeling with Mathematics

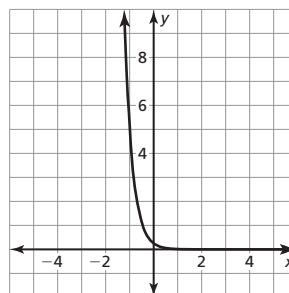
18. exponential growth



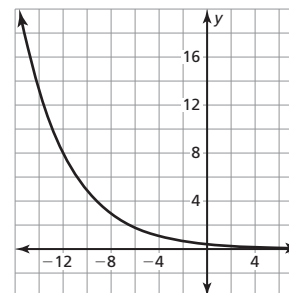
19. exponential growth



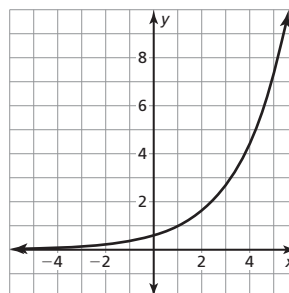
20. exponential decay



21. exponential decay



22. exponential growth



23. D; The graph shows growth and has a y -intercept of 1.

24. A; The graph shows decay and has a y -intercept of 1.

25. B; The graph shows decay and has a y -intercept of 4.

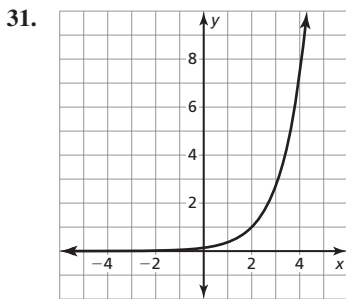
26. C; The graph shows growth and has a y -intercept of 0.75.

27. $y = (1 - 0.221)^t$; 22.1% decay

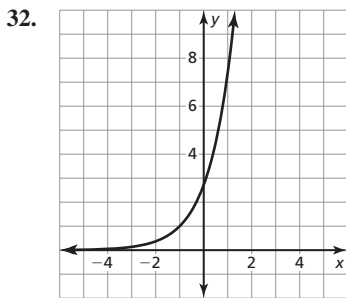
28. $y = (1 - 0.528)^t$; 52.8% decay

29. $y = 2(1 + 0.492)^t$; 49.2% growth

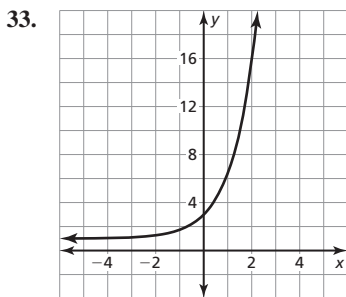
30. $y = 0.5(1 + 1.226)^t$; 122.6% growth



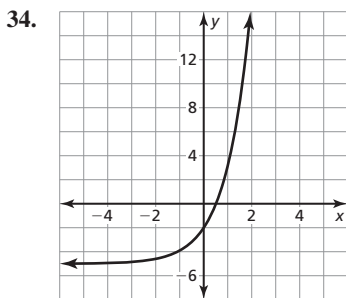
domain: all real numbers, range: $y > 0$



domain: all real numbers, range: $y > 0$



domain: all real numbers, range: $y > 1$



domain: all real numbers, range: $y > -5$

38. Let $m = \frac{n}{r}$, so $n = mr$ and $\frac{r}{n} = \frac{1}{m}$.

Substituting into $A = P\left(1 + \frac{r}{n}\right)^{nt}$ gives $A = P\left(1 + \frac{1}{m}\right)^{mrt}$

which can be written as $A = P\left[\left(1 + \frac{1}{m}\right)^m\right]^{rt}$. By definition,

$\left(1 + \frac{1}{m}\right)^m$ approaches e as m approaches $+\infty$. So, the equation becomes $A = Pe^{rt}$.

39. no; e is an irrational number. Irrational numbers cannot be expressed as a ratio of two integers.

40. no; The value of $f(x)$ at $x = 1000$ is too small for the calculator to display, so the calculator rounds the value to 0. The function $f(x) = e^{-x}$ has no x -intercept, but $f(x) \rightarrow 0$ as $x \rightarrow \infty$.

41. account 1; With account 1, the balance would be

$$A = 2500\left(1 + \frac{0.06}{4}\right)^{4 \cdot 10} \approx \$4535.05.$$

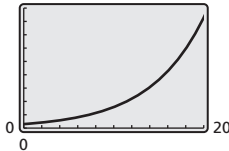
With account 2, the balance would be $A = 2500e^{0.04 \cdot 10} \approx \3729.56 .

42. a. ∞

b. -3

43. a. $N(t) = 30e^{0.166t}$

b. 900



c. At 3:45 P.M., it has been 2 hours and 45 minutes, or 2.75 hours, since 1:00 P.M. Using the *trace* feature of the calculator, type 2.75 to find the point (2.75, 47.356183). At 3:45 P.M., there are about 47 cells.

6.2 Maintaining Mathematical Proficiency

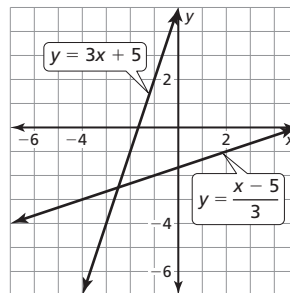
44. 6×10^{-3}

45. 5×10^3

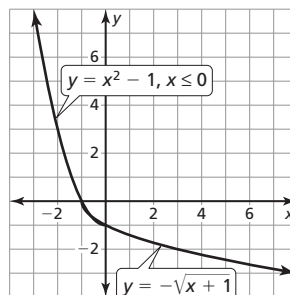
46. 2.6×10^7

47. 4.7×10^{-8}

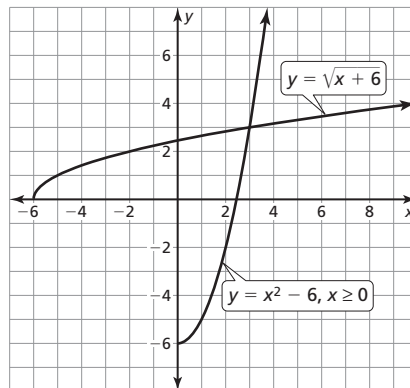
48. $y = \frac{x-5}{3}$



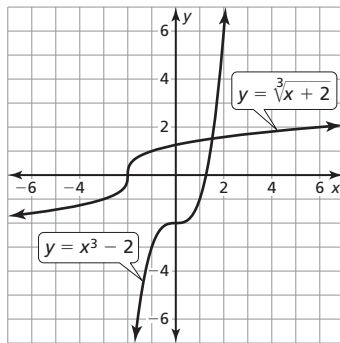
49. $y = -\sqrt{x+1}$



50. $y = x^2 - 6, x \geq 0$



51. $y = \sqrt[3]{x+2}$



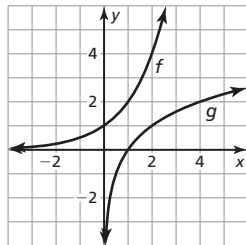
6.3 Explorations

2. a.

x	-2	-1	0	1	2
$f(x) = 2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
$g(x) = \log_2 x$	-2	-1	0	1	2

The functions f and g are inverses, so the x - and y -coordinates can be switched to create the table for $g(x)$.

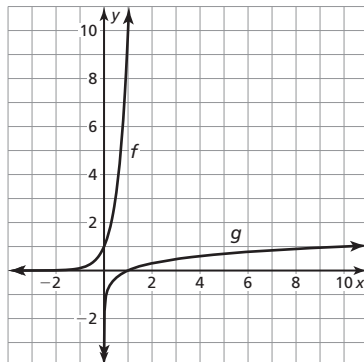


b.

x	-2	-1	0	1	2
$f(x) = 10^x$	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100

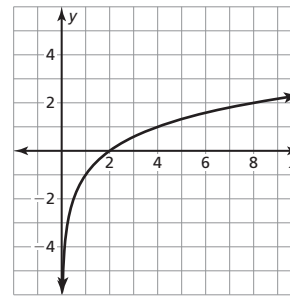
x	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100
$g(x) = \log_{10} x$	-2	-1	0	1	2

The functions f and g are inverses, so the x - and y -coordinates can be switched to create the table for $g(x)$.

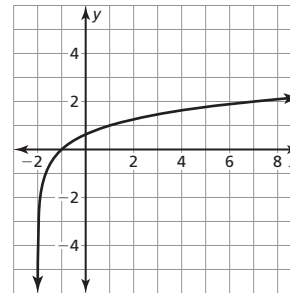


6.3 Monitoring Progress and Modeling with Mathematics

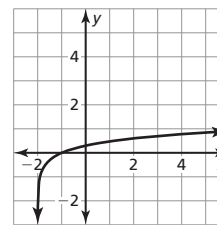
59.



60.

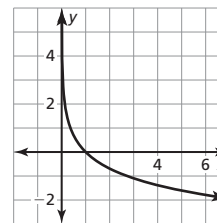


61.



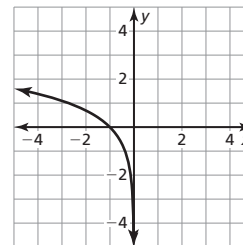
domain: $x > -2$, range: all real numbers, asymptote: $x = -2$

62.



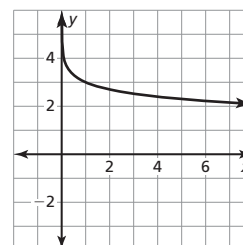
domain: $x > 0$, range: all real numbers, asymptote: $x = 0$

63.



domain: $x < 0$, range: all real numbers, asymptote: $x = 0$

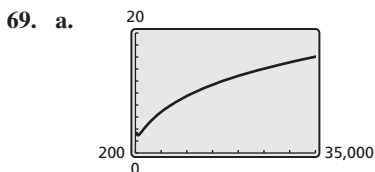
64.



domain: $x > 0$, range: all real numbers, asymptote: $x = 0$

65. no; Any logarithmic function of the form $g(x) = \log_b x$ will pass through $(1, 0)$, but if the function has been translated or reflected in the x -axis, it may not pass through $(1, 0)$.

66. b, c, a, d

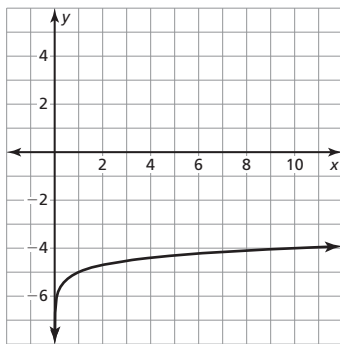


b. 15 species

c. about 3918 m²

d. The number of species of fish increases; *Sample answer:* This makes sense because in a smaller pool or lake, one species could dominate another more easily and feed on the weaker species until it became extinct.

70. *Sample answer:* $y = \log(x) - 5$



71. a. $\frac{2}{3}$
 b. $\frac{5}{3}$
 c. $\frac{4}{3}$
 d. $\frac{7}{2}$

6.3 Maintaining Mathematical Proficiency

72. $g(x) = -\sqrt[3]{x}$

73. $g(x) = \sqrt[3]{\frac{1}{2}x}$

74. $g(x) = \sqrt[3]{-x} + 3$

75. $g(x) = \sqrt[3]{x+2}$

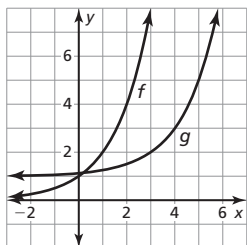
76. constant; The graph is a translation 3 units down of the parent constant function.

77. quadratic; The graph is a translation 2 units left and 1 unit down of the parent quadratic function.

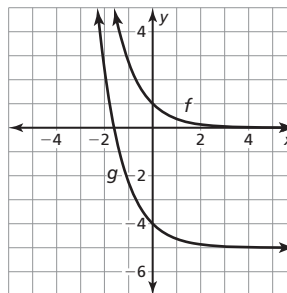
78. absolute value; The graph is a reflection in the x -axis followed by a translation 1 unit right and 2 units up of the parent absolute value function.

6.4 Lesson Monitoring Progress

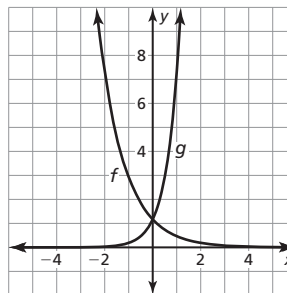
1. The graph of g is a translation 3 units right and 1 unit up of the graph of f .



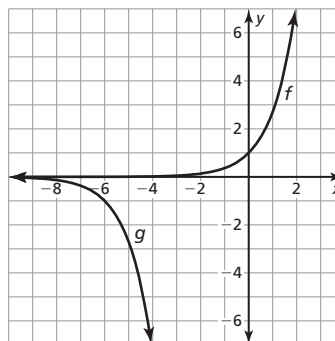
2. The graph of g is a translation 5 units down of the graph of f .



3. The graph of g is a reflection in the y -axis followed by a horizontal shrink by a factor of $\frac{1}{2}$ of the graph of f .

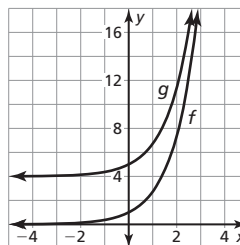


4. The graph of g is a reflection in the x -axis followed by a translation 6 units left of the graph of f .

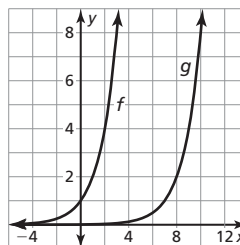


6.4 Monitoring Progress and Modeling with Mathematics

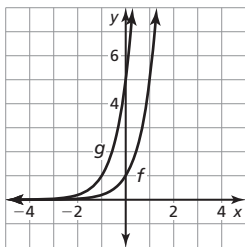
10. The graph of g is a translation 4 units up of the graph of f .



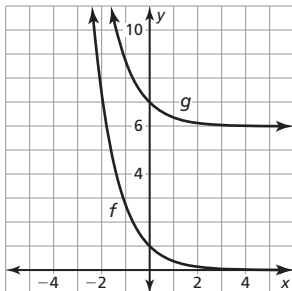
11. The graph of g is a translation 7 units right of the graph of f .



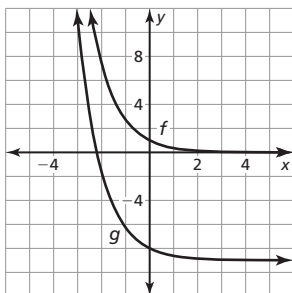
12. The graph of g is a translation 1 unit left of the graph of f .



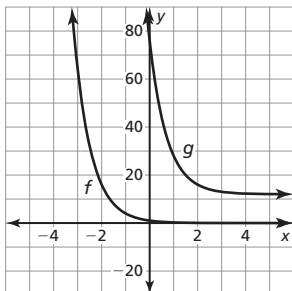
13. The graph of g is a translation 6 units up of the graph of f .



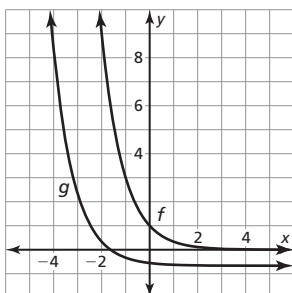
14. The graph of g is a translation 9 units down of the graph of f .



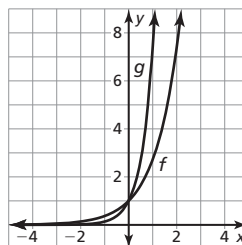
15. The graph of g is a translation 3 units right and 12 units up of the graph of f .



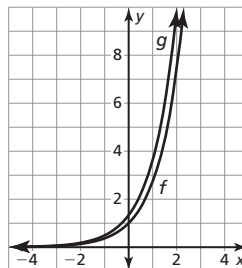
16. The graph of g is a translation 2 units left and $\frac{2}{3}$ unit down of the graph of f .



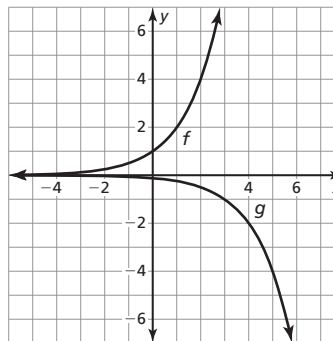
17. The graph of g is a horizontal shrink by a factor of $\frac{1}{2}$ of the graph of f .



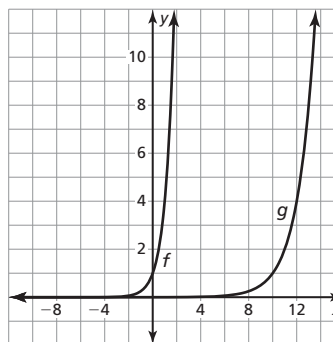
18. The graph of g is a vertical stretch by a factor of $\frac{4}{3}$ of the graph of f .



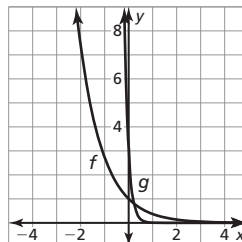
19. The graph of g is reflection in the x -axis followed by a translation 3 units right of the graph of f .



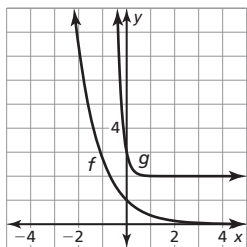
20. The graph of g is a horizontal stretch by a factor of 2 followed by a translation 5 units right of the graph of f .



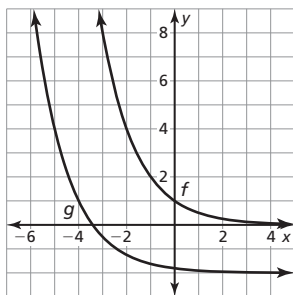
21. The graph of g is a horizontal shrink by a factor of $\frac{1}{6}$ followed by a vertical stretch by a factor of 3 of the graph of f .



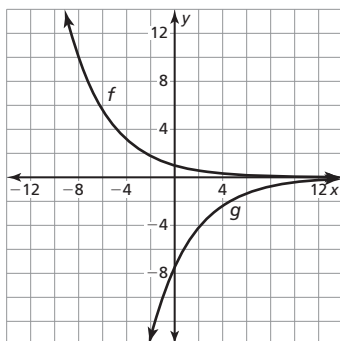
22. The graph of g is a horizontal shrink by a factor of $\frac{1}{5}$ followed by a translation 2 units up of the graph of f .



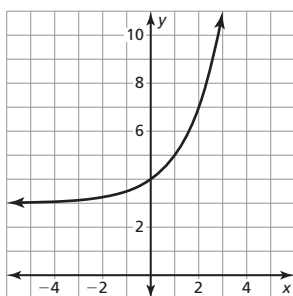
23. The graph of g is a vertical stretch by a factor of 6 followed by a translation 5 units left and 2 units down of the graph of f .



