Multiple Transformations for Absolute Value and Quadratic Functions

When finding the equation of absolute value or quadratic functions from a graph in the form \( f(x) = a(x - h)^2 + k \) or \( f(x) = a|x - h| + k \), follow these steps:

1. Figure out what kind of parent function it is:
   - V-shaped \( \rightarrow \) Absolute value function so \( f(x) = a|x - h| + k \)
   - U-shaped/parabola \( \rightarrow f(x) = a(x - h)^2 + k \)
2. Find the vertex. This will give you \( h \) and \( k \).
3. Plug the vertex into the above equation for the correct parent function. Remember, if \( h \) is negative, it will become + inside the absolute value/parentheses since two negatives equals a positive.
4. If the function is opening downward, you know it’s a reflection and there will be a negative sign in front of the absolute value/parentheses.
5. Lastly, find \( a \). To do this, find another point that’s on your graph besides the vertex. If you use the vertex, this will not work! Plug the point in for \( x \) and \( y \) (\( f(x) \)) in your equation. You should have the \( h \) and \( k \) already filled in from the vertex and you now will have \( x \) and \( y \) filled in as well. The only variable left should be \( a \). Solve your equation for \( a \).
6. In your final equation, you should have \( h \) and \( k \) from the vertex and \( a \) from the previous step filled in. You should not have anything filled in for \( x \) and \( y \) as this point is dependent on the actual graph. Voila! You’re done!

Example #1:

Step 1: Since this is u-shaped/parabola, use the general form of the function: \( f(x) = a(x - h)^2 + k \)
Step 2: Find the vertex \( \rightarrow (2,1) \). Thus, \( h=2 \) and \( k=1 \).
Step 3: Plug \( h \) and \( k \) into the equation: \( f(x) = a(x - 2)^2 + 1 \)
Step 4: Since the parabola is opening up, it is not a reflection and thus, there will be no negative sign.
Step 5: We need to pick another point on the parabola that’s not the vertex. For this, I’ll use \( (5,4) \). Now, plug this into your equation for step 3. \( x=5 \) and \( y \) or \( f(x) = 4 \). So our new equation is \( 4 = a(5 - 2)^2 + 1 \). Solve your new equation.
   
   \[
   4 = a(5 - 2)^2 + 1 \\
   4 = a(3)^2 + 1 \\
   4 = 9a + 1 \\
   3 = 9a \\
   a = \frac{1}{3}
   \]
Step 6: Plug the values for \( h \), \( k \), and \( a \) back into your general form of the equation and you’re done!

\[
f(x) = \frac{1}{3} (x - 2)^2 + 1
\]

Example #2:

Step 1: Since this is v-shaped, use the general form of the function: \( f(x) = a|x - h| + k \)
Step 2: Find the vertex \( \rightarrow (-1,2) \). Thus, \( h=-1 \) and \( k=2 \).
Step 3: Plug \( h \) and \( k \) into the equation: \( f(x) = a|x + 1| + 2 \) **Note: Since \( h \) is negative, it becomes + inside the absolute value.
Step 4: Since the parabola is opening down, it is a reflection and thus, there will be a negative sign in front of the absolute value.
Step 5: We need to pick another point on the parabola that’s not the vertex. For this, I’ll use \( (0,0) \). Now, plug this into your equation for step 3. \( x=0 \) and \( y \) or \( f(x) = 0 \). So our new equation is \( 0 = a|0 + 1| + 2 \). Solve your new equation.
   
   \[
   0 = a|0 + 1| + 2 \\
   0 = a|1| + 2 \\
   0 = 1a + 2 \\
   -2 = 1a \\
   a = -2
   \]
Step 6: Plug the values for \( h \), \( k \), and \( a \) back into your general form of the equation and you’re done!

\[
f(x) = -2|x + 1| + 2
\]