

# Chapter 1

## Chapter 1 Maintaining Mathematical Proficiency (p. 1)

$$\begin{aligned} 1. 5 \cdot 2^3 + 7 &= 5 \cdot 8 + 7 \\ &= 40 + 7 \\ &= 47 \end{aligned}$$

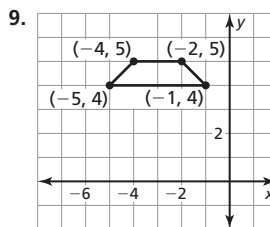
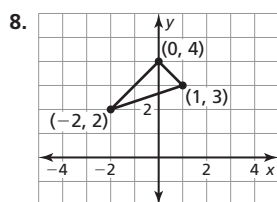
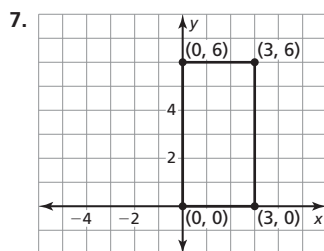
$$\begin{aligned} 2. 4 - 2(3 + 2)^2 &= 4 - 2(5)^2 \\ &= 4 - 2(25) \\ &= 4 - 50 \\ &= -46 \end{aligned}$$

$$\begin{aligned} 3. 48 \div 4^2 + \frac{3}{5} &= 48 \div 16 + \frac{3}{5} \\ &= 3 + \frac{3}{5} \\ &= \frac{15}{5} + \frac{3}{5} \\ &= \frac{18}{5} \\ &= 3\frac{3}{5} \end{aligned}$$

$$\begin{aligned} 4. 50 \div 5^2 \cdot 2 &= 50 \div 25 \cdot 2 \\ &= 2 \cdot 2 \\ &= 4 \end{aligned}$$

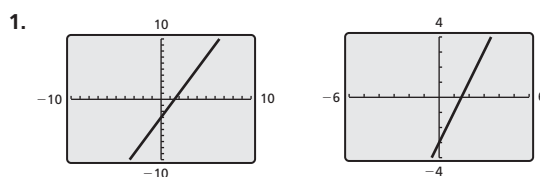
$$\begin{aligned} 5. \frac{1}{2}(2^2 + 22) &= \frac{1}{2}(4 + 22) \\ &= \frac{1}{2}(26) \\ &= 13 \end{aligned}$$

$$\begin{aligned} 6. \frac{1}{6}(6 + 18) - 2^2 &= \frac{1}{6}(24) - 2^2 \\ &= \frac{1}{6}(24) - 4 \\ &= 4 - 4 \\ &= 0 \end{aligned}$$

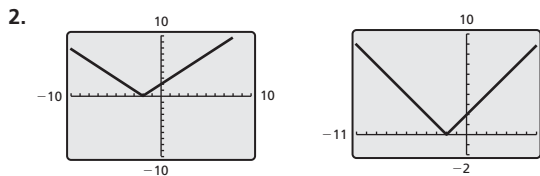


10. *Sample answer:* Consider the expression  $50 \div 25 \cdot 2$ . When the order of operations are followed (multiply and divide from left to right), the expression gives  $50 \div 25 \cdot 2 = 2 \cdot 2 = 4$ . However, when they are not followed, then the result might be  $50 \div 25 \cdot 2 = 50 \div 50 = 1$ . Following the order in which transformations are given is also important. For example, translating the point  $(3, 2)$  up 3 units and then reflecting in the  $x$ -axis, the new coordinate is  $(3, -5)$ . Reflecting in the  $x$ -axis and then translating up 3 units, the new coordinate is  $(3, 1)$ .

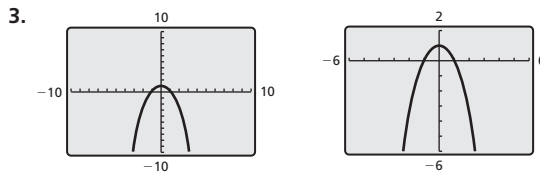
## Chapter 1 Mathematical Practices (p. 2)



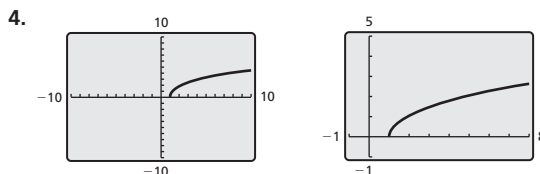
The square viewing window makes the line look steeper.



The square viewing window makes the graph look narrower.

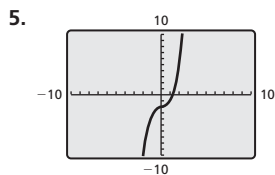


The square viewing window makes the parabola look wider.

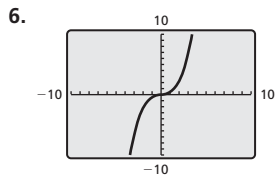
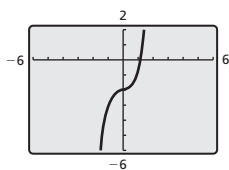


The standard viewing window makes the curve look flatter.

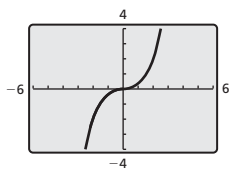
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The square viewing window makes the curve look wider.



The square viewing window makes the curve look wider.



7.  $\frac{\text{Height}}{\text{Width}} = \frac{8 - (-2)}{8 - (-8)} = \frac{10}{16} = \frac{5}{8}$

no; The height-to-width ratio is 5 to 8.

8.  $\frac{\text{Height}}{\text{Width}} = \frac{8 - (-2)}{8 - (-7)} = \frac{10}{15} = \frac{2}{3}$

yes; The height-to-width ratio is 2 to 3.

9.  $\frac{\text{Height}}{\text{Width}} = \frac{8 - (-2)}{9 - (-6)} = \frac{10}{15} = \frac{2}{3}$

yes; The height-to-width ratio is 2 to 3.

10.  $\frac{\text{Height}}{\text{Width}} = \frac{3 - (-3)}{2 - (-2)} = \frac{6}{4} = \frac{3}{2}$

no; The height-to-width ratio is 3 to 2.

11.  $\frac{\text{Height}}{\text{Width}} = \frac{3 - (-3)}{5 - (-4)} = \frac{6}{9} = \frac{2}{3}$

yes; The height-to-width ratio is 2 to 3.

12.  $\frac{\text{Height}}{\text{Width}} = \frac{3 - (-3)}{4 - (-4)} = \frac{6}{8} = \frac{3}{4}$

no; The height-to-width ratio is 3 to 4.

## 1.1 Explorations (p. 3)

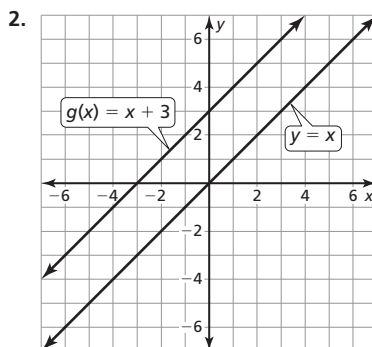
- absolute value; The graph is a "V" shape.
- square root; The domain is  $x \geq 0$  and the range is  $y \geq 0$ .
- constant; The  $y$ -value is the same for all  $x$ -values.
- exponential; The function grows faster as  $x$  increases.
- cubic; The domain and range are both all real numbers, and the function grows quickly indicating an exponent.
- linear; The function is always increasing at the same rate.
- reciprocal; The function has a vertical asymptote at the  $y$ -axis, and the  $y$ -values become smaller as  $x$  increases.
- quadratic; The graph is a parabola.

2. *Sample answer:* Most of the parent functions go through the origin. Most are either symmetric about the  $y$ -axis or the origin.

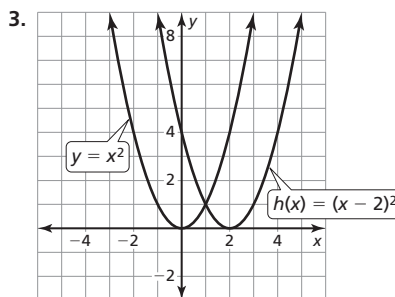
- The equation for the graph is  $y = |x|$ .
- The equation for the graph is  $y = \sqrt{x}$ .
- The equation for the graph is  $y = 1$ .
- The equation for the graph is  $y = 2^x$ .
- The equation for the graph is  $y = x^3$ .
- The equation for the graph is  $y = x$ .
- The equation for the graph is  $y = \frac{1}{x}$ .
- The equation for the graph is  $y = x^2$ .

## 1.1 Monitoring Progress (pp. 4–7)

- The function  $g$  belongs to the family of quadratics. The graph is translated right and is wider than the graph of the parent quadratic function. The domain of each function is all real numbers and the range of each function is  $y \geq 0$ .

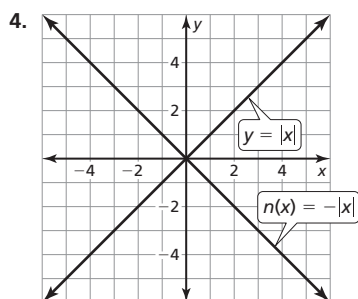


So, the graph of  $g(x) = x + 3$  is a translation 3 units up of the parent linear function.

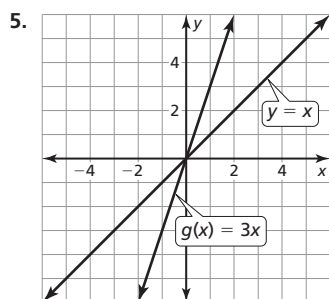


So, the graph of  $h(x) = (x - 2)^2$  is a translation 2 units right of the parent quadratic function.

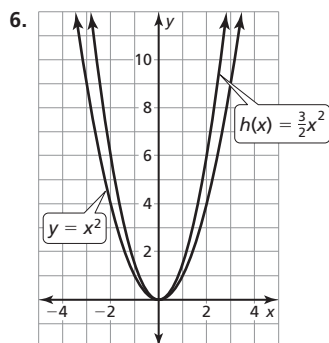
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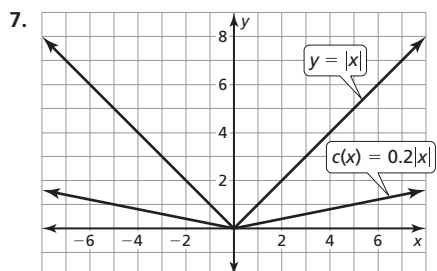
So, the graph of  $n(x) = |x|$  is a reflection in the  $x$ -axis of the parent absolute value function.



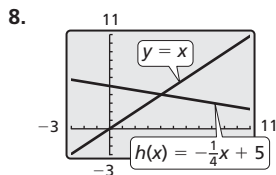
So, the graph of  $g(x) = 3x$  is a vertical stretch of the parent linear function.



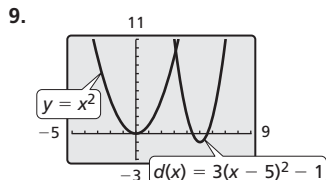
So, the graph of  $h(x) = \frac{3}{2}x^2$  is a vertical stretch of the parent quadratic function.



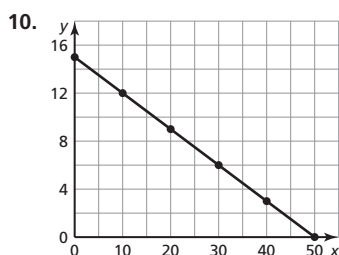
So, the graph of  $h(x) = 0.2|x|$  is a vertical shrink of the parent absolute value function.



The graph of  $h(x) = -\frac{1}{4}x + 5$  is a reflection in the  $x$ -axis followed by a vertical shrink and a translation 5 units up of the parent linear function.



The graph of  $h(x) = 3(x - 5)^2 - 1$  is a translation 5 units right followed by a vertical stretch and then a translation 1 unit down of the parent quadratic function.



The data appear to lie on a line. So, you can model the data with a linear function. The graph shows that the tank will be empty after 50 minutes.

## 1.1 Exercises (pp. 8–10)

### Vocabulary and Core Concept Check

1. The function  $f(x) = x^2$  is the parent function of  $f(x) = 2x^2 - 3$ .

2. The question that is different is “What are the vertices of the figure after a translation of 6 units up, followed by a reflection in the  $x$ -axis?” The coordinates are:

$$(1, -2) \rightarrow (1, -4)$$

$$(3, -2) \rightarrow (3, -4)$$

$$(3, -4) \rightarrow (3, -2)$$

$$(1, -4) \rightarrow (1, -2)$$

The other three questions result in the coordinates:

$$(1, -2) \rightarrow (3, 4)$$

$$(3, -2) \rightarrow (5, 4)$$

$$(3, -4) \rightarrow (5, 2)$$

$$(1, -4) \rightarrow (3, 2)$$

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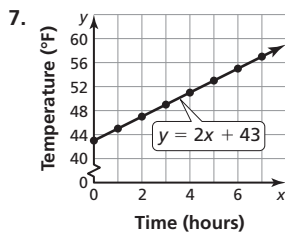
## Monitoring Progress and Modeling with Mathematics

3. The function  $f$  belongs to the family of absolute value functions. The graph of  $f(x) = 2|x + 2| - 8$  is a horizontal translation 2 units left followed by a vertical stretch and a vertical translation 8 units down of the parent absolute value function. The domain of each function is all real numbers, but the range of  $f$  is  $y \geq -8$ , and the range of the parent function is  $y \geq 0$ .

4. The function  $f$  belongs to the family of quadratic functions. The graph of  $f(x) = -2x^2 + 3$  is a reflection in the  $x$ -axis followed by a vertical stretch and a vertical translation 3 units up of the parent absolute value function. The domain of each function is all real numbers, but the range of  $f$  is  $y \leq 3$ , and the range of the parent quadratic function is  $y \geq 0$ .

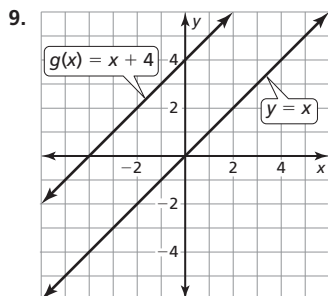
5. The function  $f$  belongs to the family of linear functions. The graph of  $f(x) = 5x - 2$  is a vertical stretch followed by a vertical translation 2 units down of the parent linear function. The domain and range of each function is all real numbers.

6. The function  $f$  belongs to the family of constant functions. The graph of  $f(x) = 3$  is a vertical translation 2 units up of the parent constant function. The domain of each function is all real numbers, but the range of  $f$  is  $y = 3$ , and the range of the parent function is  $y = 1$ .

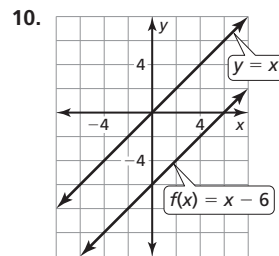


The type of function that can model the data is a linear function. The temperature is increasing by the same amount at each interval.

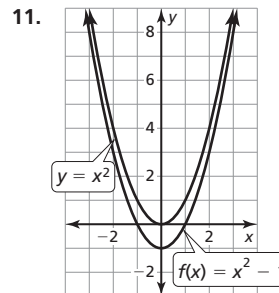
8. The type of function that is used as a model is a quadratic function.



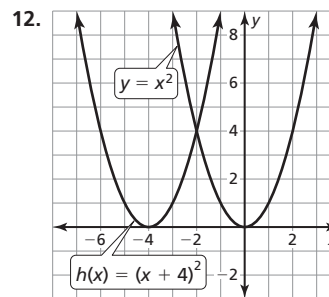
So, the graph of  $g(x) = x + 4$  is a vertical translation 4 units up of the parent linear function.



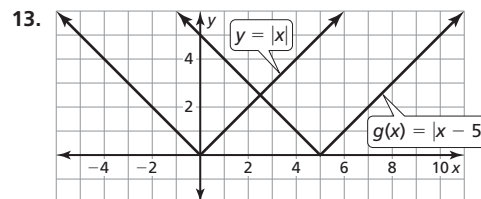
So, the graph of  $f(x) = x - 6$  is a vertical translation 6 units down of the parent linear function.



So, the graph of  $f(x) = x^2 - 1$  is a vertical translation 1 unit down of the parent quadratic function.

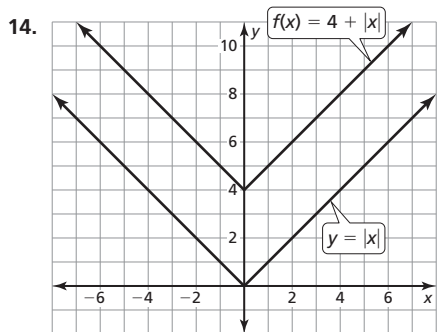


So, the graph of  $h(x) = (x + 4)^2$  is a horizontal translation 4 units left of the parent quadratic function.

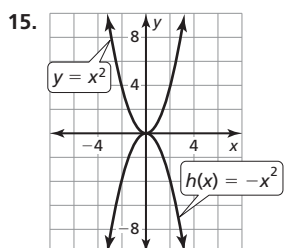


So, the graph of  $g(x) = |x - 5|$  is a horizontal translation 5 units right of the parent absolute value function.

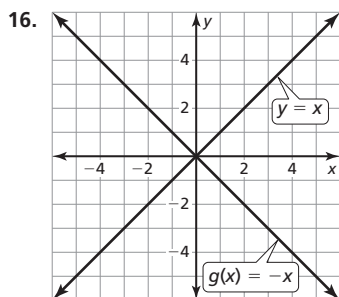
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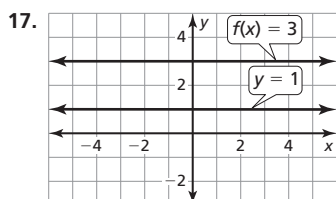
So, the graph of  $f(x) = 4 + |x|$  is a vertical translation 4 units up of the parent absolute value function.



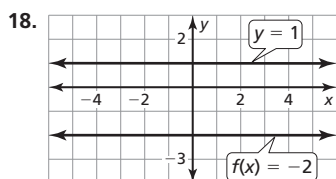
So, the graph of  $h(x) = -x^2$  is a reflection in the  $x$ -axis of the parent quadratic function.



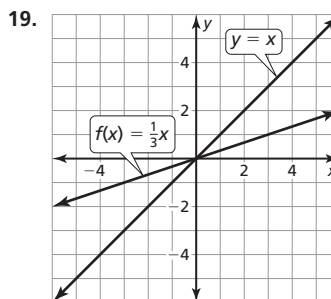
So, the graph of  $g(x) = -x$  is a reflection in the  $x$ -axis of the parent linear function.



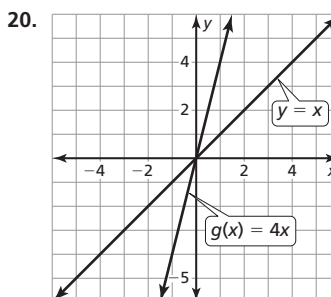
So, the graph of  $f(x) = 3$  is a vertical translation 2 units up of the parent constant function.



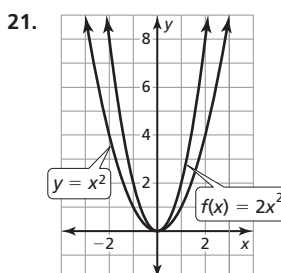
So, the graph of  $f(x) = -2$  is a vertical translation 3 units down of the parent constant function.