24. The graph of g is a reflection in the x -axis followed by a translation 7 units right and 1 unit up of the graph of f .



25. The graph of the parent function $f(x) = 2^x$ was translated 3 units left instead of up.



31. A; The graph of f has been translated 2 units right.
 32. D; The graph of f has been translated 2 units left.
 33. C; The graph of f has been stretched vertically by a factor of 2.
 34. B; The graph of f has been shrunk horizontally by a factor of $\frac{1}{2}$.
 35. $g(x) = 5^{-x} - 2$ 36. $g(x) = -6\left(\frac{2}{3}\right)^{x+4}$
 37. $g(x) = e^{2x} + 5$ 38. $g(x) = \frac{1}{3}e^{-x+4} - \frac{1}{3}$
 39. $g(x) = 6 \log_6 x - 5$ 40. $g(x) = -\log_5(x+9)$
 41. $g(x) = \log_{1/2}(-x+3) + 2$ 42. $g(x) = \ln\left(\frac{1}{8}x - 3\right) + 1$
 43. Multiply the output by -1 ; Substitute $\log_7 x$ for $f(x)$.
 Subtract 6 from the output; Substitute $-\log_7 x$ for $h(x)$.

44. Multiply output by 4; Substitute 8^x for $f(x)$.
 Add 3 to the input and add 1 to the output; Replace x with $x+3$ in $h(x)$ and add 1 to the output.
 45. The graph of g is a translation 4 units up of the graph of f ; $y = 4$
 46. The graph of g is a translation 9 units right of the graph of f ; $y = 0$
 47. The graph of g is a translation 6 units left of the graph of f ; $x = -6$
 48. The graph of g is a translation 13 units up of the graph of f ; $x = 0$
 49. The graph of S is a vertical shrink by a factor of 0.118 followed by a translation 0.159 unit up of the graph of f ; For fine sand, the slope of the beach is about 0.05. For medium sand, the slope of the beach is about 0.09. For coarse sand, the slope of the beach is about 0.12. For very coarse sand, the slope of the beach is about 0.16.
 50. a. The graph of g is a reflection in the y -axis of the graph of f .
 b. no; When $f(x) = b^x$ is reflected in the y -axis, it becomes b^{-x} , which is equal to $g(x) = \left(\frac{1}{b}\right)^x$ for any $0 < b < 1$.
 51. yes; *Sample answer:* If the graph is reflected in the y -axis, the graphs will never intersect because there are no values of x where $\log x = \log(-x)$.
 52. yes; $y = \ln x$ and $y = e^x$ are inverses. So, the graph of $y = \ln x$ is a reflection in the line $y = x$ of the graph of $y = e^x$.
 53. a. never; The asymptote of $f(x) = \log x$ is a vertical line and would not change by shifting the graph vertically.
 b. always; The asymptote of $f(x) = e^x$ is a horizontal line and would be changed by shifting the graph vertically.
 c. always; The domain of $f(x) = \log x$ is $x > 0$ and would not be changed by a horizontal shrink.
 d. sometimes; The graph of the parent exponential function does not intersect the x -axis, but if it is shifted down, the graph would intersect the x -axis.
 54. a. domain: $t \geq 0$, range: $0 < P \leq 100$
 b. 69.77 g
 c. The graph would be a vertical stretch by a factor of 5.5 of the graph of the parent function.
 d. The translation does not affect the domain but it does affect the range; It changes the range to $0 < P \leq 550$ because the initial amount is 550, but in both cases there can never be 0 grams remaining.
 55. The graph of h is a translation 2 units left of the graph of f ; The graph of h is a reflection in the y -axis followed by a translation 2 units left of the graph of g ; x has been replaced with $x+2$.
 x has been replaced with $-(x+2)$.

56. *Sample answer:* $y = 1.5 \cdot 2^{x+1} + 2$

6.4 Maintaining Mathematical Proficiency

57. $(fg)(x) = x^6$; $(fg)(3) = 729$

58. $\left(\frac{f}{g}\right)(x) = 2x^3$; $\left(\frac{f}{g}\right)(5) = 250$

59. $(f+g)(x) = 14x^3$; $(f+g)(2) = 112$

60. $(f-g)(x) = -x^2$; $(f-g)(6) = -36$

6.5 Monitoring Progress and Modeling with Mathematics

46. It is not true; By the Product Property, $\log_b M + \log_b N = \log_b(MN)$. If $\log_b(M+N) = \log_b(MN)$, then $M+N = M \cdot N$ which is not always true.

47. Rewrite each logarithm in exponential form to obtain $a = b^x$,

$$c = b^y, \text{ and } a = c^z. \text{ So, } \frac{\log_b a}{\log_b c} = \frac{\log_b c^z}{\log_b c} = \frac{z \log_b c}{\log_b c} = z = \log_c a.$$

48. *Sample answer:* The graph of g could be a translation 1 unit up of the graph of f . The graph of g could be a horizontal shrink by a factor of $\frac{1}{100}$ followed by a translation 1 unit down of the graph of f . The graph of g could be a horizontal shrink by a factor of $\frac{1}{10}$ of the graph of f ; Transformation 1:

$$g(x) = f(x) + 1 = \log x + 1 = 2 + \log x - 1$$

$$= \log 100 + \log x - 1 = \log 100x - 1;$$

Transformation 2: $g(x) = f(100x) - 1 = \log 100x - 1$;

Transformation 3: $g(x) = f(10x) = \log(10x) = \log\left(\frac{100x}{10}\right)$

$$= \log 100x - \log 10 = \log 100x - 1$$

6.5 Maintaining Mathematical Proficiency

49. $x < -2$ or $x > 2$
 50. $x \leq 6 - \sqrt{21}$ or $x \geq 6 + \sqrt{21}$
 51. $-7 < x < -6$
 52. $x \leq -6$ or $x \geq 2$
 53. $x \approx -0.76$ and $x \approx 2.36$
 54. $x \approx -0.89$ and $x \approx 3.39$
 55. $x \approx -1.79$ and $x \approx 1.12$
 56. $x \approx -9.81$ and $x \approx -7.17$

6.6 Explorations

2. a. *Sample answer:* For $e^x = 2$, use $e^x = y$:

x	0.55	0.60	0.65	0.70	0.75	0.80
y	1.7333	1.8221	1.9155	2.0138	2.1170	2.2255

Because $e^{0.70} \approx 2$, $x \approx 0.70$; For $\ln x = -1$, use $\ln x = y$:

x	0.32	0.34	0.36	0.38	0.40	0.42
y	-1.139	-1.079	-1.022	-0.968	-0.916	-0.868

Because $\ln 0.36 \approx -1$, $x \approx 0.36$.

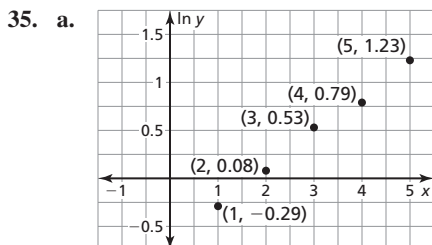
6.6 Monitoring Progress and Modeling with Mathematics

74. d. A lead apron does not need to be as thick as aluminum or copper to result in the same intensity.

6.6 Maintaining Mathematical Proficiency

75. $y + 2 = 4(x - 1)$ 76. $y - 2 = -2(x - 3)$
 77. $y + 8 = -\frac{1}{3}(x - 3)$ 78. $y - 5 = 2(x - 2)$
 79. 3; $y = 2x^3 - x + 1$
 80. 4; $y = x^4 - 2x^3 + x^2 + x - 2$
 81. 4; $y = -3x^4 + 2x^3 - x^2 + 5x - 6$

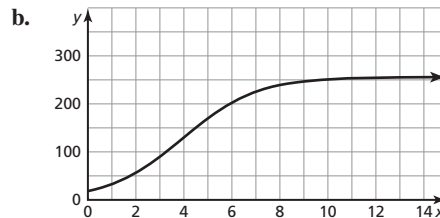
6.7 Monitoring Progress and Modeling with Mathematics



Sample answer: $y = 0.50(1.47)^x$

- b. about 47%; The base is 1.47 which means that the function shows 47% growth.
 36. yes; The points $(x, \ln y)$ are linear.

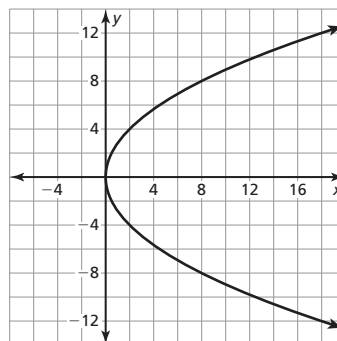
37. no; When d is the independent variable and t is the dependent variable, the data can be modeled with a logarithmic function. When the variables are switched, the data can be modeled with an exponential function.
 38. yes; The data have a common ratio of 2.
 39. a. 5.9 weeks



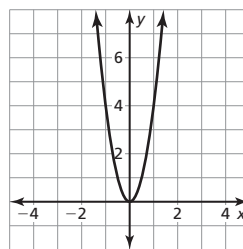
The asymptote is the line $y = 256$ and represents the maximum height of the sunflower.

6.7 Maintaining Mathematical Proficiency

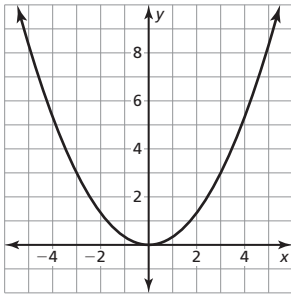
40. yes; When one variable is increased by a factor, the other variable increases by the same factor.
 41. no; When one variable is increased by a factor, the other variable does not increase by the same factor.
 42. no; When one variable is increased by a factor, the other variable does not increase by the same factor.
 43. yes; When one variable is increased by a factor, the other variable increases by the same factor.
 44. The focus is $(2, 0)$, the directrix is $x = -2$, and the axis of symmetry is $y = 0$.



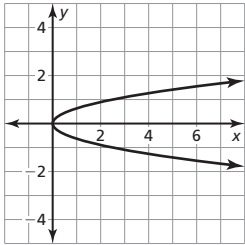
45. The focus is $(0, \frac{1}{16})$, the directrix is $y = -\frac{1}{16}$, and the axis of symmetry is $x = 0$.



46. The focus is $(0, \frac{3}{4})$, the directrix is $y = -\frac{3}{4}$, and the axis of symmetry is $x = 0$.



47. The focus is $(0.1, 0)$, the directrix is $x = -0.1$, and the axis of symmetry is $y = 0$.



Chapter 6 Test

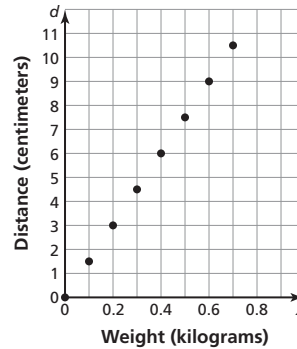
11. Both equations transform into a linear equation that is solved to find the value of x . To get the linear equation, the first equation requires that exponent rules are used to make the bases the same. In the second equation, the definition of logarithm is used to write the equation in exponential form to get the linear equation; $x = 0.8$; $x = -0.2$
12. The expressions are all equivalent; change-of-base formula
13. 39.329 billion barrels; $y = 40.473(1.085)^x$; The inverse function gives the number of wells, y , that need to be drilled to obtain x billions of barrels.
14. a. $L(x) = 100e^{-0.02x}$
 b. exponential decay; Natural base functions of the form $y = ae^{rx}$ show decay when r is negative.
 c. about 44.9%
15. *Sample answer:* To find an exponential equation in the form $y = ab^x$, the common ratio, b , can be found by dividing consecutive terms and a is the y -value when x is 0. Two points can be used to create a system of equations in the form $y = ab^x$ which can be solved for a and b . Or, use the *regression* feature of a graphing calculator to find an exponential model which fits the data; $4200(0.90)^x$; in the 4th year

Chapter 7

7.1 Explorations

1. b.

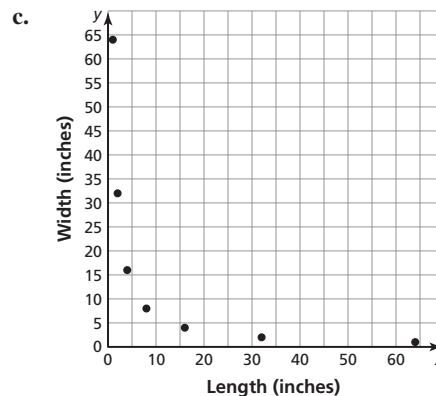
Weight, x (in kilograms)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
Distance, d (in centimeters)	0	1.5	3	4.5	6	7.5	9	10.5



As the value of x increases, the value of y also increases. The points lie on a line.

2. a.

x	1	2	4	8	16	32	64
y	64	32	16	8	4	2	1



As the values of x increase, the values of y decrease. The points lie on a curve.

7.1 Monitoring Progress and Modeling with Mathematics

25. a.

Size	2	2.5	3	5
Number of songs	5000	4000	3333	2000

- b. The number of songs decreases.

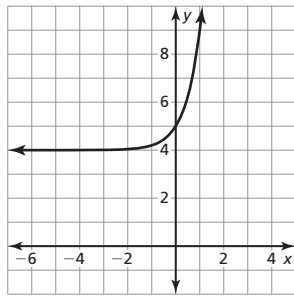
31. *Sample answer:* As the speed of your car increases, the number of minutes per mile decreases.
32. x varies directly with z ; $xy = a$, so $y = \frac{a}{x}$; $yz = b$, so $y = \frac{b}{z}$;
 $\frac{a}{x} = \frac{b}{z}$, $az = bx$, $x = \frac{a}{b}z$
33. cat: 4 ft, dog: 2 ft; The inverse equations are $d = \frac{a}{7}$ and $6 - d = \frac{a}{14}$. Because the constant is the same, solve the equation $7d = 14(6 - d)$ for d .

7.1 Maintaining Mathematical Proficiency

34. $x - 9$

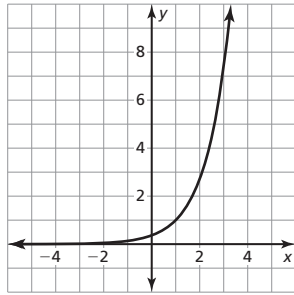
35. $x^2 - 6$

36.



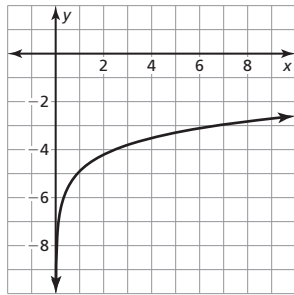
domain: all real numbers, range: $y > 4$

37.



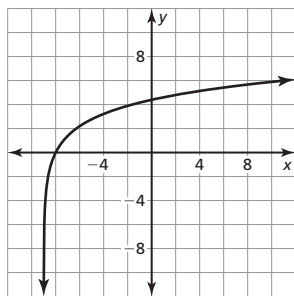
domain: all real numbers, range: $y > 0$

38.



domain: $x > 0$, range: all real numbers

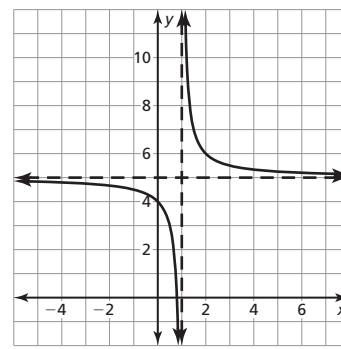
39.



domain: $x > -9$, range: all real numbers

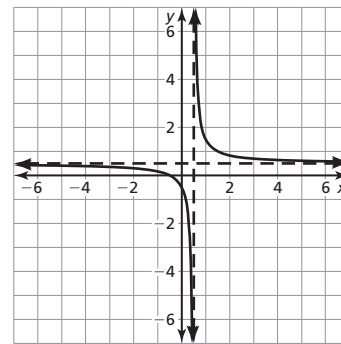
7.2 Lesson Monitoring Progress

4.



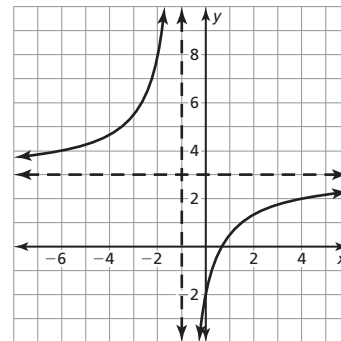
domain: all real numbers except 1; range: all real numbers except 5

6.



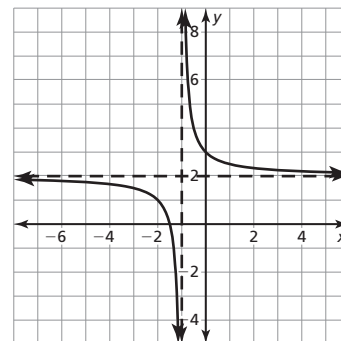
domain: all real numbers except $\frac{1}{2}$; range: all real numbers except 5

7.



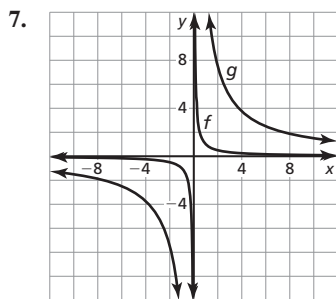
domain: all real numbers except -1 ; range: all real numbers except 3

8. $g(x) = \frac{1}{x+1} + 2$

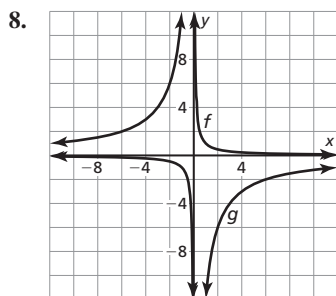


translation 1 unit left and 2 units up

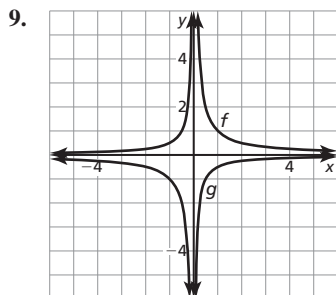
7.2 Monitoring Progress and Modeling with Mathematics



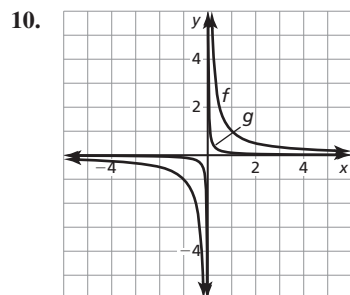
The graph of g lies farther from the axes. Both graphs lie in the first and third quadrants and have the same asymptotes, domain, and range.



