Multiple Transformations for Absolute Value and Quadratic Functions

When graphing multiple transformations in the form $f(x) = a(x - h)^2 + k$ or $f(x) = a|x - h| + k$, you must go in this order:

1.) Horizontal stretches/shrinks *(when "a" is inside parentheses/absolute value)*
2.) Horizontal translations (shifts) *("h" value moving right if – and left if +)*
3.) Reflections *(if "a" is negative, over the x-axis)*
4.) Vertical stretches/shrinks *(when "a" is not inside parentheses/absolute value)*
5.) Vertical translations (shifts) *("k" value moving up if + and down if –)*

Example #1: $f(x) = 2(x - 3)^2 + 4$

Questions to Ask Yourself
1.) Which parent function does this have?
- Answer: $f(x) = x^2$
2.) When comparing to the parent function, which transformations has this function undergone?
- Horizontal translation, vertical stretch, and vertical translation

Steps to Graph
1.) Start with the 5 points for the parent function:

x	y
-2	4
-1	1
0	0
1	1
2	4

2.) Perform the transformations in the order above:
   a. Horizontal translation: translated 3 units right → add 3 to each x-coordinate

x	y
1	4
2	1
3	0
4	1
5	4

   b. Vertical stretch: vertical stretch by a factor of 2 → multiply each y-coordinate by 2

x	y
1	8
2	2
3	0
4	2
5	8

   c. Vertical translation: translated 4 units up → add 4 to each y-coordinate

x	y
1	12
2	6
3	4
4	6
5	12

3.) Check to make sure your points back sense! You can always plug your x-coordinates back into the function to make sure you get the right corresponding y-coordinates. Remember order of operations when checking!

Example #2: $f(x) = -3|x + 4| - 2$

<u>Questions to Ask Yourself</u>
1.) Which parent function does this have?
- Answer: $f(x) = |x|$

2.) When comparing to the parent function, which transformations has this function undergone?
- Horizontal translation, reflection, vertical stretch, and vertical translation

<u>Steps to Graph</u>
1.) Start with the 5 points for parent function:

x	y
-2	2
-1	1
0	0
1	1
2	2

2.) Perform the transformations in the order above:
   a. Horizontal translation: translated 4 units left → subtract 4 from each x-coordinate

x	y
-6	2
-5	1
-4	0
-3	1
-2	2

   b. Reflection: reflected over the x-axis → multiply each y-coordinate by -1

x	y
-6	-2
-5	-1
-4	0
-3	-1
-2	-2

   c. Vertical Shrink: vertical stretch with a factor of 3 → multiply each y-coordinate by 3

x	y
-6	-6
-5	-3
-4	0
-3	-3
-2	-6

   d. Vertical translation: translated 2 units down → subtract 2 from each y-coordinate

x	y
-6	-8
-5	-5
-4	-2
-3	-5
-2	-8

3.) Check to make sure your points back sense! You can always plug your x-coordinates back into the function to make sure you get the right corresponding y-coordinates. Remember order of operations when checking!