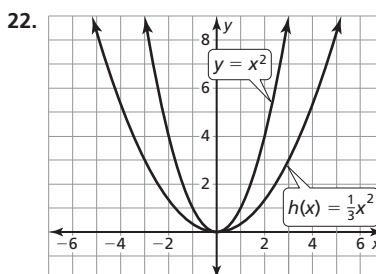
So, the graph of  $f(x) = \frac{1}{3}x$  is a vertical shrink of the parent linear function.



So, the graph of  $g(x) = 4x$  is a vertical stretch of the parent linear function.

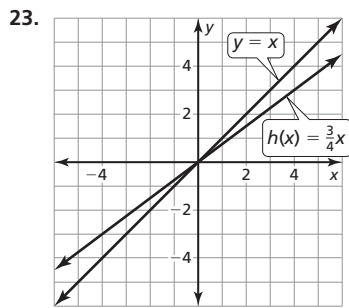


So, the graph of  $f(x) = 2x^2$  is a vertical stretch of the parent quadratic function.

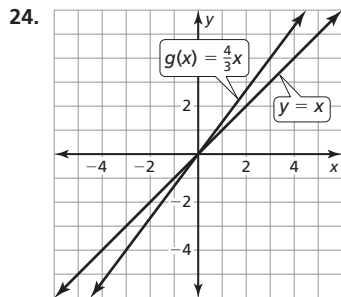


So, the graph of  $h(x) = \frac{1}{3}x^2$  is a vertical shrink of the parent quadratic function.

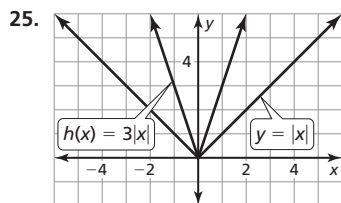
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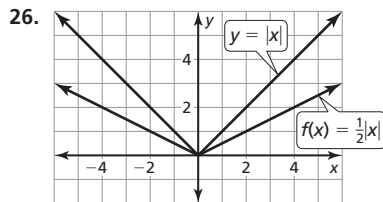
So, the graph of  $h(x) = \frac{3}{4}x$  is a vertical shrink of the parent linear function.



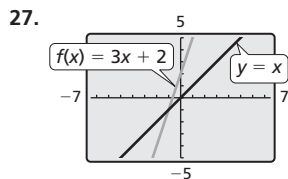
So, the graph of  $g(x) = \frac{4}{3}x$  is a vertical stretch of the parent linear function.



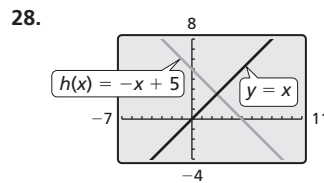
So, the graph of  $h(x) = 3|x|$  is a vertical stretch of the parent absolute value function.



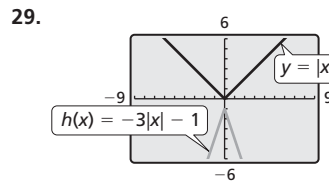
So, the graph of  $f(x) = \frac{1}{2}|x|$  is a vertical shrink of the parent absolute value function.



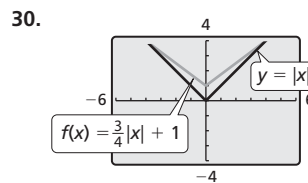
So, the graph of  $f(x) = 3x + 2$  is a vertical stretch followed by a vertical translation 2 units up of the parent linear function.



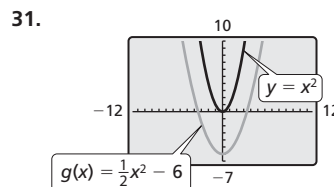
So, the graph of  $h(x) = -x + 5$  is a reflection in the  $x$ -axis followed by a vertical translation 5 units up of the parent linear function.



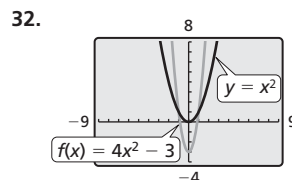
So, the graph of  $h(x) = -3|x| - 1$  is a reflection in the  $x$ -axis followed by a vertical stretch and then a vertical translation 1 unit down of the parent absolute value function.



So, the graph of  $f(x) = \frac{3}{4}|x| + 1$  is a vertical shrink followed by a vertical translation 1 unit up of the parent absolute value function.



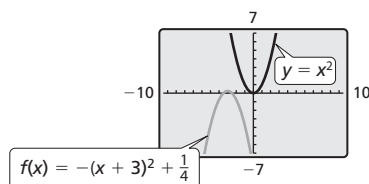
So, the graph of  $g(x) = \frac{1}{2}x^2 - 6$  is a vertical shrink followed by a vertical translation 6 units down of the parent quadratic function.



So, the graph of  $f(x) = 4x^2 - 3$  is a vertical stretch followed by a vertical translation 3 units down of the parent quadratic function.

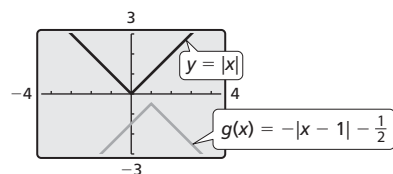
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33.



So, the graph of  $f(x) = -(x + 3)^2 + \frac{1}{4}$  is a translation 3 units left followed by a reflection in the  $x$ -axis and then a vertical translation  $\frac{1}{4}$  unit up of the parent quadratic function.

34.



So, the graph of  $g(x) = -|x - 1| - \frac{1}{2}$  is a translation 1 unit right followed by a reflection in the  $x$ -axis and then a vertical translation  $\frac{1}{2}$  unit down of the parent absolute value function.

35. The error is there is no vertical shrink of the parent quadratic function. The graph is a reflection in the  $x$ -axis followed by a vertical stretch of the parent quadratic function.

36. The error is that a translation to the right of 3 units is represented by a subtraction of 3 in the expression for the function. The graph is a translation 3 units right of the parent absolute function, so the function is  $f(x) = |x - 3|$ .

37.  $(x, y) \rightarrow (x, y - 2)$

$$A(2, 1) \rightarrow A'(2, -1)$$

$$B(-1, -2) \rightarrow B'(-1, -4)$$

$$C(2, -3) \rightarrow C'(2, -5)$$

38.  $(x, y) \rightarrow (x, -y)$

$$A(-1, 3) \rightarrow A'(-1, -3)$$

$$B(1, 3) \rightarrow B'(1, -3)$$

$$C(-1, 1) \rightarrow C'(-1, -1)$$

$$D(-3, 1) \rightarrow D'(-3, -1)$$

39. Function  $g$  is in the family of absolute value functions. The domain is all real numbers and the range is  $y \geq -1$ .

40. Function  $h$  is in the family of absolute value functions. The domain is all real numbers and the range is  $y \geq 2$ .

41. Function  $g$  is in the family of linear functions. The domain is all real numbers and the range is all real numbers.

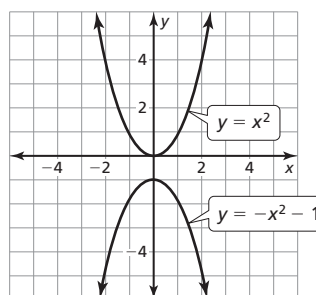
42. Function  $f$  is in the family of linear functions. The domain is all real numbers and the range is all real numbers.

43. Function  $f$  is in the family of quadratic functions. The domain is all real numbers and the range is  $y \geq -2$ .

44. Function  $f$  is in the family of quadratic functions. The domain is all real numbers and the range is  $y \leq 6$ .

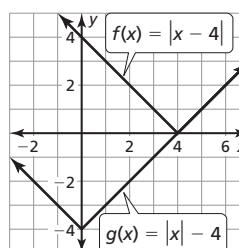
45. The type of function that can model the data is an absolute value function because the data are linear and there are positive speeds for the positive and negative displacements. The speed of the car 20 yards past the intersection is estimated to be 8 miles per hour.

46. Sample answer:



The transformation of the parent quadratic function is a reflection in the  $x$ -axis followed by a vertical translation 1 unit down.

47.



The graphs are not equivalent. The graph of  $f$  is a translation to the right, whereas the graph of  $g$  is a translation down of the parent absolute value function.

48. a. The graph of  $g$  is a vertical shrink because the factor is  $\frac{1}{2}$ .  
b. Multiply the  $y$ -coordinate of each point of  $f$  by  $-2$ .

49. Your friend is correct. Shifting the parent linear function down 2 units will create the same graph as shifting it 2 units right.

50. A linear function can be used to model the distance the swimmer travels. If the person has a 10-meter head start, the type of transformation is a vertical translation. The graph will be shifted up 10 units to represent the head start.

51. a. The type of function modeled by the equation is a quadratic function.

b. The value for  $t$  when the ball is released from the hand is zero, because 0 seconds have passed.

c. The ball is 5.2 feet above the ground when it is released from the hand, because this corresponds to  $t = 0$ .

52. An absolute value function can be used to model the data. In this situation, the  $x$ -intercept represents the number of hours that have passed at the moment the computer has 0 battery life remaining.

# Chapter 1

53. a. The function  $f(x) = 2|x| - 3$  is a vertical translation of the parent function. The graph will be shifted 3 units down.  
b. The function  $f(x) = (x - 8)^2$  is a horizontal translation of the parent function. The graph will be shifted 8 units right.  
c. The function  $f(x) = |x + 2| + 4$  is both a horizontal translation and vertical translation of the parent function. The graph will be shifted 2 units left and 4 units up.  
d. The function  $f(x) = 4x^2$  is neither a horizontal translation nor vertical translation of the parent function. The graph will have a vertical stretch.
54. a.  $f(x) = 3x + 1$ ; The graph will intersect the  $x$ -axis at  $x = -\frac{1}{3}$ .  
b.  $f(x) = |2x - 6| - 2$ ; The graph will intersect the  $x$ -axis at  $x = 2$  and  $x = 4$ .  
c.  $f(x) = -1 \cdot x^2 + 1$ ; The graph will intersect the  $x$ -axis at  $x = 1$  and  $x = -1$ .  
d.  $f(x) = 0$ ; This is the  $x$ -axis.

## Maintaining Mathematical Proficiency

55.  $f(x) = |x + 2|$   
 $-3 \stackrel{?}{=} |1 + 2|$   
 $-3 \stackrel{?}{=} |3|$   
 $-3 \neq 3$   
So,  $(1, -3)$  is not a solution.
56.  $f(x) = |x| - 3$   
 $-5 \stackrel{?}{=} |-2| - 3$   
 $-5 \stackrel{?}{=} 2 - 3$   
 $-5 \neq -1$   
So,  $(-2, -5)$  is not a solution.
57.  $f(x) = x - 3$   
 $2 \stackrel{?}{=} 5 - 3$   
 $2 = 2 \checkmark$   
So,  $(5, 2)$  is a solution.
58.  $f(x) = x - 4$   
 $8 \stackrel{?}{=} 12 - 4$   
 $8 = 8 \checkmark$   
So,  $(12, 8)$  is a solution.
59. To find the  $x$ -intercept let  $y = 0$ , then solve for  $x$ .  
 $y = x$   
 $0 = x$   
 $x = 0$   
To find the  $y$ -intercept let  $x = 0$ , then solve for  $y$ .  
 $y = x$   
 $y = 0$   
So, the  $x$ -intercept is  $(0, 0)$  and the  $y$ -intercept is  $(0, 0)$ .

60. To find the  $x$ -intercept let  $y = 0$ , then solve for  $x$ .  
 $y = x + 2$   
 $0 = x + 2$   
 $x = -2$   
To find the  $y$ -intercept let  $x = 0$ , then solve for  $y$ .  
 $y = x + 2$   
 $y = 0 + 2$   
 $y = 2$   
So, the  $x$ -intercept is  $(-2, 0)$  and the  $y$ -intercept is  $(0, 2)$ .
61. To find the  $x$ -intercept let  $y = 0$ , then solve for  $x$ .  
 $3x + y = 1$   
 $3x + 0 = 1$   
 $3x = 1$   
 $x = \frac{1}{3}$   
To find the  $y$ -intercept let  $x = 0$ , then solve for  $y$ .  
 $3x + y = 1$   
 $3(0) + y = 1$   
 $y = 1$   
So, the  $x$ -intercept is  $(\frac{1}{3}, 0)$  and the  $y$ -intercept is  $(0, 1)$ .
62. To find the  $x$ -intercept let  $y = 0$ , then solve for  $x$ .  
 $x - 2y = 8$   
 $x - 2(0) = 8$   
 $x = 8$   
To find the  $y$ -intercept let  $x = 0$ , then solve for  $y$ .  
 $x - 2y = 8$   
 $0 - 2y = 8$   
 $-2y = 8$   
 $y = -4$   
So, the  $x$ -intercept is  $(8, 0)$  and the  $y$ -intercept is  $(0, -4)$ .

## 1.2 Explorations (p. 11)

- The graph of  $y = |x| + k$  is a vertical translation of the parent function  $f(x) = |x|$ . If  $k$  is positive, the graph is shifted up  $k$  units. If  $k$  is negative, the graph is shifted down  $k$  units.
- The graph of  $y = |x + h|$  is a horizontal translation of the parent function  $f(x) = |x|$ . If  $h$  is positive, the graph is shifted right  $h$  units. If  $h$  is negative, the graph is shifted left  $h$  units.
- The graph of  $y = -|x|$  is a reflection in the  $x$ -axis of the parent function  $f(x) = |x|$ .



# Chapter 1

4. The graph of  $y = f(x) + k$  is a vertical translation of the parent function  $f$ ; the graph of  $y = f(x + h)$  is a horizontal translation of the parent function  $f$ ; and the graph of  $y = -f(x)$  is a reflection in the  $x$ -axis of the parent function  $f$ .
5. a. The graph of  $y = \sqrt{x} - 4$  is a vertical translation 4 units down of the parent function  $f(x) = \sqrt{x}$ .  
b. The graph of  $y = \sqrt{x + 4}$  is a horizontal translation 4 units left of the parent function  $f(x) = \sqrt{x}$ .  
c. The graph of  $y = -\sqrt{x}$  is a reflection in the  $x$ -axis of the parent function  $f(x) = \sqrt{x}$ .  
d. The graph of  $y = x^2 + 1$  is a vertical translation 1 unit up of the parent function  $f(x) = x^2$ .  
e. The graph of  $y = (x - 1)^2$  is a horizontal translation 1 unit right of the parent function  $f(x) = x^2$ .  
f. The graph of  $y = -x^2$  is a reflection in the  $x$ -axis of the parent function  $f(x) = x^2$ .

## 1.2 Monitoring Progress (pp. 13–15)

1. A translation 5 units up is a vertical translation that adds 5 to each output value.

$$\begin{aligned}g(x) &= f(x) + 5 \\ &= 3x + 5\end{aligned}$$

The translated function is  $g(x) = 3x + 5$ .

2. A translation 4 units to the right is a horizontal translation that subtracts 4 from each input value.

$$\begin{aligned}g(x) &= f(x - 4) \\ &= |x - 4| - 3\end{aligned}$$

The translated function is  $g(x) = |x - 4| - 3$ .

3. A reflection in the  $x$ -axis changes the sign of each output value.

$$\begin{aligned}g(x) &= -f(x) \\ &= -(-|x + 2| - 1) \\ &= |x + 2| + 1\end{aligned}$$

The reflected function is  $g(x) = |x + 2| + 1$ .

4. A reflection in the  $y$ -axis changes the sign of each input value.

$$\begin{aligned}g(x) &= f(-x) \\ &= \frac{1}{2}(-x) + 1 \\ &= -\frac{1}{2}x + 1\end{aligned}$$

The reflected function is  $g(x) = -\frac{1}{2}x + 1$ .

5. A horizontal stretch by a factor of 2 multiplies each input value by  $\frac{1}{2}$ .

$$\begin{aligned}g(x) &= f\left(\frac{1}{2}x\right) \\ &= 4\left(\frac{1}{2}x\right) + 2 \\ &= 2x + 2\end{aligned}$$

The transformed function is  $g(x) = 2x + 2$ .

6. A vertical shrink by a factor of  $\frac{1}{3}$  multiplies each output value by  $\frac{1}{3}$ .

$$\begin{aligned}g(x) &= \frac{1}{3}f(x) \\ &= \frac{1}{3}(|x| - 3) \\ &= \frac{1}{3}|x| - 1\end{aligned}$$

The transformed function is  $g(x) = \frac{1}{3}|x| - 1$ .

7. A translation 6 units down is a vertical translation that adds  $-6$  to each output value, then a reflection in the  $x$ -axis changes the sign.

$$\begin{aligned}g(x) &= -(f(x) - 6) \\ &= -(|x| - 6) \\ &= -|x| + 6\end{aligned}$$

The transformed function is  $g(x) = -|x| + 6$ .

8. The profit function is

$$\begin{aligned}p(x) &= 0.9 \cdot 3x - 50 \\ &= 2.7x - 50.\end{aligned}$$

To find the profit for 100 downloads, evaluate  $p$  when  $x = 100$ .

$$p(100) = 2.7(100) - 50 = 220$$

Your profit is \$220 for 100 downloads.

## 1.2 Exercises (pp. 16–18)

### Vocabulary and Core Concept Check

1. The function  $g(x) = |5x| - 4$  is a horizontal shrink of the function  $f(x) = |x| - 4$ .
2. The transformation “Stretch the graph of  $f(x) = x + 3$  vertically by a factor of 2” does not belong with the other three because the other three produce the same function,  $f(x) = 2x + 5$ .

### Monitoring Progress and Modeling with Mathematics

3. A translation 4 units left is a horizontal translation that subtracts  $-4$  from each input value.

$$\begin{aligned}g(x) &= f(x + 4) \\ &= (x + 4) - 5 \\ &= x - 1\end{aligned}$$

The transformed function is  $g(x) = x - 1$ .

4. A translation 2 units right is a horizontal translation that subtracts 2 from each input value.

$$\begin{aligned}g(x) &= f(x - 2) \\ &= (x - 2) + 2 \\ &= x\end{aligned}$$

The transformed function is  $g(x) = x$ .

# Chapter 1

5. A translation 2 units down is a vertical translation that adds  $-2$  to each output value.

$$\begin{aligned}g(x) &= f(x) - 2 \\ &= (|4x + 3| + 2) - 2 \\ &= |4x + 3|\end{aligned}$$

The transformed function is  $g(x) = |4x + 3|$ .

6. A translation 6 units up is a vertical translation that adds 6 to each output value.

$$\begin{aligned}g(x) &= f(x) + 6 \\ &= (2x - 9) + 6 \\ &= 2x - 3\end{aligned}$$

The transformed function is  $g(x) = 2x - 3$ .

7. A translation 3 units right is a horizontal translation that subtracts 3 from each input value.

$$\begin{aligned}g(x) &= f(x - 3) \\ &= 4 - |(x - 3) + 1| \\ &= 4 - |x - 2|\end{aligned}$$

The transformed function is  $g(x) = 4 - |x - 2|$ .

8. A translation 1 unit up is a vertical translation that adds 1 to each output value.

$$\begin{aligned}g(x) &= f(x) + 1 \\ &= (|4x| + 5) + 1 \\ &= |4x| + 6\end{aligned}$$

The transformed function is  $g(x) = |4x| + 6$ .

9. A horizontal translation 3 units right or a vertical translation 3 units up will produce the function  $g$  from the function  $f$ .