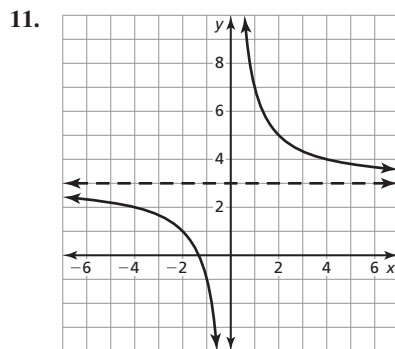
The graph of g lies farther from the axes and is reflected over the x -axis. Both graphs have the same asymptotes, domain, and range.



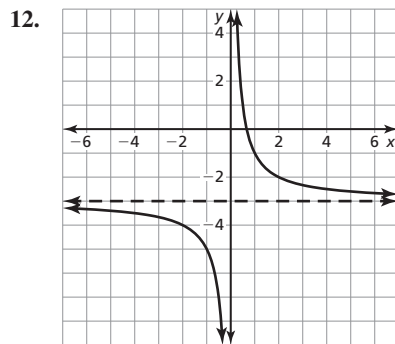
The graph of g lies closer to the axes and is reflected over the x -axis. Both graphs have the same asymptotes, domain, and range.



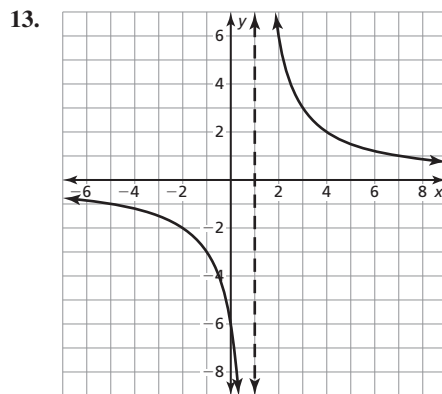
The graph of g lies closer to the axes. Both graphs lie in the first and third quadrants and have the same asymptotes, domain, and range.



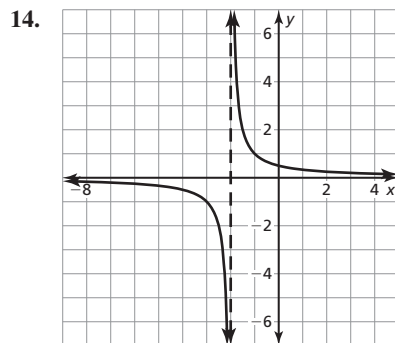
domain: all real numbers except 0; range: all real numbers except 3



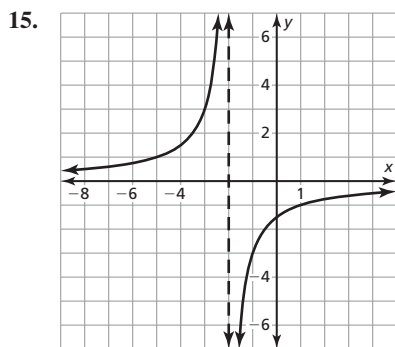
domain: all real numbers except 0; range: all real numbers except -3



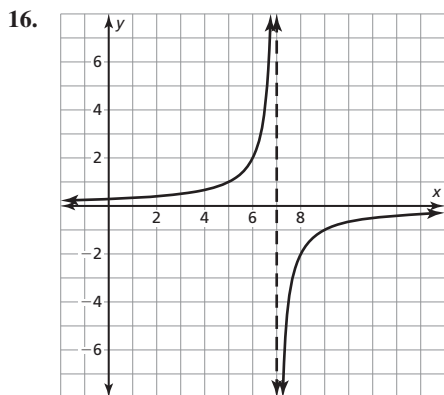
domain: all real numbers except 1; range: all real numbers except 0



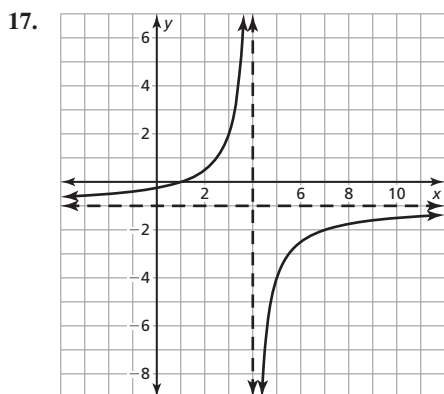
domain: all real numbers except -2 ; range: all real numbers except 0



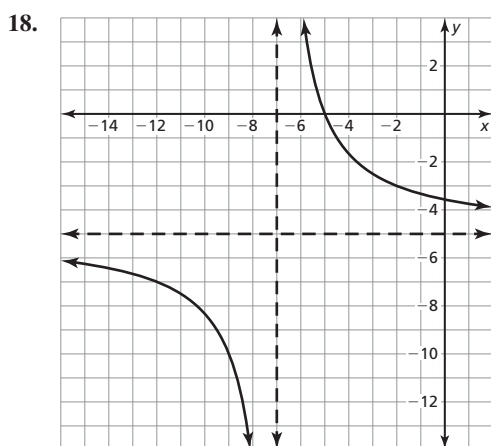
domain: all real numbers except -2 ; range: all real numbers except 0



domain: all real numbers except 7 ; range: all real numbers except 0

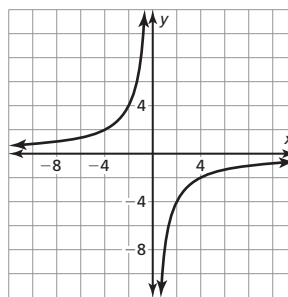


domain: all real numbers except 4 ; range: all real numbers except -1

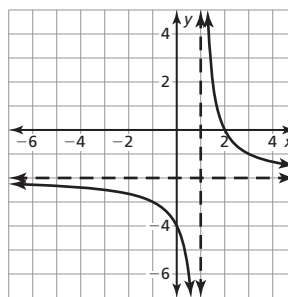


domain: all real numbers except -7 ; range: all real numbers except -5

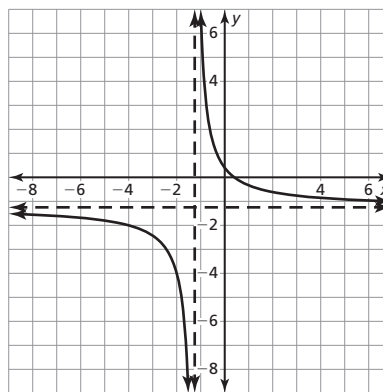
19. The graph should lie in the second and fourth quadrants instead of the first and third quadrants;



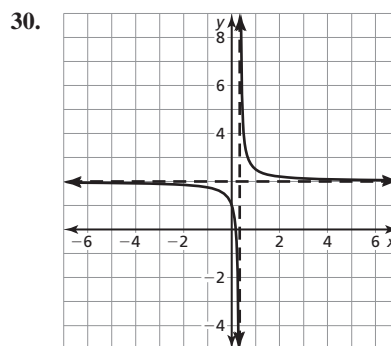
20. The vertical asymptote should be $x = 1$;



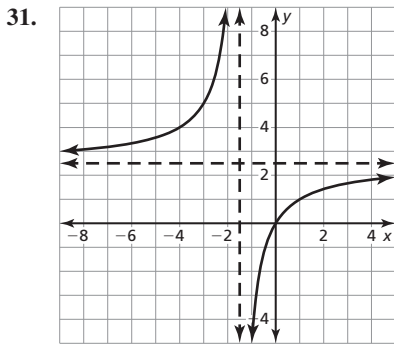
21. A; The asymptotes are $x = 3$ and $y = 1$.
 22. C; The asymptotes are $x = -3$ and $y = 1$.
 23. B; The asymptotes are $x = 3$ and $y = -1$.
 24. D; The asymptotes are $x = -3$ and $y = -1$.
 29.



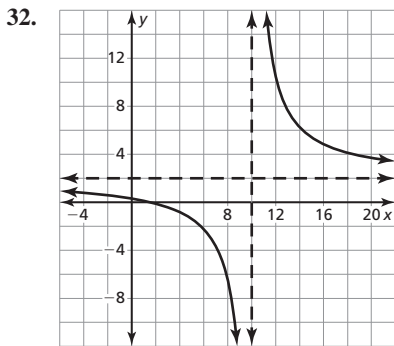
domain: all real numbers except $-\frac{5}{4}$; range: all real numbers except $-\frac{5}{4}$



domain: all real numbers except $\frac{1}{3}$; range: all real numbers except 2

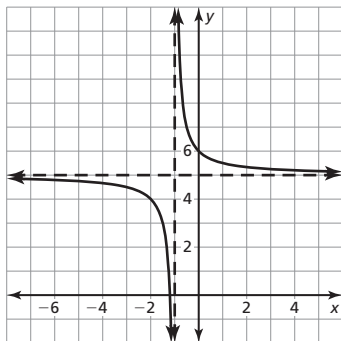


domain: all real numbers except $-\frac{3}{2}$; range: all real numbers except $\frac{5}{2}$



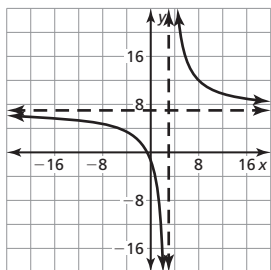
domain: all real numbers except 10; range: all real numbers except 2

33. $g(x) = \frac{1}{x+1} + 5$



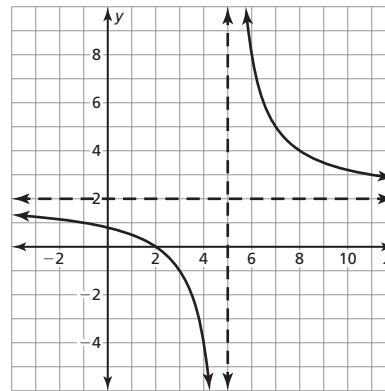
translation 1 unit left and 5 units up

34. $g(x) = \frac{25}{x-3} + 7$



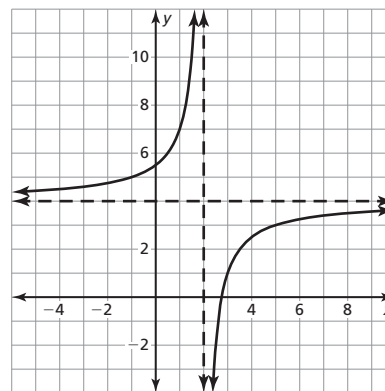
translation 3 units right and 7 units up

35. $g(x) = \frac{6}{x-5} + 2$



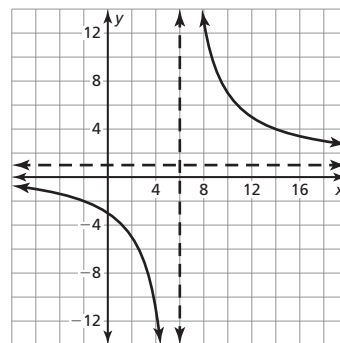
translation 5 units right and 2 units up

36. $g(x) = \frac{-3}{x-2} + 4$



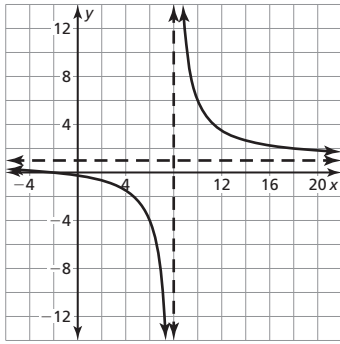
translation 2 units right and 4 units up

37. $g(x) = \frac{24}{x-6} + 1$



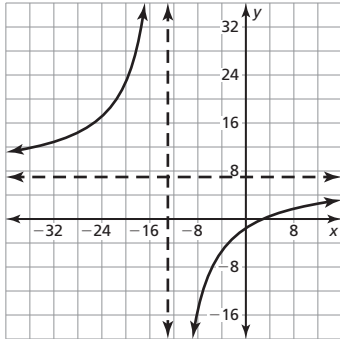
translation 6 units right and 1 unit up

38. $g(x) = \frac{10}{x-8} + 1$



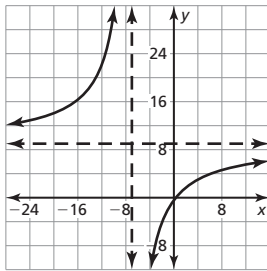
translation 8 units right and 1 unit up

39. $g(x) = \frac{-111}{x+13} + 7$



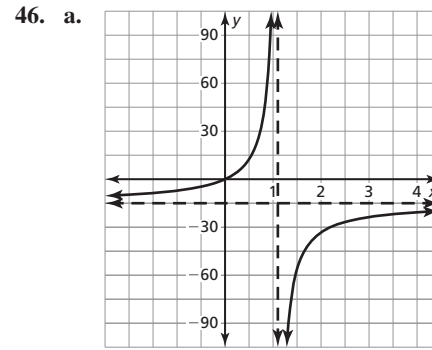
translation 13 units left and 7 units up

40. $g(x) = \frac{-66}{x+7} + 9$



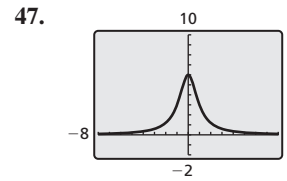
translation 7 units left and 9 units up

41. **a.** 50 students
b. The average cost per student approaches \$20.
42. **a.** 10 months
b. The average cost approaches \$59.
43. **B** **44. B**
45. **a.** about 23°C
b. -0.005 sec/°C

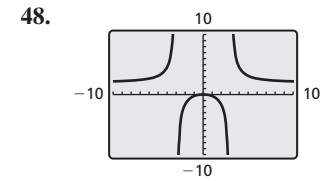


Sample answer: domain: $0 < x < 1$; range: $0 < y < 150$

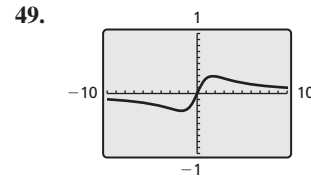
- b.** about \$3333; about \$8571; \$40,000; no; Doubling the percentage does not double the cost because this is not a linear function.



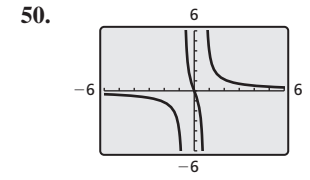
even



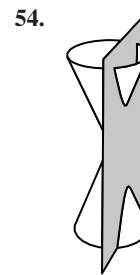
even



odd

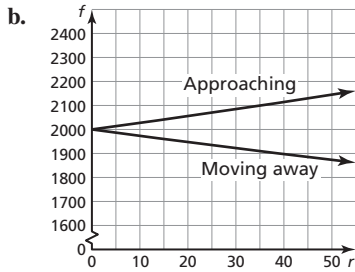


odd



55. (4, 3); The point (2, 1) is one unit left and one unit down from (3, 2), so a point on the other branch is one unit right and one unit up from (3, 2).
56. **a.** decreasing: $(-\infty, 0), (0, \infty)$; The graph falls from left to right.
b. increasing: $(-\infty, 0), (0, \infty)$; The graph rises from left to right.
57. The competitor is a better choice for less than 18 months of service; The cost of Internet service is modeled by $C = \frac{50 + 43x}{x}$. The competitor's cost is lesser when $x = 6$ and $x = 12$, and greater when $x = 18$ and $x = 24$.

58. a. $f_\ell = \frac{740(2000)}{740 - r}$, $f_r = \frac{740(2000)}{740 + r}$



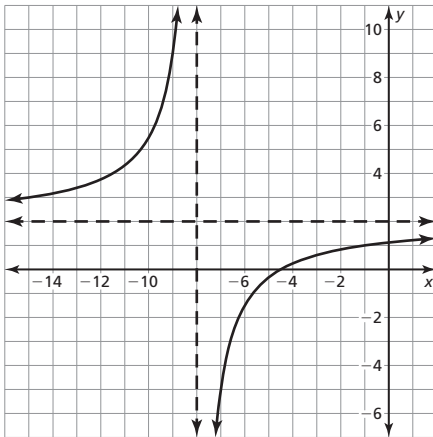
c. The frequency heard is greater for an approaching sound source.

7.2 Maintaining Mathematical Proficiency

59. $4(x - 5)(x + 4)$ 60. $3(x - 2)(x + 1)$
 61. $2(x - 3)(x + 2)$ 62. $(5x - 2)(2x + 7)$
 63. 3^6 64. $2^{11/10}$
 65. $6^{2/3}$ 66. $\frac{1}{6^2}$

7.1–7.2 Quiz

11. $g(x) = \frac{-7}{x + 8} + 2$



translation 8 units left and 2 units up

7.3 Monitoring Progress and Modeling with Mathematics

42. You are correct; The values -9 and 5 make the denominator equal to zero in the original expression, so both values are not in the domain.

43. $\frac{4}{7x}$ 44. $x - 5$

45. $9(x + 3)$, $x \neq -\frac{3}{2}$, $x \neq \frac{5}{2}$, $x \neq 7$

46. $x - 6$, $x \neq -2$, $x \neq 2$, $x \neq 6$

47. Galapagos: about 0.371, King: about 0.203; King: The King penguin has a smaller surface area to volume ratio, so it is better equipped to live in a colder environment.

48. no; *Sample answer:* When $f(x) = \sqrt{x}$ and $g(x) = \sqrt{x^3}$,
 $(fg)(x) = x^2$, $x \geq 0$, and $\left(\frac{f}{g}\right)(x) = \frac{1}{x}$, $x \geq 0$, but both graphs exclude negative values of x .

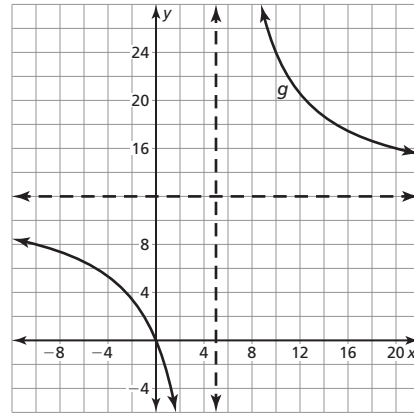
49. $f(x) = \frac{x(x - 1)}{x + 2}$, $g(x) = \frac{x(x + 2)}{x - 1}$

7.3 Maintaining Mathematical Proficiency

50. $x = -1$ 51. $x = -\frac{24}{5}$
 52. $x = \frac{4}{85}$ 53. $x = \frac{32}{15}$
 54. $2 \cdot 3 \cdot 7$ 55. $7 \cdot 13$
 56. $2^3 \cdot 3^2$ 57. prime

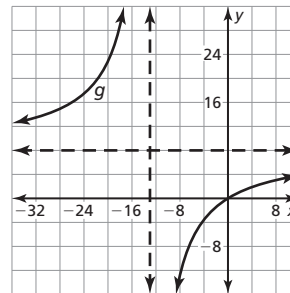
7.4 Monitoring Progress and Modeling with Mathematics

33. $g(x) = \frac{60}{x - 5} + 12$



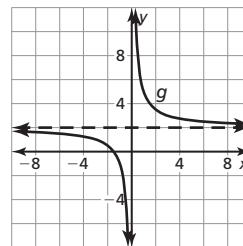
The graph of g is a translation 5 units right and 12 units up of the graph of $f(x) = \frac{60}{x}$.

