Salisbury 2CP Unit 5 You Can (No Calculator)

You should be able to demonstrate the following skills by completing the associated problems. It is highly suggested that you read over your notes before attempting this You Can. More practice problems can be found in the homework, other problems in the textbook, and on the Big Ideas website.

Can you…

- **Recognize and solve direct and inverse variation problems, including word problems**

For #1-7, tell whether x and y show *direct variation*, *inverse variation*, or neither.

1. $3xy = 1$ — Inverse
2. $\frac{5}{x} = y$ — inverse
3. $x + y = 6$ — neither
4. $\frac{x}{7} = y$ — direct
5. $\frac{4}{5}x = y$ — direct

6.

x	1.5	2.5	4	7.5	10
y	13.5	22.5	36	67.5	90

Direct (ratios are constant)

7.

x	2	4	6	8
y	24	16	12	4

Inverse (products are constant)

8. If y varies directly as x and y=35 when x=7, find y when x=11.

$y = kx$

$35 = k(7)$

$k = 5$

$y = 5x$

$y = (5)(11)$

$y = 5$

9. If y varies inversely as x and y=2 when x=2, find y when x=1.

$y = \frac{k}{x}$

$2 = \frac{k}{2}$

$k = 4$

$y = \frac{4}{x}$

$y = \frac{4}{1}$

$y = 4$

10. When temperature is held constant, the volume *V* of a gas is inversely proportional to the pressure *P* of the gas on its container. A pressure of 32 pounds per square inch results in a volume of 20 cubic feet. What is the pressure if the volume becomes 10 cubic feet?

$y = \frac{k}{x}$

$20 = \frac{k}{32}$

$k = 640$

$10 = \frac{640}{x}$

$y = 64\ lbs/in^2$

11. The length S that a spring will stretch varies directly with the weight F that is attached to the spring. If a spring stretches 20 inches with 25 pounds attached, how far will it stretch with 15 pounds attached?

$y = kx$

$20 = k(25)$

$k = \frac{4}{5}$

$y = \frac{4}{5}x$

$y = (\frac{4}{5})(15)$

$y = 12\ inches$

- **<u>Determine under what conditions a rational expression is undefined.</u>**

12. $\frac{3x^2-2x-8}{3x^2-12}$ 13. $\frac{2p}{p^2-2p+1}$ 14. $\frac{x^2-11x+24}{x^3-18x^2+80x}$

Set denominators equal to zero and solve

12. $\boxed{x=2,\ x=-2}$ 13. (x-1)(x-1)=0; $\boxed{x=1}$ 14. x(x-8)(x-10); $\boxed{x=0,\ x=8,\ x=10}$

- **<u>Multiply and divide rational expressions</u>**

15. $\frac{ab}{xy} \bullet -\frac{x}{a^3b^2}$

$-\frac{abx}{xya^3b^2}$

$\boxed{-\frac{1}{ya^2b}}$

16. $\frac{a^2b^2c^3d}{xy} \div \frac{ac^3d^2}{x^5}$

$\frac{a^2b^2c^3d}{xy} * \frac{x^5}{ac^3d^2}$

$\boxed{\frac{ab^2x^4}{dy}}$

17. $\frac{\frac{x-4}{x^2+6x+9}}{\frac{x^2-2x-8}{3+x}}$

$\frac{x-4}{x^2+6x+9} * \frac{3+x}{x^2-2x-8}$

$\frac{x-4}{(x+3)^2} * \frac{x+3}{(x-4)(x+2)}$

$\boxed{\frac{1}{(x+3)(x+2)}}$

18. $\frac{16x^2-9}{4x+3} \bullet \frac{25x^2-1}{5x+1}$

$\frac{(4x-3)(4x+3)}{(4x+3)} * \frac{(5x+1)(5x-1)}{(5x+1)}$

$\boxed{(4x-3)(5x-1)}$

19. $\frac{x^2-11x+24}{x^2-18x+80} \div \frac{x^2-9x+20}{x^2-15x+50}$

$\frac{x^2-11x+24}{x^2-18x+80} \bullet \frac{x^2-15x+50}{x^2-9x+20}$

$\frac{(x-8)(x-3)}{(x-8)(x-10)} * \frac{(x-10)(x-5)}{(x-4)(x-5)}$

$\boxed{\frac{x-3}{x-4}}$

- **<u>Add and subtract rational expressions</u>**

  Get common denominators

20. $\frac{5}{12x^4y} - \frac{1}{5x^2y^3}$

$$\frac{5y^2}{5y^2} * \frac{5}{12x^4y} - \frac{12x^2}{12x^2} * \frac{1}{5x^2y^3}$$