10. The transformation needed to model the situation using the function  $f$  is a vertical translation. The new model for the net income is  $g(x) = 4000x - 12,000$ . To find how many weeks it will take to pay off the extra expenses, set  $g$  equal to 0 and solve for  $x$ .

$$\begin{aligned}4000x - 12,000 &= 0 \\ 4000x &= 12,000 \\ x &= 3\end{aligned}$$

It will take 3 weeks to pay off the extra expenses.

11. A reflection in the  $x$ -axis changes the sign of each output value.

$$\begin{aligned}g(x) &= -f(x) \\ &= -(-5x + 2) \\ &= 5x - 2\end{aligned}$$

The transformed function is  $g(x) = 5x - 2$ .

12. A reflection in the  $x$ -axis changes the sign of each output value.

$$\begin{aligned}g(x) &= -f(x) \\ &= -\left(\frac{1}{2}x - 3\right) \\ &= -\frac{1}{2}x + 3\end{aligned}$$

The transformed function is  $g(x) = -\frac{1}{2}x + 3$ .

13. A reflection in the  $y$ -axis changes the sign of each input value.

$$\begin{aligned}g(x) &= f(-x) \\ &= |6(-x)| - 2 \\ &= |-6x| - 2 \\ &= |6x| - 2\end{aligned}$$

The transformed function is  $g(x) = |6x| - 2$ .

14. A reflection in the  $y$ -axis changes the sign of each input value.

$$\begin{aligned}g(x) &= f(-x) \\ &= |2(-x) - 1| + 3 \\ &= |-2x - 1| + 3\end{aligned}$$

The transformed function is  $g(x) = |-2x - 1| + 3$ .

15. A reflection in the  $y$ -axis changes the sign of each input value.

$$\begin{aligned}g(x) &= f(-x) \\ &= -3 + |(-x) - 11| \\ &= -3 + |-x - 11|\end{aligned}$$

The transformed function is  $g(x) = -3 + |-x - 11|$ .

16. A reflection in the  $y$ -axis changes the sign of each input value.

$$\begin{aligned}g(x) &= f(-x) \\ &= -(-x) + 1 \\ &= x + 1\end{aligned}$$

The transformed function is  $g(x) = x + 1$ .

17. A vertical stretch by a factor of 5 multiplies each output value by 5.

$$\begin{aligned}g(x) &= 5f(x) \\ &= 5(x + 2) \\ &= 5x + 10\end{aligned}$$

The transformed function is  $g(x) = 5x + 10$ .

18. A vertical shrink by a factor of  $\frac{1}{2}$  multiplies each output value by  $\frac{1}{2}$ .

$$\begin{aligned}g(x) &= \frac{1}{2}f(x) \\ &= \frac{1}{2}(2x + 6) \\ &= x + 3\end{aligned}$$

The transformed function is  $g(x) = x + 3$ .

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19. A horizontal shrink by a factor of  $\frac{1}{2}$  multiplies each input value by 2.

$$\begin{aligned} g(x) &= f(2x) \\ &= |2(2x)| + 4 \\ &= |4x| + 4 \end{aligned}$$

The transformed function is  $g(x) = |4x| + 4$ .

20. A horizontal stretch by a factor of 4 multiplies each input value by  $\frac{1}{4}$ .

$$\begin{aligned} g(x) &= f\left(\frac{1}{4}x\right) \\ &= \left|\left(\frac{1}{4}x\right) + 3\right| \\ &= \left|\frac{1}{4}x + 3\right| \end{aligned}$$

The transformed function is  $g(x) = \left|\frac{1}{4}x + 3\right|$ .

21. A vertical shrink by a factor of  $\frac{1}{2}$  multiplies each output value by  $\frac{1}{2}$ .

$$\begin{aligned} g(x) &= \frac{1}{2}f(x) \\ &= \frac{1}{2}(-2|x - 4| + 2) \\ &= -|x - 4| + 1 \end{aligned}$$

The transformed function is  $g(x) = -|x - 4| + 1$ .

22. A horizontal stretch by a factor of  $\frac{1}{3}$  multiplies each input value by 3.

$$\begin{aligned} g(x) &= f(3x) \\ &= 6 - (3x) \\ &= 6 - 3x \end{aligned}$$

The transformed function is  $g(x) = 6 - 3x$ .

23. C; There is a horizontal translation to the left.

24. A; The graph has been stretched vertically.

25. D; There is a vertical translation up.

26. B; The graph has been shrunk horizontally.

27. A vertical stretch by a factor of 2 multiplies each output value by 2 and a translation 1 unit is a vertical translation that adds 1 to each output value.

$$\begin{aligned} g(x) &= 2f(x) + 1 \\ &= 2(x) + 1 \\ &= 2x + 1 \end{aligned}$$

The transformed function is  $g(x) = 2x + 1$ .

28. A translation 3 units down is a vertical translation that adds  $-3$  to each output value and a vertical shrink by a factor of  $\frac{1}{3}$  multiplies each output value by  $\frac{1}{3}$ .

$$\begin{aligned} g(x) &= \frac{1}{3}(f(x) - 3) \\ &= \frac{1}{3}(x - 3) \\ &= \frac{1}{3}x - 1 \end{aligned}$$

The transformed function is  $g(x) = \frac{1}{3}x - 1$ .

29. A translation 2 units right is a horizontal translation that subtracts 2 from each input value and a horizontal stretch by a factor of 2 multiplies each input value by  $\frac{1}{2}$ .

$$\begin{aligned} g(x) &= f\left(\frac{1}{2}(x - 2)\right) \\ &= \left|\left(\frac{1}{2}x\right) - 2\right| \\ &= \left|\frac{1}{2}x - 2\right| \end{aligned}$$

The transformed function is  $g(x) = \left|\frac{1}{2}x - 2\right|$ .

30. A reflection in the  $y$ -axis changes the sign of each input value and a translation 3 units right is a horizontal translation that subtracts 3 from each input value.

$$\begin{aligned} g(x) &= f((-x) - 3) \\ &= |x - 3| \end{aligned}$$

The transformed function is  $g(x) = |x - 3|$ .

31. A reflection in the  $x$ -axis changes the sign of each output value and a translation 8 units down is a vertical translation that adds  $-8$  to the output value.

$$\begin{aligned} g(x) &= -f(x) - 8 \\ &= -|x| - 8 \end{aligned}$$

The transformed function is  $g(x) = -|x| - 8$ .

32. A translation 3 units down is a vertical translation that adds  $-3$  to each output value and a translation 1 unit right is a horizontal translation that subtracts 1 from each input value.

$$\begin{aligned} g(x) &= f(x - 1) - 3 \\ &= |x - 1| - 3 \end{aligned}$$

The transformed function is  $g(x) = |x - 1| - 3$ .

33. The error is that 3 was added rather than subtracted to represent the translation 3 units right. The correct function is  $g(x) = |x - 3| + 2$ .

34. A vertical stretch is given by  $y = af(x)$ , not  $y = f(ax)$ . The correct expression for the function is

$$g(x) = 5(x - 6) = 5x - 30.$$

35. Your friend is incorrect. The order in which transformations of functions are performed is important. For example, the functions  $f(x) = -x + 6$  and  $g(x) = -(x + 6)$  both represent a reflection in the  $x$ -axis and a horizontal translation by 6, but the orders in which the transformations are performed are different.

36. The transformation is a horizontal shrink by a factor of  $\frac{1}{2}$ . By using this transformation, the sales for 2010 would be cut in half according to the model.

37. The transformation is a horizontal translation 6 units left. The area of the shaded triangle is  $A = \frac{1}{2}bh = \frac{1}{2}(6)(3) = 9$  square units.

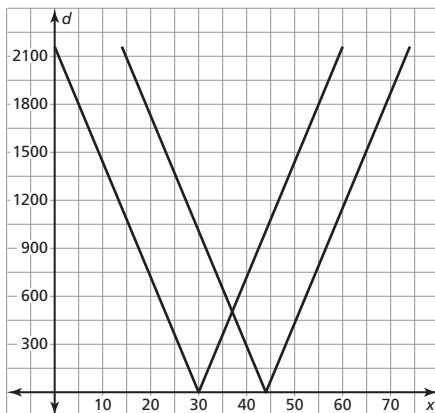
38. The transformation is a vertical translation 5 units up. The area of the shaded triangle is  $A = \frac{1}{2}bh = \frac{1}{2}(6)(3) = 9$  square units.

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39. The transformation is a reflection in the  $x$ -axis. The area of the shaded triangle is  $A = \frac{1}{2}bh = \frac{1}{2}(8)(4) = 16$  square units.
40. The transformation is a reflection in the  $y$ -axis. The area of the shaded triangle is  $A = \frac{1}{2}bh = \frac{1}{2}(10)(5) = 25$  square units.
41. a. For a vertical translation, find the difference of the  $y$ -intercepts,  $c - b$ . So, the expression is  $f(x) + (c - b)$ .  
 b. For a horizontal translation, find the difference of the  $x$ -intercepts.  

$$-\frac{c}{m} - \left(-\frac{b}{m}\right) = -\frac{c - b}{m}$$
 So, the expression is  

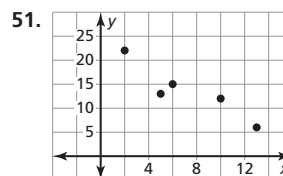
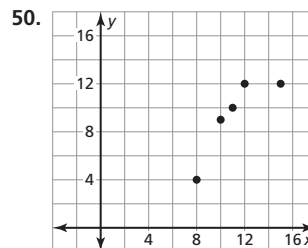
$$f\left(x - \left(-\frac{c - b}{m}\right)\right) + b = f\left(x + \frac{c - b}{m}\right) + b.$$
42. a. Reflecting the graph of  $f$  in the  $y$ -axis changes the sign of the slope and the  $x$ -intercept, and the  $y$ -intercept does not change.  
 b. Shrinking the graph of  $f$  vertically by a factor of  $\frac{1}{3}$  reduces the slope and  $y$ -intercept by a factor of  $\frac{1}{3}$ , and the  $x$ -intercept does not change.  
 c. Stretching the graph of  $f$  horizontally by a factor of 2 reduces the slope by  $\frac{1}{2}$ , doubles the value of the  $x$ -intercept, and the  $y$ -intercept does not change.
43. The order of the transformations is a vertical stretch by a factor of 4 followed by a vertical translation 2 units down, and then a reflection in the  $x$ -axis. A 4 was multiplied in only one place, so that was performed first. The vertical translation subtracts a 2, but it is added in the expression, so this had to take place second so that the reflection would cause the change in the signs.
44. The shift to the right means that you started later than the day you planned to. For example, you may have started on June 15, which is 14 days later. The function representing this change in plans is  $d = 72|x - 44|$ .



45. To have a reflection in the  $x$ -axis followed by a translation 1 unit left and 1 unit up,  $a = -2$ ,  $b = 1$ , and  $c = 0$ . The reflection will change the sign of the 2 in front of the absolute value, the translation to the left will introduce a  $-1$  on the inside of the absolute value, and the translation up will then have the 0 added to the absolute value.

## Maintaining Mathematical Proficiency

46.  $f(x) = x + 4$ ;  $x = 3$   
 $f(3) = (3) + 4 = 3 + 4 = 7$
47.  $f(x) = 4x - 1$ ;  $x = -1$   
 $f(-1) = 4(-1) - 1$   
 $= -4 - 1$   
 $= -5$
48.  $f(x) = -x + 3$ ;  $x = 5$   
 $f(5) = -(5) + 3$   
 $= -5 + 3$   
 $= -2$
49.  $f(x) = -2x - 2$ ;  $x = -1$   
 $f(-1) = -2(-1) - 2$   
 $= 2 - 2$   
 $= 0$



### 1.1–1.2 What Did You Learn? (p. 19)

- After graphing the data, the shape indicates that the absolute value function models the data.
- The number of weeks is between 3 and 4, so the value should be rounded up to 4 weeks, because 3 is not enough.

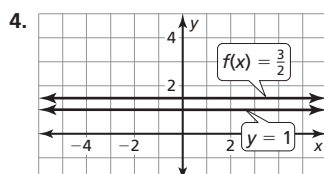
### 1.1–1.2 Quiz (p. 20)

- The function belongs to the family of linear functions. The graph of  $g(x) = \frac{1}{3}x - 1$  is a vertical shrink by a factor of  $\frac{1}{3}$  followed by a vertical translation 1 unit down of the parent linear function.

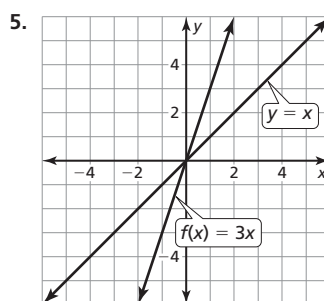
# Chapter 1

2. The function belongs to the family of quadratic functions. The graph of  $g(x) = 2(x + 1)^2$  is a horizontal translation of 1 unit left followed by a vertical stretch by a factor of 2 of the parent quadratic function.

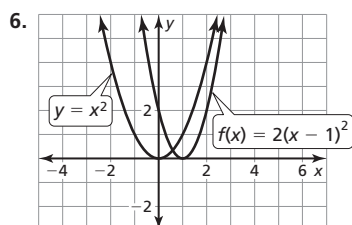
3. The function belongs to the family of absolute value functions. The graph of  $g(x) = |x + 1| - 2$  is a horizontal translation 1 unit left followed by a vertical translation 2 units down of the parent absolute value function.



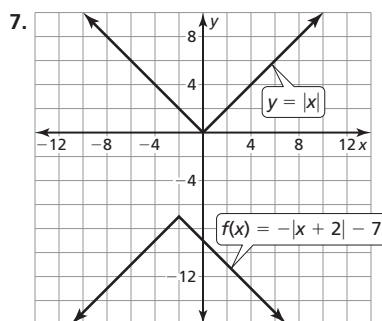
The graph of  $f(x) = \frac{3}{2}$  is a vertical translation  $\frac{1}{2}$  unit up of the parent constant function.



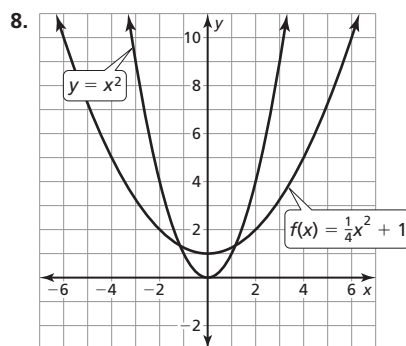
The graph of  $f(x) = 3x$  is a vertical stretch by a factor of 3 of the parent linear function.



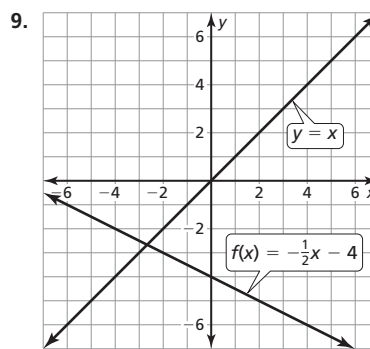
The graph of  $f(x) = 2(x - 1)^2$  is a vertical stretch by a factor of 2 followed by a horizontal translation of 1 unit right of the parent quadratic function.



The graph of  $f(x) = -|x + 2| - 7$  is a reflection in the  $x$ -axis followed by a horizontal translation 2 units left and a vertical translation 7 units down of the parent absolute value function.



The graph of  $f(x) = \frac{1}{4}x^2 + 1$  is a vertical shrink by a factor of  $\frac{1}{4}$  followed by a vertical translation 1 unit up of the parent quadratic function.



The graph of  $f(x) = -\frac{1}{2}x - 4$  is a vertical shrink by a factor of  $\frac{1}{2}$  followed by a reflection in the  $x$ -axis and then a vertical translation 4 units down of the parent linear function.

10. A translation 3 units up is a vertical translation that adds 3 to each output value.

$$\begin{aligned} g(x) &= f(x) + 3 \\ &= (2x + 1) + 3 \\ &= 2x + 4 \end{aligned}$$

The transformed function is  $g(x) = 2x + 4$ .

11. A vertical shrink by a factor of  $\frac{1}{2}$  multiplies each output value by  $\frac{1}{2}$ .

$$\begin{aligned} g(x) &= \frac{1}{2}f(x) \\ &= \frac{1}{2}(-3|x - 4|) \\ &= -\frac{3}{2}|x - 4| \end{aligned}$$

The transformed function is  $g(x) = -\frac{3}{2}|x - 4|$ .

12. A reflection in the  $x$ -axis changes the sign of each output value.

$$\begin{aligned} g(x) &= -f(x) \\ &= -3|x + 5| \end{aligned}$$

The transformed function is  $g(x) = -3|x + 5|$ .

# Chapter 1

13. A translation 4 units left is a horizontal translation that adds 4 to each input value.

$$\begin{aligned} g(x) &= f(x + 4) \\ &= \frac{1}{3}(x + 4) - \frac{2}{3} \\ &= \frac{1}{3}x + \frac{4}{3} - \frac{2}{3} \\ &= \frac{1}{3}x + \frac{2}{3} \end{aligned}$$

The transformed function is  $g(x) = \frac{1}{3}x + \frac{2}{3}$ .

14. A translation 2 units down is a vertical translation that subtracts 2 from each output value and a horizontal shrink by a factor of  $\frac{2}{3}$  multiplies each input value by  $\frac{3}{2}$ .

$$\begin{aligned} g(x) &= f\left(\frac{3}{2}x\right) - 2 \\ &= \frac{3}{2}x - 2 \end{aligned}$$

The transformed function is  $g(x) = \frac{3}{2}x - 2$ .

15. A translation 9 units down is a vertical translation that subtracts 9 from each output value and a reflection in the  $y$ -axis changes the sign of each input value.

$$\begin{aligned} g(x) &= f(-x) - 9 \\ &= -x - 9 \end{aligned}$$

The transformed function is  $g(x) = -x - 9$ .

16. A reflection in the  $x$ -axis changes the sign of each output value, a vertical stretch by a factor of 4 multiplies each output value by 4, a translation 7 units down adds  $-7$  to each output value, and a translation 1 unit right subtracts 1 from each input value.

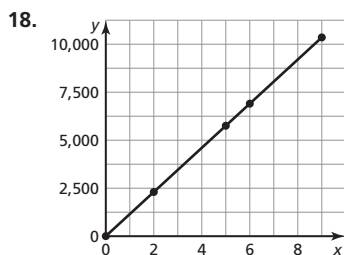
$$\begin{aligned} g(x) &= 4(-f(x - 1)) - 7 \\ &= -4|x - 1| - 7 \end{aligned}$$

The transformed function is  $g(x) = -4|x - 1| - 7$ .

17. A translation 1 unit down is a vertical translation that subtracts 1 from each output value, a translation 2 units left is a horizontal translation that adds 2 to each input value, and a vertical shrink by a factor of  $\frac{1}{2}$  multiplies each output value by  $\frac{1}{2}$ .

$$\begin{aligned} g(x) &= \frac{1}{2}[f(x + 2) - 1] \\ &= \frac{1}{2}|x + 2| - \frac{1}{2} \end{aligned}$$

The transformed function is  $g(x) = \frac{1}{2}|x + 2| - \frac{1}{2}$ .

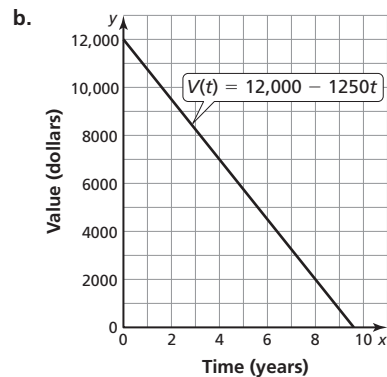


The data appear to lie on a straight line. So, a linear function can be used to model the data. The estimated mileage after 1 year is 13,800 miles.