34. $g(x) = \frac{-104}{x + 13} + 8$



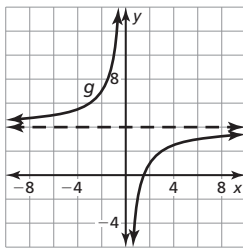
The graph of g is a translation 13 units left and 8 units up of the graph of $f(x) = \frac{-104}{x}$.

35. $g(x) = \frac{3}{x} + 2$



The graph of g is a translation 2 units up of the graph of $f(x) = \frac{3}{x}$.

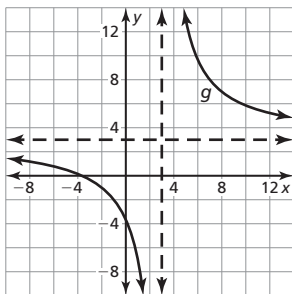
36. $g(x) = \frac{-6}{x} + 4$



The graph of g is a translation 4 units up of the graph of

$$f(x) = \frac{-6}{x}.$$

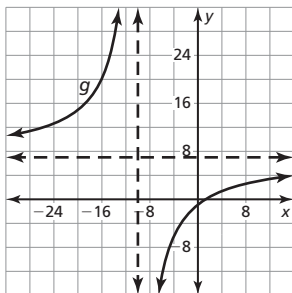
37. $g(x) = \frac{20}{x-3} + 3$



The graph of g is a translation 3 units right and 3 units

up of the graph of $f(x) = \frac{20}{x}$.

38. $g(x) = \frac{-79}{x+10} + 7$



The graph of g is a translation 10 units left and 7 units

up of the graph of $f(x) = \frac{-79}{x}$.

53. $g(x) = \frac{2.3058}{x+12.2} + 0.003$; translation 12.2 units left and 0.003 unit up of the graph of f

54. $\frac{2(2x^3 + 11x^2 + 26x + 5)}{3x(x+1)}$

55. a. $R_1 = \frac{1}{40}, R_2 = \frac{1}{x}, R_3 = \frac{1}{x+10}$

b. $R = \frac{x^2 + 90x + 400}{40x(x+10)}$

c. about 0.0758 car/min; about 4.5 cars/h; Multiply the number of cars washed per minute by the rate 60 min/h to obtain an answer in cars per hour.

56. a. $A = \frac{391(t-1)^2 + 0.112}{0.218(t-1)^4 + 0.991(t-1)^2 + 1}$

b. $A = \frac{391t^2 + 0.112}{0.218t^4 + 0.991t^2 + 1} + \frac{391(t-1)^2 + 0.112}{0.218(t-1)^4 + 0.991(t-1)^2 + 1}$

57. $1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}}}, 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}}$

1.4, about 1.4167, about 1.4138, about 1.4143, about 1.4142; $\sqrt{2}$

7.4 Maintaining Mathematical Proficiency

58. (1, 7) and (2, 10)

59. $(\frac{1}{2}, -1)$ and $(\frac{9}{4}, \frac{27}{8})$

60. no solution

61. no solution

7.5 Monitoring Progress and Modeling with Mathematics

33. a.

	Work rate	Time	Work done
You	$\frac{1 \text{ room}}{8 \text{ hours}}$	5 hours	$\frac{5}{8}$ room
Friend	$\frac{1 \text{ room}}{t \text{ hours}}$	5 hours	$\frac{5}{t}$ room

34. a.

	Work rate	Time	Work done
You	$\frac{1 \text{ park}}{2 \text{ hours}}$	1.2 hours	0.6 park
Friend	$\frac{1 \text{ park}}{t \text{ hours}}$	1.2 hours	$\frac{1.2}{t}$ park

56. a. yes; *Sample answer:* $\frac{1}{x} = \frac{1}{x+1}$

b. yes; *Sample answer:* See Example 2.

c. yes; *Sample answer:* See Example 3(b).

d. yes; *Sample answer:* $\frac{1}{x+1} = \frac{3}{3x+3}$

57. a. always true; When $x = a$, the denominators of the fractions are both zero.

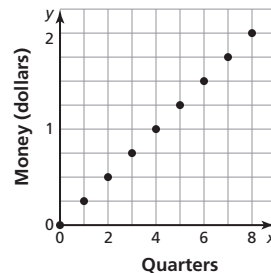
b. sometimes true; The equation will have exactly one solution except when $a = 3$.

c. always true; $x = a$ is an extraneous solution, so the equation has no solution.

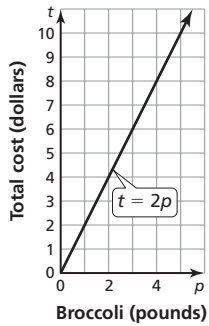
58. yes; Extraneous solutions make the denominator in one of the terms of the equation equal to zero.

7.5 Maintaining Mathematical Proficiency

59. discrete; The number of quarters in your pocket is an integer.



60. continuous; The weight of the broccoli can be any non-negative number.

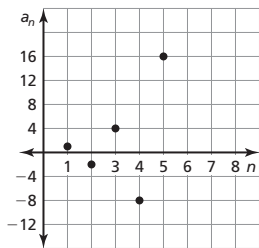


61. 3 62. 28
63. 15 64. 85

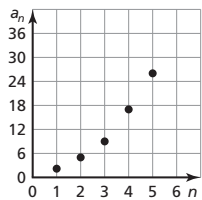
Chapter 8

8.1 Lesson Monitoring Progress

6. $(-2)^0, (-2)^1, (-2)^2, (-2)^3; a_5 = (-2)^4 = 16; a_n = (-2)^{n-1}$



7. $1^2 + 1, 2^2 + 1, 3^2 + 1, 4^2 + 1; a_5 = 5^2 + 1 = 26; a_n = n^2 + 1$



8.1 Monitoring Progress and Modeling with Mathematics

59. a. true;

$$\begin{aligned} \sum_{i=1}^n ca_i &= ca_1 + ca_2 + ca_3 + \cdots + ca_n \\ &= c(a_1 + a_2 + a_3 + \cdots + a_n) \\ &= c \sum_{i=1}^n a_i \end{aligned}$$

- b. true;

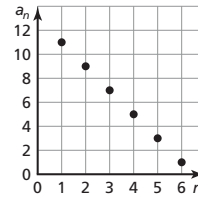
$$\begin{aligned} \sum_{i=1}^n (a_i + b_i) &= (a_1 + b_1) + (a_2 + b_2) + \cdots + (a_n + b_n) \\ &= a_1 + a_2 + \cdots + a_n + b_1 + b_2 + \cdots + b_n \\ &= \sum_{i=1}^n a_i + \sum_{i=1}^n b_i \end{aligned}$$

- c. false; $\sum_{i=1}^2 (2i)(3i) = 30, \left(\sum_{i=1}^2 2i\right)\left(\sum_{i=1}^2 3i\right) = 54$

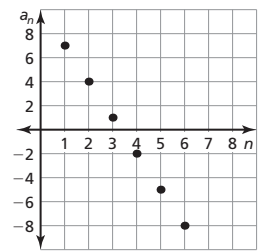
- d. false; $\sum_{i=1}^2 (2i)^2 = 20, \left(\sum_{i=1}^2 2i\right)^2 = 36$

8.2 Monitoring Progress and Modeling with Mathematics

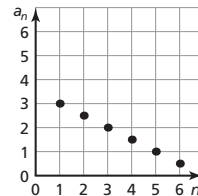
25. $a_n = -2n + 13$



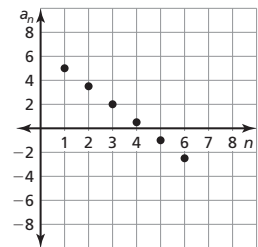
26. $a_n = -3n + 10$



27. $a_n = -\frac{1}{2}n + \frac{7}{2}$



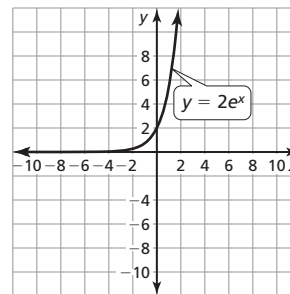
28. $a_n = -\frac{3}{2}n + \frac{13}{2}$



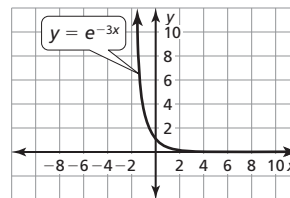
29. C

8.2 Maintaining Mathematical Proficiency

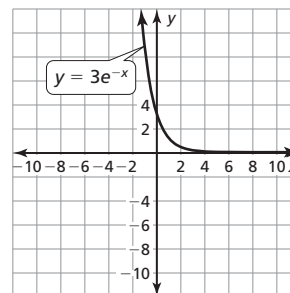
70. exponential growth



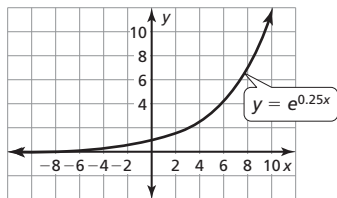
71. exponential decay



72. exponential decay

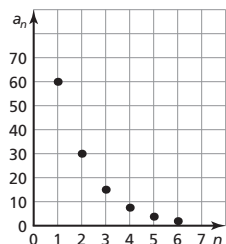


73. exponential growth

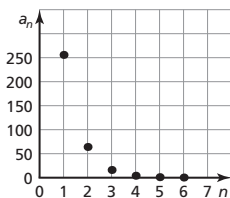


8.3 Monitoring Progress and Modeling with Mathematics

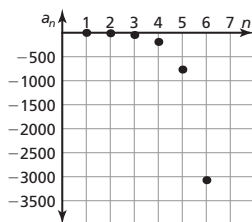
25. $a_n = 60\left(\frac{1}{2}\right)^{n-1}$



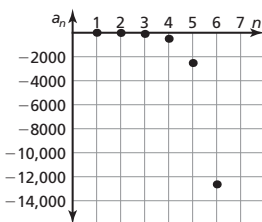
26. $a_n = 256\left(\frac{1}{4}\right)^{n-1}$



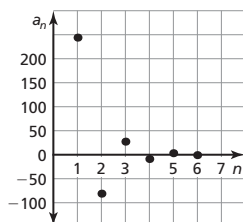
27. $a_n = -3(4)^{n-1}$



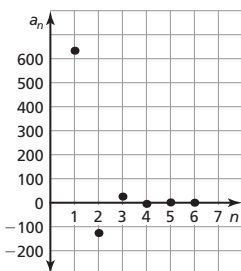
28. $a_n = -4(5)^{n-1}$



29. $a_n = 243\left(-\frac{1}{3}\right)^{n-1}$



30. $a_n = 625\left(-\frac{1}{5}\right)^{n-1}$



31. The formula should be $a_n = a_1 r^{n-1}$; $a_n = 8(6)^{n-1}$

32. The formula should be $a_n = a_1 r^{n-1}$; $a_n = 8(6)^{n-1}$

33. $a_n = 7(4)^{n-1}$

34. $a_n = 11(2)^{n-1}$

35. $a_n = -6(3)^{n-1}$ or $a_n = -6(-3)^{n-1}$

36. $a_n = -2(5)^{n-1}$ or $a_n = 2(-5)^{n-1}$

37. $a_n = 512\left(\frac{1}{8}\right)^{n-1}$ or $a_n = -512\left(-\frac{1}{8}\right)^{n-1}$

38. $a_n = (49)^{n-1}$

39. $a_n = -432\left(\frac{1}{6}\right)^{n-1}$ or $a_n = 432\left(-\frac{1}{6}\right)^{n-1}$

40. $a_n = 192\left(-\frac{1}{4}\right)^{n-1}$

66. a. $L = \frac{M}{1+i}$; $L = \frac{M}{1+i} + \frac{M}{(1+i)^2}$

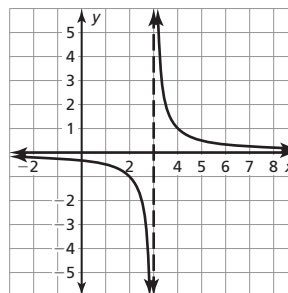
b. $L = M \sum_{k=1}^t \left(\frac{1}{1+i}\right)^k$; $M = \frac{L}{\sum_{k=1}^t \left(\frac{1}{1+i}\right)^k}$

c. $\sum_{k=1}^t \left(\frac{1}{1+i}\right)^k = \left(\frac{1}{1+i}\right) \left[\frac{1 - \left(\frac{1}{1+i}\right)^t}{1 - \left(\frac{1}{1+i}\right)} \right] = \left[\frac{1 - (1+i)^{-t}}{i} \right]$

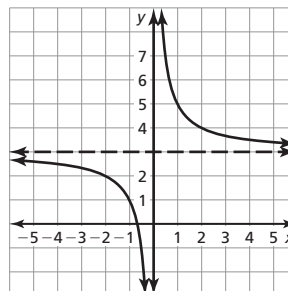
so $M = \frac{L}{\left[\frac{1 - (1+i)^{-t}}{i} \right]} = L \left[\frac{i}{1 - (1+i)^{-t}} \right]$

8.3 Maintaining Mathematical Proficiency

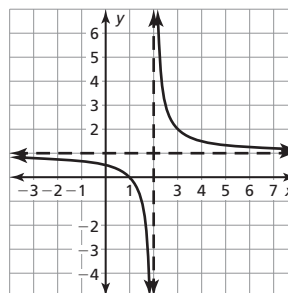
67. domain: all real numbers except 3; range: all real numbers except 0



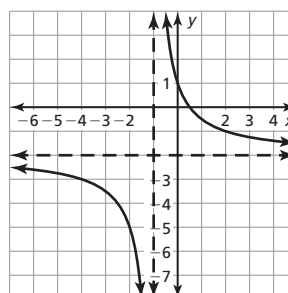
68. domain: all real numbers except 0; range: all real numbers except 3



69. domain: all real numbers except 2; range: all real numbers except 1



70. domain: all real numbers except -1; range: all real numbers except -2



8.5 Maintaining Mathematical Proficiency

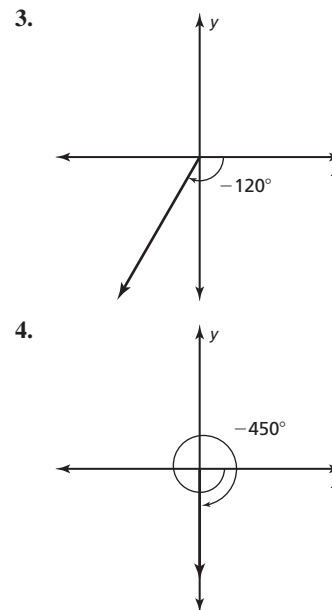
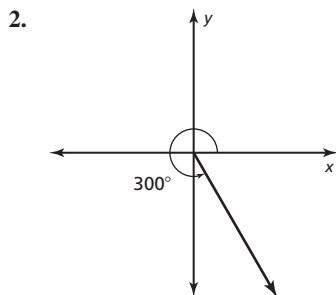
74. $x = 64$ 75. $y = \frac{18}{x}; y = \frac{9}{2}$
 76. $y = \frac{-12}{x}; y = -3$ 77. $y = \frac{320}{x}; y = 80$

Chapter 9

9.1 Monitoring Progress and Modeling with Mathematics

16. $\sin \theta = \frac{8}{15}, \cos \theta = \frac{\sqrt{161}}{15}, \tan \theta = \frac{8\sqrt{161}}{161}, \sec \theta = \frac{15\sqrt{161}}{161},$
 $\cot \theta = \frac{\sqrt{161}}{8}$
 17. $\sin \theta = \frac{\sqrt{115}}{14}, \cos \theta = \frac{9}{14}, \tan \theta = \frac{\sqrt{115}}{9}, \csc \theta = \frac{14\sqrt{115}}{115},$
 $\cot \theta = \frac{9\sqrt{115}}{115}$
 18. $\sin \theta = \frac{11\sqrt{377}}{377}, \cos \theta = \frac{16\sqrt{377}}{377}, \tan \theta = \frac{11}{16}, \csc \theta = \frac{\sqrt{377}}{11},$
 $\sec \theta = \frac{\sqrt{377}}{16}$
 19. The adjacent side was used instead of the opposite;
 $\sin \theta = \frac{\text{opp.}}{\text{hyp.}} = \frac{8}{17}$
 49. a. about 59,155 ft
 b. about 53,613 ft
 c. about 39,688 ft; Use the tangent function to find the horizontal distance, $x + y$, from the airplane to the second town to be about 93,301 ft. Subtract 53,613 ft to find the distance between the two towns.
 50. about 59 m
 51. yes; The triangle must be a 45-45-90 triangle because both acute angles would be the same and have the same cosine value.
 52. $\sin \theta = \frac{1}{2}, \cos \theta = \frac{\sqrt{3}}{2}, \tan \theta = \frac{\sqrt{3}}{3}, \csc \theta = 2, \sec \theta = \frac{2\sqrt{3}}{3},$
 $\cot \theta = \sqrt{3}$; The triangle on the right is equilateral, so all of its angles must be 60° . Using geometry, it can be shown that $\theta = 30^\circ$ and that the two triangles form a larger 30-60-90 triangle. Because the large triangle is a right triangle, the 6 trig ratios can be found. The legs of the triangle are 1 and $\sqrt{3}$ and the hypotenuse is 2.
 53. a. $x = 0.5$; 6 units
 b. *Sample answer:* Each side is part of two right triangles, with opposing angles $\left(\frac{180^\circ}{n}\right)$. So, each side length is $2 \sin\left(\frac{180^\circ}{n}\right)$, and there are n sides.
 c. $n \cdot \sin\left(\frac{180^\circ}{n}\right)$; about 3.14

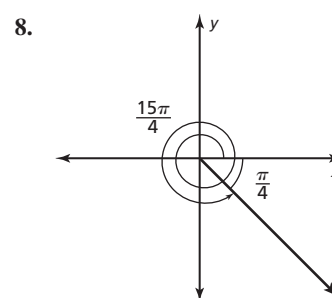
9.2 Lesson Monitoring Progress



9.2 Monitoring Progress and Modeling with Mathematics

22. $\frac{15\pi}{4}, -\frac{\pi}{4}$; *Sample answer:* 315° is equivalent to $\frac{7\pi}{4}$ radians, and $\frac{7\pi}{4} + 2\pi = \frac{15\pi}{4}$ and $\frac{7\pi}{4} - 2\pi = -\frac{\pi}{4}$.
 23. B 24. D
 25. A 26. C
 43. about 6.89 in.², about 0.76 in.², about 0.46 in.²
 44. *Sample answer:* This continued fraction (which is irrational) gives rise to a sequence of rational approximations for π . When the next fraction is added, the value gets closer to the value of $\pi = 3.1415926535\dots$, as shown.
 $3 = 3$
 $3 + \frac{1}{7} = \frac{22}{7} = 3.142857143\dots$
 $3 + \frac{1}{7 + \frac{1}{15}} = \frac{333}{106} = 3.141509434\dots$
 $3 + \frac{1}{7 + \frac{1}{15 + \frac{1}{1}}}$ $= \frac{355}{113} = 3.141592920\dots$
 45. yes; When the arc length is equal to the radius, the equation $s = r\theta$ shows that $\theta = 1$ and $A = \frac{1}{2}r^2\theta$ is equivalent to $A = \frac{s^2}{2}$ for $r = s$ and $\theta = 1$.