$$\boxed{\frac{25y^2 - 12x^2}{60x^4y^3}}$$

21. $\frac{k+3}{k^2+6k+9} - \frac{7}{2k+6}$

$$\frac{k+3}{(k+3)^2} - \frac{7}{2(k+3)}$$

$$\frac{2}{2} * \frac{1}{x+3} - \frac{7}{2(k+3)}$$

$$\boxed{\frac{-5}{2(k+3)}}$$

22. $\frac{\frac{x}{y}-1}{1+\frac{1}{x}}$

$$\frac{\frac{x}{y} - 1 * \frac{y}{y}}{\frac{x}{x} * 1 + \frac{1}{x}}$$

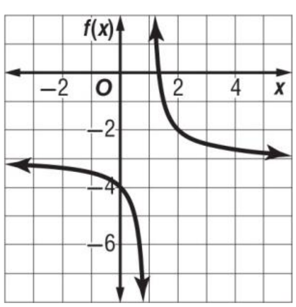
$$\frac{\frac{x-y}{y}}{\frac{x+1}{x}}$$

$$\frac{x-y}{y} * \frac{x}{x+1}$$

$$\boxed{\frac{x(x-y)}{y(x+1)}}$$

23. Given a graph of a rational function, identify characteristics of rational functions including domain and range, asymptotes, intervals of increasing and decreasing, and end behavior, using appropriate mathematical notation.

a. $f(x) = \frac{1}{x-1} - 3$



Domain $(-\infty, 1)(1, \infty)$

Range $(-\infty, -3)(-3, \infty)$

Equations for the asymptotes

$VA: x = 1 \quad HA: y = -3$

Intervals of increasing: none

Intervals of decreasing:

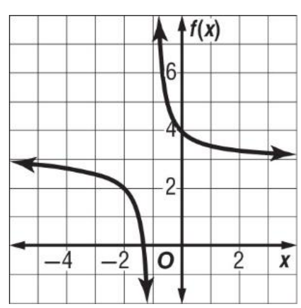
$(-\infty, 1)(1, \infty)$

End behavior

$as\ x \to -\infty, y \to -3$

$as\ x \to \infty, y \to -3$

b. $f(x) = \frac{1}{x+1} + 3$



Domain $(-\infty, -1)(-1, \infty)$

Range $(-\infty, 3)(3, \infty)$

Equations for the asymptotes

$VA: x = -1 \quad HA: y = 3$

Intervals of increasing: none

Intervals of decreasing:

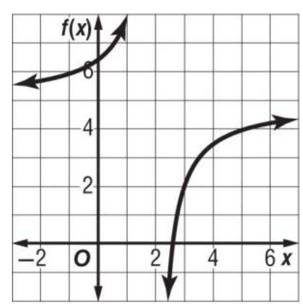
$(-\infty, -1)(-1, \infty)$

End behavior

$as\ x \to -\infty, y \to 3$

$as\ x \to \infty, y \to 3$

c. $f(x) = \frac{-3}{x-2} + 5$



Domain $(-\infty, 2)(2, \infty)$

Range $(-\infty, 5)(5, \infty)$

Equations for the asymptotes

$VA: x = 2 \quad HA: y = 5$

Intervals of increasing:

$(-\infty, 2)(2, \infty)$

Intervals of decreasing: none

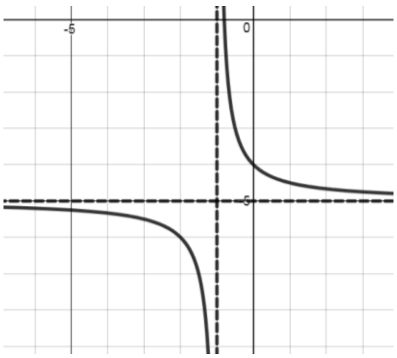
End behavior

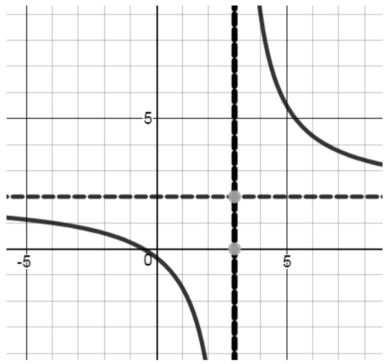
$as\ x \to -\infty, y \to 5$

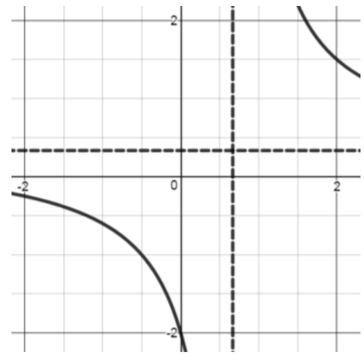
$as\ x \to \infty, y \to 5$

24. Graph a rational function using transformations. Label and show asymptotes with dashed lines.
VA: solve for denominator; HA: ratio of leading coefficients of numerator and denominator

a. $f(x) = \frac{1}{x+1} - 5$ **b.** $f(x) = \frac{2x+1}{x-3}$ **c.** $f(x) = \frac{x+4}{3x-2}$







$\boldsymbol{VA: x = -1 \quad HA: y = -5}$

$\boldsymbol{VA: x = 3 \quad HA: y = 2}$

$\boldsymbol{VA: x = 2/3 \quad HA: y = 1/3}$

25. Rewrite the function in the form $g(x) = \frac{a}{x-h} + k$. Graph the function. Describe the graph of g as a transformation of the graph $f(x) = \frac{a}{x}$.