19. Half the price relates to a vertical shrink by a factor of  $\frac{1}{2}$  and the discount of \$30 relates to a vertical translation 30 units down. The function for the amount that senior citizens pay is given by  $g(x) = \frac{1}{2}(20x + 80) - 30 = 10x + 10$ . So, if a senior citizen camped for 3 days the cost would be  $g(3) = 10(3) + 10 = \$40$ .

## 1.3 Explorations (p. 21)

1. a. A linear function that models the value  $V$  of the copier, where  $t$  is the number of years after it was purchased, is  $V(t) = -1250t + 12,000$ .



The depreciation is called straight line depreciation because the value decreases, or depreciates, at a constant rate.

- c. The slope represents the value the copier decreases each year, which is \$1250.
2. a. The corresponding graph is B. The slope,  $-20$ , is the amount the loan is reduced per week and the  $y$ -intercept is the original amount of the loan, 200.
- b. The corresponding graph is C. The slope, 2, is the amount earned per unit produced per hour and the  $y$ -intercept is the base amount earned per hour, 12.5.
- c. The corresponding graph is A. The slope, 0.565, is the amount paid for each mile driven and the  $y$ -intercept is the amount paid per day for food, 30.
- d. The corresponding graph is D. The slope,  $-100$ , is the value the computer decreased per year and the  $y$ -intercept is the amount paid when the computer was purchased, 750.
3. The  $y$ -intercept of a linear function represents the original or base amount, and the slope represents the change over time.

# Chapter 1

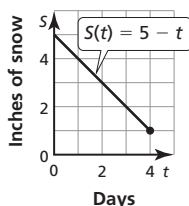
4. *Sample answer:* the amount of snow on the ground over a five-day period

a.

	A	B
1	Day	Inches of snow
2	0	5
3	1	4
4	2	3
5	3	2
6	4	1

b. A function that models the depreciation is  $S(t) = 5 - t$ , where  $t$  is the time (in days) and  $S(t)$  is the snowfall amount (in inches).

c.



## 1.3 Monitoring Progress (pp. 22–25)

1. The slope of the line is  $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{18 - 15}{0 - 10} = -\frac{3}{10}$

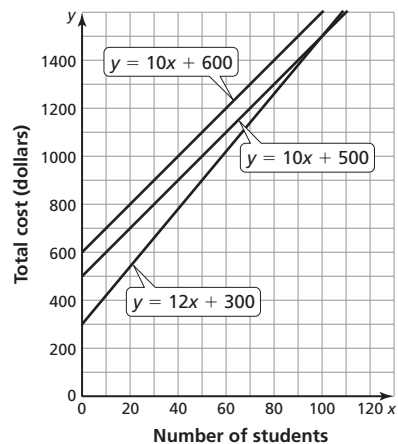
and the  $y$ -intercept is 18. So the equation of the line is

$y = -\frac{3}{10}x + 18$ . The balance decreases \$300 per payment

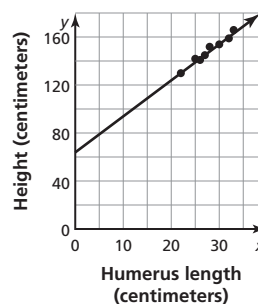
and the initial amount is \$18,000. The amount remaining

after 36 months is  $y = -\frac{3}{10}(36) + 18 = 7.2$ , or \$7200.

2. In this situation, the rental fee is \$500 and the cost for the 140 students is \$1400. An equation that models the cost for using Maple Ridge is  $y = 10x + 500$ , where  $y$  is the cost and  $x$  is the number of students. So, Maple Ridge will always cost less than Sunview Resort because the rental cost is less for Maple Ridge with the same cost per student. Maple Ridge will cost less if there are more than 100 students when compared to the cost of Lakeside Inn.



3. a.



The data show a linear relationship. Use the points (22, 130) and (30, 154) to write an equation of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{154 - 130}{30 - 22} = \frac{24}{8} = 3$$

$$y - y_1 = m(x - x_1)$$

$$y - 130 = 3(x - 22)$$

$$y - 130 = 3x - 66$$

$$y = 3x + 64$$

So, an equation of a line is  $y = 3x + 64$ . Use this equation to estimate the height of the woman.

$$y = 3(40) + 64 = 184$$

The approximate height of a woman with a 40-centimeter humerus is 184 centimeters.

b. Using the *linear regression* feature on a graphing calculator, the line of best fit for the data is given by the equation  $y = 3.05x + 63.53$ . A female that has a humerus 40 centimeters long will have an approximate height of  $y = 3.05(40) + 63.53 \approx 185.53$  centimeters. The result in part (a) is relatively close to the height given by the linear regression line.

## 1.3 Exercises (pp. 26–28)

### Vocabulary and Core Concept Check

1. The linear equation  $y = \frac{1}{2}x + 3$  is written in slope-intercept form.

2. When a line of best fit has a correlation coefficient of  $-0.98$ , this means that the slope is negative.

### Monitoring Progress and Modeling with Mathematics

3. From the graph, the slope is  $m = \frac{2}{10} = 0.2$  and the  $y$ -intercept is  $b = 0$ . Using slope-intercept form, an equation of the line is

$$y = mx + b$$

$$= 0.2x + 0.$$

The equation is  $y = 0.2x$ . The slope indicates that the tip increases \$0.20 for every dollar spent on the meal.



# Chapter 1

4. From the graph, the slope is  $m = \frac{-3}{90} = -\frac{1}{30}$  and the y-intercept is  $b = 12$ . Using slope-intercept form, an equation of the line is

$$y = mx + b$$

$$y = -\frac{1}{30}x + 12.$$

The equation is  $y = -\frac{1}{30}x + 12$ . The slope indicates that the amount of fuel in the gasoline tank decreases by  $\frac{1}{30}$  gallon per mile driven.

5. From the graph, the slope is  $m = \frac{100}{2} = 50$  and the y-intercept is  $b = 100$ . Using slope-intercept form, an equation of the line is

$$y = mx + b$$

$$y = 50x + 100.$$

The equation is  $y = 50x + 100$ . The slope indicates that the savings account balance increases by \$50 per week.

6. From the graph, the slope is  $m = \frac{6}{4} = 1.5$  and the y-intercept is  $b = 0$ . Using slope-intercept form, an equation of the line is

$$y = mx + b$$

$$y = 1.5x + 0.$$

The equation is  $y = 1.5x$ . The slope indicates that the height of the tree increases by 1.5 feet per year.

7. From the graph, the slope is  $m = \frac{165 - 55}{3 - 1} = \frac{110}{2} = 55$

and the y-intercept is  $b = 0$ . Using slope-intercept form, an equation of the line is

$$y = mx + b$$

$$y = 55x + 0.$$

The equation is  $y = 55x$ . The slope indicates that the typing rate is 55 words per minute.

8. From the graph, the slope is  $m = \frac{300 - 180}{3 - 5} = \frac{120}{-2} = -60$ .

Using slope-intercept form, an equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y - 300 = -60(x - 3)$$

$$y = -60x + 480.$$

The equation is  $y = -60x + 480$ . The slope indicates that the water level in the swimming pool decreases by 60 cubic feet per hour.

9. **1. Understand the Problem** You are given an equation that represents the total cost for an advertisement at the Greenville Journal and a table of values showing total costs for advertisements at the Daily Times. You need to compare costs.

- 2. Make a Plan** Write an equation that models the total cost of advertisements at the Daily Times. Then compare the slopes to determine which newspaper charges less per line. Finally, equate the cost expressions and solve to determine the number of lines for which the total costs are equal.

- 3. Solve the Problem** The slope is  $m = \frac{30 - 27}{5 - 4} = 3$ .

Using point-slope form, the equation to represent the total cost for advertisements at Daily Times is

$$y - y_1 = m(x - x_1)$$

$$y - 27 = 3(x - 4)$$

$$y = 3x + 15.$$

Equate the cost expressions and solve.

$$2x + 20 = 3x + 15$$

$$5 = x$$

Comparing the slopes of the equations, the Greenville Journal costs \$2 per line, which is less than the \$3 per line that the Daily Times charges. The total costs are the same if there are 5 lines in an advertisement.

10. **1. Understand the Problem** You have to write an equation that represents the linear relationship between Fahrenheit and Celsius and calculate several temperatures.

- 2. Make a Plan** Use the point-slope form to write an equation that gives degrees Fahrenheit in terms of degrees Celsius. Then, substitute the given temperature for  $x$  in the equation to calculate  $y$ . Finally, rewrite the equation by solving for  $x$ .

- 3. Solve the Problem** The slope is

$$m = \frac{32 - 212}{0 - 100} = \frac{-180}{-100} = \frac{9}{5}.$$

Using point-slope form, the equation is

$$y - y_1 = m(x - x_1)$$

$$y - 32 = \frac{9}{5}(x - 0)$$

$$y = \frac{9}{5}x + 32.$$

- (a) An equation that gives degrees Fahrenheit in terms of degrees Celsius is  $y = \frac{9}{5}x + 32$ .

- (b) Substitute 22 for  $x$ .

$$y = \frac{9}{5}(22) + 32$$

$$= 71.6$$

The outside temperature is 71.6°F.



# Chapter 1

(c) Solve the equation for  $x$ .

$$y = \frac{9}{5}x + 32$$

$$y - 32 = \frac{9}{5}x$$

$$x = \frac{5}{9}(y - 32)$$

An equation that gives degrees Celsius in terms of degrees Fahrenheit is  $x = \frac{5}{9}(y - 32)$ .

(d) Substitute 83 for  $y$ .

$$x = \frac{5}{9}(83 - 32)$$

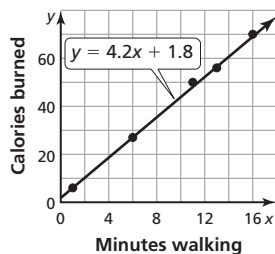
$$\approx 28.33$$

The temperature of the hotel pool is about 28.33°F.

11. The slope was correctly used in the situation, however the intercept was not used correctly. In the situation the starting balance is \$100, so after 7 years the balance is \$170.
12. The slope is incorrect in the situation. The slope is 11, so the income is \$11 per hour.
13. *Sample answer:*

**Step 1** Draw a scatter plot of the data. The data show a linear relationship.

**Step 2** Sketch the line that most closely appears to fit the data. One possible line is shown.



**Step 3** Choose two points on the line. For the line shown, you might choose (1, 6) and (6, 27).

**Step 4** Write the equation of the line. First, find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{27 - 6}{6 - 1} = \frac{21}{5} = 4.2$$

Use the point-slope form to write an equation.

Use  $(x_1, y_1) = (1, 6)$ .

$$y - y_1 = m(x - x_1)$$

$$y - 6 = 4.2(x - 1)$$

$$y - 6 = 4.2x - 4.2$$

$$y = 4.2x + 1.8$$

Use the equation to estimate the number of calories burned.

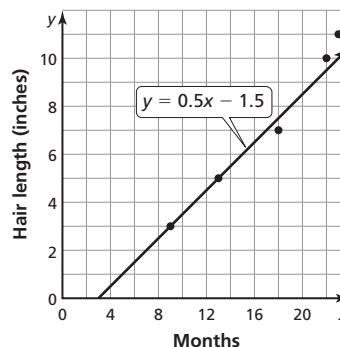
$$y = 4.2(15) + 1.8 = 64.8$$

The approximate number of calories burned when you walk for 15 minutes is 64.8 calories.

14. *Sample answer:*

**Step 1** Draw a scatter plot of the data. The data show a linear relationship.

**Step 2** Sketch the line that most closely appears to fit the data. One possible line is shown.



**Step 3** Choose two points on the line. For the line shown, you might choose (9, 3) and (13, 5).

**Step 4** Write the equation of the line. First, find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{13 - 9} = \frac{2}{4} = 0.5$$

Use point-slope form to write an equation.

Use  $(x_1, y_1) = (9, 3)$ .

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 0.5(x - 9)$$

$$y - 3 = 0.5x - 4.5$$

$$y = 0.5x - 1.5$$

Use the equation to estimate the length of hair.

$$y = 0.5(15) - 1.5 = 6$$

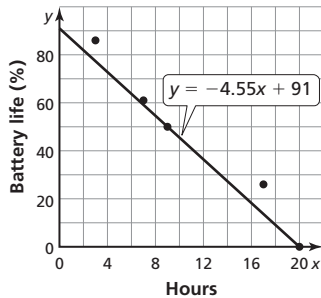
The approximate hair length after 15 months is 6 inches.

# Chapter 1

15. *Sample answer:*

**Step 1** Draw a scatter plot of the data. The data show a linear relationship.

**Step 2** Sketch the line that most closely appears to fit the data. One possible line is shown.



**Step 3** Choose two points on the line. For the line shown, you might choose (9, 50) and (20, 0).

**Step 4** Write the equation of the line. First, find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 50}{20 - 9} = \frac{-50}{11} \approx -4.55$$

Use point-slope form to write an equation.

Use  $(x_1, y_1) = (20, 0)$ .

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -4.55(x - 20)$$

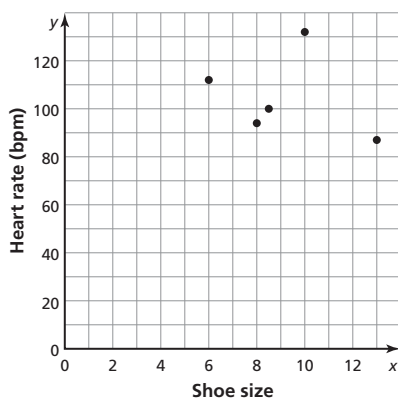
$$y = -4.55x + 91$$

Use the equation to estimate the battery life.

$$y = -4.55(15) + 91 = 22.75$$

The approximate battery life after 15 hours is 23%.

16. Draw a scatter plot of the data. The data does not show a linear relationship.



17. Enter the data into two lists. Use the *linear regression* feature. The line of best fit is  $y = 380.03x + 11,290$ .

Use the equation to estimate the annual tuition cost in 2020 ( $x = 15$ ).

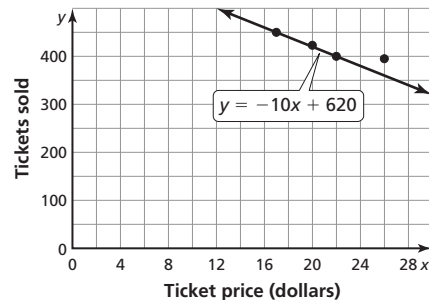
$$\begin{aligned} y &= 380.03(15) + 11,290 \\ &= 16,990.45 \end{aligned}$$

The approximate average annual tuition cost in the year 2020 is \$16,990.45. The annual tuition increases by about \$380 each year and the cost of tuition in 2005 is about \$11,290.

18. *Sample answer:*

**Step 1** Draw a scatter plot of the data. The data show a linear relationship.

**Step 2** Sketch the limit that most closely appears to fit the data. One possible line is shown.



**Step 3** Choose two points on the line. For the line shown, you might choose (17, 450) and (22, 400).

**Step 4** Write the equation of the line. First, find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{400 - 450}{22 - 17} = \frac{-50}{5} = -10$$

Use point-slope form to write an equation.

Use  $(x_1, y_1) = (17, 450)$ .

$$y - y_1 = m(x - x_1)$$

$$y - 450 = -10(x - 17)$$

$$y - 450 = -10x + 170$$

$$y = -10x + 620$$

Use the equation to estimate the number of tickets sold.

$$\begin{aligned} y &= -10(85) + 620 \\ &= -230 \end{aligned}$$

The approximate number of tickets sold when the price is \$85 is  $-230$ . This does not seem reasonable because the number of tickets sold is less than zero.

19. Enter the data into two lists. Use the *linear regression* feature. The line of best fit is  $y = 0.42x + 1.44$ .

The correlation coefficient is  $r \approx 0.61$ . This represents a weak positive correlation.

20. Enter the data into two lists. Use the *linear regression* feature. The line of best fit is  $y = 0.88x + 1.69$ .

The correlation coefficient is  $r \approx 0.88$ . This represents a strong positive correlation.

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21. Enter the data into two lists. Use the *linear regression* feature. The line of best fit is  $y = -0.45x + 4.26$ .  
The correlation coefficient is  $r \approx -0.67$ . This represents a weak negative correlation.
22. Enter the data into two lists. Use the *linear regression* feature. The line of best fit is  $y = -1.04x + 5.68$ .  
The correlation coefficient is  $r \approx -0.93$ . This represents a strong negative correlation.
23. Enter the data into two lists. Use the *linear regression* feature. The line of best fit is  $y = 0.61x + 0.10$ .  
The correlation coefficient is  $r \approx 0.95$ . This represents a strong positive correlation.
24. Enter the data into two lists. Use the *linear regression* feature. The line of best fit is  $y = -0.48x + 4.08$ .  
The correlation coefficient is  $r \approx -0.91$ . This represents a strong negative correlation.
25. a. *Sample answer:* height and weight; temperature and ice cream sales; Correlation is positive because as the first goes up, so does the second.  
b. *Sample answer:* miles driven and gas remaining; hours used and battery life remaining; Correlation is negative because as the first goes up, the second goes down.  
c. *Sample answer:* age and length of hair; typing speed and shoe size; There is no relationship between the first and second.
26. a. To determine the slope of the line, use the points (0, 30) and (24, 0). So, the slope is  

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 30}{24 - 0} = \frac{-30}{24} = -\frac{5}{4}$$
 The slope represents the amount the loan is reduced each month.  
 b. The domain of the function is  $0 \leq x \leq 24$  and the range is  $0 \leq y \leq 30$ . The domain represents the term of the balance of the loan from start to finish and the range represents the amount left to pay on the loan.  
 c. The equation of the line that models the amount left to pay on the loan is  $y = -\frac{5}{4}x + 30$ . The amount left to pay after 12 months is  $y = -\frac{5}{4}(12) + 30 = 15$ , or \$1500.
27. Your friend is incorrect. Because  $r = 0.3$  is closer to 0 than 1, the line of best fit will not make good predictions.

28. Consider the possible locations of the three points. (1)  $A$  and  $B$  are two different points on the line and  $C$  also lies on the line. (2)  $A$  and  $B$  are two different points of the line and  $C$  does not lie on the line.

*Sample answer:* Let  $A$  be (0, 4) and  $B$  be (4, 0). Then a point that is the same distance to  $A$  and to  $B$  is the midpoint of the line segment between  $A$  and  $B$ , which is

$$\left( \frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right) = \left( \frac{0 + 4}{2}, \frac{4 + 0}{2} \right) = (2, 2).$$

So,  $C$  is (2, 2).

Because points  $A$ ,  $B$ , and  $C$  all lie on the same line, the equation of the line through  $A$  and  $C$  and the equation of the line through  $B$  and  $C$  are both  $y = -x + 4$ .

Next, let  $C$  be a point that is not on the line  $y = -x + 4$ . The origin is 4 units from  $A$  and 4 units from  $B$ . So, let  $C$  be (0, 0). The equation of the line through  $A$  and  $C$  is  $x = 0$ . The equation of the line through  $B$  and  $C$  is  $y = 0$ .