9.3 Lesson Monitoring Progress



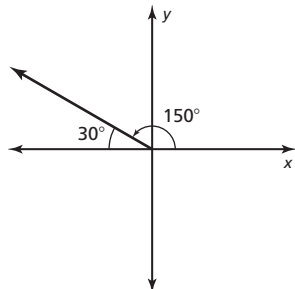
9. $-\frac{\sqrt{3}}{2}$

10. $-\sqrt{2}$

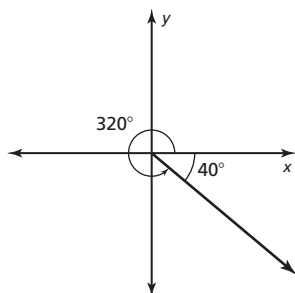
11. about 14.6 ft

9.3 Monitoring Progress and Modeling with Mathematics

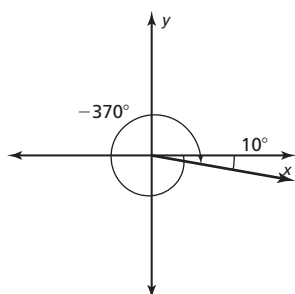
16. ; 30°



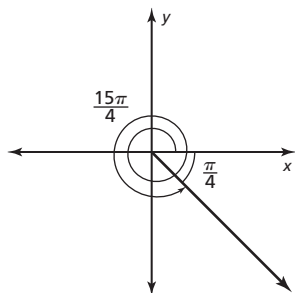
17. ; 40°



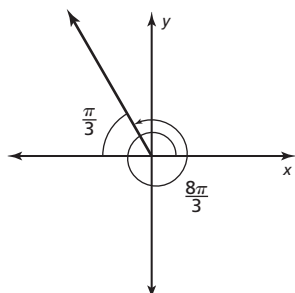
18. ; 10°



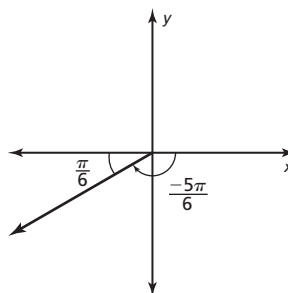
19. ; $\frac{\pi}{4}$



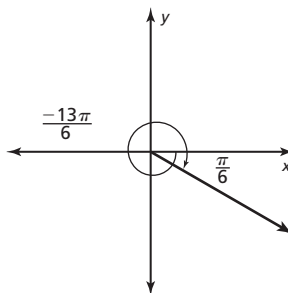
20. ; $\frac{\pi}{3}$



21. ; $\frac{\pi}{6}$



22. ; $\frac{\pi}{6}$



23. The equation for tangent is $\tan \theta = \frac{y}{x}$; $\tan \theta = \frac{y}{x} = -\frac{2}{3}$

24. The angle found is the angle between the terminal side and the y-axis instead of the x-axis; θ is coterminal with 290° , whose terminal side lies in Quadrant IV. So, $\theta' = 360^\circ - 290^\circ = 70^\circ$

25. $-\sqrt{2}$

26. $\sqrt{3}$

27. $-\frac{1}{2}$

28. $-\frac{2\sqrt{3}}{3}$

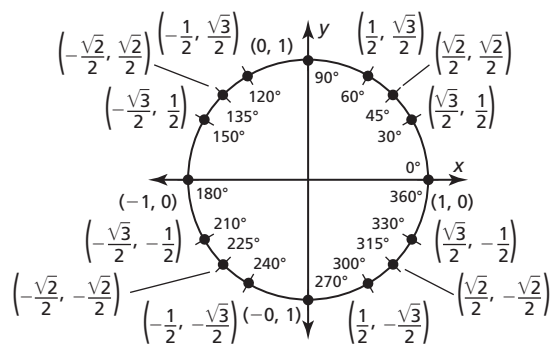
29. 1

30. $\frac{\sqrt{3}}{3}$

31. $\frac{\sqrt{2}}{2}$

32. $\frac{2\sqrt{3}}{3}$

41.



46. no; $\theta = 240^\circ$ is also a solution; any angles coterminal with 60° and 240° are also solutions.

47. a. $(-58.1, 114)$

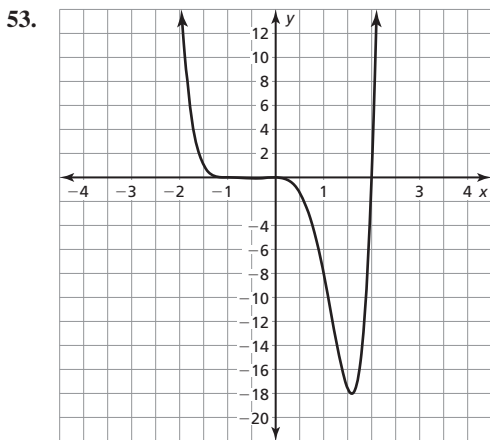
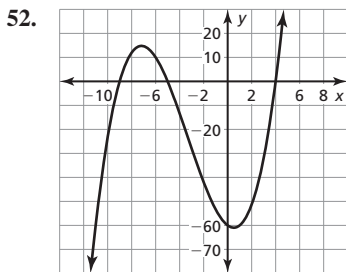
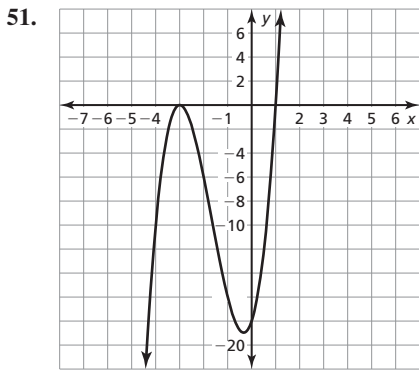
b. about 218 pm

48. 60°

9.3 Maintaining Mathematical Proficiency

49. $x = -3$ and $x = 1$

50. $x = -6, x = -1$ and $x = 3$



9.4 Explorations

1. a.

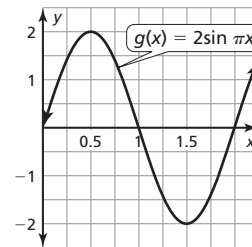
x	-2π	$-\frac{7\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	$-\pi$
$y = \sin x$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0
x	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$
$y = \sin x$	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$
x	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	
$y = \sin x$	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	
x	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$	
$y = \sin x$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	

2. a.

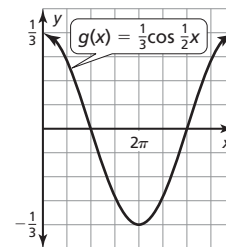
x	-2π	$-\frac{7\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	$-\pi$
$y = \cos x$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1
x	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$
$y = \cos x$	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$
x	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	
$y = \cos x$	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	
x	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$	
$y = \cos x$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	

9.4 Lesson Monitoring Progress

3. 2, 2; The graph of g is a horizontal shrink by a factor of $\frac{1}{\pi}$ and a vertical stretch by a factor of 2 of the graph of $f(x) = \sin x$.

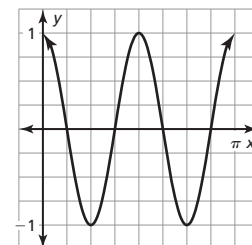


4. $\frac{1}{3}$, 4π ; The graph of g is a horizontal stretch by a factor of 2 followed by a vertical shrink by a factor of $\frac{1}{3}$ of the graph of $f(x) = \cos x$.

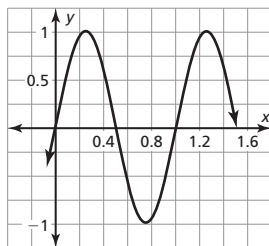


9.4 Monitoring Progress and Modeling with Mathematics

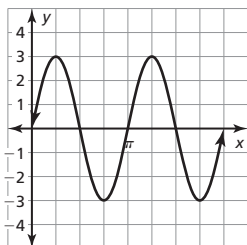
16. $1, \frac{\pi}{2}$; The graph of g is a horizontal shrink by a factor of $\frac{1}{4}$ of the graph of $f(x) = \cos x$.



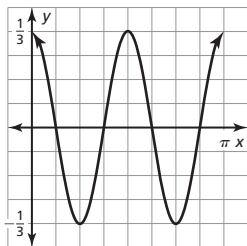
17. 1, 1; The graph of g is a horizontal shrink by a factor of $\frac{1}{2\pi}$ of the graph of $f(x) = \sin x$.



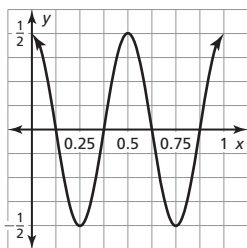
18. $3, \pi$; The graph of g is a horizontal shrink by a factor of $\frac{1}{2}$ and a vertical stretch by a factor of 3 of the graph of $f(x) = \sin x$.



19. $\frac{1}{3}, \frac{\pi}{2}$; The graph of g is a horizontal shrink by a factor of $\frac{1}{4}$ and a vertical shrink by a factor of $\frac{1}{3}$ of the graph of $f(x) = \cos x$.



20. $\frac{1}{2}, \frac{1}{2}$; The graph of g is a horizontal shrink by a factor of $\frac{1}{4\pi}$ and a vertical shrink by a factor of $\frac{1}{2}$ of the graph of $f(x) = \cos x$.



21. B, D

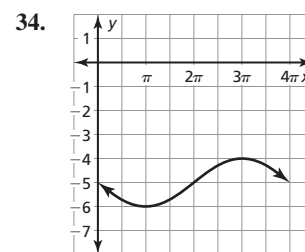
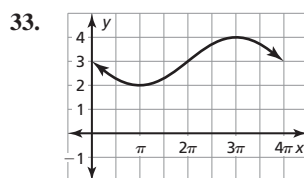
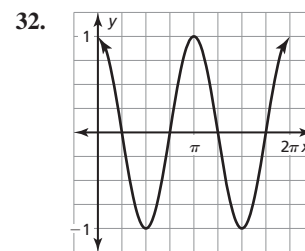
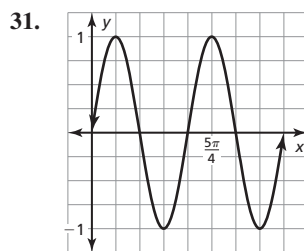
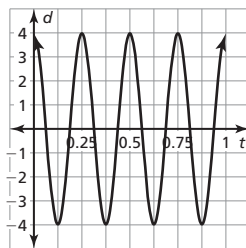
22. a. $y = \sin \frac{2\pi x}{5}$

b. $y = 10 \sin \frac{\pi}{2}x$

c. $y = 2 \sin x$

d. $y = \frac{1}{2} \sin \frac{2}{3}x$

23. The period is $\frac{1}{4}$ and represents the amount of time, in seconds, that it takes for the pendulum to go back and forth and return to the same position. The amplitude is 4 and represents the maximum distance, in inches, the pendulum will be from its resting position.



35. To find the period, use the expression $\frac{2\pi}{|b|}$; Period: $\frac{2\pi}{|b|} = \frac{2\pi}{2} = \pi$

36. $\frac{\pi}{2}$ should be added to the x -coordinate;

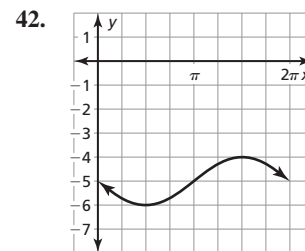
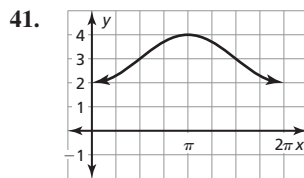
Maximum: $\left(\left(\frac{1}{4} \cdot 2\pi\right) + \frac{\pi}{2}, 2\right) = \left(\frac{\pi}{2} + \frac{\pi}{2}, 2\right) = (\pi, 2)$

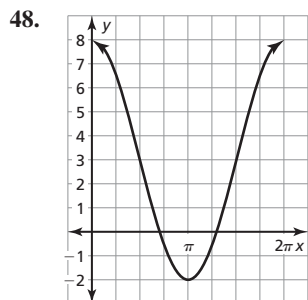
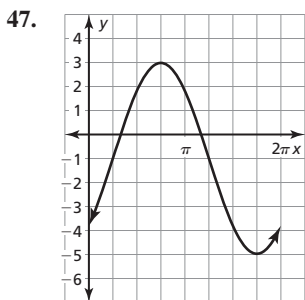
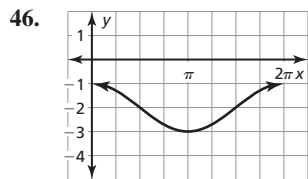
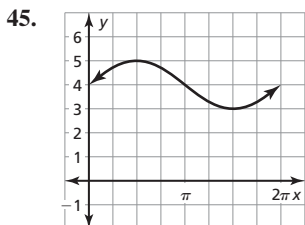
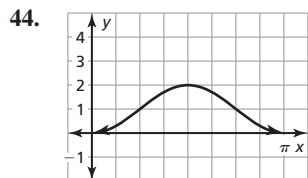
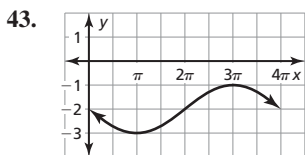
37. The graph of g is a vertical stretch by a factor of 2 followed by a translation $\frac{\pi}{2}$ units right and 1 unit up of the graph of f .

38. The graph of g is a vertical stretch by a factor of 3 followed by a translation $\frac{\pi}{4}$ units left and 2 units down of the graph of f .

39. The graph of g is a horizontal shrink by a factor of $\frac{1}{3}$ followed by a translation 3π units left and 5 units down of the graph of f .

40. The graph of g is a horizontal shrink by a factor of $\frac{1}{6}$ followed by a translation π units right and 9 units up of the graph of f .





49. A

50. a. B; The graph of sine has been translated 3 units up.
 b. C; The graph of cosine has been translated 3 units down.
 c. A; The graph of sine has been shrunk horizontally by a factor of $\frac{1}{2}$ then translated $\frac{\pi}{2}$ units right.
 d. D; The graph of cosine has been shrunk horizontally by a factor of $\frac{1}{2}$ then translated $\frac{\pi}{2}$ units right.

60. a.

x	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0
$y = \sin(-x)$	0	-1	0	1	0
$y = \cos(-x)$	1	0	-1	0	1

x	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \sin(-x)$	-1	0	1	0
$y = \cos(-x)$	0	-1	0	1

62. a. $f(x) = a \cos bx$; The y -coordinate is 5 when $x = 0$.
 b. maximum value = 5, minimum value = -5 ;
 period = π , amplitude = 5

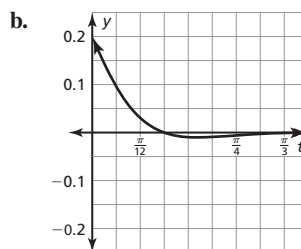
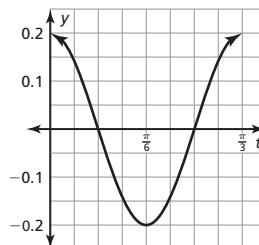
63. The x -intercepts occur when $x = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}, \dots$

Sample answer: The x -intercepts can be represented by the expression $(2n + 1)\frac{\pi}{4}$, where n is an integer.

64. no; The value of a indicates a vertical stretch or a vertical shrink and changes the amplitude of the graph. It does not affect the x -intercepts of the function. The value of b indicates a horizontal stretch of a horizontal shrink and changes the period of the graph, which is the horizontal length of each cycle. So, only the value of b affects the x -intercepts of the function.

65. The graph of $g(x) = \cos x$ is a translation $\frac{\pi}{2}$ units to the right of the graph of $f(x) = \sin x$.

68. a. $0.2 \text{ ft}, \frac{\pi}{3}$;



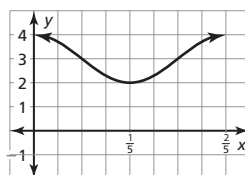
The motion becomes imperceptible as the displacement rapidly decreases towards 0.

9.4 Maintaining Mathematical Proficiency

69. $x - 2, x \neq -3$ 70. $x, x \neq -4, 6$
 71. $\frac{(x-5)(x+1)}{(x+5)(x-1)}$ 72. $\frac{x+4}{x+5}, x \neq 4$

9.1–9.4 Quiz

15. $1, \frac{2}{5}$; The graph of g is a horizontal shrink by a factor of $\frac{1}{5\pi}$ followed by a translation 3 units up of the graph of f ;



16. a. About 380 ft
 b. About 170 ft
 17. Seated at a window table, you will have traveled about 569 feet in that time. A person at a table 5 feet from the window will have traveled about 509 feet in that time.

9.5 Explorations

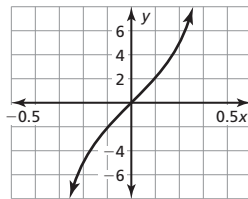
1. a.

x	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0
$y = \tan x$	undefined	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
x	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$
$y = \tan x$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$

x	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y = \tan x$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined
x	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
$y = \tan x$	1	$\sqrt{3}$	undefined	$-\sqrt{3}$

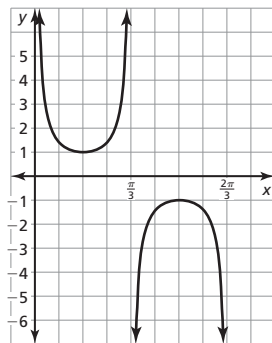
9.5 Lesson Monitoring Progress

4.



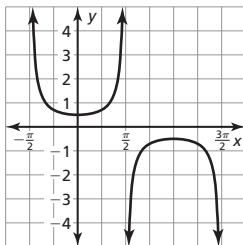
The graph of g is a vertical stretch by a factor of 5 and a horizontal shrink by a factor of $\frac{1}{\pi}$ of the graph of $f(x) = \tan x$.

5.



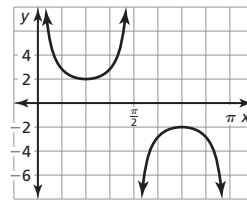
The graph of g is a horizontal shrink by a factor of $\frac{1}{3}$ of the graph of $f(x) = \csc x$.