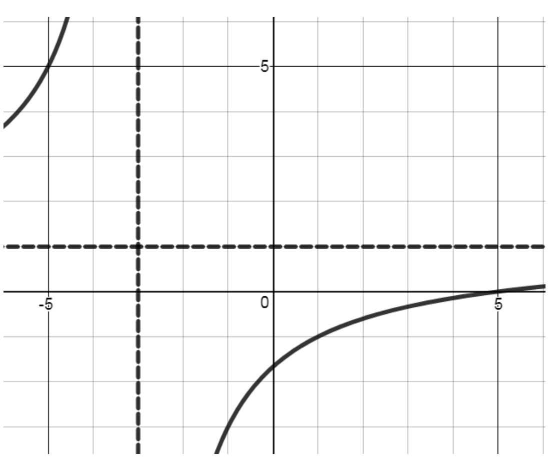
a. $f(x) = \frac{x-5}{x+3}$

b. $f(x) = \frac{3x-4}{2x+1}$

a. Use long division to get $f(x) = \frac{-8}{x+3} + 1$

one unit up, three units left, reflected over x-axis

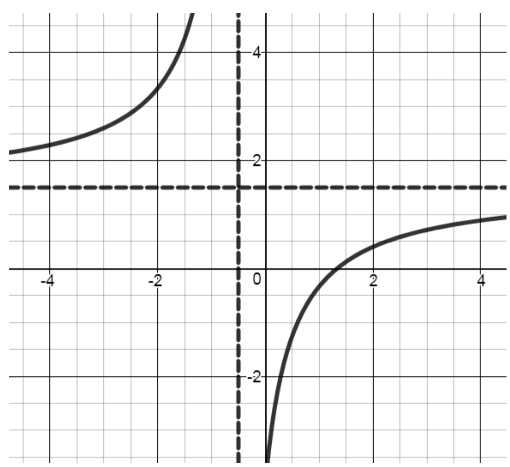
$\boldsymbol{VA: x = -3 \quad HA: y = 1}$



a. Use long division to get $f(x) = \frac{-5.5}{2x+1} + 1.5$

one unit up, three units left, reflected over x-axis

$\boldsymbol{VA: x = -\frac{1}{2} \quad HA: y = 1.5}$



26. Solve a rational equation and identify extraneous solutions.

Remember to test the solutions into the denominators to make sure they don't result in a zero in the denominator. If they do, they are extraneous solutions and should not be part of the solution set.

a. $\frac{y}{y+1} = \frac{2}{3}$

one fraction = one fraction: cross multiply

$$3(y) = 2(y+1)$$

$$3y = 2y + 2$$

$y = 2$

b. $\frac{9}{t-3} = \frac{t-4}{t-3} + \frac{1}{4}$

more than one fraction on a side, multiply both sides by LCD to get rid of fractions

$$4(t-3) * \frac{9}{t-3} = 4(t-3) * \frac{t-4}{t-3} + 4(t-3) * \frac{1}{4}$$

$$4(9) = 4(t-4) + (t-3)(1)$$

$t = 11$

c. $\frac{2}{y+2} + \frac{y}{y-2} = \frac{y^2+4}{y^2-4}$

$\frac{2}{y+2} + \frac{y}{y-2} = \frac{y^2+4}{(y+2)(y-2)}$ factor first to help find LCD: $(y-2)(y+2)$

$$(y-2)(y+2) * \frac{2}{y+2} + (y-2)(y+2) * \frac{y}{y-2} = (y-2)(y+2) * \frac{y^2+4}{(y-2)(y+2)}$$

$$2(y-2) + y(y+2) = (y^2+4)$$

$$2y - 4 + y^2 + 2y = y^2 + 4$$

$$y = 2, \text{but 2 is extraneous so}$$

no solution

27. For an application involving rational functions, write an equation representing the situation and solve. For each problem below, write an equation represent the situation and define the variables. Solve the problem.

a. Jason can water all the plants at the botanical garden in 32 minutes. Celia can water them in 25 minutes. If they work together, how long will it take for them to water the plants?

What portion of the job Jason can do in a minute	+	What portion of the job Celia can do in a minute	=	What portion of the job they can both do in a minute working together

$$\frac{1}{32}+\frac{1}{25}=\frac{1}{x}$$

x = 14.04 so about 14 minutes

b. How many liters of 20% alcohol solution should be added to 40 liters of a 50% alcohol solution to make a 30% solution?

$$Goal\ \% = \frac{orig\% * orig\ amt + new\% * New\ Amt}{Total\ Amt}$$

$$0.30 = \frac{(0.50)(40) + (0.20)(x)}{40 + x}$$

x = 80, so add 80 liters