29. As  $x$  increases,  $y$  increases, so  $z$  decreases. Therefore, the correlation between  $x$  and  $z$  is negative.
30. The equation is  $D$ . The equation of a line that is perpendicular to the graph of  $y = -4x + 1$  and passes through (8, -5) has a slope of  $m = \frac{1}{4}$ . So, the equation is given by

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-5) &= \frac{1}{4}(x - 8) \\ y + 5 &= \frac{1}{4}x - 2 \\ y &= \frac{1}{4}x - 7. \end{aligned}$$

# Chapter 1

31. The path that will give the shortest distance to the river is given by the line that is perpendicular to the graph of  $y = 3x + 2$  and that passes through the point  $(2, 1)$ . The slope of a perpendicular line to  $y = 3x + 2$  is  $m = -\frac{1}{3}$ . So, the equation of the line perpendicular to the graph of  $y = 3x + 2$  and passing through the point  $(2, 1)$  is given by

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{1}{3}(x - 2)$$

$$y - 1 = -\frac{1}{3}x + \frac{2}{3}$$

$$y = -\frac{1}{3}x + \frac{5}{3}$$

The point where the two lines intersect has an  $x$ -value given by

$$3x + 2 = -\frac{1}{3}x + \frac{5}{3}$$

$$9x + 6 = -x + 5$$

$$10x + 6 = 5$$

$$10x = -1$$

$$x = -\frac{1}{10}$$

So, the  $y$ -value of the intersection is  $y = 3\left(-\frac{1}{10}\right) + 2 = \frac{17}{10}$ .

Thus, the closest point on the line  $y = 3x + 2$  to the point  $(2, 1)$  is the point  $\left(-\frac{1}{10}, \frac{17}{10}\right)$ . Use the Distance Formula to find the shortest distance to the river.

$$d = \sqrt{\left[2 - \left(-\frac{1}{10}\right)\right]^2 + \left[1 - \frac{17}{10}\right]^2} = \frac{7}{\sqrt{10}} \approx 2.2$$

So, you must travel about 2.2 miles.

32. a. A positive correlation does make sense because the number of computers per capita and the average life expectancy have both increased over time, which would relate to a positive slope.
- b. It is not reasonable to conclude that giving residents of a country computers will lengthen their lives. There is a correlation, but there is not a causation between the two quantities.

## Maintaining Mathematical Proficiency

33. Solve the system using elimination.

$$\begin{cases} 3x + y = 7 & \text{Equation 1} \\ -2x - y = 9 & \text{Equation 2} \end{cases}$$

There is no need to change coefficients because the variable  $y$  differs only in sign.

$$3x + y = 7$$

$$-2x - y = 9$$

$$x = 16$$

So,  $x = 16$ . Now, back-substitute  $x = 16$  into one of the original equations of the system that contains the variable  $x$ . By back-substituting  $x = 16$  into Equation 1, you can solve for  $y$ .

$$3x + y = 7$$

$$3(16) + y = 7$$

$$48 + y = 7$$

$$y = -41$$

So, the solution is  $x = 16$  and  $y = -41$ .

34. Solve the system using elimination.

$$\begin{cases} 4x + 3y = 2 & \text{Equation 1} \\ 2x - 3y = 1 & \text{Equation 2} \end{cases}$$

There is no need to change coefficients because the variable  $y$  differs only in sign.

$$4x + 3y = 2$$

$$2x - 3y = 1$$

$$6x = 3$$

Solving the equation  $6x = 3$  produces  $x = \frac{1}{2}$ . Now back-substitute  $x = \frac{1}{2}$  into one of the original equations of the system that contains the variable  $y$ . By back-substituting  $x = \frac{1}{2}$  into Equation 1, you can solve for  $y$ .

$$4x + 3y = 2$$

$$4\left(\frac{1}{2}\right) + 3y = 2$$

$$2 + 3y = 2$$

$$3y = 0$$

$$y = 0$$

So, the solution is  $x = \frac{1}{2}$  and  $y = 0$ .

# Chapter 1

35. Solve the system using substitution.

$$\begin{cases} 2x + 2y = 3 & \text{Equation 1} \\ x = 4y - 1 & \text{Equation 2} \end{cases}$$

Substitute  $4y - 1$  from Equation 2 for  $x$  into Equation 1 and solve the resulting single-variable equation for  $y$ .

$$\begin{aligned} 2x + 2y &= 3 \\ 2(4y - 1) + 2y &= 3 \\ 8y - 2 + 2y &= 3 \\ 10y - 2 &= 3 \\ 10y &= 5 \\ y &= \frac{1}{2} \end{aligned}$$

Finally, you can solve for  $x$  by back-substituting  $y = \frac{1}{2}$  into the equation  $x = 4y - 1$ .

$$x = 4y - 1$$

$$x = 4\left(\frac{1}{2}\right) - 1$$

$$x = 2 - 1$$

$$x = 1$$

So, the solution is  $x = 1$  and  $y = \frac{1}{2}$ .

36. Solve the system using substitution.

$$\begin{cases} y = 1 + x & \text{Equation 1} \\ 2x + y = -2 & \text{Equation 2} \end{cases}$$

Substitute  $1 + x$  from Equation 1 for  $y$  into Equation 2 and solve the resulting single-variable equation for  $x$ .

$$\begin{aligned} 2x + y &= -2 \\ 2x + (1 + x) &= -2 \\ 3x + 1 &= -2 \\ 3x &= -3 \\ x &= -1 \end{aligned}$$

Finally, you can solve for  $y$  by back-substituting  $x = -1$  into the equation  $y = 1 + x$ .

$$y = 1 + x$$

$$y = 1 + (-1)$$

$$y = 0$$

So, the solution is  $x = -1$  and  $y = 0$ .

37. Solve the system using elimination.

$$\begin{cases} \frac{1}{2}x + 4y = 4 & \text{Equation 1} \\ 2x - y = 1 & \text{Equation 2} \end{cases}$$

You can obtain coefficients of  $x$  that differ only in sign by multiplying Equation 2 by 4.

$$\begin{aligned} \frac{1}{2}x + 4y &= 4 \rightarrow \frac{1}{2}x + 4y = 4 \\ 2x - y = 1 &\rightarrow 8x - 4y = 4 \\ \frac{17}{2}x &= 8 \end{aligned}$$

Solving the equation  $\frac{17}{2}x = 8$  produces  $x = \frac{16}{17}$ . Now,

back-substitute  $x = \frac{16}{17}$  into one of the original or revised equations of the system that contains the variable  $x$ . By back-substituting  $x = \frac{16}{17}$  into Equation 2, you can solve for  $y$ .

$$\begin{aligned} 2\left(\frac{16}{17}\right) - y &= 1 \\ \frac{32}{17} - y &= 1 \\ -y &= -\frac{15}{17} \\ y &= \frac{15}{17} \end{aligned}$$

So, the solution is  $x = \frac{16}{17}$  and  $y = \frac{15}{17}$ .

38. Solve the system using substitution.

$$\begin{cases} y = x - 4 & \text{Equation 1} \\ 4x + y = 26 & \text{Equation 2} \end{cases}$$

Substitute  $x - 4$  from Equation 1 for  $y$  into Equation 2 and solve the resulting single-variable equation for  $x$ .

$$\begin{aligned} 4x + y &= 26 \\ 4x + (x - 4) &= 26 \\ 5x - 4 &= 26 \\ 5x &= 30 \\ x &= 6 \end{aligned}$$

Finally, you can solve for  $y$  by back-substituting  $x = 6$  into the equation  $y = x - 4$ .

$$y = x - 4$$

$$y = 6 - 4$$

$$y = 2$$

So, the solution is  $x = 6$  and  $y = 2$ .

## 1.4 Explorations (p. 29)

- The system matches graph B; The two equations have the same slope but different  $y$ -intercepts, so their graphs are parallel lines. The system is inconsistent.
  - The system matches graph C; The two equations have different slopes and different  $y$ -intercepts, so there is a point that they will intersect. The system is consistent.
  - The system matches graph A; The two equations have the same slope and same  $y$ -intercept, so they will produce the same graph. The system is consistent.

# Chapter 1

2. a. Solve  $x - y = 1$  for  $x$ , which gives  $x = y + 1$ . Then substitute  $y + 1$  for  $x$  into  $2x + y = 5$  and solve the resulting equation for  $y$ .

$$\begin{aligned}2x + y &= 5 \\2(y + 1) + y &= 5 \\2y + 2 + y &= 5 \\3y + 2 &= 5 \\3y &= 3 \\y &= 1\end{aligned}$$

So,  $y = 1$ . Back-substitute  $y = 1$  into  $x = y + 1$  giving  $x = 1 + 1 = 2$ . Thus, the solution of the system is  $(2, 1)$ . The solution found graphically is  $(2, 1)$ , the point where the two lines intersect.

- b. Solve  $x + 3y = 1$  for  $x$ , which gives  $x = -3y + 1$ . Then substitute  $-3y + 1$  for  $x$  into  $-x + 2y = 4$  and solve the resulting equation for  $y$ .

$$\begin{aligned}-x + 2y &= 4 \\-(-3y + 1) + 2y &= 4 \\3y - 1 + 2y &= 4 \\5y - 1 &= 4 \\5y &= 5 \\y &= 1\end{aligned}$$

So,  $y = 1$ . Back-substitute  $y = 1$  into  $x = -3y + 1$  giving  $x = -3(1) + 1 = -2$ . Thus, the solution of the system is  $(-2, 1)$ . The solution found graphically is  $(-2, 1)$ , the point where the two lines intersect.

- c. Solve  $x + y = 0$  for  $x$ , which gives  $x = -y$ . Then substitute  $-y$  for  $x$  into  $3x + 2y = 1$  and solve the resulting equation for  $y$ .

$$\begin{aligned}3x + 2y &= 1 \\3(-y) + 2y &= 1 \\-3y + 2y &= 1 \\-y &= 1 \\y &= -1\end{aligned}$$

So,  $y = -1$ . Back-substitute  $y = -1$  into  $x = -y$  giving  $x = 1$ . Thus, the solution of the system is  $(1, -1)$ . The solution found graphically is  $(1, -1)$ , the point where the two lines intersect.

3. If the graph of a system is parallel lines, then there is no solution. If the graph of a system shows two lines that intersect or coincide, then there is at least one solution.
4. Use elimination to eliminate a variable in equations 1 and 2. Eliminate the same variable from equations 1 and 3. Using the two new resulting equations, solve that system for both variables. Substitute those values into one of the original equations to find the third variable.

5. Add Equation 1 to Equation 3.

$$\begin{array}{r}x + y + z = 1 \\-x - y + z = -1 \\ \hline 2z = 0\end{array}$$

So,  $2z = 0$ , or  $z = 0$ . This produces the system

$$\begin{aligned}x + y &= 1 \\x - y &= 3 \\-x - y &= -1.\end{aligned}$$

Now use the new forms of Equation 1 and Equation 2 with the elimination method.

$$\begin{aligned}x + y &= 1 \\x - y &= 3 \\ \hline 2x &= 4\end{aligned}$$

So,  $2x = 4$ , or  $x = 2$ . Finally, substitute the values of  $x$  and  $z$  into Equation 1 to solve for  $y$ .

$$\begin{aligned}x + y + z &= 1 \\2 + y + 0 &= 1 \\y &= -1\end{aligned}$$

So, the solution of the linear system is  $(2, -1, 0)$ .

## 1.4 Monitoring Progress (pp. 32–33)

1. **Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r}x - 2y + z = -11 \\-x + 2y + 4z = -9 \\ \hline 5z = -20 \\z = -4\end{array}$$

$$\begin{array}{r} -3x + 6y - 3z = 33 \\ 3x + 2y - z = 7 \\ \hline 8y - 4z = 40 \\ 8y - 4(-4) = 40 \\ 8y + 16 = 40 \\ y = 3\end{array}$$

- Step 2** Substitute  $y = 3$  and  $z = -4$  into an original equation and solve for  $x$ .

$$\begin{aligned}x - 2y + z &= -11 \\x - 2(3) + (-4) &= -11 \\x - 6 - 4 &= -11 \\x &= -1\end{aligned}$$

The solution is  $(-1, 3, -4)$ .

2. **Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r}-4x - 4y + 4z = 4 \\4x + 4y - 4z = -2 \\ \hline 0 = 2\end{array}$$

Because you obtain a false equation, the original system has no solution.

# Chapter 1

3. **Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} x + y + z = 8 \\ x - y + z = 8 \\ \hline 2x + 2z = 16 \\ x + z = 8 \\ x + y + z = 8 \\ -2x - y - 2z = -16 \\ \hline -x - z = -8 \end{array}$$

- Step 2** Solve the new linear system for both of its variables.

$$\begin{array}{r} x + z = 8 \\ -x - z = -8 \\ \hline 0 = 0 \end{array}$$

Because you obtain the identity  $0 = 0$ , the system has infinitely many solutions.

**Step 3** Solve new Equation 1 for  $z$  to obtain  $z = 8 - x$ . Then substitute  $8 - x$  for  $z$  in original Equation 1 to obtain  $y = 0$ . A solution of the system can be represented by any ordered triple of the form  $(x, 0, 8 - x)$ .

4. The result of Example 3 is that the solution can be expressed as  $(x, x + 3, 0)$ . In this case,  $y = x + 3$ . So to write the solution in terms of  $y$  rather than  $x$ , solve for  $x$  in  $y = x + 3$ , which gives  $x = y - 3$ . Thus, the solution can be expressed as  $(y - 3, y, 0)$ .

5. The system of equations is

$$\begin{array}{ll} y = x & \text{Equation 1} \\ x + y + z = 10,000 & \text{Equation 2} \\ 75x + 55y + 30z = 356,000 & \text{Equation 3} \end{array}$$

Rewrite the system as a linear system in two variables by substituting  $x$  for  $y$  in Equations 2 and 3.

$$\begin{array}{ll} x + y + z = 10,000 & 75x + 55y + 30z = 356,000 \\ x + x + z = 10,000 & 75x + 55(x) + 30z = 356,000 \\ 2x + z = 10,000 & 130x + 30z = 356,000 \end{array}$$

Solve the new linear system for both of its variables.

$$\begin{array}{r} -60x - 30z = 300,000 \\ 130x + 30z = 356,000 \\ \hline 70x = 56,000 \\ x = 800 \\ y = 800 \\ z = 8400 \end{array}$$

Of the 10,000 tickets sold, 8400 are lawn seats. So, there are  $17,000 - 8400 = 8600$  lawn seats still available.

## 1.4 Exercises (pp. 34–36)

### Vocabulary and Core Concept Check

1. The solution of a system of three linear equations is expressed as an ordered triple.

2. You know that a linear system in three variables has infinitely many solutions when you obtain a result of  $0 = 0$  when solving.

### Monitoring Progress and Modeling with Mathematics

3. **Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} x + y - 2z = 5 \\ -x + 2y + z = 2 \\ \hline 3y - z = 7 \\ -2x + 4y + 2z = 4 \\ 2x + 3y - z = 9 \\ \hline 7y + z = 13 \end{array}$$

- Step 2** Solve the new linear system for both of its variables.

$$\begin{array}{r} 3y - z = 7 \\ 7y + z = 13 \\ \hline 10y = 20 \\ y = 2 \\ 7y + z = 13 \\ 7(2) + z = 13 \\ z = -1 \end{array}$$

- Step 3** Substitute  $y = 2$  and  $z = -1$  into an original equation and solve for  $x$ .

$$\begin{array}{r} x + y - 2z = 5 \\ x + 2 - 2(-1) = 5 \\ x = 1 \end{array}$$

The solution is  $(1, 2, -1)$ .

4. **Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} x + 4y - 6z = -1 \\ -x + 2y - 4z = 5 \\ \hline 6y - 10z = 4 \\ 2x - y + 2z = -7 \\ -2x + 4y - 8z = 10 \\ \hline 3y - 6z = 3 \end{array}$$

- Step 2** Solve the new linear system for both of its variables.

$$\begin{array}{r} 6y - 10z = 4 \\ -6y + 12z = -6 \\ \hline 2z = -2 \\ z = -1 \\ y = -1 \end{array}$$

- Step 3** Substitute  $y = -1$  and  $z = -1$  into an original equation and solve for  $x$ .

$$\begin{array}{r} x + 4y - 6z = -1 \\ x + 4(-1) - 6(-1) = -1 \\ x - 4 + 6 = -1 \\ x = -3 \end{array}$$

The solution is  $(-3, -1, -1)$ .

# Chapter 1

**5. Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} 2x + y - z = 9 \\ 5x + 7y + z = 4 \\ \hline 7x + 8y = 13 \\ 4x + 2y - 2z = 18 \\ -x + 6y + 2z = -17 \\ \hline 3x + 8y = 1 \end{array}$$

**Step 2** Solve the new linear system for both of its variables.

$$\begin{array}{r} 7x + 8y = 13 \\ -3x - 8y = -1 \\ \hline 4x = 12 \\ x = 3 \\ y = -1 \end{array}$$

**Step 3** Substitute  $x = 3$  and  $y = -1$  into an original equation and solve for  $z$ .

$$\begin{array}{r} 2x + y - z = 9 \\ 2(3) + (-1) - z = 9 \\ 6 - 1 - z = 9 \\ z = -4 \end{array}$$

The solution is  $(3, -1, -4)$ .

**6. Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} 3x + 2y - z = 8 \\ -3x + 4y + 5z = -14 \\ \hline 6y + 4z = -6 \\ -3x + 4y + 5z = -14 \\ 3x - 9y + 12z = -42 \\ \hline -5y + 17z = -56 \end{array}$$

**Step 2** Solve the new linear system for both of its variables.

$$\begin{array}{r} 30y + 20z = -30 \\ -30y + 102z = -336 \\ \hline 122z = -366 \\ z = -3 \\ y = 1 \end{array}$$

**Step 3** Substitute  $z = -3$  and  $y = 1$  into an original equation and solve for  $x$ .

$$\begin{array}{r} x - 3y + 4z = -14 \\ x - 3(1) + 4(-3) = -14 \\ x - 3 - 12 = -14 \\ x = 1 \end{array}$$

The solution is  $(1, 1, -3)$ .

**7. Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} 2x + 2y + 5z = -1 \\ -2x + y - z = -2 \\ \hline 3y + 4z = -3 \\ 2x + 2y + 5z = -1 \\ -2x - 4y + 3z = -14 \\ \hline -2y + 8z = -15 \end{array}$$

**Step 2** Solve the new linear system for both of its variables.

$$\begin{array}{r} -6y - 8z = 6 \\ -2y + 8z = -15 \\ \hline -8y = -9 \\ y = \frac{9}{8} \\ z = -\frac{51}{32} \end{array}$$

**Step 3** Substitute  $y = \frac{9}{8}$  and  $z = -\frac{51}{32}$  into an original equation and solve for  $x$ .

$$\begin{array}{r} 2x - y + z = 2 \\ 2x - \frac{9}{8} + \left(-\frac{51}{32}\right) = 2 \\ x = \frac{151}{64} \end{array}$$

The solution is  $\left(\frac{151}{64}, \frac{9}{8}, -\frac{51}{32}\right)$ .