6.



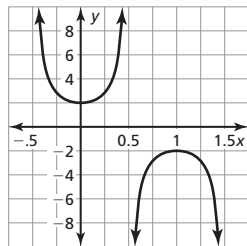
The graph of g is a vertical shrink by a factor of $\frac{1}{2}$ of the graph of $f(x) = \sec x$.

7.



The graph of g is a horizontal shrink by a factor of $\frac{1}{2}$ and a vertical stretch by a factor of 2 of the graph of $f(x) = \csc x$.

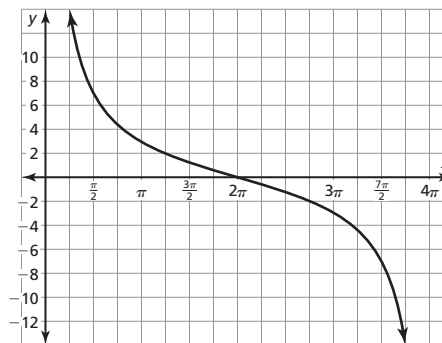
8.



The graph of g is a horizontal shrink by a factor of $\frac{1}{\pi}$ and a vertical stretch by a factor of 2 of the graph of $f(x) = \sec x$.

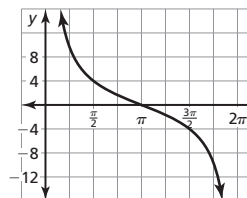
9.5 Monitoring Progress and Modeling with Mathematics

9.



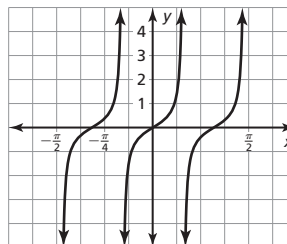
The graph of g is a horizontal stretch by a factor of 4 and a vertical stretch by a factor of 3 of the graph of $f(x) = \cot x$.

10.

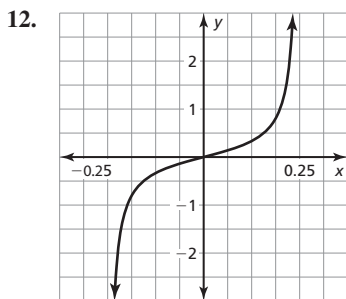


The graph of g is a horizontal stretch by a factor of 2 and a vertical stretch by a factor of 4 of the graph of $f(x) = \cot x$.

11.



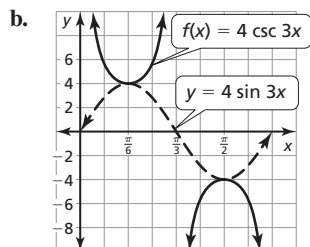
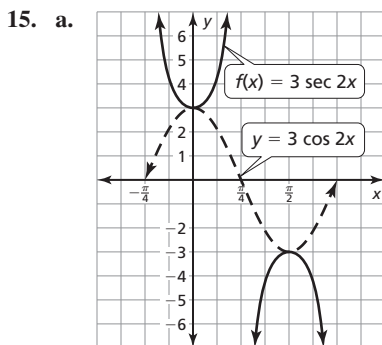
The graph of g is a horizontal shrink by a factor of $\frac{1}{\pi}$ and a vertical shrink by a factor of $\frac{1}{2}$ of the graph of $f(x) = \tan x$.



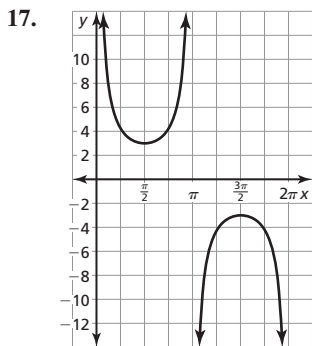
The graph of g is a horizontal shrink by a factor of $\frac{1}{2\pi}$ and a shrink by a factor of $\frac{1}{3}$ of the graph of $f(x) = \tan x$.

13. To find the period, use the expression $\frac{\pi}{|b|}$; Period: $\frac{\pi}{|b|} = \frac{\pi}{3}$

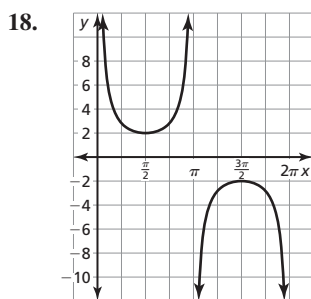
14. The horizontal and vertical shrink factors are switched; A vertical stretch by a factor of 2 and a horizontal shrink by a factor of $\frac{1}{5}$.



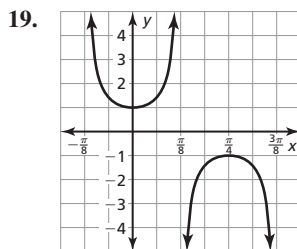
16. A



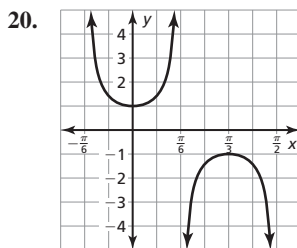
The graph of g is a vertical stretch by a factor of 3 of the graph of $f(x) = \csc x$.



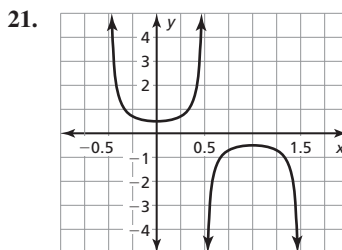
The graph of g is a vertical stretch by a factor of 2 of the graph of $f(x) = \csc x$.



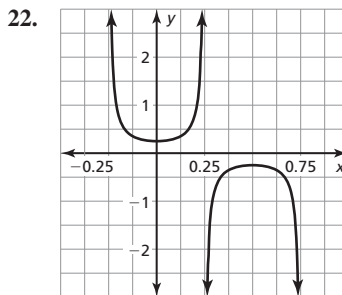
The graph of g is a horizontal shrink by a factor of $\frac{1}{4}$ of the graph of $f(x) = \sec x$.



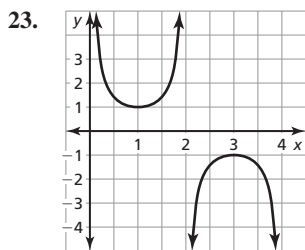
The graph of g is a horizontal shrink by a factor of $\frac{1}{3}$ of the graph of $f(x) = \sec x$.



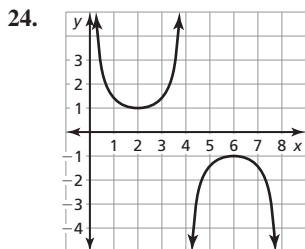
The graph of g is a horizontal shrink by a factor of $\frac{1}{\pi}$ and a vertical shrink by a factor of $\frac{1}{2}$ of the graph of $f(x) = \sec x$.



The graph of g is a horizontal shrink by a factor of $\frac{1}{2\pi}$ and a vertical shrink by a factor of $\frac{1}{4}$ of the graph of $f(x) = \sec x$.

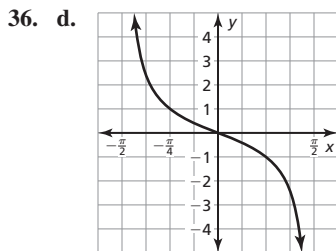


The graph of g is a horizontal stretch by a factor of $\frac{2}{\pi}$ of the graph of $f(x) = \csc x$.



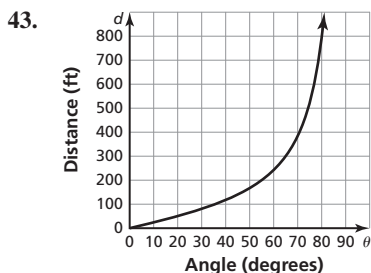
The graph of g is a horizontal stretch by a factor of $\frac{4}{\pi}$ of the graph of $f(x) = \csc x$.

25. $y = 6 \tan x$ 26. $y = \frac{1}{2} \tan x$
 27. $y = 2 \tan \pi x$ 28. $y = 5 \tan 2x$

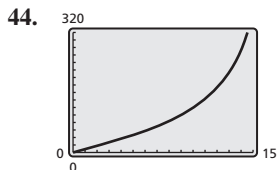


The graph of g is a reflection across the x -axis of the graph of $f(x) = \tan x$.

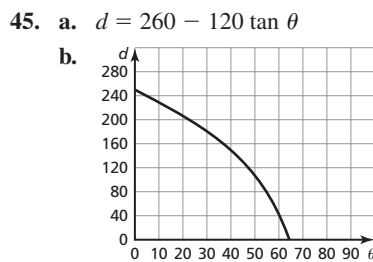
37. $g(x) = \cot\left(2x + \frac{\pi}{2}\right) + 3$
 38. $g(x) = 2 \tan(3x - \pi)$
 39. $g(x) = -5 \sec(x - \pi) + 2$
 40. $g(x) = -8 \csc x$
 41. Function B has a local maximum value of -5 so Function A's local maximum value of $-\frac{1}{4}$ is greater. Function A has a local minimum of $\frac{1}{4}$ so Function B's local minimum value of 5 is greater.
 42. D, B, C, A



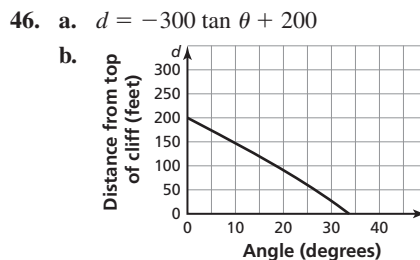
As d increases, θ increases because, as the car gets farther away, the angle required to see the car gets larger.



$0 < t < 15$ and $0 < h < 320$; The Statue of Liberty is approximately 305 feet tall so it would take almost 15 seconds to span the statue.



The graph shows a negative correlation meaning that as the angle gets larger, the distance from your friend to the top of the building gets smaller. As the angle gets smaller, the distance from your friend to the top of the building gets larger.

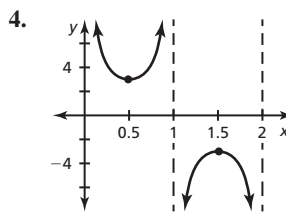


- c. About 18.4°
 47. no; The graph of cosecant can be translated $\frac{\pi}{2}$ units right to create the same graph as $y = \sec x$.
 48. a. 4
 b. $y > 2$ and $y < -2$
 c. $y = a \csc bx$; The cosecant function has an asymptote at $x = 0$.
 49. $a \sec bx = \frac{a}{\cos bx}$
 Because the cosine function is at most 1, $y = a \cos bx$ will produce a maximum when $\cos bx = 1$ and $y = a \sec bx$ will produce a minimum. When $\cos bx = -1$, $y = a \cos bx$ will produce a minimum and $y = a \sec bx$ will produce a maximum.
 50. $\csc x = \frac{1}{2}\left(\tan \frac{x}{2} + \cot \frac{x}{2}\right)$; Graphing the function produces the same graph as the cosecant function with asymptotes at $0, \pm\pi, \pm2\pi, \dots$
 51. Sample answer: $y = 5 \tan\left(\frac{1}{2}x - \frac{3\pi}{4}\right)$

9.5 Maintaining Mathematical Proficiency

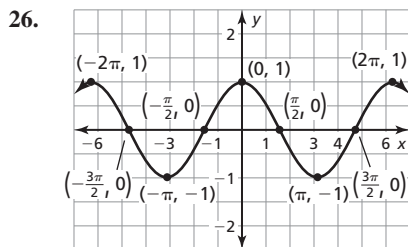
52. $y = x^3 - 3x^2 - x + 3$
 53. $y = -x^3 + 2x^2 + 5x - 6$
 54. $y = -\frac{1}{2}x^3 + 2x^2 - \frac{1}{2}x - 3$
 55. $y = \frac{1}{5}x^3 + \frac{1}{5}x^2 - \frac{9}{5}x - \frac{9}{5}$
 56. 5, 2π 57. 3, π
 58. 1, 8π

9.5 Mini-Assessment



The graph of g is a vertical stretch by a factor of 3 and a horizontal shrink by a factor of $\frac{1}{\pi}$ of the graph of f .

9.6 Monitoring Progress and Modeling with Mathematics



The slope of the graph of $y = \sin x$ is given by the function $y = \cos x$.

27. **a. and b.** A cosine function because it does not require determining a horizontal shift.
c. A sine function because it does not require determining a horizontal shift.
28. 2; The graph completes 2 full cycles in 1 unit of x .
29. $y = 2.5 \sin 4\left(x - \frac{\pi}{8}\right) + 5.5$, $y = -2.5 \cos 4x + 5.5$
30. no; The period is the reciprocal of the frequency. The reciprocal of $\frac{1}{2}$ is greater than the reciprocal of 2.
31. **a.** $d = -6.5 \cos \frac{\pi}{6}t + 10$
b. low tide: 12:00 A.M., 12:00 P.M., high tide: 6:00 A.M., 6:00 P.M.
c. It is a horizontal shift to the left by 3.

9.6 Maintaining Mathematical Proficiency

32. $\frac{17\sqrt{2}}{2}$ 33. $\frac{6 + 3\sqrt{6}}{2}$
34. $-24 + 8\sqrt{10}$ 35. $\frac{13\sqrt{11} - 13\sqrt{3}}{8}$
36. $\log_8 x - \log_8 7$ 37. $\ln 2 + \ln x$
38. $\log_3 5 + 3 \log_3 x$ 39. $\ln 4 + 6 \ln x - \ln y$

9.7 Explorations

1. **d.** Sample answer:

	θ	$\sin^2 \theta$	$\cos^2 \theta$	$\sin^2 \theta + \cos^2 \theta$
QI	$\frac{\pi}{3}$	$\frac{3}{4}$	$\frac{1}{4}$	1
QII	$\frac{3\pi}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	1
QIII	$\frac{7\pi}{6}$	$\frac{1}{4}$	$\frac{3}{4}$	1
QIV	$\frac{7\pi}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	1

9.7 Monitoring Progress and Modeling with Mathematics

21. $\sin^2 \theta = 1 - \cos^2 \theta$;
 $1 - \sin^2 \theta = 1 - (1 - \cos^2 \theta) = 1 - 1 + \cos^2 \theta = \cos^2 \theta$
22. $\tan x = \frac{\sin x}{\cos x}$;
 $\tan x \csc x = \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}$
 $= \frac{1}{\cos x}$
 $= \sec x$

23. $\sin x \csc x = \sin x \cdot \frac{1}{\sin x}$
 $= 1$
24. $\tan \theta \csc \theta \cos \theta = \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\sin \theta} \cdot \cos \theta$
 $= 1$
25. $\cos\left(\frac{\pi}{2} - x\right) \cot x = \sin x \cdot \frac{\cos x}{\sin x}$
 $= \cos x$
26. $\sin\left(\frac{\pi}{2} - x\right) \tan x = \cos x \cdot \frac{\sin x}{\cos x}$
 $= \sin x$
27. $\frac{\cos\left(\frac{\pi}{2} - \theta\right) + 1}{1 - \sin(-\theta)} = \frac{\sin \theta + 1}{1 - \sin(-\theta)}$
 $= \frac{\sin \theta + 1}{1 - (-\sin \theta)}$
 $= \frac{\sin \theta + 1}{1 + \sin \theta}$
 $= 1$
28. $\frac{\sin^2(-x)}{\tan^2 x} = \frac{(-\sin x)^2}{\tan^2 x} = \frac{\sin^2 x}{\tan^2 x}$
 $= \sin^2 x \cdot \cot^2 x$
 $= \sin^2 x \cdot \frac{\cos^2 x}{\sin^2 x}$
 $= \cos^2 x$
29. $\frac{1 + \cos x}{\sin x} + \frac{\sin x}{1 + \cos x} = \frac{1 + \cos x}{\sin x} + \frac{\sin x(1 - \cos x)}{(1 + \cos x)(1 - \cos x)}$
 $= \frac{1 + \cos x}{\sin x} + \frac{\sin x(1 - \cos x)}{1 - \cos^2 x}$
 $= \frac{1 + \cos x}{\sin x} + \frac{\sin x(1 - \cos x)}{\sin^2 x}$
 $= \frac{\sin x(1 + \cos x)}{\sin^2 x} + \frac{\sin x(1 - \cos x)}{\sin^2 x}$
 $= \frac{\sin x(1 + \cos x) + \sin x(1 - \cos x)}{\sin^2 x}$
 $= \frac{\sin x(1 + \cos x + 1 - \cos x)}{\sin^2 x}$
 $= \frac{\sin x(2)}{\sin^2 x}$
 $= \frac{2}{\sin x}$
 $= 2 \csc x$
30. $\frac{\sin x}{1 - \cos(-x)} = \frac{\sin x}{1 - \cos x}$
 $= \frac{\sin x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x}$
 $= \frac{\sin x(1 + \cos x)}{1 - \cos^2 x}$
 $= \frac{\sin x(1 + \cos x)}{\sin^2 x}$
 $= \frac{1 + \cos x}{\sin x}$
 $= \frac{1}{\sin x} + \frac{\cos x}{\sin x}$
 $= \csc x + \cot x$

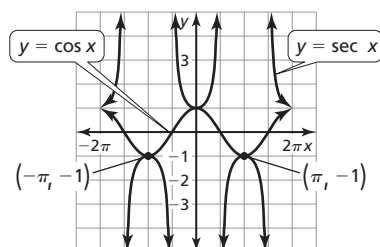
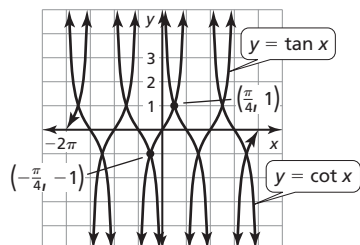
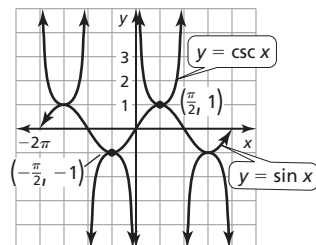
31. $\sin x, \csc x, \tan x, \cot x; \cos x, \sec x;$

$$\sin(-\theta) = -\sin \theta$$

$$\csc(-\theta) = \frac{1}{\sin(-\theta)} = -\frac{1}{\sin \theta} = -\csc \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\cot(-\theta) = \frac{1}{\tan(-\theta)} = -\frac{1}{\tan \theta} = -\cot \theta$$



32. Just as $\frac{1}{x}$ decreases as x increases, $\sec \theta = \frac{1}{\cos \theta}$ decreases as $\cos \theta$ increases; this happens on the intervals

$$\pi + 2n\pi < \theta < \frac{3\pi}{2} + 2n\pi \text{ and } \frac{3\pi}{2} + 2n\pi < \theta < 2\pi + 2n\pi$$

where n is an integer.

35. $s = \frac{h \sin(90^\circ - \theta)}{\sin \theta}$

$$s = \frac{h \cos \theta}{\sin \theta}$$

$$s = h \cot \theta$$

36. *Sample answer:* Multiply both sides by $\frac{1}{\cos x}$, so $\frac{\sin x}{\cos x} = 1$.

Then $\tan x = 1$ because $\frac{\sin x}{\cos x} = \tan x$.

37. a. $u = \tan \theta$

b. u starts at 0 and increases without bound.

38. a. $\frac{n_1}{\sqrt{\cot^2 \theta_1 + 1}} = \frac{n_2}{\sqrt{\cot^2 \theta_2 + 1}}$

$$\frac{n_1}{\sqrt{\csc^2 \theta_1}} = \frac{n_2}{\sqrt{\csc^2 \theta_2}}$$

$$\frac{n_1}{\csc \theta_1} = \frac{n_2}{\csc \theta_2}$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

b. about 1.4

c. $n_1 = n_2$; This situation could occur when the mediums have the same composition.

39. You can obtain the graph of $y = \cos x$ by reflecting the graph of $f(x) = \sin x$ in the y -axis and translating it $\frac{\pi}{2}$ units right.

40. a. $\ln |\sec \theta| = \ln \frac{1}{|\cos \theta|} = \ln |(\cos \theta)^{-1}| = -\ln |\cos \theta|$

b. $\ln |\tan \theta| = \ln \frac{|\sin \theta|}{|\cos \theta|} = \ln |\sin \theta| - \ln |\cos \theta|$

9.7 Maintaining Mathematical Proficiency

41. $x = 11$

42. $x = \frac{13\sqrt{3}}{2}$

43. $x = \frac{14\sqrt{3}}{3}$

9.8 Explorations

1. b. $d = \sqrt{(\cos b - \cos a)^2 + (\sin b + \sin a)^2}$

c. $d = \sqrt{(\cos(a-b) - 1)^2 + (\sin(a-b) - 0)^2}$

d. $\sqrt{(\cos b - \cos a)^2 + (\sin b - \sin a)^2}$

$$= \sqrt{(\cos(a-b) - 1)^2 + (\sin(a-b))^2}$$

$$(\cos b - \cos a)^2 + (\sin b - \sin a)^2$$

$$= (\cos(a-b) - 1)^2 + \sin^2(a-b)$$

$$\cos^2 b - 2 \cos a \cos b + \cos^2 a + \sin^2 b - 2 \sin a \sin b$$

$$+ \sin^2 a = (\cos(a-b) - 1)^2 + \sin^2(a-b)$$

$$- 2 \cos a \cos b - 2 \sin a \sin b + 2 = \cos^2(a-b)$$

$$- 2 \cos(a-b) + 1 + \sin^2(a-b)$$

$$- 2 \cos a \cos b - 2 \sin a \sin b + 2 = -2 \cos(a-b) + 2$$

$$- 2 \cos a \cos b - 2 \sin a \sin b = -2 \cos(a-b)$$

$$\cos a \cos b + \sin a \sin b = \cos(a-b)$$

9.8 Monitoring Progress and Modeling with Mathematics

35. $\frac{35 \tan(\theta - 45^\circ) + 35 \tan 45^\circ}{h \tan \theta}$

$$= \frac{35 \left(\frac{\tan \theta - \tan 45^\circ}{1 + \tan \theta \tan 45^\circ} \right) + 35 \tan 45^\circ}{h \tan \theta}$$

$$= \frac{35 \left(\frac{\tan \theta - 1}{1 + \tan \theta} \right) + 35}{h \tan \theta}$$

$$= \frac{35(\tan \theta - 1) + 35(1 + \tan \theta)}{h \tan \theta(1 + \tan \theta)}$$

$$= \frac{35 \tan \theta - 35 + 35 + 35 \tan \theta}{h \tan \theta(1 + \tan \theta)}$$

$$= \frac{70 \tan \theta}{h \tan \theta(1 + \tan \theta)}$$

$$= \frac{70}{h(1 + \tan \theta)}$$

$$\begin{aligned}
36. & A \cos\left(\frac{2\pi}{3} - \frac{2\pi x}{5}\right) + A \cos\left(\frac{2\pi}{3} + \frac{2\pi x}{5}\right) \\
&= A \cos \frac{2\pi}{3} \cos \frac{2\pi x}{5} + A \sin \frac{2\pi}{3} \sin \frac{2\pi x}{5} + A \cos \frac{2\pi}{3} \cos \frac{2\pi x}{5} \\
&\quad - A \sin \frac{2\pi}{3} \sin \frac{2\pi x}{5} \\
&= A \cos \frac{2\pi}{3} \cos \frac{2\pi x}{5} + A \cos \frac{2\pi}{3} \cos \frac{2\pi x}{5} \\
&= 2A \cos \frac{2\pi}{3} \cos \frac{2\pi x}{5} \\
&= 2A \left(-\frac{1}{2}\right) \cos \frac{2\pi x}{5} \\
&= -A \cos \frac{2\pi x}{5}
\end{aligned}$$

$$\begin{aligned}
37. & y_1 + y_2 = \cos 960\pi t + \cos 1240\pi t \\
&= \cos(1100\pi t - 140\pi t) + \cos(1100\pi t + 140\pi t) \\
&= \cos 1100\pi t \cos 140\pi t + \sin 1100\pi t \sin 140\pi t \\
&\quad + \cos 1100\pi t \cos 140\pi t - \sin 1100\pi t \sin 140\pi t \\
&= \cos 1100\pi t \cos 140\pi t + \cos 1100\pi t \cos 140\pi t \\
&= 2 \cos 1100\pi t \cos 140\pi t
\end{aligned}$$

38. Any point where the two graphs intersect is a solution because if $f(x) = g(x)$ then $f(x) - g(x) = 0$.

$$39. \text{ a. } \tan(\theta_2 - \theta_1) = \frac{m_2 - m_1}{1 + m_2 m_1}$$

b. 60°

40. a. $3 \sin x - 4 \sin^3 x$

b. $4 \cos^3 x - 3 \cos x$

c. $\frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$

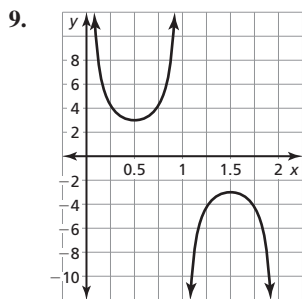
9.8 Maintaining Mathematical Proficiency

41. $x = 4$

42. $x = -\frac{16}{3}$

43. $x = -\frac{2}{3}$

Chapter 9 Test



The graph of g is a horizontal shrink by a factor of $\frac{1}{\pi}$ and a vertical stretch by a factor of 3 of the graph of $f(x) = \csc x$.

10. $-\frac{5\pi}{18}; 310^\circ; -410^\circ$

11. $144^\circ; \frac{14\pi}{5}; -\frac{6\pi}{5}$

12. $480^\circ; \frac{2\pi}{3}; -\frac{4\pi}{3}$

13. $s \approx 9.08 \text{ in.}; A \approx 59.00 \text{ in.}^2$

14. $\sin \theta = -\frac{9\sqrt{85}}{85}; \cos \theta = \frac{2\sqrt{85}}{85}; \tan \theta = -\frac{9}{2}$

$\csc \theta = -\frac{\sqrt{85}}{9}; \sec \theta = \frac{\sqrt{85}}{2}; \cot \theta = -\frac{2}{9}$

15. $\sin \theta = 0, \cos \theta = -1, \tan \theta = 0, \csc \theta = \text{undefined},$
 $\sec \theta = -1, \cot \theta = \text{undefined}$

16. Quadrant III; $\cos \theta < 0$, so θ must lie in Quadrant II or Quadrant III. Because $\tan \theta < 0$ in Quadrant II and $\tan \theta > 0$ in Quadrant III,

θ must lie in Quadrant III.

17. About 178 ft; $h = 200 \sin 60^\circ + 5$

18. $T = 23.14 \sin(0.495m - 1.95) + 63.7$; The period represents the amount of time it takes for the weather to complete one cycle, which is about 12.7 months.

Chapter 10

10.1 Monitoring Progress and Modeling with Mathematics

23. a. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

b. 2: $\frac{1}{36}$, 3: $\frac{1}{18}$, 4: $\frac{1}{12}$, 5: $\frac{1}{9}$, 6: $\frac{5}{36}$, 7: $\frac{1}{6}$, 8: $\frac{5}{36}$, 9: $\frac{1}{9}$, 10: $\frac{1}{12}$, 11: $\frac{1}{18}$, 12: $\frac{1}{36}$

c. *Sample answer:* The probabilities are similar.

24. no; Your friend calculated the experimental probability. The theoretical probability of the coin landing heads up is $\frac{1}{2}$.

25. $\frac{\pi}{6}$, or about 52%

26. $\frac{2}{3}$; $f(x) + c$ intersects the x -axis when c is 1, 2, 3, or 4.

27. $\frac{3}{400}$, or 0.75%; about 113; $(0.0075)15,000 = 112.5$

28. *Sample answer:* Box A contains three cards numbered 1, 2, and 3. Box B contains 2 cards numbered 1 and 2. One card is removed at random from each box. Find the probability that the product of the two numbers is at least 5; $\frac{1}{6}$

10.1 Maintaining Mathematical Proficiency

29. $\frac{6x^4}{y^3}$

30. $\frac{x^7}{3}, x \neq 0, y \neq 0$

31. $\frac{x^3 - 4x^2 - 15x + 18}{x^4 - 2}$

32. $\frac{12}{5}, x \neq 0, y \neq 0$

33. $\frac{15x^2}{12x^2 + x - 11}, x \neq 0$

34. $\frac{3x^2 + 2x - 13}{x^6 + 9x^4}$

10.2 Monitoring Progress and Modeling with Mathematics

25. a. *Sample answer:* Put 20 pieces of paper with each of the 20 students' names in a hat and pick one; 5%

b. *Sample answer:* Put 45 pieces of paper in a hat with each student's name appearing once for each hour the student worked. Pick one piece; about 8.9%

26. a. without

b. with

27. yes; The chance that it will be rescheduled is $(0.7)(0.75) = 0.525$, which is a greater than a 50% chance.

28. Event A represents rolling at least one 2. Event B represents the dice summing to 5; dependent; $P(A \text{ and } B) = \frac{2}{36}$ and $P(A)P(B) = \frac{11}{324}$

29. a. wins: 0%; loses: 1.99%; ties: 98.01%

b. wins: 20.25%; loses: 30.25%; ties: 49.5%

c. yes; Go for 2 points after the first touchdown, and then go for 1 point if they were successful the first time or 2 points if they were unsuccessful the first time; winning: 44.55%; losing: 30.25%

30. a. The occurrence of one event does not affect the occurrence of the other, so the probability of each event is the same whether or not the other event has occurred.

b. yes; $P(A \text{ and } B) = P(A) \cdot P(B)$ and $P(A) = P(A | B)$.

10.2 Maintaining Mathematical Proficiency

31. $x = 0.2$

32. $x = 2$

33. $x = 0.15$

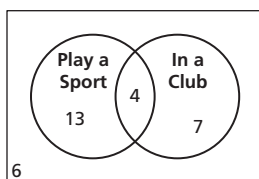
10.3 Explorations

1.

	Play an Instrument	Do Not Play an Instrument	Total
Speak a Foreign Language	16	30	46
Do Not Speak a Foreign Language	25	9	34
Total	41	39	80

- a. 41 b. 46 c. 16 d. 9 e. 25

3. *Sample answer:*



	Play a Sport	Do Not Play a Sport	Total
In a Club	4	7	11
Not in a Club	13	6	19
Total	17	13	30

Of the students in the class, 43.33% only play a sport, 13.33% play a sport and are in a club, 23.33% are only in a club, and 20% do neither activity. A total of 36.67% are in a club, and a total of 56.67% play a sport.

10.3 Lesson Monitoring Progress

3.

		Attendance	
		Attending	Not Attending
Class	Junior	0.353	0.634
	Senior	0.647	0.366

Given that a student is attending, the conditional relative frequency that he or she is a junior is about 35.3%. Given that a student is attending, the conditional relative frequency that he or she is a senior is about 64.7%. Given that a student is not attending, the conditional relative frequency that he or she is a junior is about 63.4%. Given that a student is not attending, the conditional relative frequency that he or she is a senior is about 36.6%.

4.

		Response	
		In Favor	Against
Gender	Boy	0.636	0.364
	Girl	0.808	0.192

Given that a student is a boy, the conditional relative frequency that he is in favor is about 63.6%. Given that a student is a girl, the conditional relative frequency that she is in favor is about 80.8%. Given that a student is a boy, the conditional relative frequency that he is against is about 36.4%. Given that a student is a girl, the conditional relative frequency that she is against is about 19.2%.

10.3 Monitoring Progress and Modeling with Mathematics

9.

		Gender		
		Male	Female	Total
Response	Yes	0.376	0.430	0.806
	No	0.111	0.083	0.194
Total		0.487	0.513	1

10.

		Vaccination		
		Received	Not Received	Total
Health	Flu	0.1429	0.1518	0.2947
	No Flu	0.2946	0.4107	0.7053
Total		0.4375	0.5625	1

19. *Sample answer:*

		Transportation to School			
		Rides Bus	Walks	Car	Total
Gender	Male	6	9	4	19
	Female	5	2	4	11
Total		11	11	8	30

		Transportation to School			
		Rides Bus	Walks	Car	Total
Gender	Male	0.2	0.3	0.133	0.633
	Female	0.167	0.067	0.133	0.367
Total		0.367	0.367	0.266	1

20. a. the parents surveyed that said no
 b. the total people that said yes
 c. the total people surveyed
21. Routine B is the best option, but your friend's reasoning of why is incorrect; Routine B is the best choice because there is a 66.7% chance of reaching the goal, which is higher than the chances of Routine A (62.5%) and Routine C (63.6%).

22.

		Preference		
		Math	Science	Total
Gender	Male	93	57	150
	Female	148	52	200
Total		241	109	350

23. a. about 0.438

b. about 0.387

24. *Sample answer:* Venn diagrams show a visual representation of the data, and two-way tables organize the information into rows and columns; An advantage of a Venn diagram is that people who learn visually will easily understand them. A disadvantage is that as more categories are used, the Venn diagram becomes harder to draw and interpret. An advantage of a two-way table is that it is very easy to read and interpret, even with many categories. A disadvantage is that they are not as visual as Venn diagrams.

25. a. More of the current consumers prefer the leader, so they should improve the new snack before marketing it.

b. More of the new consumers prefer the new snack than the leading snack, so there is no need to improve the snack.

26. *Sample answer:*

		Owns a Dog		
		Yes	No	Total
Gender	Male	5	3	8
	Female	7	5	12
Total		12	8	20

$$P(A | B) = \frac{P(B | A) \cdot P(A)}{P(B)}$$

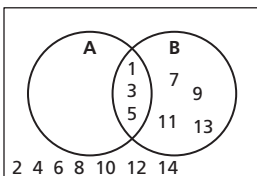
$$P(\text{Male} | \text{yes}) = \frac{P(\text{yes} | \text{Male}) \cdot P(\text{Male})}{P(\text{yes})}$$

$$= \frac{\frac{5}{8} \cdot \frac{8}{20}}{\frac{12}{20}}$$

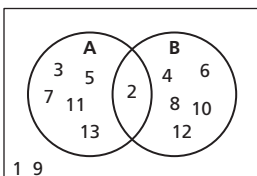
$$= \frac{5}{12}$$

10.3 Maintaining Mathematical Proficiency

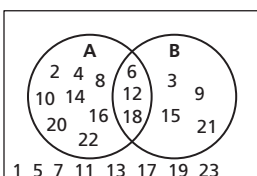
27.



28.



29.



10.4 Monitoring Progress and Modeling with Mathematics

22. a. $P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C)$

b. $P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(A \text{ and } B) - P(A \text{ and } C) - P(B \text{ and } C) + P(A \text{ and } B \text{ and } C)$

10.5 Monitoring Progress and Modeling with Mathematics

42. a. ${}_n C_n = \frac{n!}{n!0!} = 1$

b. ${}_n C_{n-r} = \frac{n!}{(n - (n - r))!(n - r)!} = \frac{n!}{(r)!(n - r)!} = {}_n C_r$

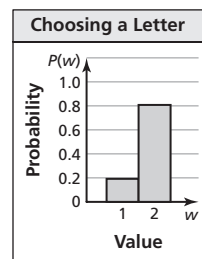
c. ${}_n C_r + {}_n C_{r-1} = \frac{n!}{(n - r)!r!} + \frac{n!}{(n - r + 1)!(r - 1)!}$
 $= \frac{n!(n - r + 1) + n!r}{(n - r + 1)!r!}$
 $= \frac{n!n + n!}{(n - r + 1)!r!}$
 $= \frac{n!(n + 1)}{(n - r + 1)!r!}$
 $= \frac{(n + 1)!}{(n + 1 - r)!r!}$

$$= {}_{n+1} C_r$$

10.6 Monitoring Progress and Modeling with Mathematics

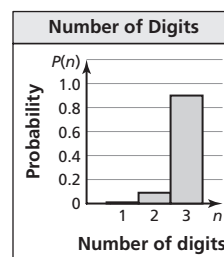
5.

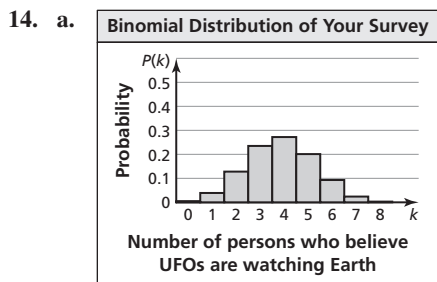
w (value)	1	2
Outcomes	5	21
$P(w)$	$\frac{5}{26}$	$\frac{21}{26}$



6.

n (value)	1	2	3
Outcomes	10	90	900
$P(n)$	$\frac{1}{100}$	$\frac{9}{100}$	$\frac{9}{10}$



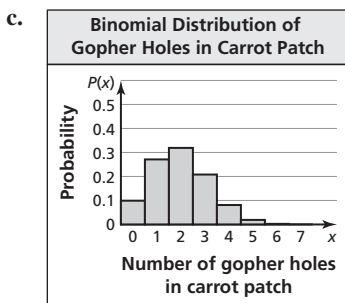


17. a. $P(0) \approx 0.099$, $P(1) \approx 0.271$, $P(2) \approx 0.319$,
 $P(3) \approx 0.208$, $P(4) \approx 0.081$, $P(5) \approx 0.019$,
 $P(6) \approx 0.0025$, $P(7) \approx 0.00014$

b.