**8. Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} 3x + 2y - 3z = -2 \\ 7x - 2y + 5z = -14 \\ \hline 10x + 2z = -16 \\ 5x + z = -8 \\ 2x + 4y + z = 6 \\ 14x - 4y + 10z = -28 \\ \hline 16x + 11z = -22 \end{array}$$

**Step 2** Solve the new linear system for both of its variables.

$$\begin{array}{r} 16x + 11z = -22 \\ -55x - 11z = 88 \\ \hline -39x = 66 \\ x = -\frac{22}{13} \\ z = \frac{6}{13} \end{array}$$

**Step 3** Substitute  $x = -\frac{22}{13}$  and  $z = \frac{6}{13}$  into an original equation and solve for  $y$ .

$$\begin{array}{r} 2x + 4y + z = 6 \\ 2\left(-\frac{22}{13}\right) + 4y + \frac{6}{13} = 6 \\ y = \frac{29}{13} \end{array}$$

The solution is  $\left(-\frac{22}{13}, \frac{29}{13}, \frac{6}{13}\right)$ .



# Chapter 1

9. The entire second equation should be multiplied by 4, not just the  $x$ -term.

$$\begin{array}{r} 4x - y + 2z = -18 \\ -4x + 8y + 4z = 44 \\ \hline 7y + 6z = 26 \end{array}$$

10. The entire first equation should be multiplied by 3, not just one side.

$$\begin{array}{r} 12x - 3y + 6z = -54 \\ 3x + 3y - 4z = 44 \\ \hline 15x + 2z = -10 \end{array}$$

11. **Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} -6x + 2y - 4z = -8 \\ 6x - 2y + 4z = -8 \\ \hline 0 = -16 \end{array}$$

Because you obtain a false equation, the original system has no solution.

12. **Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} 5x + y - z = 6 \\ x + y + z = 2 \\ \hline 6x + 2y = 8 \end{array}$$

**Step 2** Solve the new linear system for both of its variables.

$$\begin{array}{r} -12x - 4y = -16 \\ 12x + 4y = 10 \\ \hline 0 = -6 \end{array}$$

Because you obtain a false equation, the original system has no solution.

13. **Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} -x - 3y + z = -2 \\ x + y - z = 0 \\ \hline -2y = -2 \\ -3x - 3y + 3z = 0 \\ 3x + 2y - 3z = -1 \\ \hline -y = -1 \end{array}$$

**Step 2** You obtain a system in one variable. Solve the new linear system for  $y$ .

$$\begin{array}{r} -2y = -2 \\ -y = -1 \\ \hline -3y = -3 \\ y = 1 \end{array}$$

The system has infinitely many solutions.

**Step 3** Solve Equation 2 for  $x$  to obtain  $x = z - 1$ . A solution of the system can be represented by any ordered triple of the form  $(z - 1, 1, z)$ .

14. **Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} 2x + 4y - 2z = 6 \\ -2x - y + z = -1 \\ \hline 3y - z = 5 \\ -6x - 3y + 3z = -3 \\ 6x - 3y - z = -7 \\ \hline -6y + 2z = -10 \end{array}$$

**Step 2** Solve the new linear system for both of its variables.

$$\begin{array}{r} -6y - 2z = 10 \\ -6y + 2z = -10 \\ \hline 0 = 0 \end{array}$$

Because you obtain the identity  $0 = 0$ , the system has infinitely many solutions.

**Step 3** Solve new Equation 2 for  $z$  to obtain  $z = 3y - 5$ . Then substitute  $3y - 5$  for  $z$  in original Equation 1 to obtain  $y = x + 2$ . Then  $z = 3x + 1$ . A solution of the system can be represented by any ordered triple of the form  $(x, x + 2, 3x + 1)$ .

15. **Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} x + 2y + 3z = 4 \\ 3x - 2y + z = -12 \\ \hline 4x + 4z = -8 \\ x + z = -2 \\ x + 2y + 3z = 4 \\ -2x - 2y - 4z = -14 \\ -x - z = -10 \end{array}$$

**Step 2** Solve the new linear system for both of its variables.

$$\begin{array}{r} x + z = -2 \\ -x - z = -10 \\ \hline 0 = -12 \end{array}$$

Because you obtain a false equation, the system has no solution.

# Chapter 1

**16. Step 1** Rewrite the system as a linear system in two variables.

$$-2x - 3y + z = -6$$

$$\underline{2x + 2y - 2z = 10}$$

$$-y - z = 4$$

$$-7x - 7y + 7z = -35$$

$$\underline{7x + 8y - 6z = 31}$$

$$y + z = -4$$

**Step 2** Solve the new linear system for both of its variables.

$$-y - z = 4$$

$$\underline{y + z = -4}$$

$$0 = 0$$

Because you obtain the identity  $0 = 0$ , the system has infinitely many solutions.

**Step 3** Solve new Equation 3 for  $z$  to obtain  $z = -y - 4$ . Then substitute  $-y - 4$  for  $z$  in original Equation 2 to obtain  $x = -2y + 1$ . A solution of the system can be represented by any ordered triple of the form  $(-2y + 1, y, -y - 4)$ .

**17. Step 1** Write a verbal model for the situation.

$$2 \cdot \boxed{\text{Cost of pizza, } p} + \boxed{\text{Cost of soda, } d} + \boxed{\text{Cost of salad, } s} = 14$$

$$\boxed{\text{Cost of pizza, } p} + \boxed{\text{Cost of soda, } d} + 3 \cdot \boxed{\text{Cost of salad, } s} = 15$$

$$3 \cdot \boxed{\text{Cost of pizza, } p} + \boxed{\text{Cost of soda, } d} + 2 \cdot \boxed{\text{Cost of salad, } s} = 22$$

**Step 2** Write a system of equations.

$$2p + d + s = 14$$

$$p + d + 3s = 15$$

$$3p + d + 2s = 22$$

**Step 3** Rewrite the system as a linear system in two variables.

$$2p + d + s = 14$$

$$\underline{-p - d - 3s = -15}$$

$$p - 2s = -1$$

$$p + d + 3s = 15$$

$$\underline{-3p - d - 2s = -22}$$

$$-2p + s = -7$$

**Step 4** Solve the new linear system for both of its variables.

$$p - 2s = -1$$

$$\underline{-4p + 2s = -14}$$

$$-3p = -15$$

$$p = 5$$

$$s = 3$$

**Step 5** Substitute  $p = 5$  and  $s = 3$  into an original equation and solve for  $d$ .

$$p + d + 3s = 15$$

$$5 + d + 3(3) = 15$$

$$d = 1$$

So, one pizza costs \$5, a liter of soda costs \$1, and a salad costs \$3.

**18. Step 1** Write a verbal model for the situation.

$$\boxed{\text{Cost of sofa, } S} + \boxed{\text{Cost of love seat, } L} = 1300$$

$$\boxed{\text{Cost of sofa, } S} + 2 \cdot \boxed{\text{Cost of chairs, } C} = 1400$$

$$\boxed{\text{Cost of sofa, } S} + \boxed{\text{Cost of love seat, } L} + \boxed{\text{Cost of chair, } C} = 1600$$

**Step 2** Write a system of equations.

$$S + L = 1300$$

$$S + 2C = 1400$$

$$S + L + C = 1600$$

**Step 3** Rewrite the system as a linear system in two variables.

$$S + L = 1300$$

$$\underline{-S - 2C = -1400}$$

$$L - 2C = -100$$

$$S + 2C = 1400$$

$$\underline{-S - L - C = -1600}$$

$$-L + C = -200$$

**Step 4** Solve the new linear system for both of its variables.

$$L - 2C = -100$$

$$\underline{-L + C = -200}$$

$$-C = -300$$

$$C = 300$$

$$L = 500$$

**Step 5** Substitute  $L = 500$  into an original equation and solve for  $S$ .

$$S + L = 1300$$

$$S + 500 = 1300$$

$$S = 800$$

So, a sofa costs \$800, a love seat costs \$500, and a chair costs \$300.

# Chapter 1

**19. Step 1** Solve the first equation for  $y$ .

$$y = 1 + 2x - 6z$$

**Step 2** Rewrite the system as a linear system in two variables by substituting for  $y$  from Step 1 into Equations 2 and 3.

$$3x + 2y + 5z = 16$$

$$3x + 2(1 + 2x - 6z) + 5z = 16$$

$$3x + 2 + 4x - 12z + 5z = 16$$

$$7x - 7z = 14$$

$$x - z = 2$$

$$7x + 3y - 4z = 11$$

$$7x + 3(1 + 2x - 6z) - 4z = 11$$

$$7x + 3 + 6x - 18z - 4z = 11$$

$$13x - 22z = 8$$

**Step 3** Solve the new linear system for both of its variables,  $x$  and  $z$ , and then use the values to solve for  $y$ .

$$-13x + 13z = -26$$

$$\underline{13x - 22z = 8}$$

$$-9z = -18$$

$$z = 2$$

$$x = 4$$

$$y = -3$$

The solution is  $(4, -3, 2)$ .

**20. Step 1** Solve the first equation for  $x$ .

$$x = 6y + 2z - 8$$

**Step 2** Rewrite the system as a linear system in two variables by substituting for  $x$  from Step 1 into Equations 2 and 3.

$$-x + 5y + 3z = 2$$

$$-(6y + 2z - 8) + 5y + 3z = 2$$

$$-6y - 2z + 8 + 5y + 3z = 2$$

$$-y + z = -6$$

$$3x - 2y - 4z = 18$$

$$3(6y + 2z - 8) - 2y - 4z = 18$$

$$18y + 6z - 24 - 2y - 4z = 18$$

$$16y + 2z = 42$$

$$8y + z = 21$$

**Step 3** Solve the new linear system for both of its variables,  $y$  and  $z$ , and then use the values to solve for  $x$ .

$$y - z = 6$$

$$\underline{8y + z = 21}$$

$$9y = 27$$

$$y = 3$$

$$z = -3$$

$$x = 4$$

The solution is  $(4, 3, -3)$ .

**21. Step 1** Solve the first equation for  $y$ .

$$y = 4 - x - z$$

**Step 2** Rewrite the system as a linear system in two variables by substituting for  $y$  from Step 1 into Equations 2 and 3.

$$5x + 5y + 5z = 12$$

$$5x + 5(4 - x - z) + 5z = 12$$

$$5x + 20 - 5x - 5z + 5z = 12$$

$$20 = 12$$

Because you obtain a false equation, the original system has no solution.

**22. Step 1** Solve the first equation for  $x$ .

$$x = -2y - 1$$

**Step 2** Rewrite the system as a linear system in two variables by substituting for  $x$  from Step 1 into Equations 2 and 3.

$$-x + 3y + 2z = -4$$

$$-(-2y - 1) + 3y + 2z = -4$$

$$2y + 1 + 3y + 2z = -4$$

$$5y + 2z = -5$$

$$-x + y - 4z = 10$$

$$-(-2y - 1) + y - 4z = 10$$

$$2y + 1 + y - 4z = 10$$

$$3y - 4z = 9$$

**Step 3** Solve the new linear system for both of its variables,  $y$  and  $z$ , and then use the values to solve for  $x$ .

$$10y + 4z = -10$$

$$\underline{3y - 4z = 9}$$

$$13y = -1$$

$$y = -\frac{1}{13}$$

$$z = -\frac{30}{13}$$

$$x = -\frac{11}{13}$$

The solution is  $(-\frac{11}{13}, -\frac{1}{13}, -\frac{30}{13})$ .

# Chapter 1

**23. Step 1** Substitute 5 for  $z$  from Equation 3 into Equation 2 to solve for  $y$ .

$$\begin{aligned}y + 2z &= 13 \\y + 2(5) &= 13 \\y + 10 &= 13 \\y &= 3\end{aligned}$$

**Step 2** Substitute 5 for  $z$  and 3 for  $y$  into Equation 1 to solve for  $x$ .

$$\begin{aligned}2x - 3y + z &= 10 \\2x - 3(3) + 5 &= 10 \\2x - 9 + 5 &= 10 \\2x &= 14 \\x &= 7\end{aligned}$$

The solution is  $(7, 3, 5)$ .

**24. Step 1** Substitute 4 for  $x$  from Equation 1 into Equation 2 to solve for  $y$ .

$$\begin{aligned}x + y &= -6 \\4 + y &= -6 \\y &= -10\end{aligned}$$

**Step 2** Substitute 4 for  $x$  and  $-10$  for  $y$  into Equation 3 to solve for  $z$ .

$$\begin{aligned}-4x - 3y + 2z &= 26 \\4(4) - 3(-10) + 2z &= 26 \\16 + 30 + 2z &= 26 \\2z &= -20 \\z &= -10\end{aligned}$$

The solution is  $(4, -10, -10)$ .

**25. Step 1** Solve the first equation for  $x$ .

$$x = 4 - y + z$$

**Step 2** Rewrite the system as a linear system in two variables by substituting for  $x$  from Step 1 into Equations 2 and 3.

$$\begin{aligned}3x + 2y + 4z &= 17 \\3(4 - y + z) + 2y + 4z &= 17 \\12 - 3y + 3z + 2y + 4z &= 17 \\-y + 7z &= 5 \\-x + 5y + z &= 8 \\-(4 - y + z) + 5y + z &= 8 \\-4 + y - z + 5y + z &= 8 \\6y &= 12 \\y &= 2\end{aligned}$$

**Step 3** Solve the new linear system for the variables  $x$  and  $z$ .

$$\begin{aligned}y &= 2 \\-y + 7z &= 5 \\-2 + 7z &= 5 \\7z &= 7 \\z &= 1 \\x &= 3\end{aligned}$$

The solution is  $(3, 2, 1)$ .

**26. Step 1** Solve the first equation for  $y$ .

$$y = 2x - z - 15$$

**Step 2** Rewrite the system as a linear system in two variables by substituting for  $y$  from Step 1 into Equations 2 and 3.

$$\begin{aligned}4x + 5y + 2z &= 10 \\4x + 5(2x - z - 15) + 2z &= 10 \\4x + 10x - 5z - 75 + 2z &= 10 \\14x - 3z &= 85 \\-x - 4y + 3z &= -20 \\-x - 4(2x - z - 15) + 3z &= -20 \\-x - 8x + 4z + 60 + 3z &= -20 \\-9x + 7z &= -80\end{aligned}$$

**Step 3** Solve the new linear system for both of its variables,  $x$  and  $z$ , and then use the values to solve for  $y$ .

$$\begin{aligned}98x - 21z &= 595 \\-27x + 21z &= -240 \\71x &= 355 \\x &= 5 \\z &= -5 \\y &= 0\end{aligned}$$

The solution is  $(5, 0, -5)$ .

# Chapter 1

**27. Step 1** Solve the first equation for  $y$ .

$$y = 5 - 4x - 5z$$

**Step 2** Rewrite the system as a linear system in two variables by substituting for  $y$  from Step 1 into Equations 2 and 3.

$$8x + 2y + 10z = 10$$

$$8x + 2(5 - 4x - 5z) + 10z = 10$$

$$8x + 10 - 8x - 10z + 10z = 10$$

$$0 = 0$$

Because you obtain the identity  $0 = 0$ , the system has infinitely many solutions.

$$-x - y - 2z = -2$$

$$-x - (5 - 4x - 5z) - 2z = -2$$

$$-x - 5 + 4x + 5z - 2z = -2$$

$$5x + 3z = 3$$

$$z = 1 - \frac{5}{3}x$$

**Step 3** Substitute  $1 - \frac{5}{3}x$  for  $z$  in revised Equation 1 to obtain  $y = \frac{13}{3}x$ . A solution of the system can be represented by any ordered triple of the form  $(x, \frac{13}{3}x, 1 - \frac{5}{3}x)$ .

**28. Step 1** Solve the first equation for  $z$ .

$$z = x + 2y - 3$$

**Step 2** Rewrite the system as a linear system in two variables by substituting for  $z$  from Step 1 into Equations 2 and 3.

$$2x + 4y - 2z = 6$$

$$2x + 4y - 2(x + 2y - 3) = 6$$

$$6y + 6 = 6$$

$$6y = 0$$

$$y = 0$$

$$-x - 2y + z = -6$$

$$-x - 2y + (x + 2y - 3) = -6$$

$$-3 = -6$$

Because you obtain a false equation, the system has no solution.

**29. Step 1** Write a verbal model for the situation.

$$\begin{aligned} \boxed{\text{Left-handed, } \ell} + \boxed{\text{Right-handed, } r} + \boxed{\text{Ambidextrous, } a} &= 1 \\ \frac{1}{10} \cdot \boxed{\text{Right-handed, } r} &= \boxed{\text{Left-handed, } \ell} \\ \boxed{\text{Right-handed, } r} &= 9 \cdot (\boxed{\text{Left-handed, } \ell} + \boxed{\text{Ambidextrous, } a}) \end{aligned}$$

**Step 2** Write a system of equations.

$$\ell + r + a = 1$$

$$\frac{r}{10} = \ell$$

$$r = 9(\ell + a)$$

**Step 3** Rewrite the system as a linear system in two variables by substituting for  $\ell$  from Equation 2 into Equations 1 and 3.

$$\ell + r + a = 1$$

$$\frac{r}{10} + r + a = 1$$

$$\frac{11}{10}r + a = 1$$

$$11r + 10a = 10$$

$$r = 9(\ell + a)$$

$$r = 9\ell + 9a$$

$$r = 9\left(\frac{r}{10}\right) + 9a$$

$$9a - \frac{1}{10}r = 0$$

$$90a - r = 0$$

**Step 4** Solve the new linear system for both of its variables,  $r$  and  $a$ , and then use the values to solve for  $\ell$ .

$$-99r - 90a = -90$$

$$\underline{-r + 90a = 0}$$

$$-100r = -90$$

$$r = \frac{9}{10}$$

$$a = \frac{1}{100}$$

$$\ell = \frac{9}{100}$$

So, the percent of people who are ambidextrous is 1%.

# Chapter 1

30. **Step 1** Write a verbal model for the situation.

$$\begin{aligned} \boxed{\text{First place, } f} + \boxed{\text{Second place, } s} + \boxed{\text{Third place, } t} &= 20 \\ 5 \cdot \boxed{\text{First place, } f} + 3 \cdot \boxed{\text{Second place, } s} \\ &\quad + 1 \cdot \boxed{\text{Third place, } t} = 68 \\ \boxed{\text{Second place, } s} &= \boxed{\text{First place, } f} + \boxed{\text{Third place, } t} \end{aligned}$$

**Step 2** Write a system of equations.

$$\begin{aligned} f + s + t &= 20 \\ 5f + 3s + t &= 68 \\ s &= f + t \end{aligned}$$

**Step 3** Rewrite the system as a linear system in two variables by substituting for  $s$  from Equation 3 into Equations 1 and 2.

$$\begin{aligned} f + s + t &= 20 \\ f + (f + t) + t &= 20 \\ 2f + 2t &= 20 \\ f + t &= 10 \\ 5f + 3s + t &= 68 \\ 5f + 3(f + t) + t &= 68 \\ 8f + 4t &= 68 \\ 2f + t &= 17 \end{aligned}$$

**Step 4** Solve the new linear system for both of its variables,  $f$  and  $t$ , and then use the values to solve for  $s$ .

$$\begin{aligned} -f - t &= -10 \\ \underline{2f + t} &= 17 \\ f &= 7 \\ t &= 3 \\ s &= 10 \end{aligned}$$

So, 7 athletes finished in first place, 10 athletes finished in second place, and 3 athletes finished in third place.

31. *Sample answer:* It is more convenient to use the elimination method rather than the substitution method when there are more than two equations, in particular when one variable has the same coefficient or its opposite in each equation. An example is

$$\begin{aligned} 3x + 2y - 4z &= -5 \\ 2x + 2y + 3z &= 8 \\ 5x - 2y - 7z &= -9. \end{aligned}$$

32. *Sample answer:* When solving a system that has four linear equations and four variables, use the same method of elimination to rewrite the system in three linear equations using three variables. Then use the method of elimination that is used to solve a system of three linear equations in three variables.