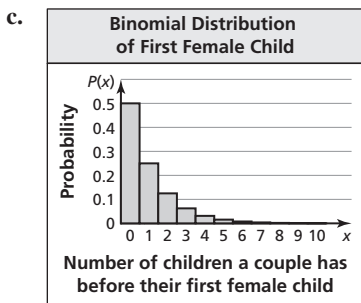
x	0	1	2	3	4
$P(x)$	0.099	0.271	0.319	0.208	0.081

x	5	6	7
$P(x)$	0.019	0.0025	0.00014



18. 0.2; 0.6
 19. no; The data is skewed right, so the probability of failure is greater.
 20. no; The probability of not choosing the coin 100 times is $(\frac{99}{100})^{100} \approx 0.366$.
 21. a. The statement is not valid, because having a male and having a female are independent events.

b. 0.03125



skewed right

22. $p > 0.5$

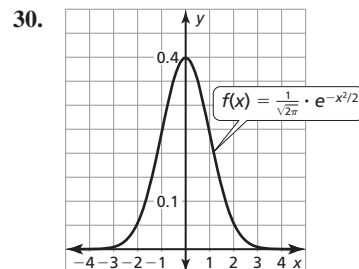
10.6 Maintaining Mathematical Proficiency

23. FFF, FFM, FMF, FMM, MMM, MMF, MFM, MFF
 24. D_1C_1 , D_1C_2 , D_1C_3 , D_2C_1 , D_2C_2 , D_2C_3

Chapter 11

11.1 Monitoring Progress and Modeling with Mathematics

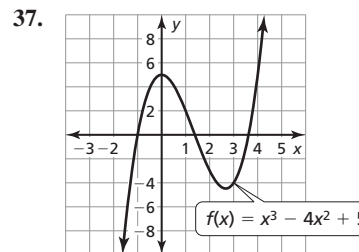
29. a. about 4.52×10^{-9}
 b. yes; The probability that a box contains an amount of cereal significantly less than the mean is very small.



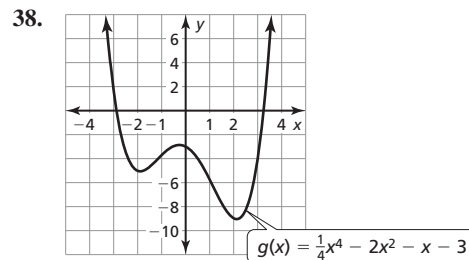
Sample answer: 0.996

31. one standard deviation above the mean
 32. $\mu = 10$, $\sigma = 3$
 33. a. 88th percentile
 b. 93rd percentile
 c. ACT; Your percentile on the ACT was higher than your percentile on the SAT.
 34. Find the z-score for the ACT. Substitute this value along with the mean and the standard deviation for the SAT in the formula for z-score, and solve for x . This value is the equivalent SAT score.
 35. no; When the mean is greater than the median, the distribution is skewed right.
 36. 0.1336

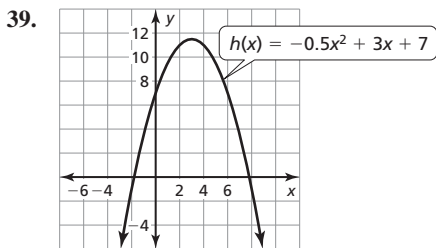
11.1 Maintaining Mathematical Proficiency



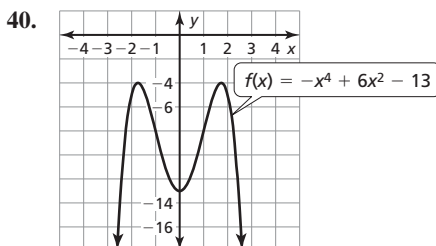
x -intercepts: -1 , about 1.4, and about 3.6; local maximum: $(0, 5)$; local minimum: $(2.67, -4.48)$; increasing when $x < 0$ and $x > 2.67$; decreasing when $0 < x < 2.67$



x -intercepts: about -2.8 and about 3.2 ; local maximum: $(-0.25, -2.87)$; local minima: $(-1.86, -5.07)$ and $(2.11, -9.06)$; increasing when $-1.86 < x < -0.25$ and $x > 2.11$; decreasing when $x < -1.86$ and $-0.25 < x < 2.11$



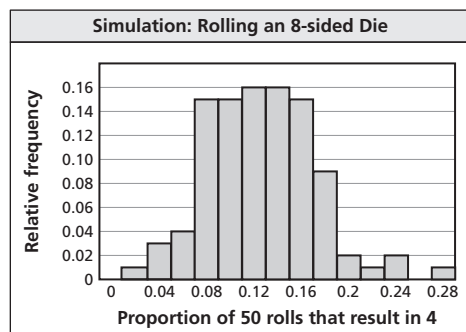
x -intercepts: about -1.8 and about 7.8 ; maximum: $(3, 11.5)$;
no local minimum; increasing when $x < 3$; decreasing when $x > 3$



no x -intercepts; maximums: $(-1.73, -4)$ and $(1.73, -4)$;
local minimum: $(0, -13)$; increasing when $x < -1.73$ and
 $0 < x < 1.73$; decreasing when $-1.73 < x < 0$ and $x > 1.73$

11.2 Monitoring Progress and Modeling with Mathematics

- Because all the players in a group were used, the mean is a parameter; Because the mean age of 26 is based on every member of a football team, it is a parameter.
- possibly, but extremely unlikely; The result is unlikely to occur by chance. The sample size of the population is too small to make such a conclusion.
- population: all the majors of students at a university, sample: majors of students at the university who take chemistry; All the students' majors at the university make up the entire group, and the majors of students who take chemistry make up a subset of the entire group.
- Sample answer:* population: all American adults, sample: the 801 American adults surveyed; The sample consists of 606 American adults who say the world's temperature will go up over the next 100 years, 174 American adults who say it will go down, and 21 American adults who have no opinion.
- yes; The sample mean is an estimate of the population mean, and 7.5 hours is relatively close to 8 hours.
- simulation 2; Simulation 2 gives a better indication of outcomes that are not likely to occur by chance.
- a.** *Sample answer:*



- The maker's claim is most likely false; The maker's claim is most likely true.

11.2 Maintaining Mathematical Proficiency

- $x = 5 \pm \sqrt{29}$ or $x \approx 10.39, x \approx -0.39$
- $t = -1 \pm \sqrt{7}$ or $t \approx 1.65, t \approx -3.65$
- $s = -5 \pm \sqrt{17}$ or $s \approx -0.88, s \approx -9.12$
- $n = -1 \pm i$
- $z = \frac{1}{2}, z = -\frac{15}{2}$
- $w = \frac{5 \pm \sqrt{69}}{2}$ or $w \approx 6.65, w \approx -1.65$

11.3 Lesson Monitoring Progress

- cluster sample
- Sample answer:* Students are divided into groups by grade, and a random sample is selected from each grade.

11.3 Monitoring Progress and Modeling with Mathematics

- stratified sample
 - Because the counties (groups) are very different in population (size), selecting the same number of people from each county will underrepresent people living in counties with a large population, and overrepresent people living in counties with a small population.
- self-selected sample and convenience sample; In a self-selected sample, only people with strong opinions are likely to respond. In a convenience sample, parts of the population have no chance of being selected for the survey.
- In a blind experiment, the people administering the treatment know which treatment each participant is receiving. In a double-blind experiment, they do not; *Sample answer:* An advantage of the double-blind experiment is that a person administering the treatment cannot accidentally influence the person receiving the treatment.
- to determine the employment rate of graduates in their field of study
 - all graduating seniors of the college
 - Sample answer:* Are you employed? If yes, is your job in your field of study?
- the second sample; In a larger sample, unusual responses will have less of an effect.
- no; *Sample answer:* Some groups in the population, like the homeless, are difficult to contact.
- survey
 - Sample answer:* Put an optional survey card in each back seat pocket for passengers to complete and mail back to the airline; Passengers with enough leg room are less likely to complete and return the survey.
 - Sample answer:* Group passengers by type of plane and class of seat. Select a random sample from each group, and have a flight attendant hand a survey to the selected passengers which they may complete anonymously and return to a box the flight attendant brings around; All segments of the population have a chance of being selected, and passengers are likely to give honest responses on an anonymous survey.
 - Sample answer:* biased: Do you feel you do not have enough leg room in a cramped seat area?; unbiased: Do you have enough leg room?
- self-selected sample
 - people who spend a lot of time on the Internet and visit that particular site; The survey is probably biased.

11.3 Maintaining Mathematical Proficiency

42. 32 43. 9
 44. -4 45. $\frac{1}{4}$
 46. 4^7 or 16,384 47. $\frac{\sqrt[3]{18}}{18}$
 48. 4 49. 3


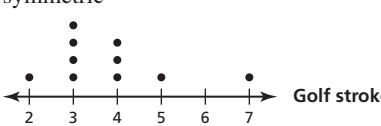
11.3 Mini-Assessment

5. The question encourages a “yes” response, because the coach is asking the question. One improvement may be to have players go online and complete an anonymous survey about their summer workouts.

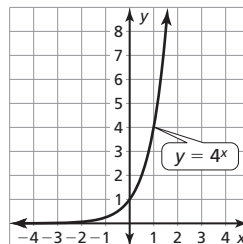
11.4 Monitoring Progress and Modeling with Mathematics

11. a. *Sample answer:* Because the heart rates are monitored for two different types of exercise, the groups cannot be compared. Running on a treadmill may have a different effect on heart rate than lifting weights; Check the heart rates of all the athletes after the same type of exercise.
 b. no potential problems
12. a. no potential problems
 b. *Sample answer:* Because the subjects volunteer for a particular group, the groups are not similar. Someone who volunteers to read may also have other habits that could raise IQ scores; Randomly assign the volunteers to the treatment group or the control group.
13. *Sample answer:* The sample size is not large enough to provide valid results; Increase the sample size.
14. It is not possible to control variables of objects in space.
15. no; Your friend would have to perform an observational study, and an observational study can show correlation, but not causality.
16. a. yes; *Sample answer:* The individuals were randomly assigned to a control group with no ads, and a treatment group with ads.
 b. watching TV with ads for candidate B
 c. no; *Sample answer:* The increase in preference for candidate B with the ads may be too small to be significant.
17. *Sample answer:* The placebo effect is response to a dummy treatment that may result from the trust in the researcher or the expectation of a cure; It can be minimized by comparing two groups so the placebo effect has the same effect on both groups.
18. *Sample answer:* Plants treated with a growth supplement will grow taller than plants that are not treated; Randomly select the same number of plants to be put in each of two groups. Use the growth supplement on the plants in one group, and do not use the growth supplement on the plants in the other group. Keep all other variables constant and record the height of the plants each day.
19. yes; Repetition reduces the effect of unusual results that may occur by chance.

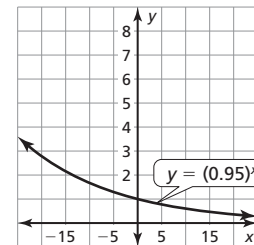
11.4 Maintaining Mathematical Proficiency

20. 
 symmetric
21. 
 skewed right

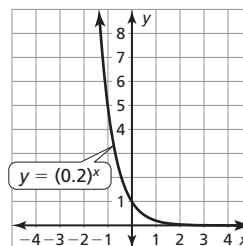
22. exponential growth



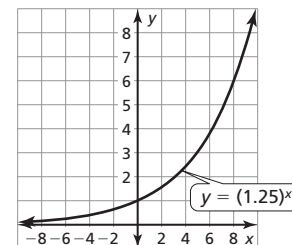
23. exponential decay



24. exponential decay



25. exponential growth



11.4 Mini-Assessment

- The study is a randomized comparative experiment. The treatment is watching reality TV at least 5 hours a week. The treatment group is the patients who watch reality TV. The control group is patients who do not watch reality TV.
- Use an experiment. Randomly select the same number of bean seeds to be placed in each of two identical planters. One planter will be set outdoors, while the other remains indoors. Keep all other variables, such as soil and water, the same. Record the growth of the plants every week.
- The sample size is not large enough to produce valid results. To improve the validity of the experiment, the sample size must be larger and the experiment must be replicated.

11.5 Monitoring Progress and Modeling with Mathematics

- simulation *a*; simulation *c*; Simulation *a* has the smallest standard deviation, and simulation *c* has the largest standard deviation.
- about 453 residents
- $4n$; 4 times; The margin of error is inversely proportional to the square root of the sample size, so cutting the margin of error in half requires multiplying the sample size by 2^2 , or 4.
- a. 500 voters b. about $\pm 4.5\%$
 c. candidate A: between 42.5% and 51.5%, candidate B: between 48.5% and 57.5%
 d. no; 273 voters
- a. $n > 9$ b. $n > 891$
 c. The smaller the percentage of a large population that have a certain characteristic, the larger the sample must be to reasonably represent the population.
- more than 2500; To be confident that sports drink X is preferred, the margin of error would need to be less than 2%.

11.5 Maintaining Mathematical Proficiency

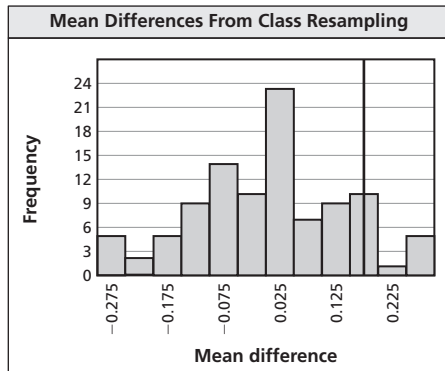
28. $y = \log x + 3$ 29. $y = \log_2(x + 5)$
 30. $y = e^x - 5$ 31. $y = 6^{x+1}$
 32. arithmetic; $a_n = -3n + 20$
 33. geometric; $a_n = 3(2)^{n-1}$
 34. geometric; $a_n = 32\left(\frac{1}{2}\right)^{n-1}$

11.5 Mini-Assessment

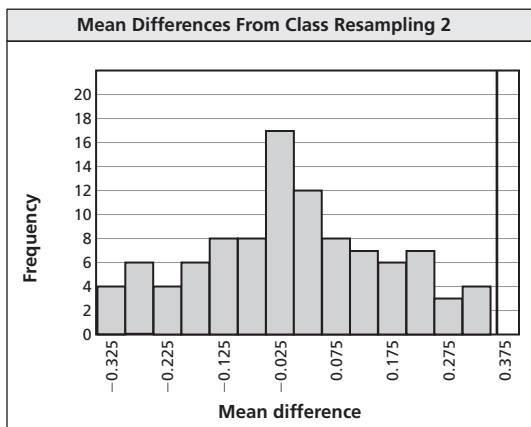
2. $\frac{14}{50} = 0.28$ occurred in 9 of the 70 random samples. It is *likely* that 14 adults out of 50 would say P.E. should be a daily requirement when the true population percentage is 28%. The company's claim is probably accurate.

11.6 Explorations

2. a. *Sample answer:*



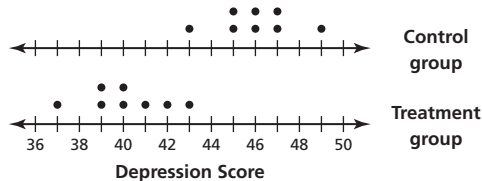
4. $\bar{x}_{\text{control}} = 0.97$; $\bar{x}_{\text{treatment}} = 1.33$; $\bar{x}_{\text{treatment}} - \bar{x}_{\text{control}} = 0.36$;
Sample answer:



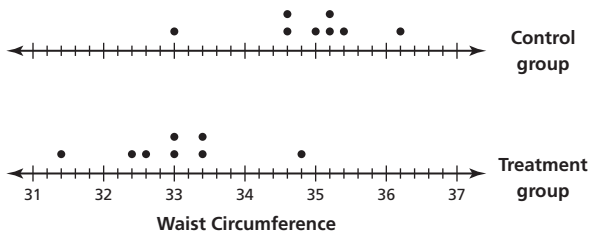
All of the values are below 0.36, so you can reject the hypothesis.

11.6 Monitoring Progress and Modeling with Mathematics

3. d.



4. d.



6. The treatment has an effect on the waist circumference of adults. The effect increases the waist circumference; The treatment has an effect on the waist circumference of adults. The effect decreases the waist circumference; The treatment has no effect on the waist circumference of adults.

7. *Sample answer:* -1.75 8. *Sample answer:* 1.15
9. The hypothesis is most likely false; Music therapy decreases depression scores.
10. The hypothesis is probably true; Music therapy does not have an effect on depression scores.
11. The histogram in Exercise 9 has a roughly normal distribution and shows the mean differences from 200 resamplings. The histogram in Exercise 11 is random and shows the mean differences from 20 resamplings; the histogram in Exercise 9 because it uses a large number of resamplings and the roughly normal distribution suggests music therapy decreases depression scores
12. negative; slightly effective; Most of the treatment values are less than most of the control values.
13. yes; As the number of samplings increase, the individual values should end up in each group approximately the same number of times, so the positive and negative differences in the means should balance out to 0.
14. *Sample answer:* the mean increase in the height of a plant given a growth supplement; The growth supplement has no effect on the growth of the plants, The growth supplement causes a decrease in the height of the plants, The growth supplement causes a slight increase in the height of the plants, The growth supplement causes a significant increase in the height of the plants.
15. 12,870; The number of combinations of 16 items in groups of 8 amounts to 12,870.

11.6 Maintaining Mathematical Proficiency

16. $5x^2(x - 3)$ 17. $(y - 2)(y^2 + 2y + 4)$
18. $(z + 3)(z - 3)(z + 5)$ 19. $(9w^2 + 4)(3w + 2)(3w - 2)$
20. yes; $g(x) = \frac{3}{x} - 5$ 21. yes; $g(x) = \frac{1}{2x} + \frac{1}{2}$
22. yes; $g(x) = \frac{2}{x + 4}$ 23. no; $y = \pm \sqrt{\frac{3}{x - 1}}$

11.6 Mini-Assessment

3. The experimental difference 0.2 falls within the middle 90%. You can conclude that the hypothesis is most likely true. So, hydroponics does not have an effect on the yield of strawberry plants.

Chapter 11 Standards Assessment

Equation	Is the inverse a function?		Is the function its own inverse?	
	Yes	No	Yes	No
$y = -x$	✓		✓	
$y = 3 \ln x + 2$	✓			✓
$y = \left(\frac{1}{x}\right)^2$		✓		✓
$y = \frac{x}{x - 1}$	✓		✓	

$y = -x$: Substituting x for y and y for x , then solving for y results in the original equation; $y = 3 \ln x + 2$: Substituting x for y and y for x , then solving for y results in $y = e^{(x - 2)/3}$, which is an exponential function; $y = \left(\frac{1}{x}\right)^2$: Substituting x for y and y for x , then solving for y results in an equation that gives 2 output values for every input value; $y = \frac{x}{x - 1}$: Substituting x for y and y for x , then solving for y results in the original equation.