33. Because the perimeter is 65 feet, the following system of equations provides the relationships between the three sides of the triangle.

$$\begin{aligned} \ell &= \frac{1}{3}m \\ n &= \ell + m - 15 \end{aligned}$$

$$\ell + m + n = 65$$

**Step 1** Rewrite the system of equations.

$$\begin{aligned} \ell - \frac{1}{3}m &= 0 \\ -\ell - m + n &= -15 \\ \ell + m + n &= 65 \end{aligned}$$

**Step 2** Add Equation 2 to Equation 3.

$$\begin{aligned} -\ell - m + n &= -15 \\ \ell + m + n &= 65 \\ \hline 2n &= 50 \\ n &= 25 \end{aligned}$$

**Step 3** Substitute  $n = 25$  into the third equation to form a system of two equations and solve.

$$\begin{aligned} \ell - \frac{1}{3}m &= 0 \\ -\ell - m &= -40 \\ \hline -\frac{4}{3}m &= -40 \\ m &= 30 \\ \ell &= 10 \end{aligned}$$

So, the lengths of the three sides of the triangle are  $\ell = 10$  feet,  $m = 30$  feet, and  $n = 25$  feet.

# Chapter 1

34. Because the sum of the angle measures in a triangle is  $180^\circ$ , the following system of equations provides the relationship between the three interior angle measures of a triangle.

$$A + B + C = 180$$

$$2A - C = B$$

$$A + B = C$$

**Step 1** Rewrite the linear system.

$$A + B + C = 180$$

$$2A - B - C = 0$$

$$A + B - C = 0$$

**Step 2** Add Equation 1 to Equation 2, then solve for  $A$ .

$$A + B + C = 180$$

$$2A - B - C = 0$$

$$\hline 3A = 180$$

$$A = 60$$

**Step 3** Add Equation 1 to Equation 3, then substitute  $A = 60$  into the result.

$$A + B + C = 180$$

$$A + B - C = 0$$

$$\hline 2A + 2B = 180$$

$$2(60) + 2B = 180$$

$$B = 30$$

**Step 4** Substitute  $A = 60$  and  $B = 30$  into an original equation and solve for  $C$ .

$$A + B + C = 180$$

$$60 + 30 + C = 180$$

$$C = 90$$

So,  $m\angle A = 60^\circ$ ,  $m\angle B = 30^\circ$ , and  $m\angle C = 90^\circ$ .

35. a. *Sample answer:* If  $a = 1$ ,  $b = 1$ , and  $c = 1$ , then the system has no solution. In this case, use elimination of Equation 1 and Equation 2, which results in a false equation.
- b. *Sample answer:* If  $a = 10$ ,  $b = 9$ , and  $c = 8$ , then the system has one solution. In this case, no equations are equivalent and no combinations of equations will lead to a false equation.
- c. *Sample answer:* If  $a = 5$ ,  $b = 5$ , and  $c = 5$ , then the system has infinitely many solutions. In this case, Equation 1 and Equation 2 are equivalent.
36. Your friend is incorrect. It is possible for two of the equations to intersect at a line while the third forms a parallel plane to one of the others. Therefore, the point  $(x, y, z)$  satisfies two of the equations but not the third.

37. **1. Understand the Problem** Find the number of square feet that each bedroom, kitchen, and bathroom will have.
- 2. Make a Plan** Write a verbal model using the information so that you can write a system of linear equations to solve.

**3. Solve the Problem**

**Step 1** Write a verbal model.

$$\boxed{\text{Bedroom, } b} + \boxed{\text{Kitchen, } k} + \boxed{\text{Bathroom, } t} = 840$$

$$\boxed{\text{Bedroom, } b} = \boxed{\text{Kitchen, } k}$$

$$2 \cdot (\boxed{\text{Kitchen, } k} + \boxed{\text{Bathroom, } t}) = 980$$

**Step 2** Write a linear system.

$$b + k + t = 840$$

$$b = k$$

$$2(k + t) = 980$$

**Step 3** Rewrite the system as a linear system in two variables by substituting for  $b$  from Equation 2 into Equation 1.

$$b + k + t = 840$$

$$k + k + t = 840$$

$$2k + t = 840$$

**Step 4** Solve the new linear system for both of its variables,  $k$  and  $t$ , and then use the values to solve for  $b$ .

$$2k + t = 840$$

$$\hline -2k - 2t = -980$$

$$t = 140$$

$$k = 350$$

$$b = 350$$

So, 350 square feet of carpet is needed for each bedroom.

38. By inspection, because the constants in all three equations are 0, one solution is  $(0, 0, 0)$ . Use the method of elimination to see whether there are other solutions.

Add  $-2$  times Equation 2 to Equation 1.

$$4x + y + z = 0$$

$$\hline -4x - y - 4z = 0$$

$$-3z = 0$$

$$z = 0$$

Substitute  $z = 0$  into the first and second equations.

$$4x + y = 0$$

$$\hline -4x - y = 0$$

$$0 = 0$$

Because you obtain the identity  $0 = 0$ , the system has infinitely many solutions. Solve new Equation 1 for  $x$  to obtain  $x = -\frac{1}{4}y$ . So, a solution of the system can be represented by any ordered triple of the form  $(-\frac{1}{4}y, y, 0)$ .

# Chapter 1

39. a. **Step 1** Write a verbal model.

$$5 \cdot (2.5 \cdot \boxed{\text{Roses, } r} + 4 \cdot \boxed{\text{Lilies, } \ell} + 2 \cdot \boxed{\text{Irises, } i}) = 160$$

$$\boxed{\text{Roses, } r} + \boxed{\text{Lilies, } \ell} + \boxed{\text{Irises, } i} = 12$$

$$\boxed{\text{Roses, } r} = 2 \cdot (\boxed{\text{Lilies, } \ell} + \boxed{\text{Irises, } i})$$

**Step 2** Write a system of equations.

$$5(2.5r + 4\ell + 2i) = 160$$

$$r + \ell + i = 12$$

$$r = 2(\ell + i)$$

b. **Step 3** Rewrite the system as a linear system in two variables by substituting for  $r$  from Equation 3 into Equations 1 and 2.

$$5(2.5r + 4\ell + 2i) = 160$$

$$2.5r + 4\ell + 2i = 32$$

$$2.5[2(\ell + i)] + 4\ell + 2i = 32$$

$$5\ell + 5i + 4\ell + 2i = 32$$

$$9\ell + 7i = 32$$

$$r + \ell + i = 12$$

$$2\ell + 2i + \ell + i = 12$$

$$3\ell + 3i = 12$$

$$\ell + i = 4$$

**Step 4** Solve the new linear system for both of its variables,  $\ell$  and  $i$ , and then use the values to solve for  $r$ .

$$9\ell + 7i = 32$$

$$\underline{-9\ell - 9i = -36}$$

$$-2i = -4$$

$$i = 2$$

$$\ell = 2$$

$$r = 8$$

So, there should be 8 roses, 2 lilies, and 2 irises in each bouquet.

c. If there is no limitation on the total cost, then there are five possible solutions, each of which is based on the total cost. The solutions are

8 roses, 0 lilies, and 4 irises at a cost of \$140

8 roses, 1 lily, and 3 irises at a cost of \$150

8 roses, 2 lilies, and 2 irises at a cost of \$160

8 roses, 3 lilies, and 1 iris at a cost of \$170

8 roses, 4 lilies, and 0 irises at a cost of \$180

40. a. Because all three circles intersect at one single point, the system of equations has one solution.

b. Because all three circles do not intersect at any point, the system of equations has no solution.

41. In order for the system of equations to have only one solution,  $(-1, 2, -3)$  will satisfy all three equations. So,  $a$ ,  $b$ , and  $c$  can be determined by substituting  $-1$  for  $x$ ,  $2$  for  $y$ , and  $-3$  for  $z$  in all three equations.

$$x + 2y - 3z = a$$

$$-1 + 2(2) - 3(-3) = a$$

$$12 = a$$

$$-x - y + z = b$$

$$-(-1) - 2 + (-3) = b$$

$$-4 = b$$

$$2x + 3y - 2z = c$$

$$2(-1) + 3(2) - 2(-3) = c$$

$$10 = c$$

So,  $a = 12$ ,  $b = -4$ , and  $c = 10$ .

42. The arrangement  $3x + 2y + (-5)z = -30$  produces the solution  $(-3, 2, 5)$  and the arrangement  $2x + (-5)y + 3z = -30$  produces the solution  $(-699, -288, -24)$ .

First arrangement:

$$x - 3y + 6z = 21 \quad \text{Equation 1}$$

$$3x + 2y - 5z = -30 \quad \text{Equation 2}$$

$$2x - 5y + 2z = -6 \quad \text{Equation 3}$$

$$-3x + 9y - 18z = -63$$

$$\underline{3x + 2y - 5z = -30}$$

$$11y - 23z = -93 \quad \text{New Equation 2}$$

$$-2x + 6y - 12z = -42$$

$$\underline{2x - 5y + 2z = -6}$$

$$y - 10z = -48 \quad \text{New Equation 3}$$

$$11y - 23z = -93$$

$$\underline{-11y + 110z = 528}$$

$$87z = 435$$

$$z = 5$$

$$y - 10(5) = -48$$

$$y = 2$$

$$x - 3(2) + 6(5) = 21$$

$$x = -3$$

The solution is  $(-3, 2, 5)$ .



# Chapter 1

Second arrangement:

$$x - 3y + 6z = 21 \quad \text{Equation 1}$$

$$2x - 5y + 3z = -30 \quad \text{Equation 2}$$

$$2x - 5y + 2z = -6 \quad \text{Equation 3}$$

$$-2x + 6y - 12z = -42$$

$$\underline{2x - 5y + 3z = -30}$$

$$y - 9z = -72 \quad \text{New Equation 2}$$

$$-2x + 5y - 3z = 30$$

$$\underline{2x - 5y + 2z = -6}$$

$$-z = 24$$

$$z = -24 \quad \text{New Equation 3}$$

$$y - 9(-24) = -72$$

$$y = -288$$

$$x - 3(-288) + 6(-24) = 21$$

$$x = -699$$

The solution is  $(-699, -288, -24)$ .

43. The linear system represented by the three pictures is

$$a + t = g$$

$$t + b = a$$

$$2g = 3b.$$

Solve for  $b$  in Equation 2.

$$b = a - t$$

Substitute  $b$  into Equation 3.

$$g = \frac{3}{2}b$$

$$= \frac{3}{2}(a - t)$$

Substitute  $g$  into Equation 1 and express  $a$  in terms of  $t$ .

$$a + t = g$$

$$a + t = \frac{3}{2}(a - t)$$

$$2a + 2t = 3a - 3t$$

$$5t = a$$

So, 5 tangerines are required to balance one apple.

## Maintaining Mathematical Proficiency

44.  $(x - 2)^2 = (x - 2)(x - 2)$

$$= x^2 - 2x - 2x + 4$$

$$= x^2 - 4x + 4$$

45.  $(3m + 1)^2 = (3m + 1)(3m + 1)$

$$= (3m)^2 + 3m + 3m + 1$$

$$= 9m^2 + 6m + 1$$

46.  $(2z - 5)^2 = (2z - 5)(2z - 5)$

$$= (2z)^2 - 10z - 10z + 25$$

$$= 4z^2 - 20z + 25$$

47.  $(4 - y)^2 = (4 - y)(4 - y)$   
 $= 16 - 4y - 4y + y^2$   
 $= y^2 - 8y + 16$

48. A translation 2 units left is a horizontal translation that subtracts  $-2$  from each input value.

$$g(x) = f(x + 2)$$

$$= |x + 2| - 5$$

The transformed function is  $g(x) = |x + 2| - 5$ .

49. A reflection in the  $x$ -axis changes the sign of each output value.

$$g(x) = -f(x)$$

$$= -(|x| - 5)$$

$$= -|x| + 5$$

The transformed function is  $g(x) = -|x| + 5$ .

50. A translation 4 units up is a vertical translation that adds 4 to each output value.

$$g(x) = f(x) + 4$$

$$= (|x| - 5) + 4$$

$$= |x| - 1$$

The transformed function is  $g(x) = |x| - 1$ .

51. A vertical stretch by a factor of 3 multiplies each output value by 3.

$$g(x) = 3f(x)$$

$$= 3(|x| - 5)$$

$$= 3|x| - 15$$

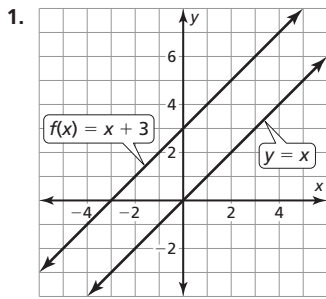
The transformed function is  $g(x) = 3|x| - 15$ .

## 1.3–1.4 What Did You Learn? (p. 37)

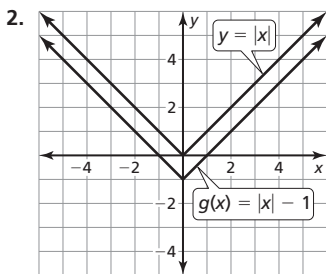
- Sample answer:* Because the  $y$ -intercept is 0, use one of the points in the equation  $y = mx$ .
- Sample answer:* An equation can be written stating that the total number of placers was 20. A second equation can be written stating that the total number of points was 68. A third equation can be written stating that the number of second place finishers was equal to the number of first and third place finishers combined.
- Sample answer:* If  $a = b = c = 5$ , the first two equations will be the same and a solution will exist. So, choose any number so long as  $a$ ,  $b$ , and  $c$  are not 5 for the system to have no solution.

# Chapter 1

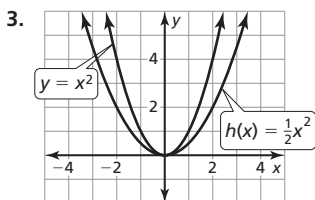
## Chapter 1 Review (pp. 38–40)



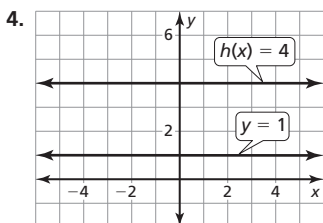
The graph of  $f$  is a translation 3 units up of the parent linear function.



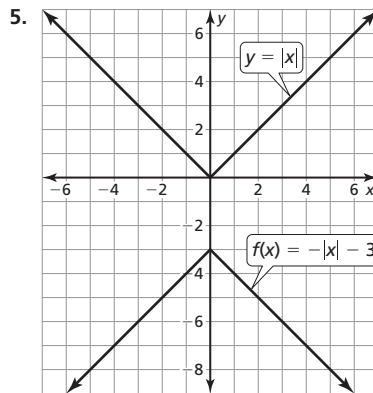
The graph of  $g$  is a translation 1 unit down of the parent absolute value function.



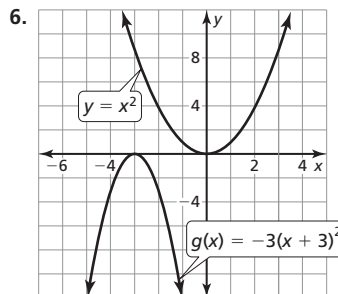
The graph of  $h$  is a vertical shrink by a factor of  $\frac{1}{2}$  of the parent quadratic function.



The graph of  $h$  is a translation 3 units up of the parent constant function.



The graph of  $f$  is a reflection in the  $x$ -axis and a translation 3 units down of the parent absolute value function.



The graph of  $g$  is a vertical stretch by a factor of 3 followed by a reflection in the  $x$ -axis and translation 3 units left of the parent quadratic function.

**7. Step 1** First write a function  $h$  that represents the reflection of  $f$ .

$$\begin{aligned} h(x) &= -f(x) \\ &= -|x| \end{aligned}$$

**Step 2** Then write a function  $g$  that represents the translation 4 units left of  $h$ .

$$\begin{aligned} g(x) &= h(x + 4) \\ &= -|x + 4| \end{aligned}$$

The transformed function is  $g(x) = -|x + 4|$ .

**8. Step 1** First write a function  $h$  that represents the vertical shrink of  $f$ .

$$\begin{aligned} h(x) &= \frac{1}{2}f(x) \\ &= \frac{1}{2}|x| \end{aligned}$$

**Step 2** Then write a function  $g$  that represents the translation 2 units up of  $h$ .

$$\begin{aligned} g(x) &= h(x) + 2 \\ &= \frac{1}{2}|x| + 2 \end{aligned}$$

The transformed function is  $g(x) = \frac{1}{2}|x| + 2$ .

# Chapter 1

- 9. Step 1** First write a function  $h$  that represents the translation 3 units down of  $f$ .

$$\begin{aligned} h(x) &= f(x) - 3 \\ &= x - 3 \end{aligned}$$

- Step 2** Then write a function  $g$  that represents a reflection in the  $y$ -axis of  $h$ .

$$\begin{aligned} g(x) &= h(-x) \\ &= -x - 3 \end{aligned}$$

The transformed function is  $g(x) = -x - 3$ .

- 10.** Enter the data into two lists. Use the *linear regression* feature. The line of best fit is  $y = 0.03x + 1.23$ .
- 11.** Having traveled 3.5 miles in 10 minutes corresponds to the point (10, 3.5) and traveling 10.5 miles in 30 minutes corresponds to the point (30, 10.5). Write an equation of the line that passes through the points (10, 3.5) and (30, 10.5). First, find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10.5 - 3.5}{30 - 10} = \frac{7}{20} = 0.35$$

Use point-slope form to write an equation.

Use  $(x_1, y_1) = (10, 3.5)$ .

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 3.5 &= 0.35(x - 10) \\ y - 3.5 &= 0.35x - 3.5 \end{aligned}$$

$$y = 0.35x$$

Use the equation to estimate how far you ride your bike in 45 minutes.

$$\begin{aligned} y &= 0.35(45) \\ &= 15.75 \end{aligned}$$

After 45 minutes of riding your bike, you have traveled 15.75 miles.

- 12. Step 1** Rewrite the system as a linear system in two variables by substituting for  $x$  from Equation 3 into Equations 1 and 2.

$$\begin{aligned} x + y + z &= 3 \\ 4z + y + z &= 3 \\ y + 5z &= 3 \\ -x + 3y + 2z &= -8 \\ -4z + 3y + 2z &= -8 \\ 3y - 2z &= -8 \end{aligned}$$

- Step 2** Solve the new linear system for both of its variables.

$$\begin{aligned} 3y + 15z &= 9 \\ -3y + 2z &= 8 \\ \hline 17z &= 17 \\ z &= 1 \end{aligned}$$

$$\begin{aligned} y + 5(1) &= 3 \\ y &= -2 \end{aligned}$$

- Step 3** Substitute  $y = -2$  and  $z = 1$  into an original equation and solve for  $x$ .

$$\begin{aligned} x + y + z &= 3 \\ x - 2 + 1 &= 3 \\ x &= 4 \end{aligned}$$

The solution is (4, -2, 1).

- 13. Step 1** Rewrite the system as a linear system in two variables.

$$\begin{aligned} 2x - 5y - z &= 17 \\ 5x + 5y + 15z &= 95 \\ \hline 7x + 14z &= 112 \\ x + 2z &= 16 \\ -6x - 6y - 18z &= -114 \\ -4x + 6y + z &= -20 \\ \hline -10x - 17z &= -134 \end{aligned}$$

- Step 2** Solve the new linear system for both of its variables.

$$\begin{aligned} 10x + 20z &= 160 \\ -10x - 17z &= -134 \\ \hline 3z &= 26 \\ z &= \frac{26}{3} \end{aligned}$$

$$\begin{aligned} x + 2\left(\frac{26}{3}\right) &= 16 \\ x &= -\frac{4}{3} \end{aligned}$$

- Step 3** Substitute  $x = -\frac{4}{3}$  and  $z = \frac{26}{3}$  into an original equation and solve for  $y$ .

$$\begin{aligned} x + y + 3z &= 19 \\ -\frac{4}{3} + y + 3 \cdot \frac{26}{3} &= 19 \\ y &= -\frac{17}{3} \end{aligned}$$

The solution is  $\left(-\frac{4}{3}, -\frac{17}{3}, \frac{26}{3}\right)$ .

- 14. Step 1** Rewrite the system as a linear system in two variables.

$$\begin{aligned} -x - y - z &= -2 \\ 2x - 3y + z &= 11 \\ \hline x - 4y &= 9 \\ 4x - 6y + 2z &= 22 \\ -3x + 2y - 2z &= -13 \\ \hline x - 4y &= 9 \end{aligned}$$

- Step 2** Solve the new linear system for both of its variables.

$$\begin{aligned} x - 4y &= 9 \\ -x + 4y &= -9 \\ \hline 0 &= 0 \end{aligned}$$

Because you obtain the identity  $0 = 0$ , the system has infinite solutions.

# Chapter 1

- 15. Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} x + 4y - 2z = 3 \\ -x - 3y - 7z = -1 \\ \hline y - 9z = 2 \\ -2x - 8y + 4z = -6 \\ 2x + 9y - 13z = 2 \\ \hline y - 9z = -4 \end{array}$$

- Step 2** Solve the new linear system for both of its variables.

$$\begin{array}{r} y - 9z = 2 \\ -y + 9z = 4 \\ \hline 0 = 6 \end{array}$$

Because you obtain a false equation, the system has no solution.

- 16. Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} 5x - 5y + 15z = 30 \\ -6x + 6y - 15z = -27 \\ \hline -x + y = 3 \end{array}$$

- Step 2** Solve the new linear system for both of its variables.

$$\begin{array}{r} -x + y = 3 \\ x - 2y = 5 \\ \hline -y = 8 \\ y = -8 \\ x - 2(-8) = 5 \\ x = -11 \end{array}$$

- Step 3** Substitute  $x = -11$  and  $y = -8$  into an original equation and solve for  $z$ .

$$\begin{array}{r} x - y + 3z = 6 \\ -11 + 8 + 3z = 6 \\ 3z = 9 \\ z = 3 \end{array}$$

The solution is  $(-11, -8, 3)$ .

- 17. Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} -3x - 3y - 3z = -18 \\ 3x + 3y + 4z = 28 \\ \hline z = 10 \\ x + 2z = 4 \\ x + 2(10) = 4 \\ x = -16 \end{array}$$

- Step 2** Substitute  $x = -16$  and  $z = 10$  into an original equation and solve for  $y$ .

$$\begin{array}{r} x + y + z = 6 \\ -16 + y + 10 = 6 \\ y = 12 \end{array}$$

The solution is  $(-16, 12, 10)$ .

- 18. Step 1** Write a verbal model for the situation.

$$\begin{array}{l} \boxed{\text{Students, } s} + \boxed{\text{Adults, } a} + \boxed{\text{Children, } c} = 600 \\ 3 \cdot \boxed{\text{Students, } s} + 7 \cdot \boxed{\text{Adults, } a} + 2 \cdot \boxed{\text{Children, } c} = 3150 \\ \boxed{\text{Adults, } a} = \boxed{\text{Students, } s} + 150 \end{array}$$

- Step 2** Write a system of equations.

$$\begin{array}{r} s + a + c = 600 \\ 3s + 7a + 2c = 3150 \\ a = s + 150 \end{array}$$

- Step 3** Rewrite the system as a linear system in two variables by substituting for  $a$  from Equation 3 into Equations 1 and 2.

$$\begin{array}{r} s + a + c = 600 \\ s + (s + 150) + c = 600 \\ 2s + c = 450 \\ 3s + 7a + 2c = 3150 \\ 3s + 7(s + 150) + 2c = 3150 \\ 10s + 2c = 2100 \\ 5s + c = 1050 \end{array}$$

- Step 4** Solve the new linear system for both of its variables,  $s$  and  $c$ , and then use the values to solve for  $a$ .

$$\begin{array}{r} -2s - c = -450 \\ 5s + c = 1050 \\ \hline 3s = 600 \\ s = 200 \\ c = 50 \end{array}$$

$$\begin{array}{r} a = 200 + 150 \\ a = 350 \end{array}$$

So, 200 student tickets, 350 adult tickets, and 50 children under 12 tickets were sold.

# Chapter 1

## Chapter 1 Test (p. 41)

1. Using the points (2, 400) and (3, 600), write the equation of the line passing through the points. First find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{600 - 400}{3 - 2} = \frac{200}{1} = 200$$

Use point-slope form to write an equation.

Use  $(x_1, y_1) = (2, 400)$ .

$$y - y_1 = m(x - x_1)$$

$$y - 400 = 200(x - 2)$$

$$y - 400 = 200x - 400$$

$$y = 200x$$

The slope, \$200, is the amount the balance increases each week and the y-intercept is the original amount in the account, \$0.

2. Using the points (0, 50) and (20, 40), write the equation of the line passing through the points. First find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{50 - 40}{0 - 20} = \frac{10}{-20} = -0.5$$

Use point-slope form to write an equation.

Use  $(x_1, y_1) = (0, 50)$ .

$$y - y_1 = m(x - x_1)$$

$$y - 50 = -0.5(x - 0)$$

$$y - 50 = -0.5x$$

$$y = -0.5x + 50$$

The slope,  $-\$0.50$ , is the amount the price decreases for each 1% of discount and the y-intercept is the initial price, \$50.

3. **Step 1** Rewrite the system as a linear system in two variables.

$$-2x + y + 4z = 5$$

$$2x + 6y - 2z = 4$$

$$7y + 2z = 9$$

$$-4x + 2y + 8z = 10$$

$$4x + y - 6z = 11$$

$$3y + 2z = 21$$

- Step 2** Solve the new linear system for both of its variables.

$$-7y - 2z = -9$$

$$3y + 2z = 21$$

$$-4y = 12$$

$$y = -3$$

$$z = 15$$

- Step 3** Substitute  $y = -3$  and  $z = 15$  into an original equation and solve for  $x$ .

$$-2x + y + 4z = 5$$

$$-2x + (-3) + 4(15) = 5$$

$$x = 26$$

The solution is (26,  $-3$ , 15).

4. **Step 1** Rewrite the system as a linear system in two variables.

$$-3x - 6y - 15z = -6$$

$$3x + 6y - 3z = 9$$

$$-18z = 3$$

$$z = -\frac{1}{6}$$

- Step 2** Substitute  $z = -\frac{1}{6}$  into the first equation and solve for  $y$ .

$$y = \frac{1}{2}\left(-\frac{1}{6}\right)$$

$$= -\frac{1}{12}$$

- Step 3** Substitute  $y = -\frac{1}{12}$  and  $z = -\frac{1}{6}$  into an original equation and solve for  $x$ .

$$x + 2y + 5z = 2$$

$$x + 2\left(-\frac{1}{12}\right) + 5\left(-\frac{1}{6}\right) = 2$$

$$x = 3$$

The solution is  $\left(3, -\frac{1}{12}, -\frac{1}{6}\right)$ .

5. **Step 1** Rewrite the system as a linear system in two variables.

$$-2x + 2y - 10z = -6$$

$$2x + 3y - z = 2$$

$$5y - 11z = -4$$

$$4x + 6y - 2z = 4$$

$$-4x - y - 9z = -8$$

$$5y - 11z = -4$$

- Step 2** Solve the new linear system for both of its variables.

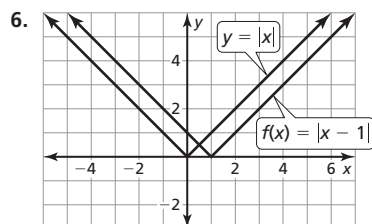
$$5y - 11z = -4$$

$$-5y + 11z = 4$$

$$0 = 0$$

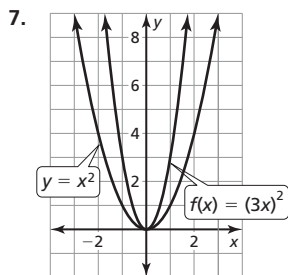
Because you obtain the identity  $0 = 0$ , the system has infinitely many solutions.

- Step 3** Solve new Equation 2 for  $y$  to obtain  $y = \frac{11}{5}z - \frac{4}{5}$ . Substitute  $\frac{11}{5}z - \frac{4}{5}$  for  $y$  into original Equation 1 to obtain  $x = -\frac{14}{5}z + \frac{11}{5}$ . A solution of the system can be represented by any ordered triple of the form  $\left(-\frac{14}{5}z + \frac{11}{5}, \frac{11}{5}z - \frac{4}{5}, z\right)$ .

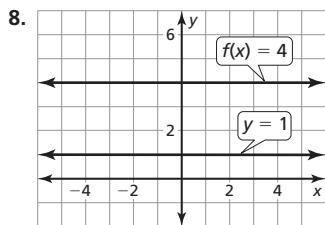


The graph of  $f$  is the graph of the parent absolute value function translated right 1 unit.

# Chapter 1



The graph of  $f$  is the graph of the parent quadratic value function with a horizontal shrink by a factor of  $\frac{1}{3}$ .



The graph of  $f$  is the graph of the parent constant function translated up 3 units.

9. The graph of  $g$  is B;  $g(x) = 2x + 3$
10. The graph of  $g$  is C;  $g(x) = 3x - 2$
11. The graph of  $g$  is A;  $g(x) = -2x - 3$

12. **Step 1** Write a verbal model for the situation.

$$\boxed{\text{Doughnuts, } d} = 3 \cdot \boxed{\text{Bagels, } b}$$

$$1 \cdot \boxed{\text{Doughnuts, } d} + 1.5 \cdot \boxed{\text{Muffins, } m} + 1.2 \cdot \boxed{\text{Bagels, } b} = 150$$

$$\boxed{\text{Doughnuts, } d} + \boxed{\text{Muffins, } m} + \boxed{\text{Bagels, } b} = 130$$

**Step 2** Write a system of equations.

$$d = 3b$$

$$d + 1.5m + 1.2b = 150$$

$$d + m + b = 130$$

**Step 3** Rewrite the system as a linear system in two variables by substituting  $d = 3b$  from Equation 1 into Equations 2 and 3.

$$3b + 1.5m + 1.2b = 150$$

$$3b + m + b = 130$$

$$1.5m + 4.2b = 150$$

$$m + 4b = 130$$

**Step 4** Solve the new linear system for both of its variables.

$$1.5m + 4.2b = 150$$

$$-1.5m - 6b = -195$$

$$-1.8b = -45$$

$$b = 25$$

$$m = 30$$

**Step 5** Substitute  $b = 25$  into the first original equation and solve for  $d$ .

$$d = 3(25)$$

$$= 75$$

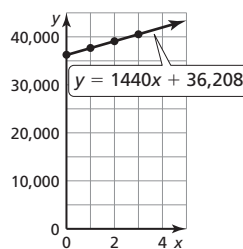
So, 75 doughnuts, 30 muffins, and 25 bagels are made.

13. The graph that will model the second fountain will have a horizontal shrink of the graph of  $f$  by a factor of  $\frac{1}{2}$ . So,  $g(t) = f(2t) = \frac{1}{4}|2t - 20|$ . The depth of the first fountain after 4 minutes is  $f(4) = \frac{1}{4}|4 - 20| = 4$  feet. The depth of the second fountain after 4 minutes is  $g(4) = \frac{1}{4}|2(4) - 20| = 3$  feet.

## Chapter 1 Standards Assessment (pp. 42–43)

1. a. The graph of  $g$  is a vertical translation 5 units up of the graph of  $f$ .
  - b. The graph of  $g$  is a vertical shrink by a factor of  $\frac{1}{2}$  of the graph of  $f$ .
  - c. The graph of  $g$  is a horizontal shrink by a factor of  $\frac{1}{3}$  of the graph of  $f$ .
  - d. The graph of  $g$  is a vertical translation 8 units down of the graph of  $f$ .
  - e. The graph of  $g$  is a reflection in the  $x$ -axis of the graph of  $f$ .
  - f. The graph of  $g$  is a reflection in the  $y$ -axis of the graph of  $f$ .
2. a. **Step 1** Draw a scatter plot of the data. The data shows a linear relationship.

**Step 2** Sketch a line that most closely appears to fit the data. One possibility is shown.



Choose two points on the line. For the line shown, you might choose  $(0, 36,208)$  and  $(2, 39,088)$ .

**Step 4** Write the equation of the line. First, find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{39,088 - 36,208}{2 - 0} = \frac{2,880}{2} = 1440$$

Use the point-slope form of an equation.

Use  $(x_1, y_1) = (0, 36,208)$ .

$$y - y_1 = m(x - x_1)$$

$$y - 36,208 = 1440(x - 0)$$

$$y - 36,208 = 1440x$$

$$y = 1440x + 36,208$$

- b. The slope, \$1440, is the amount the tuition increases each year and the  $y$ -intercept is the tuition cost in 2010, \$36,208.

# Chapter 1

- c. Use the equation in part (a) to estimate the cost of tuition in 2015.

$$y = 1440(5) + 36,208 = 43,408$$

The approximate cost of tuition for a private school in 2015 is \$43,408.

3. Your friend is incorrect. The slope of the line of best fit is negative, thus the correlation coefficient is negative.

#### 4. System A

**Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} 2x + 4y - z = 7 \\ -2x + 12y + 24z = 26 \\ \hline 16y + 23z = 33 \\ -14x - 28y + 7z = -49 \\ 14x + 28y - 7z = 49 \\ \hline 0 = 0 \end{array}$$

Because you have obtained the identity  $0 = 0$ , the system has infinite solutions.

#### System B

**Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} 3x - 3y + 3z = 5 \\ -3x + 3y - 3z = 24 \\ \hline 0 = 29 \end{array}$$

Because you have obtained a false equation, the system has no solution.

#### System C

**Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} 4x - y + 2z = 18 \\ -4x + 8y + 4z = 44 \\ \hline 7y + 6z = 62 \\ -3x + 6y + 3z = 33 \\ 3x + 3y - 4z = 44 \\ \hline 9y - z = 77 \end{array}$$

**Step 2** Solve the new linear system for both of its variables.

$$\begin{array}{r} 7y + 6z = 62 \\ 54y - 6z = 462 \\ \hline 61y = 524 \\ y = \frac{524}{61} \\ z = \frac{19}{61} \end{array}$$

**Step 3** Substitute  $y = \frac{524}{61}$  and  $z = \frac{19}{61}$  into an original equation and solve for  $x$ .

$$\begin{array}{r} -x + 2y + z = 11 \\ -x + 2\left(\frac{524}{61}\right) + \frac{19}{61} = 11 \\ x = \frac{396}{61} \end{array}$$

The solution is  $\left(\frac{396}{61}, \frac{524}{61}, \frac{19}{61}\right)$ .

So, the linear systems are ordered as B, C, A, according to the number of solutions from least to greatest.

5. a. Write a verbal model for the situation.

$$\begin{array}{r} 30 \cdot \boxed{\text{Comedy, } c} + 60 \cdot \boxed{\text{Drama, } d} \\ + 60 \cdot \boxed{\text{Reality-based, } r} = 360 \\ \boxed{\text{Comedy, } c} + \boxed{\text{Drama, } d} + \boxed{\text{Reality-based, } r} = 7 \\ \boxed{\text{Drama, } d} = 2 \cdot \boxed{\text{Comedy, } c} \end{array}$$

The system of equations that models the situation is

$$\begin{array}{r} 30c + 60d + 60r = 360 \\ c + d + r = 7 \\ d = 2c \end{array}$$

- b. **Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} 30c + 60d + 60r = 360 \\ -60c - 60d - 60r = -420 \\ \hline -30c = -60 \end{array}$$

Solve the equation  $-30c = -60$  for  $c$  to obtain  $c = 2$ .

**Step 2** Substitute  $c = 2$  into the equation  $d = 2c$ .  
So,  $d = 2(2) = 4$ .

**Step 3** Substitute  $c = 2$  and  $d = 4$  into  $c + d + r = 7$ .  
So,  $2 + 4 + r = 7$ , which gives  $r = 1$ . So, there are 2 episodes of comedies, 4 episodes of dramas, and 1 reality-based episode.

- c. Use a method similar to part (a) to write a system of equations that models the situation.

$$\begin{array}{r} 30c + 60d + 60r = 360 \\ c + d + r = 6 \end{array}$$

**Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} 30c + 60d + 60r = 360 \\ -60c - 60d - 60r = -360 \\ \hline -30c = 0 \\ c = 0 \end{array}$$

Substitute  $c = 0$  to write a new linear system.

$$\begin{array}{r} 60d + 60r = 360 \\ d + r = 6 \end{array}$$

# Chapter 1

**Step 2** Solve the new linear system for both of its variables.

$$\begin{array}{r} 60d + 60r = 360 \\ -60d - 60r = -360 \\ \hline 0 = 0 \end{array}$$

Because you obtain the identity  $0 = 0$ , the system has infinitely many solutions.

**Step 3** Describe the solutions of the system using an ordered triple of the form  $(c, d, r)$ . One way to do this is to solve the equation  $d + r = 6$  for  $r$  to obtain  $r = 6 - d$ . So, the solutions of the system are of the form  $(0, d, 6 - d)$ .

The two DVDs have a different number of comedies because the first DVD has 2 episodes of comedy and the second DVD has none. The two DVDs may have a different number of dramas and reality-based episodes because the second DVD can have from 0 to 6 dramas and from 0 to 6 reality-based episodes.

6. Equation D models the situation. From the graph, the  $y$ -intercept is  $(0, 450)$ . Another point on the line is  $(5, 400)$ . So, the slope is

$$y = mx + 450$$

$$400 = m(5) + 450$$

$$-50 = 5m$$

$$-10 = m.$$

Therefore, an equation of the line is  $y = -10x + 450$ , or  $10x + y = 450$ .

7. *Sample answer:* The order of the transformations is F followed by A, then followed by C.

8. For every 1 unit increase in  $x$ ,  $f(x)$  increases by 3. So, the data values for  $(x, f(x))$  can be modeled by a linear equation with a slope of 3. Use the point  $(-2, -14)$  to determine the equation.

$$y + 14 = 3(x + 2)$$

$$y + 14 = 3x + 6$$

$$y = 3x - 8$$

An equation relating  $x$  and  $f(x)$  is  $f(x) = 3x - 8$  and  $f(22) = 3(22) - 8 = 58$ .

For every 1 unit increase in  $x$ ,  $g(x)$  increases by 4. So, the data values for  $(x, g(x))$  can be modeled by a linear equation with a slope of 4. Use the point  $(0, -10)$  to determine the equation.

$$y + 10 = 4(x - 0)$$

$$y + 10 = 4x$$

$$y = 4x - 10$$

An equation relating  $x$  and  $g(x)$  is  $g(x) = 4x - 10$  and  $g(22) = 4(22) - 10 = 78$ .

So,  $f(22) < g(22)$ .